# Algebraic approach to cluster states in odd-mass nuclei. II. Electromagnetic and other properties

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The SU(3)×U(2) limit of the vibron-fermion model introduced recently by us as an algebraic approach to cluster states in odd-mass nuclei is developed further. The electromagnetic transition operators $T^{(E2)}$, $T^{(M1)}$ and $T^{(E1)}$ are constructed, together with the one-nucleon transfer operators. Experimental data available for the $\alpha$-cluster states of the $^{19}$F nucleus are analyzed in terms of our model to test its applicability. It is found that, although some quantities are not reproduced very well, the overall performance of the model is satisfactory. Furthermore, the phenomenological operators introduced here are able to describe the most intensive transitions in the lowest order; while introducing higher-order (two- or three-body) terms they also qualitatively give account of the less intensive transitions.

## I. INTRODUCTION

In the preceding paper [1] we have introduced an algebraic approach to cluster states of a class of nuclei in which fermionic degrees of freedom also play an important role in addition to the dipole ones describing the relative motion of two clusters. This new model, the SU(3)×U(2) limit of the vibron-fermion model, is the fermionic extension of the U(3) limit of the vibron model [2], which has already been applied to describe cluster states of light even-even nuclei [3]. Our model is able to handle the interplay between the collective (bosonic) and single-particle (fermionic) degrees of freedom using algebraic techniques. The collective degrees of freedom (i.e., the relative motion of the clusters) are described in terms of the U(3) limit of the vibron model, while the single-particle degrees of freedom (i.e., nucleonic excitation of one of the clusters) are treated by allowing nucleons (or holes) to occupy the states of an oscillator shell. (This shell is not necessarily a real nuclear one.) The group structure of the model introduced this way contains the $SU^B(3)$ bosonic group of the vibron model and the $SU_I^F(3)$ fermionic orbital angular momentum group, and the coupling of the bosonic and fermionic degrees of freedom is represented by an SU(3) coupling of the two group structures.

Our model was introduced in analogy with the vibron-electron model [4] of diatomic chemical molecules, which can be considered another limit [with O(4)×U(2) dynamical symmetry] of the general vibron-fermion model, built on the O(4) dynamical symmetry of the vibron model. The mathematical formulation of the SU(3)×U(2) limit of the vibron-fermion model was partly done following the methods used in the analogous limit of the interacting boson-fermion model (IBFM) [5].

After identifying the group structure of the model we introduced the coupled vibron-fermion basis associated with the SU(3)×U(2) dynamical symmetry and determined the eigenvalues of the corresponding Hamiltonian in this basis. We tried to identify the range of nuclei to which our model may be applicable, and suggested the $\alpha$-cluster states of the $^{19}$F nucleus as a possible example.

Now we develop the model further and construct the electromagnetic transition operators $T^{(E2)}$, $T^{(M1)}$, and $T^{(E1)}$ and determine their matrix elements in the SU(3)×U(2) basis. We also study the one-nucleon transfer operators, which, in the presence of only one nucleon (or hole), link SU(3)×U(2) basis states to vibron model states. These will be done in Sec. II. In Sec. III we apply our model to the $\alpha$-cluster states of the $^{19}$F nucleus to test its predictions by comparing them with the experimental data; finally, we summarize the results in Sec. IV. Part of the mathematics necessary in the calculations is presented in the Appendix.

## II. CONSTRUCTION OF FURTHER PHYSICAL OPERATORS

Here we shall construct the operators of some physical quantities not discussed in our preceding paper [1]. These operators can be sandwiched between the most general vibron-fermion basis states, including those with $M=0$ (i.e., no fermions), belonging to the vibron model. In the next section we shall use the SU(3)×U(2) basis [1] in the simplest case, i.e., with $M=1$, which corresponds to the coupling of one nucleon (or hole) to the relative motion of the clusters. The operators to be introduced here consist of bosonic and fermionic parts, acting on the bosonic and fermionic parts of the coupled wave functions [1]. The formulas necessary in the calculations and the reduced matrix elements of the operators occuring in this section are listed in the Appendix.

### A. Electromagnetic transitions

In the phenomenologic algebraic models the electromagnetic transition operators are written as Hermitian combinations of group generators with appropriate tensorial character. In the usual treatment it is enough to consider only one-body terms; nevertheless, higher-order terms can also be taken into account if some physical considerations require them. The application of the U(3) limit of the vibron model to $\alpha$-cluster states in light nuclei [3] showed that the $n_\pi$ quantum number can be used

 

to distinguish different cluster bands. Interband transitions can be described only with operators not conserving the number of $\pi$ bosons. Since $E2$ and $M1$ transitions of this kind are quite common in these nuclei, we have to consider higher-order terms in the $T^{(E2)}$ and $T^{(M1)}$ transition operators. The situation is the same with the odd-mass light nuclei with $\alpha$-cluster states, to which we want to apply the SU(3)×U(2) limit of the vibron-fermion model. Therefore we shall take higher-order bosonic terms into account in the $T^{(E2)}$ and $T^{(M1)}$ operators. Since we consider only one nucleon (or hole) at the moment, there is no need for higher-order fermionic terms.

### 1. E2 transitions

The most general one-body $E2$ operator in the vibron-fermion model is

$$T^{(E2)}_{\mu}=q_2[\pi^{\dagger}\times\tilde{\pi}]^{(2)}_{\mu}+\sum_{jj'} t^{(2)}_{jj'}[a^{\dagger}\times\tilde{a}_{j'}]^{(2)}_{\mu} , \tag{2.1}$$

consisting of a collective and a single-particle part. The matrix elements of $T^{(E2)}$ are related to the $B(E2)$ values and the quadrupole momenta in the usual way,

$$B(E2;\alpha J\rightarrow\alpha' J')=\frac{1}{2J+1}\langle\alpha' J'||T^{(E2)}||\alpha J\rangle^2 , \tag{2.2}$$

$$Q(\alpha,J)=\left[\frac{16\pi}{5}\right]^{1/2}\langle\alpha JM=J|T^{(E2)}_0|\alpha JM=J\rangle . \tag{2.3}$$

Calculations become simpler if the coefficients $t^{(2)}_{jj'}$ are chosen in a special way, namely, if the $T^{(E2)}$ operator can be written as the linear combination of the bosonic and fermionic quadrupole momentum operators:

$$T^{(E2)}_{\mu}=\tilde{q}_2 Q^{(2)}_{B,\mu}+\tilde{t}_2 Q^{(2)}_{F,\mu} . \tag{2.4}$$

An even more special case is obtained if $\tilde{t}_2=\pm\tilde{q}_2$ holds (+ for particle, − for hole coupling), since in this case $T^{(E2)}$ becomes proportional to the united quadrupole momentum operator $Q^{(2)}$:

$$T^{(E2)}_{\mu}=\tilde{q}_2 Q^{(2)}_{\mu} . \tag{2.5}$$

This operator allows transitions only between states with the same $(\lambda,\mu)$, while the more general operator in Eq. (2.4) connects states with the same $n_{\pi}$, and allows the change of $(\lambda,\mu)$. But this operator is still not general enough, since the $E2$ transitions changing $n_{\pi}$ with two units may also occur. In order to describe transitions of this kind we have to add new terms to the $T^{(E2)}$ operator. These new terms should conserve the total boson number $N$, while changing $n_{\pi}$ with 2 units. The simplest two-body terms with this character are

$$P^{(2)}_{\mu}=\tilde{p}[[[\sigma^{\dagger}\times\tilde{\pi}]^{(1)}\times[\sigma^{\dagger}\times\tilde{\pi}]^{(1)}]^{(2)}_{\mu}+\text{H.c.}] . \tag{2.6}$$

The matrix elements of this operator depend explicitly on $N$, the total number of bosons, but this dependence can be absorbed in $\tilde{p}$.

### 2. M1 transitions

The most general one-body $M1$ operator is

$$T^{(M1)}_{\mu}=\left[\frac{3}{4\pi}\right]^{1/2}\left[c_1[\pi^{\dagger}\times\tilde{\pi}]^{(1)}_{\mu}+\sum_{jj'} t^{(1)}_{jj'}[a^{\dagger}_{j}\times\tilde{a}_{j'}]^{(1)}_{\mu}\right] . \tag{2.7}$$

With a special choice of the $t^{(1)}_{jj'}$ parameters the fermion orbital angular momentum and fermion spin operators, i.e., generators of the product group $\mathrm{SU}^F_l(3)\times\mathrm{SU}^F_s(2)$, can be obtained. Replacing $L^{(1)}_B$, $L^{(1)}_F$, and $S^{(1)}_F$ with another set of independent angular momenta and redefining the parameters, $T^{(M1)}$ can be written in a different form:

$$T^{(1)}_{\mu}=\left[\frac{3}{4\pi}\right]^{1/2}[g_B J^{(1)}_{\mu}+(g_l-g_B)L^{(1)}_{F\mu}+(g_s-g_B)S^{(1)}_{F\mu}] , \tag{2.8}$$

where $S^{(1)}_{F\mu}=-(1/\sqrt{2})S^{(1)}_{\mu}(\frac{1}{2},\frac{1}{2})$ is the generator of the fermionic spin group $\mathrm{SU}^F_s(2)$. Here $J^{(1)}$ is diagonal in the SU(3)×U(2) basis, so it will not contribute to the transitions, only to the magnetic dipole momenta, $S^{(1)}_F$ links states with the same quantum numbers $(\lambda,\mu)$ and $L$, while $L^{(1)}_F$ is able to describe transitions between bands with different $(\lambda,\mu)$ quantum numbers. Besides the one-body operators, two-body terms are also often necessary to give a more complete description of $M1$ transitions in the phenomenologic algebraic models. A possible choice is to follow the usual treatment of the interacting boson model (IBM) [6] and the IBFM [5] introducing the new term $[Q^{(2)}\times J^{(1)}]^{(1)}_{\mu}$. This operator allows intraband transitions with $|\Delta L|\leq 2$. In order to generalize this term to interband transitions we can replace it with the operators $[Q^{(2)}_B\times J^{(1)}]^{(1)}_{\mu}$ and $[Q^{(2)}_F\times J^{(1)}]^{(1)}_{\mu}$.

