Approximate treatment of the nucleon-nucleus interaction in the resonating-group formulation

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The nucleon-nucleus interaction is considered within the framework of the resonating-group method (RGM), but with the simplifying assumptions of omitting target-recoil effects and core-exchange contributions. The resulting model, called model K, contains then only direct and knock-on-exchange terms. Comparisons between RGM and model-K results in $n+\alpha$, $n+{}^{16}$O, and $n+{}^{40}$Ca systems for bound-state energies, phase shifts, differential scattering cross sections, and polarizations show that model K works very well when the target nucleus has a nucleon number greater than about ten and when the scattering energy is higher than about 10 MeV. Because of the adoption of these simplifying assumptions, general expressions for the nuclear-central, Coulomb, and spin-orbit parts of the nucleon-nucleus nonlocal potential can be analytically derived without much difficulty. These expressions have simple forms and are suitable for a systematic and large-scale analysis of existing scattering data even in cases where heavy target nuclei are involved.

I. INTRODUCTION

The resonating-group method (RGM) has been successfully employed to study the properties of many nuclear systems [1]. In light nuclei where resonating-group calculations have been performed with multiple cluster configurations [2,3], it was especially found that good agreements with experimental results on bound-state energies, scattering angular distributions, and reaction cross sections can be obtained. On the other hand, because of computational complexities arising from the necessity of using totally antisymmetric wave functions and correctly taking into account the motions of the total centers of mass, systems with nucleon numbers greater than about 20 have not been extensively treated with this method [4]. Until now, there generally exist only single-configuration calculations for a rather small number of selected heavier systems [5]. Indeed, it is our viewpoint that, at the present stage of technical development, the practical difficulty of analytically deriving the kernel function, representing the nonlocal interaction between the clusters involved, has prevented the undertaking of systematic and large-scale considerations of many medium-weight and heavy nuclei, thus significantly reducing the usefulness of the RGM in achieving the major objective of understanding the general features of structures and reactions in nuclear systems.

Even for the comparatively simpler problem of nucleon scattering by a nucleus with nucleon number N_A (the nucleon number of the incident particle, denoted as N_B, is equal to 1 in this particular case), resonating-group studies have mainly been carried out for selected nuclei with N_A between 2 and 19 [6]. The only calculation that we know of, where N_A has a larger value outside of this range, is for the case of $N+{}^{40}$Ca scattering (N stands for nucleon) [7]. In this latter case, it was found that one can obtain very satisfactory results for both differential scattering cross sections and polarizations when a phenomenological imaginary potential is introduced into the formulation to take into account the effects of open reaction channels.

It seems clear that, to substantially enhance the utility of the RGM, one must introduce some simplifications into the formulation. In this investigation, we shall take the first step by microscopically considering the $N+$ nucleus problem in the basic RGM approach, but with the following simplifying assumptions.

(1) Recoil effects of the target nucleus will be omitted. Intuitively, one feels that this should be a good approximation when N_A/N_B is much larger than 1.

(2) Among the one-exchange terms, only the knock-on-exchange term will be taken into consideration. From a number of previous investigations [8], it has been determined that this particular term plays an important role in determining the essential behavior of the system.

The resultant calculation, incorporating these two simplifications, will be referred to as the model-K calculation (K stands for knock-on-exchange kernel), in distinction from the exact RGM calculation which takes into account not only recoil effects but also all nucleon-exchange terms.

The rationale for adopting the above simplifications

will be carefully discussed in Sec. II. With these simplifications, the analytical derivation of the $N+$ nucleus interaction, consisting of direct (i.e., no-exchange) and knock-on-exchange terms, becomes quite simple. In the next section, we shall present the general expressions for these terms in the case where both neutrons and protons fill up various (nl) harmonic-oscillator subshells, with n and l denoting the number of oscillator quanta and the orbital angular momentum quantum number, respectively [9].

To determine the domain of validity for model K regarding the center-of-mass scattering energy E and the nucleon number N_A of the target nucleus, we compare in Sec. III results obtained by model-K and exact RGM calculations in $n+\alpha$, $n+{}^{16}\mathrm{O}$, and $n+{}^{40}\mathrm{Ca}$ systems. The physical quantities to be compared are bound-state energies, phase shifts, differential scattering cross sections, and polarizations. Since it has already been established [7,10,11] that the RGM results agree well with experiment, such a comparison will give us information regarding the situations in which model K can well represent the microscopic features of the system under consideration.

A summary of the findings of this investigation is given in Sec. IV. Here also, we shall indicate directions in which model K can be used to study the important properties of nuclear systems.

II. FORMULATION OF MODEL K

A. Justification of simplifying assumptions

As is mentioned in the Introduction, model K is achieved by omitting recoil effects of the target nucleus and by taking into account only no-exchange and knock-on-exchange terms. In this subsection, we shall discuss the reasons for adopting these simplifications in the nucleon + nucleus problem.

It is an intuitive feeling that target-recoil effects should be rather unimportant when the target nucleus is much heavier than the incident particle. Indeed, this was found to be true by Philpott [12] in a careful study based on the continuum shell model. By comparing phase shifts and differential scattering cross sections at relatively high energies obtained in the with-recoil and no-recoil calculations, it was shown in that study that recoil effects are important in the $n+\alpha$ case [13], but rather insignificant in the $n+{}^{16}\mathrm{O}$ and $n+{}^{40}\mathrm{Ca}$ cases. In the lower-energy region where bound and rather sharp resonance states exist, Philpott further discovered that the effects of target recoil seem to become somewhat more important; however, even here, the omission of these effects was found not to lead to serious consequences when N_A/N_B is large.

The adoption of the no-recoil approximation greatly simplifies the analytical derivation of the kernel function representing the nonlocal part of the $N+$ nucleus interaction. The reason is that, with this approximation, the introduction of generator coordinates into the formulation is no longer necessary. One can now simply fix the center of mass of the target nucleus at the original and perform the derivation by conveniently using the spatial coordinates of the incident nucleon and the nucleons in the target nucleus.

