Coulomb sum rule in heavier nuclei

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The Coulomb sum rule is evaluated for a large class of nuclei up to the heavy ones. Beyond mean-field effects, short-range correlations are taken into account. The latter decrease the sum rule in the whole considered momentum range. The size of the reduction increases with the average proton density reaching its maximum for medium heavy nuclei.

A longstanding problem in quasielastic electron scattering is the discrepancy between the nonrelativistic Coulomb sum rule (CSR) and the integrated experimental longitudinal response function. The basic result of the CSR is that at momentum transfers $|\mathbf{q}|$, where the effects of correlations vanish ($|\mathbf{q}|>2\ \mathrm{fm}^{-1}$), only the proton number is counted. At present, however, experiments confirm such results only for light nuclei [1], while for medium-weight ones the integrated strength is suppressed up to about 40% [2]. The situation for heavy nuclei is not yet settled, since rather different results exist for ^{238}U and ^{208}Pb [3,4].

There are two classes of possible explanations for the discrepancy between theory and experiment [4]. One is based on the assumption that the suppression is due to conventional nuclear structure effects: correlations might shift strength above the highest experimental energy without violating the CSR which implies an integration up to infinity. Other explanations consider effects beyond classical nuclear physics, like off-shell nucleon form factor, quark and relativistic dynamical effects. In both cases the different results for light and medium heavy nuclei require a dependence on the nuclear environment; however, a systematic study of the medium effects on the CSR is still missing. This work is the first attempt to cover a great part of the nuclear table within a conventional nuclear physics approach.

Up to the present there are CSR calculations for light nuclei [5,6], for medium-weight ones [7–9], and for nuclear matter (NM) [6]. The various methods, however, are very different and heavy nuclei have not yet been considered at all. We have used a unique model, which takes into account both short-range NN correlations and finite-size effects. It is based on the assumption that at the same density correlations beyond the mean field are similar in NM and in finite nuclei. Such a model has been recently successfully applied to evaluate nucleon momentum distributions in nuclei [10].

The integral in the energy transfer ω of the inelastic longitudinal form factor $R^L_{\mathrm{inel}}(|\mathbf{q}|,\omega)$ is related to the CSR in the following way:

$$\int_0^\infty d\omega \frac{R^L_{\mathrm{inel}}(|\mathbf{q}|,\omega)}{|F_p(q^2_\mu)|^2} = Z + Z(Z-1)f(|\mathbf{q}|) - Z^2|F(|\mathbf{q}|)|^2 \equiv \mathrm{CSR}\ , \tag{1}$$

where $F(|\mathbf{q}|)$ is the elastic nuclear charge form factor for pointlike protons and $F_p(q^2_\mu)$ is the proton form factor as function of the four-momentum transfer. The function $f(|\mathbf{q}|)$ represents the Fourier transform of the p-p density $\rho_{pp}(\mathbf{s})$

$$f(|\mathbf{q}|) = \int d\mathbf{s}\, \rho_{pp}(\mathbf{s}) e^{i\mathbf{q}\cdot\mathbf{s}}\ , \tag{2}$$

where $\mathbf{s}$ is the displacement between two protons. This is the crucial quantity one has to study to understand the role of the nuclear medium on the CSR. Since the elastic form factor $F(|\mathbf{q}|)$ dies out much faster than $f(|\mathbf{q}|)$, the latter governs the way the asymptotic limit Z of the CSR is reached for large $|\mathbf{q}|$.

In the case of NM the function $f(|\mathbf{q}|)$ can be separated in two independent parts

$$f_{\mathrm{NM}}(|\mathbf{q}|,k_F) = f_{\mathrm{FG}}(|\mathbf{q}|,k_F) + \Delta f_{\mathrm{NM}}(|\mathbf{q}|,k_F)\ , \tag{3}$$

where $f_{\mathrm{FG}}(|\mathbf{q}|,k_F)$ is the Fermi-gas (FG) contribution and $\Delta f_{\mathrm{NM}}(|\mathbf{q}|,k_F)$ contains the correlations beyond the single-particle scheme. For a finite nucleus a similar separation can be performed:

$$f(|\mathbf{q}|) = f_{\mathrm{HF}}(|\mathbf{q}|) + \Delta f(|\mathbf{q}|)\ . \tag{4}$$