Operators mentioned here are able to link states with the same $n_{\pi}$ quantum numbers. In order to describe transitions with $\Delta n_{\pi}=\pm 2$ even third-order terms may be necessary. Based on the arguments similar to those presented in connection with the $E2$ transitions, the introduction of $[P^{(2)}\times J^{(1)}]^{(1)}_{\mu}$ seems to be the best choice, where $P^{(2)}_{\mu}$ is the operator describing $E2$ transitions with $\Delta n_{\pi}=\pm 2$. The $M1$ operator completed with these higher-order terms can be written as

$$T^{(M1)}_{\mu}=\left[\frac{3}{4\pi}\right]^{1/2}\{g_B J^{(1)}_{\mu}+(g_l-g_B)L^{(1)}_{F\mu}+(g_s-g_B)S^{(1)}_{F\mu}+g_{QB}[Q^{(2)}_B\times J^{(1)}]^{(1)}_{\mu}+g_{QF}[Q^{(2)}_F\times J^{(1)}]^{(1)}_{\mu}+g_P[P^{(2)}\times J^{(1)}]^{(1)}_{\mu}\} . \tag{2.9}$$

The $M1$ transitions and magnetic dipole momenta can be calculated in the usual way:

$$B(M1;\alpha J\rightarrow\alpha' J')=\frac{1}{2J+1}\langle\alpha' J'||T^{(M1)}||\alpha J\rangle^2 , \quad (2.10)$$

$$\mu(\alpha,J)=\left[\frac{4\pi}{3}\right]^{1/2}\langle\alpha JM=J|T_0^{(M1)}|\alpha JM=J\rangle . \quad (2.11)$$

### 3. *E1 transitions*

Electric dipole transitions connect states with different parity, so it is impossible to construct pure fermionic terms in $T^{(E1)}$ in the SU(3)×U(2) limit of the vibron-fermion model. This could only be possible if there were single-particle fermionic states with opposite parity in the model space. [This is the case in the O(4)×U(2) limit of the vibron-fermion model.] Considering these arguments, one-body terms in the $E1$ transition operator can only be purely bosonic. There are two Hermitian operators with the required tensorial character, $D_\mu^{(1)}=[\pi^\dagger\times\tilde{\sigma}-\sigma^\dagger\times\tilde{\pi}]_\mu^{(1)}$, which is usually considered the dipole operator, and $R_\mu^{(1)}=i[\pi^\dagger\times\tilde{\sigma}+\sigma^\dagger\times\tilde{\pi}]_\mu^{(1)}$. (Here we follow the definition of Ref. [4], which slightly differs from the original definitions in Ref. [2].) The most general one-body $E1$ operator can be written as the linear combination of $D_\mu^{(1)}$ and $R_\mu^{(1)}$,

$$\begin{aligned}T_\mu^{(E1)}&=d[\pi^\dagger\times\tilde{\sigma}-\sigma^\dagger\times\tilde{\pi}]_\mu^{(1)}+ri[\pi^\dagger\times\tilde{\sigma}+\sigma^\dagger\times\tilde{\pi}]_\mu^{(1)}\\&=(-d+ri)[\sigma^\dagger\times\tilde{\pi}]_\mu^{(1)}+(d+ri)[\pi^\dagger\times\tilde{\sigma}]_\mu^{(1)}\\&=c[\sigma^\dagger\times\tilde{\pi}]_\mu^{(1)}-c^*[\pi^\dagger\times\tilde{\sigma}]_\mu^{(1)} . \quad (2.12)\end{aligned}$$

This result shows that the same real parameter ($|c|=|c^*|$) has to be used in the $n_\pi\rightarrow n_\pi+1$ and $n_\pi\rightarrow n_\pi-1$ transitions. Since the ordering of the levels is such that states with higher $n_\pi$ usually lie higher, only the second kind of these transitions will be present in most cases.

There is no need for higher-order bosonic terms either (in the sense we introduced them in the previous subsections) to bridge states with $|\Delta n_\pi|>1$, since such transitions seem to be unimportant. In principle, we could add two-body terms which change $n_\pi$ with one unit and contain another bilinear product of the boson or fermion operators to give a more detailed description of the $E1$ transitions, but it seems unnecessary in the first step.

The $B(E1)$ values are written in the usual way:

$$B(E1;\alpha J\rightarrow\alpha' J')=\frac{1}{2J+1}|\langle\alpha' J'||T^{(E1)}||\alpha J\rangle|^2 , \quad (2.13)$$

now taking into account the possibility of complex parameters in Eq. (2.12).

### B. One-nucleon transfer reactions

The U(3) limit of the vibron model has already been applied in the description of $\alpha$-cluster states in even-even light nuclei [3]. As we want to use the SU(3)×U(2) limit of the vibron-fermion model to describe such states in odd-mass light nuclei, the study of one-nucleon transfer reactions offer a convenient way to connect the model spaces of the vibron model and the vibron-fermion model.

The simplest phenomenologic one-nucleon transfer operators contain one-body terms. Since such operators create or annihilate one nucleon in the specified state, these one-body operators can be written as [5]

$$\begin{aligned}P_{+,\mu}^{(j)}&=\xi_j a_{j\mu}^\dagger ,\\P_{-,\mu}^{(j)}&=\xi_j\tilde{a}_{j\mu}=(-1)^{j-\mu}(P_{+,\mu}^{(j)})^\dagger .\end{aligned} \quad (2.14)$$

Since the fermions are allowed to occupy states in a given oscillator shell, these operators preserve or change the parity of the model states depending on the parity of the fermionic shell taken into account. Therefore, one-body terms can describe only either parity-conserving or parity-changing reactions, so higher-order terms are required in order to describe both processes.

The general expression of the phenomenologic two-body operators is

$$P_{+,\mu}^{(j)}(\text{2-body})=\sum_{kj'}\xi_{(kj')j}[[b^\dagger\times\tilde{b}]^{(k)}\times a_{j'}^\dagger]_\mu^{(j)} \quad (2.15a)$$

for the nucleon adding and

$$P_{-,\mu}^{(j)}(\text{2-body})=(-1)^{j-\mu}[P_{+,-\mu}^{(j)}(\text{2-body})]^\dagger \quad (2.15b)$$

for the nucleon subtracting processes, where $b^\dagger=\sigma^\dagger$, $\pi^\dagger$ and $\tilde{b}=\tilde{\sigma}$, $\tilde{\pi}$, and we take bosonic terms with definite parity only.

The matrix elements of the one- and two-body nucleon transfer operators for a transition going from the ground state $|N(n_{\pi\min},0)L=0\rangle$ of an even-even nucleus to a vibron-fermion state denoted by $|N(n_\pi,0)(\lambda_F,\mu_F);(\lambda,\mu)L\frac{1}{2}J\rangle$ can be calculated using the formulas presented in the Appendix:

$$\langle N(n_\pi,0)(\lambda_F,\mu_F);(\lambda,\mu)L\tfrac{1}{2}J||P_+^{(j)}(\text{1-body}||N(n_{\pi,\min},0)L=0\rangle$$
$$=-\xi_j\delta_{jJ}\delta_{n_\pi n_{\pi,\min}}\delta_{Ll}(2j+1)^{1/2}\langle(n_\pi,0)0;(\lambda_F\mu_F)l||(\lambda,\mu)l\rangle , \quad (2.16a)$$

$$\langle N(n_\pi,0)(\lambda_F,\mu_F);(\lambda,\mu)L\tfrac{1}{2}J||P_+^{(j)}(\text{2-body})||N(n_{\pi,\min},0)L=0\rangle$$
$$=\delta_{jJ}\sum_{kj'}\xi_{(kj')j}\langle(n_\pi,0)k;(\lambda_F\mu_F)l||(\lambda,\mu)L\rangle(-1)^{j'+1/2}\left[\frac{(2j+1)(2j'+1)(2L+1)}{2k+1}\right]^{1/2}\begin{Bmatrix}k & j & j'\\ \frac{1}{2} & l & L\end{Bmatrix}$$
$$\times\langle N(n_\pi,0)k||[b^\dagger\times\tilde{b}]^{(k)}||N(n_{\pi,\min},0)0\rangle . \quad (2.16b)$$

Examining the reduced matrix elements of $[b^\dagger \times \tilde{b}]^{(k)}$ in Eq. (2.16b) we can establish the following. Taking $[\pi^\dagger \times \tilde{\pi}]^{(k)}$, $k=1$ cannot occur, since $k$ must be even if $n_\pi = n_{\pi,\min}$ = even. $k=0$ means only a redefinition of the coefficient $\xi_j$ in the one-body term, so the only significant term of this type is $[\pi^\dagger \times \tilde{\pi}]^{(2)}$. Since the one-nucleon transfer operators are not Hermitian, we can vary the coefficients of the terms $[\sigma^\dagger \times \tilde{\pi}]^{(1)}$ and $[\pi^\dagger \times \tilde{\sigma}]^{(1)}$ independently, contrary to the $T^{(E1)}$ operators (2.12), which also contain these terms. Contrary to the terms with $[\pi^\dagger \times \tilde{\pi}]^{(k)}$, these terms change the parity of the bosonic state.

