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Proton-neutron multiplet structure of ^{104}In

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The structure of the ^{104}In nucleus was studied by means of interacting boson-fermion-fermion model. All the experimentally known states below 1.2 MeV were assigned to proton-neutron multiplets on the basis of their electromagnetic properties. A *W*-like energy splitting of the $\pi g_{9/2}^{-1}\nu\tilde{d}_{5/2}$ and a slightly distorted, open-up parabolic splitting of the $\pi g_{9/2}^{-1}\nu\tilde{g}_{7/2}$ multiplet have been predicted by the model, in agreement with the experiment.

In a series of recent studies, we have investigated the structure of $^{106-112}$In nuclei from both experimental and theoretical points of view [1–4]. Our systematic investigations showed that the interacting boson-fermion-fermion model (IBFFM) describes the states of the low-lying proton-neutron multiplets in odd-odd In nuclei reasonably well. Using the systematics of the parameters obtained, one can extrapolate the structure of lighter In nuclei, too.

The structure of ^{104}In may be particularly interesting for two reasons. One reason is that experiments searching for the doubly magic nucleus ^{100}Sn are based on extrapolations. To make better extrapolations the structure of the odd-odd In daughter nuclei must be known, as pointed out by Walters [5].

The other reason is that understanding of the structure of ^{104}In may also be useful for the investigation of an interesting phenomenon, the mixing of $\Delta I=2$ states in a strong external electric-field gradient. Such a phenomenon has not been observed yet in nuclei, but may be possible in case of the very-close-lying (3^+) and (5^+) states of ^{104}In [6]. Szerypo *et al.* proposed to search for the above effect by measuring the change of the lifetime of the (3^+) isomeric state in the electric-field gradient generated by specific atomic electrons [6]. To estimate the probability of the mixing and its effect on the lifetime of the 3_1^+ state, the wave functions of the three lowest-lying levels are required.

The experimental information on the structure of ^{104}In is taken mainly from the novel investigations of the decay of ^{104}Sn by Barden *et al.* [7] and Szerypo *et al.* [6] (see Ref. [8] for earlier results). These investigations lead to a series of positive-parity levels with unique spin assignments, well-established gamma branching ratios, and gamma-ray multipolarities. The electromagnetic moments of the ground state were determined by Eberz *et al.* [9]. The configuration of the ground state was determined from its magnetic moments and of the lowest-lying 1^+ state from the log *ft* value.

The odd-odd indium states are expected to arise from the angular momentum coupling of the proton and neutron states of the neighboring odd indium and odd tin isotopes. These states can be described as being of particle plus boson or 1+3 quasiparticle nature. Approximating the collective and/or broken pair degree of freedom with bosons, the structure of the odd-odd In nuclei is described by the Hamiltonian of the interacting boson-fermion-fermion model [10]:

$$H_{\mathrm{IBFFM}}=H_{\mathrm{IBFM}}(\pi)+H_{\mathrm{IBFM}}(\nu)-H_{\mathrm{IBM}}+H_{\mathrm{EFF}}\ ,$$

where $H_{\mathrm{IBFM}}(\pi)$ and $H_{\mathrm{IBFM}}(\nu)$ denote the IBFM Hamiltonians for the neighboring odd-even nuclei with an odd proton and odd neutron, respectively [11]. H_{IBM} denotes the IBM Hamiltonian [12] for the even-even core nucleus. H_{EFF} denotes the residual proton-neutron interaction. Depending on whether one uses the Schwinger or the Holstein-Primakoff representation of the SU(6) boson Hamiltonian, one can distinguish between the interacting boson-fermion-fermion and the odd-odd truncated quadrupole phonon representations, respectively [4]. The two representations are equivalent on the phenomenological level.

In this work the core Hamiltonian was approximated with its SU(5) limit, which is reasonable for spherical nuclei in this energy region. A spin-dependent delta interaction with an additional spin-polarization term was taken as the residual proton-neutron interaction:

$$H_{\mathrm{EFF}}=V_0\delta(\mathbf{r}_\pi-\mathbf{r}_\nu)(1+\alpha\sigma_\pi\sigma_\nu)+V_{\mathrm{sp}}[\sigma_\pi\sigma_\nu]_0\ .$$

The IBFFM Hamiltonian was diagonalized in the proton-neutron-boson basis:

$$|(j_\pi,j_\nu)J,n_dR;I\rangle\ ,$$

where j_π and j_ν stand for the proton and neutron angular momenta coupled to J, n_d is the number of d bosons, R is their total angular momentum, and I is the spin of the state. The computer code IBFFM, used for the calculations, was written by Brant, Paar, and Vretenar [13].