Now $f_{\mathrm{HF}}(|\mathbf{q}|)$ is the result of a Hartree-Fock (HF) calculation describing finite-size and shell effects, while $\Delta f(|\mathbf{q}|)$ embodies the contribution due to the residual NN correlations. Assuming that at the same density correlations beyond the mean field are similar in NM and in finite nuclei, our ansatz consists in applying the local-density approximation to the second term in Eq. (4), namely,

$$\Delta f(|\mathbf{q}|) = \int d\mathbf{r} \frac{[k_F(\mathbf{r})]^3}{3\pi^2 Z} \Delta f_{\mathrm{NM}}(|\mathbf{q}|,k_F(\mathbf{r}))\ . \tag{5}$$

For the $\mathbf{r}$ dependence of the proton Fermi momentum k_F we utilize the Thomas-Fermi approximation, i.e., $k_F(\mathbf{r})=[3\pi^2\rho(\mathbf{r})]^{1/3}$. For the calculation of $\rho(\mathbf{r})$ we use a standard HF code with the Skyrme force (SKM) [11]. In order to evaluate the correlation contribution $\Delta f(|\mathbf{q}|)$ of Eq. (5) one needs the nuclear matter p-p density in momentum space [Eqs. (2) and (3)] as a function of k_F. This could be provided by NM calculations, but we avoid complicated numerics using a parametrization based on the lowest-order expansion [12]

$$\rho_{pp}(\mathbf{s})=g^2(\mathbf{s})\rho_{pp}^{\mathrm{FG}}(\mathbf{s}) \ , \tag{6}$$

where $g(\mathbf{s})$ is an analytical function that reflects the effect of dynamical correlations. For our purpose we have found it convenient to choose

$$g(\mathbf{s})=1-ae^{-\beta s^2}-(1-a)e^{-\gamma s^2} \ . \tag{7}$$

The parameters have been fixed fitting the $\Delta f_{\mathrm{NM}}(|\mathbf{q}|,k_F=1.33\ \mathrm{fm}^{-1})$ of Refs. [6,13]. The fit was only performed in the $|\mathbf{q}|$ range beyond 1 fm^{-1}, since at lower $|\mathbf{q}|$ dynamical correlations are of minor importance and in addition the Δf_{NM} of the NM calculation is somewhat problematic (see Appendix B of Ref. [6]). We obtain the following values of the parameters: $a=1.15$, $\beta=2.58\ \mathrm{fm}^{-2}$, $\gamma=0.77\ \mathrm{fm}^{-2}$.

The good quality of the fit is shown in Fig. 1(a). In Fig. 1(b) we also compare the $\rho_{pp}(\mathbf{s})$ of Eq. (6) with the NM [6,13] and the FG ones. The strong effect of $g(\mathbf{s})$ and the good description of the NM p-p density are readily seen

At this point we would like to comment on our ansatz. By implying that correlations beyond the mean field are similar in NM and in finite nuclei at the same density, our model allows us to calculate the CSR for a large number of nuclei. Through the term $f_{\mathrm{HF}}(|\mathbf{q}|)$ we take into account finite-size effects, which are completely missing in NM calculations, but which are important even in the heaviest nuclei. We remind that such effects are crucial at low $|\mathbf{q}|$, where they are responsible for the $|\mathbf{q}|^2$ behavior of the CSR, while NM calculations lead to a linear $|\mathbf{q}|$ dependence.

In Fig. 2 we show the results of Δf relative to the fitted Δf_{NM} [see Fig. 1(a)] for various nuclei. The almost constant ratios demonstrate that the shapes of Δf are very similar to that of NM; the strengths, however, are different varying from about 70% (^{60}Ni) to about 55% (^{16}O). Moreover, one finds an interesting mass dependence. There is a monotonous decrease of Δf from ^{60}Ni both for lighter and heavier nuclei; only the calcium isotopes as well as ^{120}Sn and ^{140}Ce are interchanged. This mass dependence will be commented on later.