The results of the preceding paragraph can be summarized as follows. If the fermionic shell taken into account has even parity ($n$ = even), the parity-conserving one-nucleon transfer operator can be written as

$$P^{(j^+)}_{+,\mu} = \xi^{(+)}_j a^\dagger_{j\mu} + \sum_{j'} \xi^{(+)}_{(2j')j} [[\pi^\dagger \times \tilde{\pi}]^{(2)} \times a^\dagger_{j'}]^{(j)}_\mu \tag{2.17a}$$

and the parity-changing one-nucleon transfer operators are

$$P^{(j^-)}_{+,\mu} = \sum_{j'} \xi^{(-)}_{(1j')j} [[\sigma^\dagger \times \tilde{\pi}]^{(1)} \times a^\dagger_{j'}]^{(j)}_\mu + \sum_{j'} \xi^{(-)}_{(1j')j} [[\pi^\dagger \times \tilde{\sigma}]^{(1)} \times a^\dagger_{j'}]^{(j)}_\mu , \tag{2.17b}$$

containing only two-body terms. In the $n$ = odd case, however, the situation will change:

$$P^{(j^+)}_{+,\mu} = \sum_{j'} \xi^{(+)}_{(1j')j} [[\sigma^\dagger \times \tilde{\pi}]^{(1)} \times a^\dagger_{j'}]^{(j)}_\mu + \sum_{j'} \xi^{(+)}_{(1j')j} [[\pi^\dagger \times \tilde{\sigma}]^{(1)} \times a^\dagger_{j'}]^{(j)}_\mu \tag{2.18a}$$

is the parity-conserving transfer operator, and

$$P^{(j^-)}_{+,\mu} = \xi^{(-)}_j a^\dagger_{j\mu} + \sum_{j'} \xi^{(-)}_{(2j')j} [[\pi^\dagger \times \tilde{\pi}]^{(2)} \times a^\dagger_{j'}]^{(j)}_\mu . \tag{2.18b}$$

is the parity-changing transfer operator, now with one- and two-body terms. Matrix elements of the inverse processes can be calculated using the operators

$$P^{(j^\pi)}_{-,\mu} = (-1)^{j-\mu} (P^{(j^\pi)}_{+,-\mu})^\dagger . \tag{2.19}$$

To measure the strength of the transfer reaction the spectroscopic strength can be introduced, similarly to other phenomenologic models, like the IBFM [5]:

$$S(\alpha' J' \to \alpha j) = \frac{1}{2J'+1} |\langle \alpha J || P^{(j)} || \alpha' J' \rangle|^2 . \tag{2.20}$$

## III. APPLICATION OF THE MODEL TO REALISTIC PROBLEMS

In our preceding paper [1] we suggested that the $\alpha$-cluster states of the $^{19}$F nucleus may serve as a good example for the approximate realization of the SU(3)×U(2) limit of the vibron-fermion model. We identified 25 $^{19}$F states of this kind with SU(3)×U(2) model states comparing the structure of the experimental and the model spectra. Here we discuss whether the available information of the electric quadrupole, magnetic dipole, and electric dipole transition probabilities support this assignment. We also study the available data on the one-nucleon transfer reaction $^{20}$Ne(t,$\alpha$)$^{19}$F and try to interpret them in terms of our model. We expect our model to be able to give account of the close relationship of certain cluster bands of the $^{20}$Ne and $^{19}$F nuclei. Here we accepted the band assignment proposed by Descouvemont and Baye [7], which differs from other schemes [8] at some places (both in the assignment and in the interpretation of energy levels and bands), and also introduces new bands in addition to the old ones.

As discussed in Ref [1], the physical picture behind our model is the coupling of the relative motion of the $\alpha$ and the $^{15}$N clusters to a hole on the $p$ shell of the $^{15}$N core. This allows cluster bands built on the excited $J^\pi = \frac{3}{2}^-$ nitrogenic state, the importance of which has been emphasized by several authors [7,9]. In fact, the basis states of our model contain contributions from the $\alpha + {}^{15}\mathrm{N}_{\mathrm{g.s.}}$ and the $\alpha + {}^{15}\mathrm{N}(\frac{3}{2}^-)$ configurations alike. [The relative weight of the two configurations in the model states is determined by the SU(3)⊃O(3) Wigner coefficients [1].]

In these first calculations we assume that the $\alpha$-cluster states of $^{19}$F have an exact SU(3)×U(2) dynamical symmetry. Of course, this is a simplification of the physical problem: In a more realistic description symmetry-breaking terms should also be taken into account, and the model states would contain contributions from several SU(3)×U(2) states, etc. However, our aim was the test of the validity of our new model and not the realistic description of any nucleus. This could be the next step later on.

### A. Electromagnetic transitions

Here we shall use two different methods to test our model by comparing the calculated electromagnetic transition probabilities with the available experimental data. First we shall select a number of transitions (preferably those with the lowest relative error) sufficient to fix the model parameters by equating their probabilities to the values calculated from our model. Following this procedure the other transition rates can be "predicted" and the performance of our model can be judged on the basis of the overall "goodness" of this prediction. (We shall call this approach method 1.) The second method (method 2) is a least-squares fit of all the transition probabilities with respect to the parameters appearing in the transition operators. We shall take into account all experimental data with equal weight (no matter how large their relative errors are), but we shall ignore those which represent only a lower or upper bound of the given transition probability.

We shall display the results from both approaches, but, since our aim is not the description of the $^{19}$F nucleus, we shall present only those theoretical values which have experimental correspondents. In order to develop a more complete picture of the performance of our model we shall also display the results obtained from another phenomenologic cluster model, the local potential model of Buck *et al.* [8]. It has to be noted, however, that the interpretation and level assignment of this model differ at

some places from the conventions (given in Ref. [7]) we followed in our model. The $K^\pi=\frac{1}{2}_1^-$ and $\frac{1}{2}_2^+$ bands are interpreted as $\alpha$-cluster bands with $2N+L=8$ and 9, respectively, while the $K^\pi=\frac{1}{2}_1^+$ (ground-state) band is described as a $t+^{16}$O cluster band with $2N+L=6$. Similarly, triton-cluster structure with $2N+L=7$ is assigned to a set of negative-parity states, some of which we interpreted as members of the $K^\pi=\frac{1}{2}_3^-$ or $\frac{3}{2}^-$ bands, or as states with different structure. It is also worth mentioning that Buck and Pilt presented $B(E2)$ and $B(M1)$ values for intraband transitions only and their $B(E1)$ values are also restricted to transitions connecting states with similar cluster structure.

### 1. *E2 transitions*

There are 16 transitions (see Table I) belonging to two intraband and two interband sets, for which experimental data are available [7,10]. We have three model parameters to fix: $\tilde{q}_2$ and $\tilde{t}_2$ from Eq. (2.4) and $\tilde{p}$ from Eq. (2.6). The operators $Q_B^{(2)}$ and $Q_F^{(2)}$ corresponding to the first two parameters link states with the same $n_\pi$, while operator $P^{(2)}$ connects states with $|\Delta n_\pi|=2$. This means that the two transitions going from the $K^\pi=\frac{1}{2}_2^+$ band to the $K^\pi=\frac{1}{2}_1^+$ ground-state band can be treated separately, and the parameters $\tilde{p}$ can be fixed to these independently from the others. These transitions seem to be generally weaker than the others, which is in agreement with our expectations, since these transitions are described by a two-body operator in our approach.

It is also worth mentioning that fitting transitions within only one band requires only one parameter ($\tilde{q}_2$ or $\tilde{t}_2$), because these parameters appear in the same combination in the corresponding matrix elements. Both parameters are needed only when there are two or more bands with intraband transitions.