The model parameters were as follows. *The d-boson energy* of the core excitation was 1.21 MeV, taken from the energy of the 2_1^+ state of ^{104}Sn. Since the introduction of the anharmonicities, allowed in the SU(5) limit, had negligible effects on the states investigated, they were neglected. The maximal number of bosons was equal to the number of valence nucleon pairs, $N=2$.

The *shell-model space* in the present calculation consisted of the $g_{9/2}$ subshell for the proton hole and the $s_{1/2}$, $d_{3/2}$, $d_{5/2}$ and $g_{7/2}$ subshells for the neutron quasiparticles. The neutron *occupation probabilities* were determined from BCS calculations. The single-particle energies were taken from the systematics of Kissliger and Sorensen [14], while the BCS gap parameter was taken from the odd-even mass difference $\Delta=1.38$ MeV. For most cases we could apply the BCS *quasiparticle energies,* except the $\tilde{g}_{7/2}$ energy, which was fitted to ^{104}In. (The tilde denotes the quasiparticle states.) The fitted $\tilde{g}_{7/2}$ neutron energy is ≈ 200 keV lower then the BCS prediction, as it was in the heavier In nuclei, too. In this connection we note that the BCS approximation applies only to the representation employing the particle-hole channel. In the particle-particle channel, which is assumed in the standard IBM representation, the occupation probabilities and quasiparticle energies can differ from the BCS result. The quasiparticle parameters used in the calculations are

$$\begin{array}{ll} V^2(\tilde{s}_{1/2})=0.125\ , & E(\tilde{s}_{1/2})=0.7\ \text{MeV}\ , \\ V^2(\tilde{d}_{3/2})=0.053\ , & E(\tilde{d}_{3/2})=1.7\ \text{MeV}\ , \\ V^2(\tilde{d}_{5/2})=0.407\ , & E(\tilde{d}_{5/2})=0.0\ \text{MeV}\ , \\ V^2(\tilde{g}_{7/2})=0.195\ , & E(\tilde{g}_{7/2})=0.1\ \text{MeV}\ . \end{array}$$

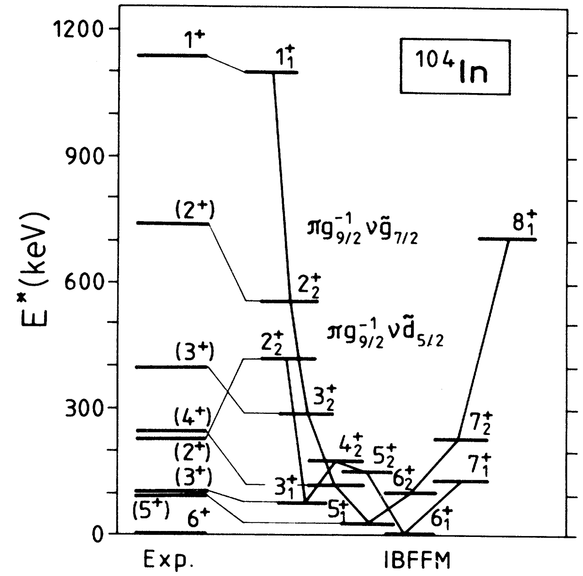

FIG. 1. Levels of ^{104}In. The experimental and theoretical spectra are denoted by Exp. and IBFFM, respectively. Only the proton-neutron multiplet-like states are shown in the IBFFM column. The abcissa of the theoretical part is scaled according to $I(I+1)$.

The *boson-fermion coupling strengths* could not be taken from the neighboring odd nuclei because of lack of experimental data. Instead, the strengths typical for the whole range of the odd In and Sn nuclei [15] were used. For the protons the dynamical interaction strength was 1.0 MeV and $\chi_p=-0.4$, as done also in the heavier odd-odd In nuclei. The exchange interaction was neglected, since the d boson consists basically of neutron excitations. The proton-boson monopole interaction was 0.05 MeV. In the case of neutrons, the dynamical interaction strength

TABLE I. Wave functions of the low-lying ^{104}In states. The components of the wave functions are characterized by the notation $|(j_\pi,j_\nu)J;n_d,R\rangle$. Only amplitudes, corresponding to weights, larger than 6% are given.