Next, we consider the approximation of omitting all nucleon-exchange terms except the knock-on-exchange term. In the $N+$ nucleus case, the number of nucleons which can be interchanged between the two nuclei is equal to one. As has been discussed previously, the resultant one-exchange terms can be classified into three types, namely, types 1a, 1c, and 1d [14]. In addition, it was found to be advantageous to further divide the exchange terms into two classes [15]. For terms belonging to class A, the Born scattering amplitudes are forward peaked and can be exactly reproduced by equivalent local, energy-dependent potentials having a Wigner character. On the other hand, for terms belonging to class B, the Born scattering amplitudes are backward peaked and can be exactly reproduced by equivalent local, energy-dependent potentials having a Majorana character.

The type-1c term is the one commonly referred to as the knock-on-exchange term. In this term, the nucleon-nucleon interaction occurs between the two interchanged nucleons. It is a class-A term and has been found to be generally important in all scattering systems and over a wide energy range. In contrast to this, the type-1a and type-1d terms are class-B terms (hence, they should more properly be called core-exchange type-a and type-d terms), which have been shown to be generally important only when the nucleon-number difference (N_A-N_B) of the interacting nuclei is rather small [16]. Indeed, from explicit studies in a number of nuclear systems [8,15], it has been determined that the effects of core-exchange class-B terms do become quite insignificant when (N_A-N_B) has a value larger than about 10.

For the $N+$ nucleus case where $N_B=1$, one finds that the conditions for the validity of the two simplifying assumptions are both met when the target nucleus has a nucleon number N_A much larger than one (say, $N_A \gtrsim 10$). In fact, it is seen that, in this special case, these two conditions are entirely correlated; that is, a large value for the ratio N_A/N_B also implies a large value for the difference (N_A-N_B), and vice versa. It should be mentioned, however, that such a close correlation no longer exists in the more general case of nucleus + nucleus scattering with N_B not equal to one. Here, N_A/N_B having a large value does imply that (N_A-N_B) is also large, but the converse is not generally true. For example, in the case of $\alpha+{}^{16}\mathrm{O}$ scattering, (N_A-N_B) has a large value equal to 12, but the ratio N_A/N_B, being equal to 4, is comparatively small. Thus, for this latter scattering problem, one can safely omit the core-exchange terms (i.e., four-exchange terms) in the calculation [8], but the recoil effects must be properly taken into consideration.

There is another aspect which indicates that the $N+$ nucleus case is a special situation. In this particular case, the adoption of the two simplifying assumptions reduces by a large extent the analytical work required to derive the kernel functions. On the other hand, in the case of the nucleus + nucleus scattering, the corresponding reduction in computational effort will be appreciably smaller. Let us consider the problem of $\alpha+{}^{90}\mathrm{Zr}$ scatter-

ing as an example. In this problem, the conditions on N_A/N_B and (N_A-N_B) are both well satisfied; hence, it is appropriate to omit both core-exchange terms and target-recoil effects in the calculation. However, even though one can now conveniently fix the center of mass of ^{90}Zr at the origin, the composite nature of the α particle still makes it necessary to introduce a generator coordinate into the formulation. In addition, it is noted that, although the class-A, type-1c term is the dominant exchange term, other exchange terms, such as 1a, 1b, 1d terms, and even two-exchange terms, have significant effects and must also be properly considered. Therefore, the general case of nucleus + nucleus scattering represents a more formidable challenge, and additional simplifying assumptions may have to be sought in order to substantially alleviate the computational difficulty which one faces in dealing with a heavy nuclear system.

B. Direct and knock-on-exchange terms

For a general formulation, we use a local nucleon-nucleon potential that consists of nuclear-central, spin-orbit, and Coulomb parts. It has the basic form

$$V_{ij}=-V_0\exp(-\kappa r_{ij}^2)(w-mP_{ij}^{\sigma}P_{ij}^{\tau}+bP_{ij}^{\sigma}-hP_{ij}^{\tau}) -\frac{1}{2\hbar}V_{\lambda}\exp(-\lambda r_{ij}^2)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot(\mathbf{r}_i-\mathbf{r}_j)\times(\mathbf{p}_i-\mathbf{p}_j) +\frac{e^2}{4r_{ij}}(1+\tau_{iz})(1+\tau_{jz}) . \quad (1)$$

In the no-recoil approximation, the wave function for the N + nucleus system is written as

$$\Psi=\mathcal{A}\left[\phi_A\delta(t_0,\pm\tfrac{1}{2})\sum_{J=1/2}^{\infty}\sum_{L=J-1/2}^{J+1/2}\frac{1}{R}f_{JL}(R)\mathcal{Y}_{JL\frac{1}{2}}^{1/2}(\hat{R},s_0)\right] , \quad (2)$$

where $\mathcal{A}$ is an antisymmetrization operator, and the plus and minus signs are for the p + nucleus and n + nucleus cases, respectively. The variables $\mathbf{R}$, s_0, and t_0 denote, respectively, the spatial, spin, and isospin coordinates of the incident nucleon. The function $\mathcal{Y}_{JL\frac{1}{2}}^{M}$ is a normalized spin-angle function belonging to a state of total angular momentum J whose z component is M and which is a combination of a relative orbital angular momentum L and a spin angular momentum $S=\frac{1}{2}$. The target nucleus is described by the wave function ϕ_A, constructed by using single-nucleon spatial wave functions

$$\psi_{nlm}(\mathbf{r})=\chi_{nl}(r)\exp(-\alpha r^2/2)Y_{lm}(\hat{\mathbf{r}}) \quad (3)$$

in a harmonic-oscillator well of width parameter α. For clarity, we shall consider only target nuclei with filled (nl) subshells for both protons and neutrons. As has been mentioned already, the adoption of this simplification is to substantially simplify the expressions and the discussion of the direct and knock-on-exchange terms, but will not compromise the objective of this investigation which is to determine the domain of validity for model K.