In Fig. 3 we show the CSR for ^{16}O, ^{60}Ni, and ^{238}U. We already mentioned the different $|\mathbf{q}|$ behavior between the HF and the FG results at $|\mathbf{q}|\rightarrow 0$, but discrepancies are also evident at higher $|\mathbf{q}|$. In the $|\mathbf{q}|$ range between 1 and 2 fm^{-1} one has differences up to 20%. The residual correlations reduce the HF results in the whole considered momentum range. Contrary to the mean-field effects they influence the CSR even beyond 2.5 fm^{-1}. In Fig. 3(a) we also show the results of a variational Monte Carlo calculation [8]. In the region beyond 2 fm^{-1} there

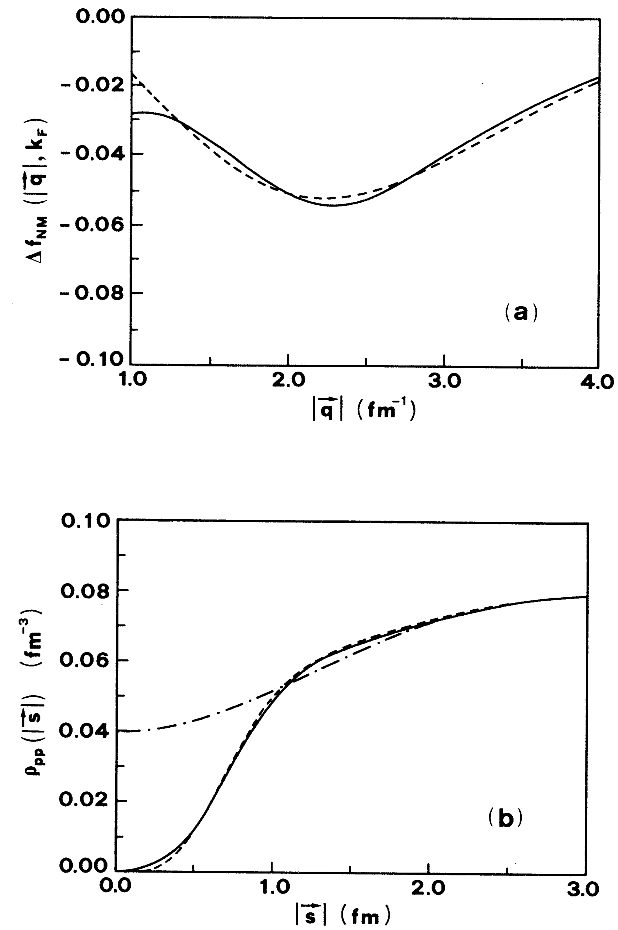

FIG. 1. (a) The $\Delta f_{\mathrm{NM}}(|\mathbf{q}|,k_F=1.33)$ of Refs. [6,13] (full curve) and the fit according to Eqs. (2), (6), and (7) (dashed curve); (b) $\rho_{pp}(|s|)$ of Refs. [6,13] (full curve), from Eq. (6) (dashed curve) and $\rho_{pp}^{\mathrm{FG}}(|s|)$ (dot-dashed curve).

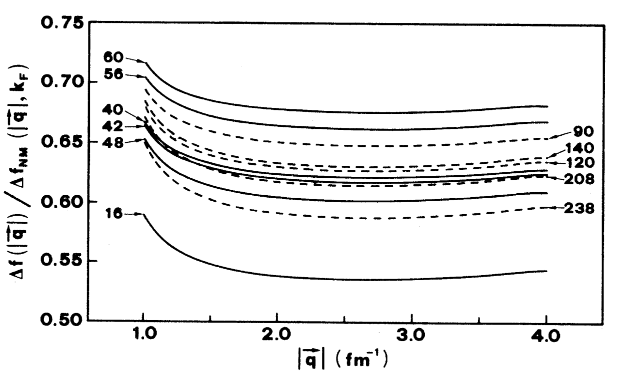

FIG. 2. $\Delta f(|\mathbf{q}|)$ of various nuclei relative to the fitted $\Delta f_{\mathrm{NM}}(|\mathbf{q}|,k_F=1.33)$: ^{16}O, ^{40}Ca, ^{42}Ca, ^{48}Ca, ^{56}Fe, ^{60}Ni (full curves) and ^{90}Zr, ^{120}Sn, ^{140}Ce, ^{208}Pb, ^{238}U (dashed curves). Curves are labeled with mass number A.

is a rather good agreement with our full calculation. There are some differences at lower $|\mathbf{q}|$, presumably caused by the application of HF to the rather light ^{16}O and/or nonsufficient long-range correlations in Δf.