1_1		3_2	
$\lvert(\frac{9}{2},\frac{7}{2})1;0,0\rangle$	-0.687	$\lvert(\frac{9}{2},\frac{7}{2})3;0,0\rangle$	0.633
$(\frac{9}{2},\frac{7}{2})2;1,2\rangle$	0.557	$\lvert(\frac{9}{2},\frac{7}{2})4;1,2\rangle$	-0.513
$\lvert(\frac{9}{2},\frac{7}{2})3;1,2\rangle$	-0.245	$\lvert(\frac{9}{2},\frac{7}{2})2;1,2\rangle$	0.265
2_1		4_1	
$\lvert(\frac{9}{2},\frac{5}{2})2;0,0\rangle$	-0.549	$\lvert(\frac{9}{2},\frac{7}{2})4;0,0\rangle$	-0.473
$\lvert(\frac{9}{2},\frac{5}{2})3;1,2\rangle$	0.394	$\lvert(\frac{9}{2},\frac{5}{2})4;0,0\rangle$	0.447
$\lvert(\frac{9}{2},\frac{5}{2})2;1,2\rangle$	-0.351	$\lvert(\frac{9}{2},\frac{7}{2})5;1,2\rangle$	0.401
$\lvert(\frac{9}{2},\frac{7}{2})3;1,2\rangle$	0.350	$\lvert(\frac{9}{2},\frac{5}{2})5;1,2\rangle$	-0.307
$\lvert(\frac{9}{2},\frac{7}{2})2;0,0\rangle$	-0.285		
2_2		5_1	
$\lvert(\frac{9}{2},\frac{7}{2})2;0,0\rangle$	0.577	$\lvert(\frac{9}{2},\frac{7}{2})5;0,0\rangle$	-0.684
$\lvert(\frac{9}{2},\frac{7}{2})3;1,2\rangle$	-0.443	$\lvert(\frac{9}{2},\frac{7}{2})6;1,2\rangle$	0.464
$\lvert(\frac{9}{2},\frac{5}{2})2;0,0\rangle$	-0.424	$\lvert(\frac{9}{2},\frac{7}{s})4;1,2\rangle$	-0.358
3_1		6_1	
$\lvert(\frac{9}{2},\frac{5}{2})3;0,0\rangle$	-0.623	$\lvert(\frac{9}{2},\frac{5}{2})6;0,0\rangle$	0.624
$\lvert(\frac{9}{2},\frac{5}{2})4;1,2\rangle$	0.469	$\lvert(\frac{9}{2},\frac{5}{2})6;1,2\rangle$	0.481
$\lvert(\frac{9}{2},\frac{5}{2})3;1,2\rangle$	-0.378	$\lvert(\frac{9}{2},\frac{5}{2})7;1,2\rangle$	-0.360

was 0.8 MeV and the exchange interaction strength was 1.5 MeV. For the monopole interaction strength, the value of 0.1 MeV was used.

The short-range *proton-neutron effective interaction strengths* $V_0=-500$ MeV fm^3 and $\alpha=0.15$ are deduced from the double closed-shell nuclei [16]. The radial matrix elements were calculated using harmonic-oscillator wave functions with oscillator parameter $b=2.218$ fm. The spin-polarization interaction with a strength of $V_{\rm sp}=0.11$ MeV was used. The strength of the effective spin-spin interaction, which simulates the admixing of $M1$ giant resonance to the low-lying states, was estimated from the effective spin gyromagnetic ratios according to the relations given by Bohr and Mottelson [17]. The calculated *energy spectrum* of ^{104}In is presented in Fig. 1 in comparison with the experimental data. Only the states which correspond to $\pi g_{9/2}^{-1}\nu\tilde{g}_{7/2}$ and $\pi g_{9/2}^{-1}\nu\tilde{d}_{5/2}$ proton-neutron multiplets are shown. The calculated level energies are in a reasonably good agreement with the experimental ones.

The *wave functions* of the experimentally known states are presented in Table I. The IBFFM calculation preserves the approximate multiplet classification for most of the low-lying states. The dominant components of the 1_1^+, 2_2^+, 3_2^+, 4_1^+, and 5_1^+ states arise from the $\pi g_{9/2}^{-1}\nu\tilde{g}_{7/2}$ multiplet, while the main components of 2_1^+, 3_1^+, and 6_1^+ states come from the $\pi g_{9/2}^{-1}\nu\tilde{d}_{5/2}$ multiplet. Although the IBFFM wave functions are quite complex, they are based mainly on one multiplet, except the 4^+ states. The latter ones can be presented approximately as

$$|4_1\rangle \approx (1/\sqrt{2})(-|4_1^0\rangle + |4_2^0\rangle)\ ,$$

$$|4_2\rangle \approx (1/\sqrt{2})(|4_1^0\rangle + |4_2^0\rangle)\ ,$$

where $|4_1^0\rangle$ and $|4_2^0\rangle$ denote the $\pi g_{9/2}^{-1}\nu\tilde{g}_{7/2}$ and $\pi g_{9/2}^{-1}\nu\tilde{d}_{5/2}$ two-quasiparticle multiplet states, respectively.

Employing the wave functions obtained from diagonalization, the electromagnetic properties were calculated, too. For effective proton and neutron charges and gyromagnetic ratios, the following values have been used: $e_p=1.5e$, $e_n=0.5e$, $g_{lp}=1$, $g_{ln}=0$, $g_{sp}=0.5g_{sp}^{\rm free}$, $g_{sn}=0.5g_{sn}^{\rm free}$, $g_R=Z/A$, which are the standard values in the region [18]. For the boson charge the value $e_{\rm VIB}=2.0e$ was applied in accordance with the values applied previously in the heavier odd-odd In nuclei.