With the usual resonating-group projection procedure [1], one finds the following integro-differential equation satisfied by the radial wave function $f_{JL}(R)$:

$$\left[-\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}\right]-E+V^N(R)+V^C(R)+\eta_{JL}V^{\text{s.o.}}(R)\right]f_{JL}(R) +\int_0^{\infty}[k_L^N(R,R')+k_L^C(R,R')+\eta_{JL}k_L^{\text{s.o.}}(R,R')]f_{JL}(R')dR'=0 , \quad (4)$$

where η_{JL} is given by

$$\eta_{L+1/2,L}=L , \quad \eta_{L-1/2,L}=-(L+1) . \quad (5)$$

In Eq. (4), V^N, V^C, and $V^{\text{s.o.}}$ represent the N + nucleus direct nuclear-central, direct Coulomb, and direct spin-orbit potentials, respectively, while k_L^N, k_L^C, and $k_L^{\text{s.o.}}$ are partial-wave kernel functions representing the various knock-on-exchange contributions. These latter functions are related to the corresponding full kernel functions $K^N(\mathbf{R},\mathbf{R}')$, $K^C(\mathbf{R},\mathbf{R}')$, and $K^{\text{s.o.}}(\mathbf{R},\mathbf{R}')$ by the equation

$$k_L^N(R,R')=2\pi RR'\int_{-1}^{1}K^N(\mathbf{R},\mathbf{R}')P_L(z)dz , \quad (6)$$

with $z=\hat{\mathbf{R}}\cdot\hat{\mathbf{R}}'$, and similar equations for k_L^C and $k_L^{\text{s.o.}}$. As has been emphasized in the preceding subsection, the adoption of the two simplifying assumptions makes it very easy to analytically derive these kernel functions. In the following, we shall present their explicit expressions, together with the expressions for the various direct potentials.

1. Direct nuclear-central potential

The direct nuclear-central potential $V^N(R)$ is easily obtained by a folding procedure. For the p + nucleus or the n + nucleus case, the expression of this potential is as follows.

(a) The p + nucleus case:

$$V^N(R)=V_p^N(R)+V_n^N(R) , \quad (7)$$

where V_p^N and V_n^N represent contributions arising from the interactions of the incident proton with the protons and the neutrons in the target nucleus, respectively.

They are given by

$$V_p^N = C_p \tilde{V}_p(R) , \tag{8}$$

$$V_n^N = C_n \tilde{V}_n(R) , \tag{9}$$

with

$$C_p(p+\text{nucleus}) = 2w - m + b - 2h , \tag{10}$$

$$C_n(p+\text{nucleus}) = 2w + b . \tag{11}$$

The function $\tilde{V}_p(R)$ in Eq. (8) has the form

$$\tilde{V}_p(R) = -V_0 \exp\left[-\frac{\alpha\kappa}{\alpha+\kappa}R^2\right] P_p(R) , \tag{12}$$

where $P_p(R)$ is a polynomial function which can be written as a summation over filled (nl) subshells of the target protons, i.e.,

$$P_p(R) = \sum_{nl} P_{nl}(R) , \tag{13}$$

with

$$P_{nl}(R) = \frac{2l+1}{4\pi} \int \chi_{nl}^2(|\mathbf{s} + [\kappa/(\alpha+\kappa)]\mathbf{R}|) e^{-(\alpha+\kappa)s^2} d\mathbf{s} . \tag{14}$$

The function $\tilde{V}_n(R)$ in Eq. (9) has a similar form, except that P_p in Eq. (12) is replaced by P_n which is a polynomial function written as a summation over filled (nl) subshells of the target neutrons [see Eq. (13)].

The integral in Eq. (14), involving single-nucleon functions χ_{nl}, can be easily performed. For the lower subshells, the explicit expressions for $P_{nl}(R)$ are [17]

$$P_{00}(R) = \gamma , \tag{15}$$

$$P_{11}(R) = \gamma \left[\frac{3\alpha}{\alpha+\kappa} + \frac{2\alpha\kappa^2}{(\alpha+\kappa)^2}R^2\right] , \tag{16}$$

$$P_{20}(R) = \gamma \left[\frac{3}{2} - \frac{\alpha(\alpha+6\kappa)}{2(\alpha+\kappa)^2} + \frac{2\alpha(2\alpha-3\kappa)\kappa^2}{3(\alpha+\kappa)^3}R^2 + \frac{2\alpha^2\kappa^4}{3(\alpha+\kappa)^4}R^4\right] , \tag{17}$$

$$P_{22}(R) = \gamma \left[\frac{5\alpha^2}{(\alpha+\kappa)^2} + \frac{20\alpha^2\kappa^2}{3(\alpha+\kappa)^3}R^2 + \frac{4\alpha^2\kappa^4}{3(\alpha+\kappa)^4}R^4\right] , \tag{18}$$

with

$$\gamma = \left[\frac{\alpha}{\alpha+\kappa}\right]^{3/2} . \tag{19}$$

(b) The n + nucleus case. The expression for V^N in the n + nucleus case is similar to that in the p + nucleus case, except that the values of C_p and C_n in Eqs. (10) and (11) are interchanged; that is,

$$C_p(n+\text{nucleus}) = 2w + b , \tag{20}$$

$$C_n(n+\text{nucleus}) = 2w - m + b - 2h . \tag{21}$$

2. Direct Coulomb potential

The direct Coulomb potential is, of course, equal to zero in the n + nucleus case. In the p + nucleus case, its expression can again be simply derived by a folding procedure. The result is

$$V^C = \sum_{nl} V_{nl}^C , \tag{22}$$

where the summation is over all the filled (nl) subshells of the target protons, and

$$V_{nl}^C = \frac{2l+1}{4\pi} \int \chi_{nl}^2(s) e^{-\alpha s^2} \frac{2e^2}{|\mathbf{R}-\mathbf{s}|} d\mathbf{s} . \tag{23}$$