The results of the fitting procedures are displayed in Table I. Following method 1 we have selected the transi-

TABLE I. $E2$ reduced transition probabilities in Weisskopf units.

| Expt. [7,10] | | | Ref. [8] | SU(3)×U(2) | | | |
|---|---|---|---|---|---|---|---|
| $J_i^\pi(E_{xi})$ | $J_f^\pi(E_{xf})$ | $B(E2)_{\text{exp}}$ | $B(E2)_B$ | $B(E2)_{\text{th1}}$[a] | $B(E2)_{\text{th2}}$[a] | Band | Labels |
| $K^\pi=\frac{1}{2}_1^+$ | $\rightarrow K^\pi=\frac{1}{2}_1^+$ | | | | | 7(6,0) | $\rightarrow$7(6,0) |
| $\frac{5}{2}^+(0.20)$ | $\frac{1}{2}^+(0.0)$ | 6.95±0.08[b] | 6.42 | 6.95[b] | 10.68 | 2 | 0 |
| $\frac{3}{2}^+(1.55)$ | $\frac{1}{2}^+(0.0)$ | 6.8±0.7 | 6.21 | 6.95 | 10.68 | 2 | 0 |
| | $\frac{5}{2}^+(0.20)$ | <127[c] | 2.83 | 3.10 | 4.77[c] | 2 | 2 |
| $\frac{9}{2}^+(2.78)$ | $\frac{5}{2}^+(0.20)$ | 8.2±0.9 | 7.30 | 8.09 | 12.47 | 4 | 2 |
| $\frac{13}{2}^+(4.65)$ | $\frac{9}{2}^+(2.78)$ | 5.3±0.9 | 4.75 | 5.27 | 8.09 | 6 | 4 |
| $\frac{7}{2}^+(5.46)$ | $\frac{5}{2}^+(0.20)$ | 2.0±0.5 | 0.85 | 0.81 | 1.24 | 4 | 2 |
| | $\frac{3}{2}^+(1.55)$ | 14±4 | 7.90 | 7.28 | 11.18 | 4 | 2 |
| | $\frac{9}{2}^+(2.78)$ | 3±2 | 0.63 | 0.78 | 1.20 | 4 | 4 |
| $K^\pi=\frac{1}{2}_2^+$ | $\rightarrow K^\pi=\frac{1}{2}_1^+$ | | | | | 9(8,0)$\rightarrow$ | 7(6,0) |
| $\frac{1}{2}^+(5.34)$ | $\frac{5}{2}^+(0.20)$ | <2[c] | | 0.63 | 0.83[c] | 0 | 2 |
| $\frac{3}{2}^+(5.50)$ | $\frac{5}{2}^+(0.20)$ | 2.4 | | 0.91 | 1.19 | 2 | 2 |
| $\frac{5}{2}^+(6.29)$ | $\frac{1}{2}^+(0.0)$ | 1.9±0.7[b] | | 1.90[b] | 2.48 | 2 | 0 |
| $K^\pi=\frac{1}{2}_1^-$ | $\rightarrow K^\pi=\frac{1}{2}_1^-$ | | | | | 8(7,0)$\rightarrow$ | 8(7,0) |
| $\frac{5}{2}^-(1.35)$ | $\frac{1}{2}^-(0.11)$ | 20±3[b] | 15.60 | 20.00[b] | 14.13 | 3 | 1 |
| $\frac{3}{2}^-(1.46)$ | $\frac{1}{2}^-(0.11)$ | 25±11 | 15.60 | 21.89 | 15.46 | 1 | 1 |
| $\frac{7}{2}^-(4.00)$ | $\frac{5}{2}^-(1.35)$ | >4[c] | 2.00 | 3.13 | 2.21[c] | 3 | 3 |
| | $\frac{3}{2}^-(1.46)$ | $10^{+9}_{-4}$ | 18.07 | 25.71 | 18.16 | 3 | 1 |
| $\frac{9}{2}^-(4.03)$ | $\frac{5}{2}^-(1.35)$ | 28±6 | 20.08 | 22.51 | 15.90 | 5 | 3 |
| $\frac{11}{2}^-(8.95)$ | $\frac{7}{2}^-(4.00)$ | 8.1±1.2 | 17.18 | 23.88 | 16.86 | 5 | 3 |
| | $\frac{9}{2}^-(4.03)$ | $0.5^{+0.9}_{-0.4}$ | 0.66 | 1.28 | 0.90 | 5 | 5 |
| $K^\pi=\frac{1}{2}_3^-$ | $\rightarrow K^\pi=\frac{1}{2}_1^-$ | | | | | 8(8,1)$\rightarrow$ | 8(7,0) |
| $\frac{3}{2}^-(6.79)$ | $\frac{1}{2}^-(0.11)$ | 0.7±0.3 | | 294.78 | 2.60 | 1 | 1 |

[a] $B(E2)_{\text{th1}}$: fitting selected transitions (method 1); $B(E2)_{\text{th2}}$: fitting all the transitions (method 2).
[b] Used to fit model parameters (method 1).
[c] Not fitted (method 2).

tions $\frac{5}{2}^+ \rightarrow \frac{1}{2}^+$ and $\frac{5}{2}^- \rightarrow \frac{1}{2}^-$ within the $K^\pi = \frac{1}{2}_1^+$ and $K^\pi = \frac{1}{2}_1^-$ bands, respectively, to fix the parameters $\tilde{q}_2$ and $\tilde{t}_2$, while $\tilde{p}$ was fixed separately using the $K^\pi = \frac{1}{2}_2^+$, $J^\pi = \frac{5}{2}^+ \rightarrow K^\pi = \frac{1}{2}_1^+$, $J^\pi = \frac{1}{2}^+$ transition. As it can be seen from Table I, the calculated values agree remarkably well with the experimental data for the $K^\pi = \frac{1}{2}_1^+$ and $\frac{1}{2}_1^-$ bands; only the single $K^\pi = \frac{1}{2}_3^- \rightarrow K^\pi = \frac{1}{2}_1^-$ interband transition is overestimated considerably. The parameters in this case are $\tilde{q}_2 = 7.198$, $\tilde{t}_2 = 61.764$, and $\tilde{p} = 0.013$. Using method 2, the least-squares-fitting procedure, we arrive at the parameter set $\tilde{q}_2 = 1.975$, $\tilde{t}_2 = 7.098$, and $\tilde{p} = 0.015$ (and $\chi^2 = 487.1$). We can see that the general trend of the transition rates is reasonably well reproduced in this case too. A large fraction of $\chi^2$ originates from transitions within the $K^\pi = \frac{1}{2}_1^-$ band with large error bars.

It is interesting to see that our results obtained from method 1 are close to those of Buck and Pilt [8] for the $\frac{1}{2}_1^+$ band, while the same holds for our results from method 2 and for the $\frac{1}{2}_1^-$ band. (Of the two data sets in Ref. [8] here we presented the one calculated with nonzero effective charge.)

As a test of the parameters obtained from the fitting procedures, we can calculate the quadrupole momenta of some states using Eq. (2.3). Experimental value is available only for the $K^\pi = \frac{1}{2}_1^+$, $J^\pi = \frac{5}{2}^+$ state: $Q = \pm(11 \pm 2)$ $e$ fm$^2$. The negative sign is generally accepted, since other models give negative values: $Q = -8.26$ $e$ fm$^2$ (generator coordinate method (GCM) [7]); $Q = -8.83$ $e$ fm$^2$ (Buck and Pilt [8]). Using the parameters fixed previously by methods 1 and 2 we get $Q = -5.45$ and $-6.75$ $e$ fm$^2$, respectively, which are smaller in magnitude than the experimental value, but are close to the prediction of the other models.

### 2. *M1 transitions*

There are 17 $M1$ transitions for which experimental information is available (see Table II). We shall use four

TABLE II. $M1$ reduced transition probabilities in Weisskopf units.