TABLE II. Magnetic dipole (μ in μ_N) and electric quadrupole (Q in e b) moments of the long-lived ^{104}In states.

	(6_1^+)	(3_1^+)
$T_{1/2}$ (s)	115	15.7
$\mu_{\rm EXP}$$^{\rm a}$	4.435(22)	
$\mu_{\rm BFFM}$	4.149	4.549
$Q_{\rm EXP}$$^{\rm a}$	0.659(106)	
$Q_{\rm IBFFM}$	0.685	0.445

$^{\rm a}$Reference [9].

The calculated *electromagnetic moments* of the isomeric states are presented in Table II in comparison with the experimental data. The magnetic dipole and electric quadrupole moments of the ground state agree with the experimental values within 7%. The IBFFM calculations show that the contribution of the collective $M1$ operator to the magnetic moments is small, while the quadrupole moments arise mainly from the off-diagonal collective contributions.

The experimental and theoretical $M1$ *reduced transition probabilities* for the transitions between the low-lying states are presented in Table III. As seen, the IBFFM calculations reproduce the experimental data with in a factor of 2 in all but one case.

The splitting of the $\pi g_{9/2}^{-1}\nu\tilde{d}_{5/2}$ multiplet is interesting, since its shape is very sensitive to the admixture of d-boson components. As the $d_{5/2}$ subshell is approximately half filled, only the spin-dependent residual interactions and the boson-neutron-exchange interaction affects the energies of the states. In the two-quasiparticle model (corresponding to the IBFFM with zero bosons), the residual interactions give a linear decrease of the energies as a function of $I(I+1)$, modified with an odd-even staggering: the even-spin states are pushed up and the odd-spin states down. However, the full IBFFM description, leading to $\approx 40\%$ boson admixture, predicts a W-like shape of the $\pi g_{9/2}^{-1}\nu\tilde{d}_{5/2}$ multiplet (see Fig. 1).

The fact that the ground state has the spin 6^+ means that both the 5^+ and 7^+ members of the multiplet must lie higher than the 6^+ state. This is in agreement with

TABLE III. Transitions between low-lying ^{104}In states. The experimental data were taken from Ref. [6]. The index C means calculated in the frame of IBFFM values.

E_i (keV)	I_i	E_f (keV)	I_f	I_γ/E_γ^3 (Rel.)	$B(M1)_C$ (Rel.)	$\lvert\delta_c\rvert$
1139	1_1^+	226	(2_1^+)	22	33	0.11
		738	(2_2^+)	100	100	0.11
738	(2_2^+)	93	(3_1^+)	13	16	0.04
		226	(2_1^+)	16	16	0.03
		396	(3_2^+)	100	100	0.04
396	(3_2^+)	226	(2_1^+)	11	12	0.07
		93	(3_1^+)	11	2	0.03
		241	(4_1^+)	100	100	0.02
241	(4_1^+)	93	(3_1^+)	80	71	0.01
		93	(5_1^+)	100	100	0.02

the IBFFM description, but contradicts the expectations based on the two-quasiparticle model, indicating that the role of the collective (or multiquasiparticle) excitations is significant even in the vicinity of ^{100}Sn.

It is interesting to mention that the splitting of the same multiplet in the ^{94}Nb, having a $g_{9/2}$ proton instead of the proton hole and an approximately half-filled $d_{5/2}$ neutron subshell, has quite a similar shape [19].

The *mixing of the 3_1^+ and 5_1^+ states* in the hyperfine electric field of the atomic electrons is treated as a consequence of the interaction of the nuclear quadrupole moment with the electric-field gradient. Thus the mixing amplitude is proportional to the $B(E2;3_1\rightarrow 5_1)$ transition probability [6]. The admixing of the short-lived 5_1^+ state to the isomeric 3_1^+ one leads to the reduction of the lifetime of the mixed state. The deviation is proportional to the $B(M1;5_1\rightarrow 6_1)$ transition probability [6]. According to our calculations, the inter multiplet $B(E2)$ value is 10.4 $e^2\,\mathrm{fm}^4$ (0.36 W.u.), having mainly collective character, and the also intermultiplet $B(M1)$ value is $0.08\mu_N^2$ (0.04 W.u.). Both values practically arise from the mixing of the multiplets. Using the above transition probabilities, one gets a few hundredth percent change of the (3_1^+) state's lifetime in the electric field generated by a $p_{3/2}$ electron, instead of the few-percent estimation of Szerypo *et al.* [6] obtained by using the Weisskopf estimate for the transition probabilities.

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