For lower subshells, the explicit expressions for V_{nl}^C are

$$V_{00}^C(R) = 2e^2 \frac{1}{R}\Phi(\sqrt{\alpha}R) , \tag{24}$$

$$V_{11}^C(R) = 2e^2 \left[\frac{3}{R}\Phi(\sqrt{\alpha}R) - 2\left[\frac{\alpha}{\pi}\right]^{1/2} e^{-\alpha R^2}\right] , \tag{25}$$

$$V_{20}^C(R) = 2e^2 \left[\frac{1}{R}\Phi(\sqrt{\alpha}R) - \frac{1}{3}\left[\frac{\alpha}{\pi}\right]^{1/2}(1+2\alpha R^2)e^{-\alpha R^2}\right] , \tag{26}$$

$$V_{22}^C(R) = 2e^2 \left[\frac{5}{R}\Phi(\sqrt{\alpha}R) - \frac{2}{3}\left[\frac{\alpha}{\pi}\right]^{1/2}(7+2\alpha R^2)e^{-\alpha R^2}\right] , \tag{27}$$

with $\Phi(x)$ being an error function given by

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt . \tag{28}$$

3. Direct spin-orbit potential

Because our chosen two-nucleon spin-orbit potential has no isospin dependence, the direct spin-orbit potential $V^{\text{s.o.}}(R)$ in the p + nucleus case is the same as that in the n + nucleus case. Similar to the direct nuclear-central potential, it can again be divided into two parts, i.e.,

$$V^{\text{s.o.}}(R) = V_p^{\text{s.o.}}(R) + V_n^{\text{s.o.}}(R) , \tag{29}$$

with $V_p^{\text{s.o.}}$ and $V_n^{\text{s.o.}}$ representing contributions arising from the interactions of the incident nucleon with the target protons and neutrons, respectively.

Since the two-nucleon spin-orbit potential is commonly considered to be very short ranged [18], we have, for ease in computation, adopted the simplification of letting the spin-orbit range parameter λ approaching infinity. In this limit, the N + nucleus direct spin-orbit potential can then be simply obtained by using Eq. (38) of Ref. [19],

and we find that

$$V_p^{\text{s.o.}}(V_n^{\text{s.o.}})=\sum_{nl} V_{nl}^{\text{s.o.}} , \tag{30}$$

where the summation is over the filled (nl) subshells of the target protons (neutrons), and

$$V_{nl}^{\text{s.o.}}=\frac{(2l+1)\sqrt{\pi}}{8}J_\lambda\frac{1}{R}\frac{d}{dR}[\chi_{nl}^2(R)e^{-\alpha R^2}] , \tag{31}$$

which, due to our adoption of the zero-range approximation, involves only a single parameter J_λ given by

$$J_\lambda=V_\lambda\lambda^{-5/2} . \tag{32}$$

For lower subshells, the explicit expressions of $V_{nl}^{\text{s.o.}}$ are as follows:

$$V_{00}^{\text{s.o.}}=-J_\lambda\alpha^{5/2}e^{-\alpha R^2} , \tag{33}$$

$$V_{11}^{\text{s.o.}}=-J_\lambda\alpha^{5/2}(2\alpha R^2-2)e^{-\alpha R^2} , \tag{34}$$

$$V_{20}^{\text{s.o.}}=-J_\lambda\alpha^{5/2}(\tfrac{2}{3}\alpha^2R^4-\tfrac{10}{3}\alpha R^2+\tfrac{7}{2})e^{-\alpha R^2} , \tag{35}$$

$$V_{22}^{\text{s.o.}}=-J_\lambda\alpha^{5/2}(\tfrac{4}{3}\alpha^2R^4-\tfrac{8}{3}\alpha R^2)e^{-\alpha R^2} . \tag{36}$$

4. *Knock-on-exchange nuclear-central kernel function*

Knock-on-exchange nuclear-central kernel functions $K^N(\mathbf{R},\mathbf{R}')$ in the $p+$ nucleus and $n+$ nucleus cases are somewhat different. Hence, they will be separately considered in the following.

(a) The $p+$ nucleus case. Here again, the kernel function $K^N(\mathbf{R},\mathbf{R}')$ consists of a proton part and a neutron part, i.e.,

$$K^N(\mathbf{R},\mathbf{R}')=K_p^N(\mathbf{R},\mathbf{R}')+K_n^N(\mathbf{R},\mathbf{R}') , \tag{37}$$

where

$$K_p^N(\mathbf{R},\mathbf{R}')=D_p\tilde{K}_p(\mathbf{R},\mathbf{R}') , \tag{38}$$

$$K_n^N(\mathbf{R},\mathbf{R}')=D_n\tilde{K}_n(\mathbf{R},\mathbf{R}') , \tag{39}$$

with

$$D_p(p+\text{nucleus})=-w+2m-2b+h , \tag{40}$$

$$D_n(p+\text{nucleus})=2m+h . \tag{41}$$

With the no-recoil approximation, the function $\tilde{K}_p(\mathbf{R},\mathbf{R}')$ in Eq. (38) can be easily derived and the result is

$$\tilde{K}_p(\mathbf{R},\mathbf{R}')=-V_0\exp\left[-\left[\frac{\alpha}{2}+\kappa\right](R^2+R'^2)+2\kappa\mathbf{R}\cdot\mathbf{R}'\right]Q_p(\mathbf{R},\mathbf{R}') , \tag{42}$$

with

$$Q_p(\mathbf{R},\mathbf{R}')=\sum_{nl}Q_{nl}(\mathbf{R},\mathbf{R}') , \tag{43}$$

where the summation is over filled (nl) subshells of the target protons, and

$$Q_{nl}(\mathbf{R},\mathbf{R}')=\frac{2l+1}{4\pi}\chi_{nl}(R)\chi_{nl}(R')P_l(\hat{\mathbf{R}}\cdot\hat{\mathbf{R}}') . \tag{44}$$

The function $\tilde{K}_n(\mathbf{R},\mathbf{R}')$ in Eq. (39) has a similar form, except that Q_p in Eq. (42) is replaced by Q_n which is given by a summation of Q_{nl} over filled (nl) subshells of the target neutrons.