To discuss the mass dependence of the CSR we depict the results of the various nuclei relative to the ^{60}Ni result

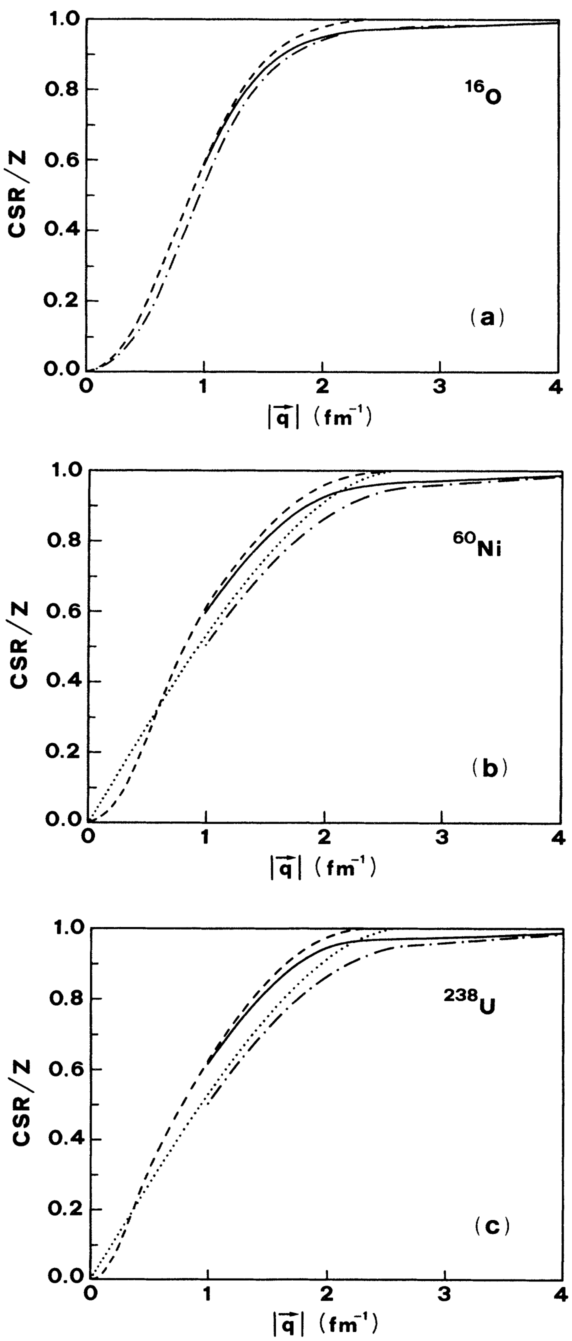

FIG. 3. The CSR normalized to 1 for ^{16}O (a), ^{60}Ni (b), and ^{238}U (c): HF results (dashed curves) and with correlations (full curves). The dot-dashed curve in (a) is the result of Ref. [8]. In (b) and (c) the NM results of Refs. [6,13] (dot-dashed curves) and the FG ones (dotted curves) are also shown.

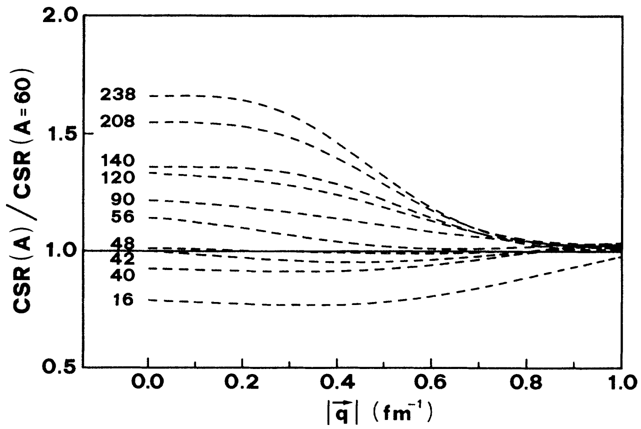

FIG. 4. The HF-CSR of various nuclei relative to that of ^{60}Ni in the low $|\mathbf{q}|$ region. Notations as in Fig. 2.