| $J_i^\pi(E_{xi})$ | Expt. [7,10] $J_f^\pi(E_{xf})$ | $B(M1)_{\text{exp}}$ | Ref. [8] $B(M1)_B$ | SU(3)×U(2) $B(M1)_{\text{th1}}$[a] | $B(M1)_{\text{th2}}$[a] | Band | Labels |
|---|---|---|---|---|---|---|---|
| $K^\pi = \frac{1}{2}_1^+ \rightarrow K^\pi = \frac{1}{2}_1^+$ | | | | | | 7(6,0)→ | 7(6,0) |
| $\frac{3}{2}^+(1.55)$ | $\frac{1}{2}^+(0.0)$ | $(4.3\pm2.5)\times10^{-2}$ | 0 | 0.001 | 0.227 | 2 | 0 |
| | $\frac{5}{2}^+(0.20)$ | $2.3\pm1.4$ | 2.49 | 0.972 | 1.110 | 2 | 2 |
| $\frac{7}{2}^+(5.46)$ | $\frac{5}{2}^+(0.20)$ | $(1.5\pm0.5)\times10^{-3}$[b] | 0 | 0.0015[b] | 0.238 | 4 | 2 |
| | $\frac{9}{2}^+(2.78)$ | $0.9\pm0.2$[b] | 2.29 | 0.900[b] | 1.028 | 4 | 4 |
| $K^\pi = \frac{1}{2}_2^+ \rightarrow K^\pi = \frac{1}{2}_1^+$ | | | | | | 9(8,0)→ | 7(6,0) |
| $\frac{1}{2}^+(5.34)$ | $\frac{1}{2}^+(0.0)$ | $0.19\pm0.03$ | | 0 | 0 | 0 | 0 |
| | $\frac{3}{2}^+(1.55)$ | $<0.03$[c] | | 0.005 | 0.002[c] | 0 | 2 |
| $\frac{3}{2}^+(5.50)$ | $\frac{5}{2}^+(0.20)$ | 0.33 | | 0.046 | 0.020 | 2 | 2 |
| | $\frac{3}{2}^+(1.55)$ | 0.18 | | 0.083 | 0.036 | 2 | 2 |
| $\frac{5}{2}^+(6.29)$ | $\frac{5}{2}^+(0.20)$ | $(3.0\pm1.1)\times10^{-3}$ | | 0.252 | 0.109 | 2 | 2 |
| | $\frac{3}{2}^+(1.55)$ | $(3.0\pm0.8)\times10^{-2}$[b] | | 0.030[b] | 0.013 | 2 | 2 |
| $K^\pi = \frac{1}{2}_1^- \rightarrow K^\pi = \frac{1}{2}_1^-$ | | | | | | 8(7,0)→ | 8(7,0) |
| $\frac{3}{2}^-(1.46)$ | $\frac{1}{2}^-(0.11)$ | $(9.1\pm1.9)\times10^{-2}$ | 0 | 0.518 | 0.510 | 1 | 1 |
| $\frac{7}{2}^-(4.00)$ | $\frac{5}{2}^-(1.35)$ | $(6.2\pm2.4)\times10^{-2}$ | 0 | 0.666 | 0.656 | 3 | 3 |
| $\frac{11}{2}^-(8.95)$ | $\frac{9}{2}^-(4.03)$ | $(8.3\pm1.4)\times10^{-3}$ | 0 | 0.707 | 0.696 | 5 | 5 |
| $K^\pi = \frac{1}{2}_3^- \rightarrow K^\pi = \frac{1}{2}_1^-$ | | | | | | 8(8,1)→ | 8(7,0) |
| $\frac{3}{2}^-(6.79)$ | $\frac{1}{2}^-(0.11)$ | $0.34\pm0.06$[b] | | 0.340[b] | 0.126 | 1 | 1 |
| | $\frac{5}{2}^-(1.35)$ | $(8.6\pm2.1)\times10^{-2}$ | | 0 | 0 | 1 | 3 |
| | $\frac{3}{2}^-(1.46)$ | $0.44\pm0.09$ | | 1.702 | 0.629 | 1 | 1 |
| $K^\pi = \frac{3}{2}^- \rightarrow K^\pi = \frac{1}{2}_1^-$ | | | | | | 8(8,1)→ | 8(7,0) |
| $\frac{3}{2}^-(6.89)$ | $\frac{5}{2}^-(1.35)$ | $0.53\pm0.14$ | | 0.557 | 0.206 | 2 | 3 |
| | $\frac{3}{2}^-(1.46)$ | $0.27\pm0.09$ | | 0.255 | 0.094 | 2 | 1 |

[a] $B(M1)_{\text{th1}}$: fitting selected transitions (method 1); $B(m1)_{\text{th2}}$: fitting all the transitions (method 2).
[b] Used to fit model parameters (method 1).
[c] Not fitted (method 2).

parameters to fit these: $g_l-g_B$, $g_s-g_B$, $g_Q=g_{Q_B}-g_{Q_F}$, and $g_P$ from Eq. (2.9). The third one of these is the parameter of the two-body operator $[Q^{(2)}\times J^{(1)}]^{(1)}$, which is able to describe intraband transitions only. The same holds for the operator related to $g_s-g_B(S_F^{(1)})$, while $L_F^{(1)}$, the fermionic orbital angular momentum operator, is able to link states with different $(\lambda,\mu)$ (and the same $n_\pi$).

Without any calculations we can estimate the importance of various terms of the $T^{(M1)}$ operator, since there are transitions which can be described by only $L_F^{(1)}$, $[Q^{(2)}\times J^{(1)}]^{(1)}$, or $[P^{(2)}\times J^{(1)}]^{(1)}$ in our approach. The one-body term $L_F^{(1)}$ is expected to be significant, since it is the only operator which is able to link states with $(\lambda,\mu)=(8,1)$ and (7,0), which we assigned to the strong $K^\pi=\frac{1}{2}_3^-\rightarrow K^\pi=\frac{1}{2}_1^-$ and $K^\pi=\frac{3}{2}^-\rightarrow K^\pi=\frac{1}{2}_1^-$ transitions. We expect the two-body term $[Q^{(2)}\times J^{(1)}]^{(1)}$ to be less important: the only transitions it describes alone are the two weak ones within the $K^\pi=\frac{1}{2}_1^+$ band. The three-body term $[P^{(2)}\times J^{(1)}]^{(1)}$ describes only transitions with $\Delta n_\pi=2$ (corresponding to $K^\pi=\frac{1}{2}_2^+\rightarrow K^\pi=\frac{1}{2}_1^+$ in our approach), so the related parameter can be fixed independently from the others, similarly to the $P^{(2)}$ operator of the $E2$ transitions. This term is also expected to be less significant, as the corresponding transitions are not too strong.

Following method 1 we used transitions $K^\pi=\frac{1}{2}_3^-$, $J^\pi=\frac{3}{2}^-\rightarrow K^\pi=\frac{1}{2}_1^-$, $J^\pi=\frac{1}{2}^-$ and $K^\pi=\frac{1}{2}_1^+$, $J^\pi=\frac{7}{2}^+\rightarrow K^\pi=\frac{1}{2}_1^+$, $J^\pi=\frac{5}{2}^+$ to fix parameters $g_l-g_B$ and $g_Q$, respectively, as these are the most precisely measured transitions among the ones that can be described by the operators $L_F^{(1)}$ and $[Q^{(2)}\times J^{(1)}]^{(1)}$ alone. In order to fix the third parameter, $g_s-g_B$, we chose transition $J^\pi=\frac{7}{2}^+\rightarrow J^\pi=\frac{9}{2}^+$ within the $K^\pi=\frac{1}{2}_1^+$ band, which is the strongest among those with comparable relative error. Unfortunately, the best candidate to fix $g_P$ to the $K^\pi=\frac{1}{2}_2^+\rightarrow K^\pi=\frac{1}{2}_1^+$ transitions (with $|\Delta n_\pi|=2$) belongs to a vanishing matrix element in our approach ($J^\pi=\frac{1}{2}^+\rightarrow\frac{1}{2}^+$), so we used transition $J^\pi=\frac{5}{2}^+\rightarrow\frac{3}{2}^+$ with the next smallest relative error. The parameter set obtained from method 1 is as follows: $g_l-g_B=3.118$, $g_s-g_B=2.396$, $g_Q=0.055$, and $g_P=0.005$. The least-squares fit (method 2) leads to the parameter set $g_l-g_B=1.895$, $g_s-g_B=4.298$, $g_Q=0.686$, and $g_P=0.003$ (with $\chi^2=2.913$). The agreement between the experimental and calculated data is not really satisfactory, especially not for the transitions within the $K^\pi=\frac{1}{2}_1^-$ band, where the transition rates are overestimated in both approaches (methods 1 and 2). Transitions to this band from other bands, however, are relatively well reproduced by using method 1. $K^\pi=\frac{1}{2}_2^+\rightarrow K^\pi=\frac{1}{2}_1^+$ transitions, to which we assigned $|\Delta n_\pi|=2$, are poorly reproduced, which shows that operator $[P^{(2)}\times J^{(1)}]^{(2)}$ may not be the best choice here.

There are two states for which we know the magnetic dipole momenta from experiments, both of them belonging to the $K^\pi=\frac{1}{2}_1^+$ ground-state band: $\mu(\frac{1}{2}^+)=2.63\mu_N$ and $\mu(\frac{5}{2}^+)=3.61\mu_N$. The corresponding values are $2.55\mu_N$ and $3.39\mu_N$ in the GCM [7] treatment, while Buck and Pilt [8] got $2.98\mu_N$ and $3.70\mu_N$. When we calculate magnetic dipole momenta in our model, the diagonal $g_BJ^{(1)}$ term in Eq. (2.9) appears in addition to the ones used already in determining the $M1$ transition rates. If we fix $g_B$ by a least-squares fit of the $\mu(\frac{1}{2}^+)$ and $\mu(\frac{5}{2}^+)$ values, we get the following results: method 1, $\mu(\frac{1}{2}^+)=1.80\mu_N$ and $\mu(\frac{5}{2}^+)=3.78\mu_N$ (with $g_B=1.226$); method 2, $\mu(\frac{1}{2}^+)=2.02\mu_N$ and $\mu(\frac{5}{2}^+)=3.73\mu_N$ (with $g_B=-0.252$). These results are somewhat worse than those obtained from other models, but are still in good agreement with experimental data.