For lower subshells, the explicit expressions of $Q_{nl}(\mathbf{R},\mathbf{R}')$ are

$$Q_{00}(\mathbf{R},\mathbf{R}')=\beta , \tag{45}$$

$$Q_{11}(\mathbf{R},\mathbf{R}')=\beta(2\alpha\mathbf{R}\cdot\mathbf{R}') , \tag{46}$$

$$Q_{20}(\mathbf{R},\mathbf{R}')=\beta\frac{2\alpha^2}{3}\left[R^2R'^2-\frac{3}{2\alpha}(R^2+R'^2)+\frac{9}{4\alpha^2}\right] , \tag{47}$$

$$Q_{22}(\mathbf{R},\mathbf{R}')=\beta\frac{2\alpha^2}{3}[3(\mathbf{R}\cdot\mathbf{R}')^2-R^2R'^2] , \tag{48}$$

with

$$\beta=\left[\frac{\alpha}{\pi}\right]^{3/2} . \tag{49}$$

(b) The $n+$ nucleus case. The expression for $K^N(\mathbf{R},\mathbf{R}')$ in the $n+$ nucleus case is similar to that in the $p+$ nucleus case, except that the values of D_p and D_n in Eqs. (40) and (41) are interchanged; that is,

$$D_p(n+\text{nucleus})=2m+h , \tag{50}$$

$$D_n(n+\text{nucleus})=-w+2m-2b+h . \tag{51}$$

5. *Knock-on-exchange Coulomb kernel function*

The knock-on-exchange Coulomb kernel function equals zero in the $n+$ nucleus case. In the $p+$ nucleus case, it has the form

$$K^C(\mathbf{R},\mathbf{R}')=\sum_{nl}K_{nl}^C(\mathbf{R},\mathbf{R}') , \tag{52}$$

where the summation is over all filled (nl) subshells of the target protons, and

$$K_{nl}^C(\mathbf{R},\mathbf{R}')=-\exp\left[-\frac{\alpha}{2}(R^2+R'^2)\right]\frac{e^2}{|\mathbf{R}-\mathbf{R}'|}Q_{nl}(\mathbf{R},\mathbf{R}') , \tag{53}$$

with $Q_{nl}(\mathbf{R},\mathbf{R}')$ given by Eq. (44).

6. *Knock-on-exchange spin-orbit kernel function*

In the zero-range limit (i.e., λ approaches infinity), it can be readily seen that the knock-on-exchange spin-orbit kernel function $k_L^{\text{s.o.}}$ contains a factor $\delta(R-R')$ and the following relation holds:

$$\int_0^\infty k_{L,nl}^{\text{s.o.}}(R,R')f_{JL}(R')dR'=\zeta_{nl}V_{nl}^{\text{s.o.}}(R)f_{JL}(R) , \tag{54}$$

where $k_{L,nl}^{\text{s.o.}}$ represents the partial-wave kernel function

arising from the spin-orbit interaction of the incident nucleon with the nucleons in a filled (nl) subshell. The value of ζ_{nl} for the subshell with $(nl)=(00)$ or (11) was found [10,12] to be equal to 0.5. As for the higher subshells, its value has not been explicitly determined; however, by using information gained from an analogous study on the structures of $P_{nl}(R)$ and $Q_{nl}(\mathbf{R},\mathbf{R}')$ [see Eqs. (14) and (44)] in the $\kappa\rightarrow\infty$ limit of the nuclear-central interaction, one can confidently infer that ζ_{nl} is, in fact, a constant, independent of the quantum numbers n and l which characterize the subshell.

Based on the above discussion, one can now analyze the experimental data by simply using the expressions for the direct potential $V^{\text{s.o.}}(R)$, given above in Sec. II B 3. The resultant, phenomenologically determined value of J_λ will also contain, of course, the effects of the exchange part of the intercluster spin-orbit interaction.

C. Intercluster interactions in the $n+\alpha$, ^{16}O, and ^{40}Ca cases

To gain information about the domain of validity of model K, we shall explicitly consider the nuclear systems $n+\alpha$, $n+^{16}$O, and $n+^{40}$Ca. For these systems, we shall assume that protons and neutrons fill the $(nl)=(00)$ subshell for the α particle, the $(nl)=(00)$ and (11) subshells for ^{16}O, and the $(nl)=(00)$, (11), (20), and (22) subshells for ^{40}Ca. Then, by using the expressions given in the preceding subsection, the $n+$ nucleus interactions in these three cases can be easily obtained. The results are as follows.

(1) The $n+\alpha$ system:

$$V^N(R)=-V_0\gamma(4w-m+2b-2h)\exp\left[-\frac{\alpha\kappa}{\alpha+\kappa}R^2\right], \tag{55}$$

$$K^N(\mathbf{R},\mathbf{R}')=-V_0\beta(-w+4m-2b+2h)\times\exp\left[-\left[\frac{\alpha}{2}+\kappa\right](R^2+R'^2)+2\kappa\mathbf{R}\cdot\mathbf{R}'\right], \tag{56}$$

$$V^{\text{s.o.}}(R)=-2J_\lambda\alpha^{5/2}\exp(-\alpha R^2)\ . \tag{57}$$

(2) The $n+^{16}$O system:

$$V^N(R)=-V_0\gamma(4w-m+2b-2h)\times\left[\frac{4\alpha+\kappa}{\alpha+\kappa}+\frac{2\alpha\kappa^2}{(\alpha+\kappa)^2}R^2\right]\exp\left[-\frac{\alpha\kappa}{\alpha+\kappa}R^2\right], \tag{58}$$

$$K^N(\mathbf{R},\mathbf{R}')=-V_0\beta(-w+4m-2b+2h)(1+2\alpha\mathbf{R}\cdot\mathbf{R}')\times\exp\left[-\left[\frac{\alpha}{2}+\kappa\right](R^2+R'^2)+2\kappa\mathbf{R}\cdot\mathbf{R}'\right], \tag{59}$$

$$V^{\text{s.o.}}(R)=-2J_\lambda\alpha^{5/2}(2\alpha R^2-1)\exp(-\alpha R^2)\ . \tag{60}$$