in Figs. 4 and 5. At low $|\mathbf{q}|$ (Fig. 4) the CSR grows with increasing mass number according to the relation $\mathrm{CSR}(A)/|\mathbf{q}|^2=\sigma_{-1}(A)/4\pi^2\alpha \propto A^{4/3}$ [14,15], where σ_{-1} is the bremsstrahlung-weighted cross section. This relation seems to be valid up to about 0.3 fm^{-1}, while there are already various crossing overs in the region of 1 fm^{-1}. For higher $|\mathbf{q}|$ (Fig. 5) one finds the same mass dependence as in Fig. 2: the CSR decreases with A up to ^{60}Ni [Fig. 5(a)], but increases again for heavier nuclei [Fig. 5(b)]. This mass dependence is already present for the HF-CSR; however, correlations amplify the differences among the various nuclei. They have their strongest effects on ^{60}Ni, while, for example, there are rather similar influences on ^{40}Ca and ^{208}Pb. This A dependence is presumably related to the average point proton density as can be argued from Table I, where we

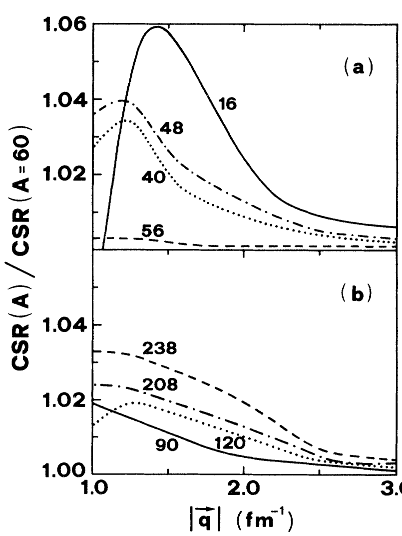

FIG. 5. The correlated CSR for various nuclei relative to that of ^{60}Ni. In (a) and (b) the nuclei with $A<60$ and $A>60$ are shown, respectively. Notations as in Fig. 2.

TABLE I. rms charge radii [16] and the various densities as explained in the text. The experimental radii [16] have been corrected for the finite size of the proton by assuming, for each nucleus, the same differences between the charge and the proton radii as in the HF calculation.

Nucleus	$\langle r^2 \rangle_{ch}^{1/2}$ (fm)	ρ_{box}^{exp} (fm^{-3})	ρ_{box}^{HF} (fm^{-3})	ρ_{av}^{HF} (fm^{-3})
^{16}O	2.71	0.0508	0.0463	0.0418
^{40}Ca	3.46	0.0580	0.0559	0.0491
^{42}Ca	3.50	0.0560	0.0558	0.0488
^{48}Ca	3.46	0.0580	0.0550	0.0478
^{56}Fe	3.76	0.0582	0.0601	0.0526
^{60}Ni	3.82	0.0598	0.0612	0.0537
^{90}Zr	4.26	0.0605	0.0595	0.0518
^{120}Sn	4.64	0.0581	0.0576	0.0506
^{140}Ce			0.0570	0.0510
^{208}Pb	5.50	0.0564	0.0564	0.0498
^{238}U	5.84	0.0527	0.0534	0.0479

report the average density (ρ_{av}) and the density of a hard sphere which has the HF rms radius (ρ_{box}^{HF}) or the experimental one (ρ_{box}^{exp})

$$\rho_{av} = \frac{\int d\mathbf{r}\, \rho^2(\mathbf{r})}{\int d\mathbf{r}\, \rho(\mathbf{r})}, \quad \rho_{box} = \frac{3}{4\pi}\left[\frac{3}{5}\right]^{3/2} \frac{Z}{\langle r^2 \rangle^{3/2}} . \tag{8}$$

The table shows that ^{60}Ni has the highest HF densities and that the results of the other nuclei group in a similar way as in Fig. 5. A very similar mass dependence is also found for ρ_{box}^{exp}.

We conclude that the $|\mathbf{q}|$ dependence of the CSR in NM and in finite nuclei is rather different. However, the relative effects of correlations are small in both cases leading to reductions of the CSR by about 5–10 % for 2 fm$^{-1} < |\mathbf{q}| < 3$ fm^{-1}. For $|\mathbf{q}| > 1$ fm^{-1} there is a mass dependence of the HF-CSR related to the average density, which is amplified by correlations. The reduction is maximal in the medium-weight region. Though the present calculation does not give information about the distribution of the strength, more detailed studies [17] suggest that correlations might shift some strength to an energy range that is not easily available in experiment. Higher energy weighted sum rules are more appropriate to study this hypothesis and a careful analysis for a large class of nuclei might lead to interesting results.

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