#### 3. *E1 transitions*

There are 16 transitions between $^{19}$F states for which experimental information is available (see Table III). In our approach all of them correspond to transitions with $\Delta n_\pi=1$; so we use only parameter $c$ [from Eq. (2.12)] to fit their probabilities. Here the matrix elements of $T^{(E1)}$ explicitly depend on $N$, the total number of bosons (see the Appendix), and this may slightly influence the relative strength of different interband transitions. Here we used $N=16$. Following method 1 we chose transition $K^\pi=\frac{1}{2}_1^-$, $J^\pi=\frac{1}{2}^-\rightarrow K^\pi=\frac{1}{2}_1^+$, $J^\pi=\frac{1}{2}^+$ to fix parameter $c$. Since this is the only parameter now, the results from method 2 differ only in an overall scaling factor. Our recults are $c=0.429$ (method 1) and $c=0.364$, with $\chi^2=502.7$ (method 2). The results are not very satisfactory. It has to be added, though, that we used only one parameter here, and $E1$ transitions are generally poorly reproduced by other models too [7,8]. $K^\pi=\frac{1}{2}_1^-\rightarrow K^\pi=\frac{1}{2}_1^+$ transitions are overestimated by an order of magnitude in both the GCM and orthogonality condition model (OCM) treatment [7]. (According to Buck and Pilt [8], these transitions connect $\alpha$-cluster states with triton-cluster ones, and no calculated values are given in Ref. [8].) These models also overestimate the $K^\pi=\frac{1}{2}_2^+\rightarrow K^\pi=\frac{1}{2}_1^-$ transitions, while our model underestimates them. Our results could be improved if we introduced two-body terms containing the $n_\pi$ operator (e.g., $n_\pi[\pi^\dagger\times\tilde{\sigma}-\sigma^\dagger\times\tilde{\pi}]^{(1)}$) which could increase the difference between the average strength of the transitions with $n_\pi=8\rightarrow n_\pi=7$ and $n_\pi=9\rightarrow n_\pi=8$.

### B. One-nucleon transfer reactions

In order to test the predictions of the model we used spectroscopic information obtained from the $^{20}$Ne$(t,\alpha)^{19}$F proton pickup reaction by Garrett and Hansen [11]. These authors presented data for the states of $^{19}$F below $E_x=6.1$ MeV and for a single state at $E_x=6.79$ MeV. Their results are presented in Table IV, in which we included all the states which have corresponding model states in our approach. (Besides these there are 13 other states in Ref. [11], which are not interpreted as $\alpha$-cluster states [7].)

In Ref. [11] there are nine $^{19}$F states for which the spectroscopic factor ($C^2S$) is given numerically, six of which have corresponding model states in Table IV. The authors also refer to nine states as being weakly populated in this reaction and also nine states as not having angular distribution characteristic of an orbital angular momentum transfer. There are four states of both kinds

included in Table IV.

If we assign the vibron model quantum numbers $n_\pi=8$ and $R^\pi=0^+$ to the ground state of the $^{20}$Ne nucleus, there are only four SU(3)×U(2)vibron-fermion states which give nonvanishing matrix elements with the one-body transfer operator introduced in Sec. II B. We expect these states to be strongly populated in this reaction. This seems to be the case, since three of them have numerical spectroscopic information ($C^2S$) in Table IV (the only negative-parity states which have at all), and the fourth (with $E_x=6.99$ MeV) is just outside the energy range studied in Ref. [11]. (It has to be added, though, that there is a peak at $E_x\approx 7$ MeV in Fig. 1 of Ref. [11], which is not analyzed by the authors, but which may arise from the $J^\pi=\frac{1}{2}^-$ state of $E_x=6.99$ MeV.)

If we neglect the two-body terms in $P^{(3/2^-)}_{-,\mu}$ [the inverse of (2.18b)] at the moment, we have a chance to test the predictions of our model for the $J^\pi=\frac{3}{2}^-$ states of the $K^\pi=\frac{1}{2}_1^-$ and $K^\pi=\frac{1}{2}_3^-$ bands. Using (2.18b), (2.19), and (2.20) we get $S=\frac{16}{15}(\xi^{(-)}_{(1,1/2)3/2})^2$ and $S=\frac{44}{15}(\xi^{(-)}_{(1,1/2)3/2})^2$ for the two states. The ratio of the two is $\frac{16}{44}=0.364$. At the same time $C^2S=0.30$ and 0.96 for the same two states from the experiment (see Table IV), so their ratio is 0.30/0.96 = 0.313, which is close to the prediction of the model. Unfortunately no similar check can be done for the $J^\pi=\frac{1}{2}^-$ states of the same bands.

There are no more numerical data for the other negative-parity states, so there is no point in trying to fix the parameters of the two-body operators. Nevertheless, we expect that their contribution is not significant. There are six other negative-parity states giving nonzero matrix elements with the two-body transfer operator (2.18b). Four of them are outside the energy range studied in Ref. [11], one is referred to as being weakly populated in the reaction, and one has angular distribution not characteristic of an orbital angular momentum transfer.

As for the positive-parity states, there are three of them with well-defined $C^2S$ in Table IV, all three belonging to the ground-state band $K^\pi=\frac{1}{2}_1^+$. They are just the same three states the model correspondents of which can be reached from the ground state of $^{20}$Ne with the lowest-order transfer operator, the inverse of (2.18a), which now has two-body terms. Three states of the $K^\pi=\frac{1}{2}_2^+$ band can also be linked with the ground state of $^{20}$Ne by two-body transfer operators, but these transitions are weakly populated in the $^{20}$Ne$(t,\alpha)^{19}$F reaction (see

TABLE III. $E1$ reduced transition probabilities in $10^{-3}$ Weisskopf units.

| | Expt. [7,10] | | Ref. [8] | SU(3)×U(2) | | | |
|---|---|---|---|---|---|---|---|
| $J_i^\pi(E_{xi})$ | $J_f^\pi(E_{xf})$ | $B(E1)_{\text{exp}}$ | $B(E1)_B$ | $B(Eq)_{\text{th1}}$[a] | $B(E1)_{\text{th2}}$[a] | Band | Labels |
| $K^\pi=\frac{1}{2}_1^-$ | $\rightarrow K^\pi=\frac{1}{2}_1^+$ | | | | | 8(7,0)→ | 7(6,0) |
| $\frac{1}{2}^-(0.11)$ | $\frac{1}{2}^+(0.0)$ | 1.2±0.1[b] | | 1.20[b] | 0.86 | 1 | 0 |
| $\frac{3}{2}^-(1.46)$ | $\frac{1}{2}^+(0.0)$ | 1.0±0.2 | | 1.20 | 0.86 | 1 | 0 |
| | $\frac{5}{2}^+(0.20)$ | 0.81±0.19 | | 4.82 | 3.46 | 1 | 2 |
| $\frac{5}{2}^-(1.35)$ | $\frac{5}{2}^+(0.20)$ | $(6.5\pm2.2)\times10^{-3}$ | | 0.54 | 0.39 | 3 | 2 |
| $\frac{7}{2}^-(4.00)$ | $\frac{5}{2}^+(0.20)$ | 0.23±0.10 | | 8.08 | 5.81 | 3 | 2 |
| $K^\pi=\frac{1}{2}_3^-$ | $\rightarrow K^\pi=\frac{1}{2}_1^+$ | | | | | 8(8,1)→ | 7(6.0) |
| $\frac{3}{2}^-(6.79)$[c] | $\frac{1}{2}^+(0.0)$ | 5.4±1.5 | 309 | 4.31 | 3.1 | 1 | 0 |
| | $\frac{5}{2}^+(0.20)$ | 5.1±1.3 | 404 | 0.05 | 0.04 | 1 | 2 |
| $\frac{7}{2}^-(6.93)$ | $\frac{5}{2}^+(0.20)$ | 12±2 | 444 | 0.03 | 0.02 | 3 | 2 |
| | $\frac{9}{2}^+(2.78)$ | 1.7±0.5 | 2.86 | 0.002 | 0.002 | 3 | 4 |
| $K^\pi=\frac{3}{2}_2^-$ | $\rightarrow K^\pi=\frac{1}{2}_1^+$ | | | | | 8(8,1)→ | 7(6,0) |
| $\frac{3}{2}^-(6.89)$ | $\frac{1}{2}^+(0.0)$ | 1.7±0.6 | | 0 | 0 | 2 | 0 |
| $K^\pi=\frac{1}{2}_2^+$ | $\rightarrow K^\pi=\frac{1}{2}_1^-$ | | | | | 9(8,0)→ | 8(7,0) |
| $\frac{1}{2}^+(5.34)$ | $\frac{1}{2}^-(0.11)$ | 10±2 | 15 | 0.48 | 0.34 | 0 | 1 |
| | $\frac{3}{2}^-(1.46)$ | 12±2 | 25.4 | 0.96 | 0.69 | 0 | 1 |
| $\frac{3}{2}^+(5.50)$ | $\frac{1}{2}^-(0.11)$ | 7.0 | 15 | 4.83 | 3.48 | 2 | 1 |
| | $\frac{5}{2}^-(1.35)$ | 9.8 | 22 | 5.06 | 3.64 | 2 | 3 |
| $\frac{5}{2}^+(6.28)$ | $\frac{5}{2}^-(1.35)$ | 2.1±0.5 | 1.1 | 0.24 | 0.17 | 2 | 3 |
| | $\frac{3}{2}^-(1.46)$ | 1.2±0.4 | 15.4 | 5.80 | 4.17 | 2 | 1 |

[a] $B(M1)_{\text{th1}}$: fitting selected transitions (method 1); $B(M1)_{\text{th2}}$: fitting all the transitions (method 2).
[b] Used to fit model parameters (method 1).
[c] In Ref. [8] another $J^\pi=\frac{3}{2}^-$ state is taken here with $E_x=6.088$ MeV.