(3) The $n+^{40}$Ca system:

$$V^N(R)=-V_0\gamma(4w-m+2b-2h)\times\left[\frac{5}{2}+\frac{15\alpha^2}{2(\alpha+\kappa)^2}+\frac{10\alpha^2\kappa^2}{(\alpha+\kappa)^3}R^2+\frac{2\alpha^2\kappa^4}{(\alpha+\kappa)^4}R^4\right]\exp\left[-\frac{\alpha\kappa}{\alpha+\kappa}R^2\right], \tag{61}$$

$$K^N(\mathbf{R},\mathbf{R}')=-V_0\beta(-w+4m-2b+2h)\times[\tfrac{5}{2}-\alpha(\mathbf{R}-\mathbf{R}')^2+2\alpha^2(\mathbf{R}\cdot\mathbf{R}')^2]\times\exp\left[-\left[\frac{\alpha}{2}+\kappa\right](R^2+R'^2)+2\kappa\mathbf{R}\cdot\mathbf{R}'\right], \tag{62}$$

$$V^{\text{s.o.}}(R)=-2J_\lambda\alpha^{5/2}(2\alpha^2R^4-4\alpha R^2+\tfrac{5}{2})\exp(-\alpha R^2)\ . \tag{63}$$

One can now readily understand the advantage of adopting model K. In comparison with the corresponding RGM kernel functions given in the appendices of Refs. [7] and [10] for these three systems, the expressions for $K^N(\mathbf{R},\mathbf{R}')$ in model K are much simpler. Thus, while the RGM is suitable mainly for the study of a few selected systems, model K can be used, without much computational difficulty, to systematically investigate many $N+$ nucleus systems even when the target nucleus involved contains a large number of nucleons.

III. COMPARISON OF RGM AND MODEL-K RESULTS

To examine whether model K is a useful model or not, we make a comparison between RGM and model-K results for bound-state energies, phase shifts, differential scattering cross sections, and polarizations. This comparison will be performed, with no imaginary potentials, in both the $n+\alpha$ case, which involves a light target nucleus, and the $n+^{16}$O and $n+^{40}$Ca cases, where the target nucleus contains a much larger number of nucleons. The results of this comparison should not only provide us with information concerning the conditions on energy and target mass number under which the simplifying assumptions adopted in model K are valid, but also give us guidance in our future attempt to carry out systematic investigations of $N+$ nucleus systems using this model.

For a meaningful comparison, it is, of course, necessary to use the same nucleon-nucleon potential and the same rms radius $\tilde{R}$ for the matter distribution of the target nucleus in both calculations. In the RGM formulation with a harmonic-oscillator width parameter α_{RGM} and the model-K formulation with a corresponding width parameter α, $\tilde{R}^2$ is given by

$$\tilde{R}^2=\frac{1}{A\alpha_{\text{RGM}}}\left[\left[\sum_{nl}c_{nl}(n+\tfrac{3}{2})\right]-\tfrac{3}{2}\right]=\frac{1}{A\alpha}\sum_{nl}c_{nl}(n+\tfrac{3}{2})\ , \tag{64}$$

where c_{nl} is the occupation number of an (nl) subshell and the summation is over all filled (nl) subshells of the nucleons. With $\alpha_{\rm RGM}$ equal to 0.514, 0.32, and 0.25 fm^{-2} for the nuclei α, ^{16}O, and ^{40}Ca, respectively, which are chosen to yield the correct empirical values for $\tilde{R}$ [20], we find that the corresponding values of α should be taken to be 0.685, 0.334, and 0.253 fm^{-2} for these three target nuclei.

A. $n+\alpha$ system

The nucleon-nucleon potential used in the $n+\alpha$ case is given by Eq. (1) with $V_0=72.98$ MeV, $\kappa=0.46$ fm^{-2}, $w=m=0.4075$, and $b=h=0.0925$. For simplicity, the spin-orbit interaction is not considered by setting J_λ in Eq. (57) equal to zero. In Fig. 1, phase shifts up to a c.m. energy E of 50 MeV are presented for $L=0$–3. The model-K results are shown by solid curves, while the RGM results are indicated by solid circles. As can be seen from this figure, there are substantial quantitative differences between the RGM and model-K results. In particular, it is noted that the $L=1$ resonance energies are appreciably different. Additionally, we should point out one feature which is especially important. This feature is that the model-K phase shifts are generally larger than the RGM phase shifts in even-L states, but generally smaller in odd-L states. Thus, the odd-even L-dependent effect [1], which is well known to exist in nuclear systems where the nucleon-number difference of the interacting nuclei is small, turns out to be no longer existent in model K.

The above findings are, of course, not surprising. In the $n+\alpha$ case, the nucleon-number ratio N_A/N_B and the nucleon-number difference (N_A-N_B) are both small. Thus, the simplifying assumptions of model K are not expected to be valid. For a case like this where the target nucleus has a small mass number, it is clear that an exact RGM calculation must be carried out in order to obtain reliable results.

That model K is inadequate in the $n+\alpha$ case can be seen more clearly from Fig. 2, where RGM and model-K differential scattering cross sections at 20 and 50 MeV are compared. Here one finds that, because of the lack of odd-even L-dependent effects in model K, the rapid RGM cross-section rise in the backward angular region can no longer be reproduced.