Table IV). Therefore we conclude that the parameters $\bar{\xi}^{(+)}_{(1j')j}$ are much smaller than the parameters $\xi^{(+)}_{(1j')j}$ in Eq. (2.18a).

It is not our aim to give a precise description of the $^{19}$F nucleus, so we are not going to analyze the data further by fixing model parameters. As a conclusion we can say that the phenomenological one-nucleon transfer operators of the vibron-fermion model give a reasonably good prediction of the spectroscopic factors in the $^{20}$Ne$(t,\alpha)^{19}$F reaction. They give large matrix elements in the lowest order for those $^{19}$F states which are strongly populated in the reaction, while using higher-order terms they predict reasonably well the range of states which are less strongly populated.

## IV. SUMMARY AND CONCLUSIONS

Here we have developed further the SU(3)×U(2) limit of the vibron-fermion model introduced recently by us [1], as an algebraic approach to cluster states in odd-mass nuclei. We introduced the electric quadrupole, magnetic dipole, and electric dipole transition operators, and the one-nucleon transfer operator. These operators contain bosonic and fermionic parts acting on the corresponding constituent of the coupled vibron-fermion basis states. They may include one-, two-, or even higher-order terms to describe transitions between vibron-fermion states (including vibron model states as a special case), but our expectation is that they should be able to describe the most important transitions in the lowest order.

We used the basis states assigned to the SU(3)×U(2) dynamical symmetry to calculate the matrix elements of these operators in the simplest case, in which we take only one nucleon (or hole) into account on an oscillator shell. In this case the one-nucleon transfer operators link SU(3)×U(2) states with basis states of the vibron model assigned to the U(3) dynamical symmetry.

In order to test the general performance of our model we applied it to the $\alpha$-cluster states of the $^{19}$F nucleus, proposed previously [1] as an example for the approximate realization of the SU(3)×U(2) dynamical symmetry. The physical picture behind our model in this case is the

TABLE IV. Spectroscopic information on the $^{20}$Ne$(t,\alpha)^{19}$F reaction and its interpretation in the SU(3)×U(2) limit of the vibron-fermion model.

| $K^\pi$ | $J^\pi$ | $E_x$ | $C^2S_{\text{exp}}$ (Ref. [11]) | Order of the transfer operator | $n_\pi(\lambda,\mu)L$ |
|---|---|---|---|---|---|
| $\frac{1}{2}^+_1$ | $\frac{1}{2}^+$ | 0.0 | 0.12 | 2 | 7 (6,0) 0 |
| | $\frac{3}{2}^+$ | 1.55 | 0.31 | 2 | 2 |
| | $\frac{5}{2}^+$ | 0.20 | 1.6 | 2 | 2 |
| | $\frac{7}{2}^+$ | 5.46 | b | 3 | 4 |
| | $\frac{9}{2}^+$ | 2.78 | c | 3 | 4 |
| | $\frac{13}{2}^+$ | 4.65 | c | 3 | 6 |
| $\frac{1}{2}^-_1$ | $\frac{1}{2}^-$ | 0.11 | 1.7 | 1, 2 | 8 (7,0) 1 |
| | $\frac{3}{2}^-$ | 1.46 | 0.30 | 1,2 | 1 |
| | $\frac{5}{2}^-$ | 1.35 | b | 2 | 3 |
| | $\frac{7}{2}^-$ | 4.00 | c | 2 | 3 |
| | $\frac{9}{2}^-$ | 4.03 | c | 3 | 5 |
| | $\frac{11}{2}^-$ | 8.95 | a | 3 | 5 |
| | $\frac{13}{2}^-$ | 8.29 | a | 4 | 7 |
| $\frac{1}{2}^+_2$ | $\frac{1}{2}^+$ | 5.34 | b | 2 | 9 (8,0) 0 |
| | $\frac{3}{2}^+$ | 5.50 | b | 2 | 2 |
| | $\frac{5}{2}^+$ | 6.28 | a | 2 | 2 |
| | $\frac{7}{2}^+$ | 7.11 | a | 3 | 4 |
| $\frac{1}{2}^-_3$ | $\frac{1}{2}^-$ | 6.99 | a | 1,2 | 8 (8,1) 1 |
| | $\frac{3}{2}^-$ | 6.69 | 0.96 | 1,2 | 1 |
| | $\frac{5}{2}^-$ | 7.74 | a | 2 | 3 |
| | $\frac{7}{2}^-$ | 6.93 | a | 2 | 3 |
| $\frac{3}{2}^-_2$ | $\frac{3}{2}^-$ | 6.89 | a | 2 | 8 (8,1) 2 |
| | $\frac{5}{2}^-$ | 9.82 | a | 2 | 2 |
| $\frac{1}{2}^-_2$ | $\frac{1}{2}^-$ | 6.43 | a | 3 | 10, (9,0) 1 |
| | $\frac{3}{2}^-$ | 7.90 | a | 3 | 1 |

[a] $E_x$ is outside the energy range studied in Ref. [11].
[b] Not strongly populated.
[c] Angular distribution not characteristic of an orbital angular momentum transfer.

coupling of a hole on the $p$ shell to the relative motion of an $\alpha$ cluster and the $^{15}$N core. Besides the ground state of the core nucleus, this approach also takes into account the excited state of $^{15}$N with $J^{\pi}=\frac{3}{2}^{-}$; in fact, the basis states of our model contain contributions from both configurations, so they are not "pure" $\alpha+{}^{15}\mathrm{N}_{\mathrm{g.s.}}$ or $\alpha+{}^{15}\mathrm{N}(\frac{3}{2}^{-})$ states. We identified the SU(3)×U(2) states with experimental ones comparing the band structure of our model and that proposed by Descouvemont and Baye [7] to classify the $\alpha$-cluster states of the $^{19}$F nucleus.

We used two different methods to fix the model parameters of the electromagnetic transition operators. [In the first one we selected the minimal number of transitions (preferably those with the smallest relative error) necessary to fix the model parameters, while in the second one we performed a least-squares fit of all the transition rates with respect to the model parameters.] It was found that generally there was only a difference of less than 30% between the transition rates calculated from the two methods. We found that although the experimental values are not always reproduced very well, our model is able to describe qualitatively these transitions, and it can account for the most important ones in the lowest order (i.e., using only one-body terms in the transition operators). Very good results were obtained for the $E2$ transitions (especially from method 1), while the agreement between the experimental and calculated $B(M1)$ values was not very satisfactory in some cases. In case of the electric dipole transitions the overall agreement was not very good either, but it was still not worse than the prediction of other models [7,8], which generally overestimate many of the $B(E1)$ values. We used the fixed parameters to calculate quadrupole momenta and magnetic dipole momenta of the model states, and it was found that these values are in good agreement with the available experimental data, and are close to the predictions of other models [7,8] as well.

We studied the one-nucleon transfer reaction $^{20}$Ne$(t,\alpha)$ and tried to interpret the available experimental data in terms of our model. We again found that the lowest-order operators are able to reproduce qualitatively the most important transitions.

These calculations were not meant to give a detailed description of the $^{19}$F nucleus; rather, our aim was to test the applicability of our model to realistic problems. Our results indicate that this simple phenomenological model can serve as a starting point for further studies either by introducing new degrees of freedom, or by taking into account symmetry breaking.

## APPENDIX

Here we collect the formulas necessary to calculate electromagnetic transition rates and other quantities with physical importance, such as matrix elements of symmetry-conserving or -breaking interaction terms in the Hamiltonian. We use the SU(3)×U(2) basis and present the formulas separately for operators with different tensorial character. Here we consider only one fermion ($M=1$), so the $(\lambda_F,\mu_F)$ $\mathrm{SU}_I^F(3)$ representations are either $(n,0)$ or $(0,n)$.