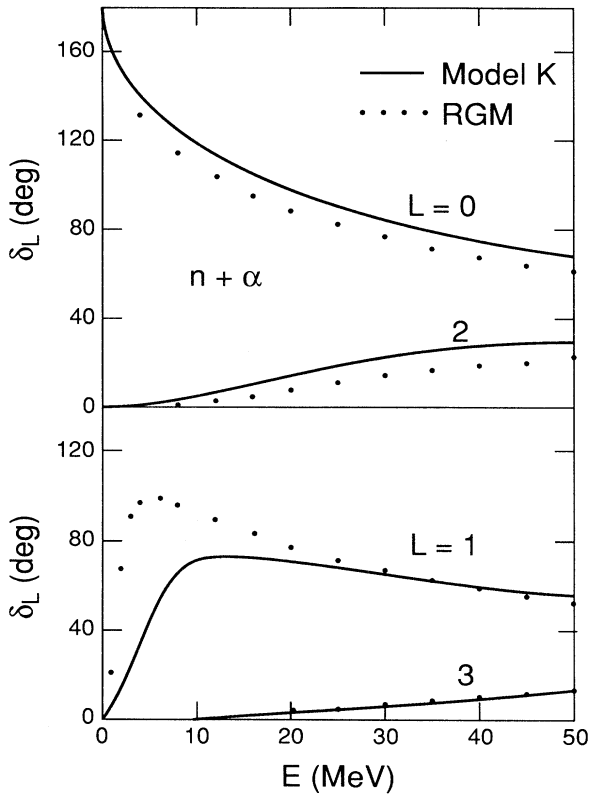

FIG. 1. Comparison of RGM (solid circles) and model-K (solid curves) results for phase shifts in the $n+\alpha$ system.

B. $n+^{16}$O system

The nucleon-nucleon potential used in the $n+^{16}$O case is different from that given by Eq. (1). It has the form [11]

$$V_{ij}=\left[V_R+\frac{1+P^\sigma_{ij}}{2}V_t+\frac{1-P^\sigma_{ij}}{2}V_s\right]\left[\frac{u}{2}-\frac{2-u}{2}P^\sigma_{ij}P^\tau_{ij}\right]$$
$$-\frac{1}{2\hbar}V_\lambda\exp(-\lambda r_{ij}^2)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot(\mathbf{r}_i-\mathbf{r}_j)\times(\mathbf{p}_i-\mathbf{p}_j)+\frac{e^2}{4r_{ij}}(1+\tau_{iz})(1+\tau_{jz})\,, \tag{65}$$

where

$$V_R=V_{0R}\exp(-\kappa_R r_{ij}^2)\,,$$
$$V_t=-V_{0t}\exp(-\kappa_t r_{ij}^2)\,, \tag{66}$$
$$V_s=-V_{0s}\exp(-\kappa_s r_{ij}^2)\,,$$

with

$$V_{0R}=200.0\text{ MeV}\,,\quad \kappa_R=1.487\text{ fm}^{-2}\,,$$
$$V_{0t}=178.0\text{ MeV}\,,\quad \kappa_t=0.639\text{ fm}^{-2}\,, \tag{67}$$
$$V_{0s}=91.85\text{ MeV}\,,\quad \kappa_s=0.465\text{ fm}^{-2}\,.$$

This particular nucleon-nucleon potential, commonly referred to as the Minnesota or MN potential, has been extensively and successfully employed in many RGM stud-

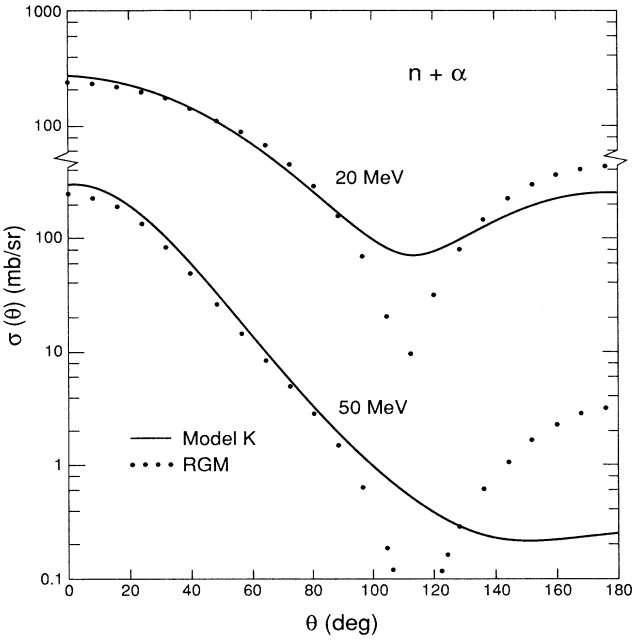

FIG. 2. Comparison of RGM and model-K results for differential scattering cross sections at 20 and 50 MeV in the $n+\alpha$ system.

ies of light nuclear systems (see, e.g., Ref. [2]).

The MN potential has a somewhat more complicated central part than the nucleon-nucleon potential of Eq. (1). However, we should emphasize that this does not cause any problem. The calculation can still be readily carried out by making only trivial modifications of the expressions given in Sec. II C for the direct potentials and the knock-on-exchange kernel functions.

The exchange-mixture parameter u in the MN potential was determined to be 0.924 in the $n+{}^{16}\mathrm{O}$ system by a previous RGM calculation [11]. As for the spin-orbit interaction, we shall again omit it here for the sake of simplicity in making comparisons between RGM and model-K results.

Bound states are found to exist for $L=0$ and 2 in both the RGM and the model-K calculations. In the RGM calculation, the energies of these two states, measured with respect to the $n+{}^{16}\mathrm{O}$ threshold, are equal to -3.26 and -1.50 MeV, respectively. These are to be compared with the corresponding model-K results of -2.04 and -0.31 MeV. For both of these L states, we note that there is a discrepancy of about 1.2 MeV. This indicates that model K is not too accurate as far as bound states are concerned. However, considering the fact that a great deal of computational effort can be saved by employing model K, one can still view a discrepancy of this magnitude as reasonably tolerable.

A comparison between RGM and model-K phase-shift results is shown in Fig. 3. Here one sees that there is a good quantitative agreement. The main discrepancy seems to occur in the $L=3$ state where the resonance energies from these two calculations differ by about 0.5 MeV. This is, however, not a serious discrepancy, because the $L=3$ state is a broad state with a very large level width.