Reduced matrix elements of boson number conserving tensor operators with $\mathrm{O}^B(3)$ tensorial character, $B_{\mu}^{(k)}$:

$$\begin{aligned}
&\langle (n_\pi,0),(\lambda_F,\mu_F);(\lambda,\mu)\kappa,L\tfrac{1}{2}J||B^{(k)}||(n'_\pi,0),(\lambda'_F,\mu'_F);(\lambda',\mu')\kappa',L'\tfrac{1}{2}J'\rangle \\
&\quad=\delta_{(\lambda_F,\mu_F),(\lambda'_F,\mu'_F)}(-1)^{L+1/2+J'+k}\sqrt{(2J+1)(2J'+1)}\begin{Bmatrix} J' & k & J \\ L & \frac{1}{2} & L' \end{Bmatrix} \\
&\qquad\times\sum_{RR'l}(-1)^{R+l+k+L'}\sqrt{(2L+1)(2L'+1)}\begin{Bmatrix} L' & k & L \\ R & l & R' \end{Bmatrix} \\
&\qquad\times\langle (n_\pi,0),R;(\lambda_F,\mu_F)l||(\lambda,\mu)\kappa L\rangle\langle (n'_\pi,0),R';(\lambda_F,\mu_F)l||(\lambda',\mu')\kappa' L'\rangle\langle (n_\pi,0)R||B^{(k)}||(n'_\pi,0)R'\rangle\ . \qquad \text{(A1)}
\end{aligned}$$

$\mathrm{O}_I^F(3)$ tensor operators $F_{\mu}^{(k)}$:

$$\begin{aligned}
&\langle (n_\pi,0),(\lambda_F,\mu_F);(\lambda,\mu)\kappa,L\tfrac{1}{2}J||F^{(k)}||(n'_\pi,0),(\lambda'_F,\mu'_F);(\lambda',\mu')\kappa',L'\tfrac{1}{2}J'\rangle \\
&\quad=\delta_{n_\pi,n'_\pi}(-1)^{L+1/2+J'+k}\sqrt{(2J+1)(2J'+1)}\begin{Bmatrix} J' & k & J \\ L & \frac{1}{2} & L' \end{Bmatrix} \\
&\qquad\times\sum_{ll'R}(-1)^{R+l'+k+L}\sqrt{(2L+1)(2L'+1)}\begin{Bmatrix} L' & k & L \\ l & R & l' \end{Bmatrix} \\
&\qquad\times\langle (n_\pi,0),R;(\lambda_F,\mu_F)l||(\lambda,\mu)\kappa L\rangle\langle (n_\pi,0),R;(\lambda'_F,\mu'_F)l'||(\lambda',\mu')\kappa' L'\rangle\langle (\lambda_F,\mu_F)l||F^{(k)}||(\lambda'_F,\mu'_F)l'\rangle\ . \qquad \text{(A2)}
\end{aligned}$$

$\mathrm{SU}_S^F(2)$ tensor operators $S_{\mu}^{(k)}$:

$$\langle (n_\pi,0),(\lambda_F,\mu_F);(\lambda,\mu)\kappa,L\tfrac{1}{2}J||S^{(k)}||(n'_\pi,0),(\lambda'_F,\mu'_F);(\lambda',\mu')\kappa',L'\tfrac{1}{2}J'\rangle$$
$$=\delta_{\alpha,\alpha'}\delta_{L,L'}(-1)^{L+1/2+J+k}\sqrt{(2J+1)(2J'+1)}\begin{Bmatrix} J' & k & J \\ \frac{1}{2} & L & \frac{1}{2}\end{Bmatrix}\langle \tfrac{1}{2}||S^{(k)}||\tfrac{1}{2}\rangle \ , \quad \text{(A3)}$$

where $\alpha=n_\pi,(\lambda_F,\mu_F),(\lambda,\mu)\kappa$.

O(3) tensor operators $A^{(k)}_\mu$:

$$\langle (\lambda,\mu)\kappa,L\tfrac{1}{2}J||A^{(k)}||(\lambda',\mu')\kappa',L'\tfrac{1}{2}J'\rangle$$
$$=(-1)^{L+1/2+J'+k}\sqrt{(2J+1)(2J'+1)}\begin{Bmatrix} J' & k & J \\ L & \frac{1}{2} & L'\end{Bmatrix}\langle (\lambda,\mu)\kappa L||A^{(k)}||(\lambda',\mu')\kappa' L'\rangle \ . \quad \text{(A4)}$$

Further relations can be derived for SU(3)⊃O(3)⊃O(2) tensor operators, such as the quadrupole and angular momentum operators $Q^{(2)}$ and $L^{(1)}$. The $T^{(\lambda,\mu)}_{\kappa LM}$ tensorial character of these operators is [12,13]

$$L^{(1)}_\mu=T^{(1,1)}_{11\mu} \ , \quad \text{(A5a)}$$
$$Q^{(2)}_\mu=\sqrt{\tfrac{3}{8}}T^{(1,1)}_{12\mu} \ , \quad \text{(A5b)}$$

and they form the generator set of the SU(3) group. [There are several conventions known for the choice of the numerical factor in (A5b). Here we followed the one used in Ref. [5].] The reduced matrix elements $Q^{(2)}$ can be determined from the formula

$$\langle (\lambda,\mu)\kappa L||T^{(1,1)}_{1l}||(\lambda',\mu')\kappa' L'\rangle=\delta_{(\lambda,\mu),(\lambda',\mu')}\sqrt{L(L+1)(2L+1)}\frac{\langle (\lambda,\mu)\kappa' L';(1,1)1l||(\lambda,\mu)\kappa L\rangle_{\rho=1}}{\langle (\lambda,\mu)\kappa L;(1,1)11||(\lambda,\mu)\kappa L\rangle_{\rho=1}} \ , \quad \text{(A6)}$$

which is obtained from the requirement that it should reproduce the matrix elements of $L^{(1)}$ in the $l=1$ case [5]. Similar relations hold for the reduced matrix elements of the $SU^B(3)$ and $SU^F_l(3)$ tensor operators in the corresponding bases, with the difference that the $(\lambda,\mu)$ representations are to be replaced with the simpler representations $(n_\pi,0)$ and $(n,0)$ [or $(0,n)$], respectively. Below we displayed the reduced matrix elements of $Q^{(2)}_B$. These can be used to determine the reduced matrix elements of $Q^{(2)}_F$ as well for the $(n,0)$ (particle coupling) case, while the reduced matrix elements for the $(0,n)$ (hole coupling) case can be calculated from the formula

$$\langle (0,n)l||Q^{(2)}_F||(0,n)l'\rangle=-\langle (n,0)l||Q^{(2)}_F||(n,0)l'\rangle \ , \quad \text{(A7)}$$

obtained from the symmetry properties of the SU(3)⊃O(3) Wigner coefficients.

Reduced matrix elements of some (boson) operators in the vibron-fermion model follow. Some of these formulas can be used to determine the reduced matrix elements of fermionic operators as well (see above):

$$\langle N(n_\pi,0)R||\hat{n}_\pi||N(n_\pi,0)R\rangle=n_\pi\sqrt{2R+1} \ ,$$
$$\langle N(n_\pi,0)R||L^{(1)}_B||N(n_\pi,0)R\rangle=\sqrt{R(R+1)(2R+1)} \ ,$$
$$\langle N(n_\pi,0)R+2||Q^{(2)}_B||N(n_\pi,0)R\rangle=\frac{1}{2}\left[\frac{2(n_\pi+R+3)(n_\pi-R)(R+1)(R+2)}{2R+3}\right]^{1/2} \ ,$$
$$\langle N(n_\pi,0)R||Q^{(2)}_B||N(n_\pi,0)R\rangle=-\frac{1}{2}(2n_\pi+3)\left[\frac{R(R+1)(2R+1)}{2(2R-1)(2R+3)}\right]^{1/2} \ ,$$
$$\langle N(n_\pi-1,0)R-1||D^{(1)}_B||N(n_\pi,0)R\rangle=-\sqrt{(N-n_\pi+1)(n_\pi+R+1)R} \ ,$$
$$\langle N(n_\pi-1,0)R+1||D^{(1)}_B||N(n_\pi,0)R\rangle=\sqrt{(N-n_\pi+1)(n_\pi-R)(R+1)} \ ,$$
$$\langle N(n_\pi-2,0)R-2||P^{(2)}_B||N(n_\pi,0)R\rangle=\left[\frac{(N-n_\pi-2)(N-n_\pi+1)(n_\pi+R+1)(n_\pi+R-1)R(R-1)}{2R-1}\right]^{1/2} \ ,$$
$$\langle N(n_\pi-2,0)R||P^{(2)}_B||N(n_\pi,0)R\rangle=-\left[\frac{2(N-n_\pi+2)(N-n_\pi+1)(n_\pi+R+1)(n_\pi-R)R(R+1)(2R+1)}{3(2R-1)(2R+3)}\right]^{1/2} \ ,$$
$$\langle N(n_\pi-2,0)R+2||P^{(2)}_B||N(n_\pi,0)R\rangle=\left[\frac{(N-n_\pi+2)(N-n_\pi+1)(n_\pi-R-2)(n_\pi-R)(R+1)(R+2)}{2R+3}\right]^{1/2} \ .$$

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