Differential cross sections for $n+{}^{16}\mathrm{O}$ scattering calculated with the RGM and model K are compared at 5 and 10 MeV in Fig. 4, and at 20 and 30 MeV in Fig. 5. From these figures, one readily concludes that model K is generally quite satisfactory. Except at large backward angles, the agreement between these two calculations is seen to be somewhat better at higher energies. This can be understood as related to the fact that there exist broad resonances with $L=1$ and 3 in the low-energy region, and model K tends to work less well at energies where resonance effects are significant. In the present case, the situation is not too unfavorable, however, because these two resonances have large widths and, hence, their presence does not affect the accuracy of model K to a large extent

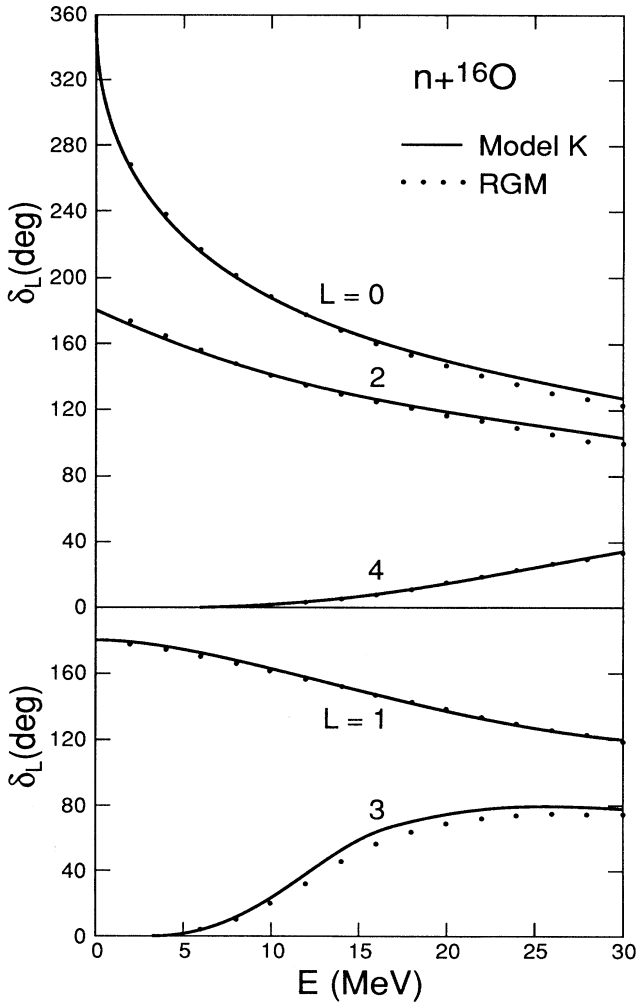

FIG. 3. Comparison of RGM and model-K results for phase shifts in the $n+{}^{16}\mathrm{O}$ system.

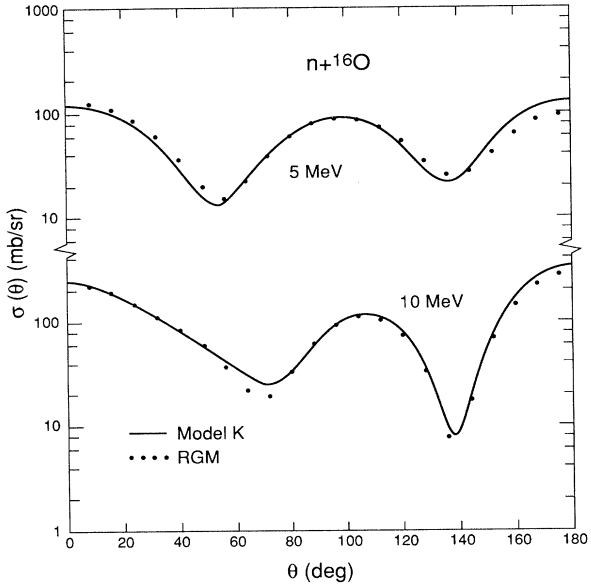

FIG. 4. Comparison of RGM and model-K results for differential scattering cross sections at 5 and 10 MeV in the $n+{}^{16}$O system.

even when the scattering energy involved is relatively low.

The minor discrepancy between RGM and model-K results at large backward angles can also be readily explained. In the formulation of model K, we have, in particular, made the assumption of omitting core-exchange type-a and type-d terms. As is explained in Sec. II A, these are terms which, at relatively high energies, contribute mainly in the backward angular region.

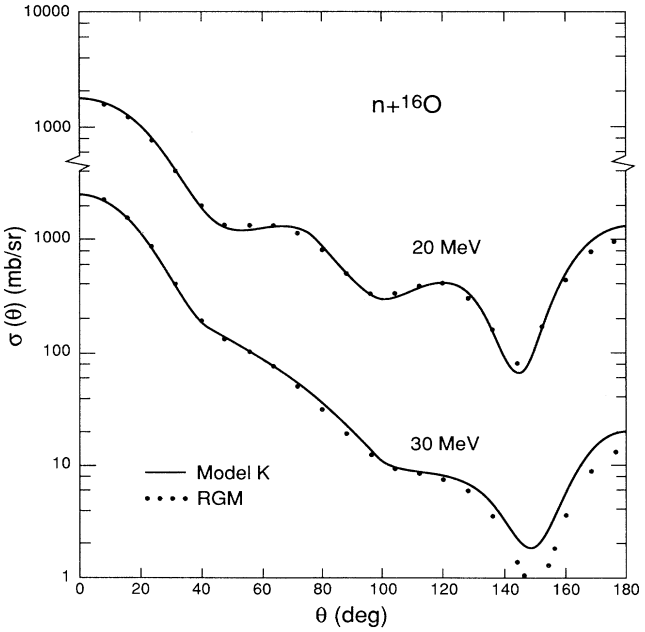

FIG. 5. Comparison of RGM and model-K results for differential scattering cross sections at 20 and 30 MeV in the $n+{}^{16}$O system.

C. $n+{}^{40}$Ca system

For the RGM and model-K calculations in the $n+{}^{40}$Ca system, the nucleon-nucleon potential used is

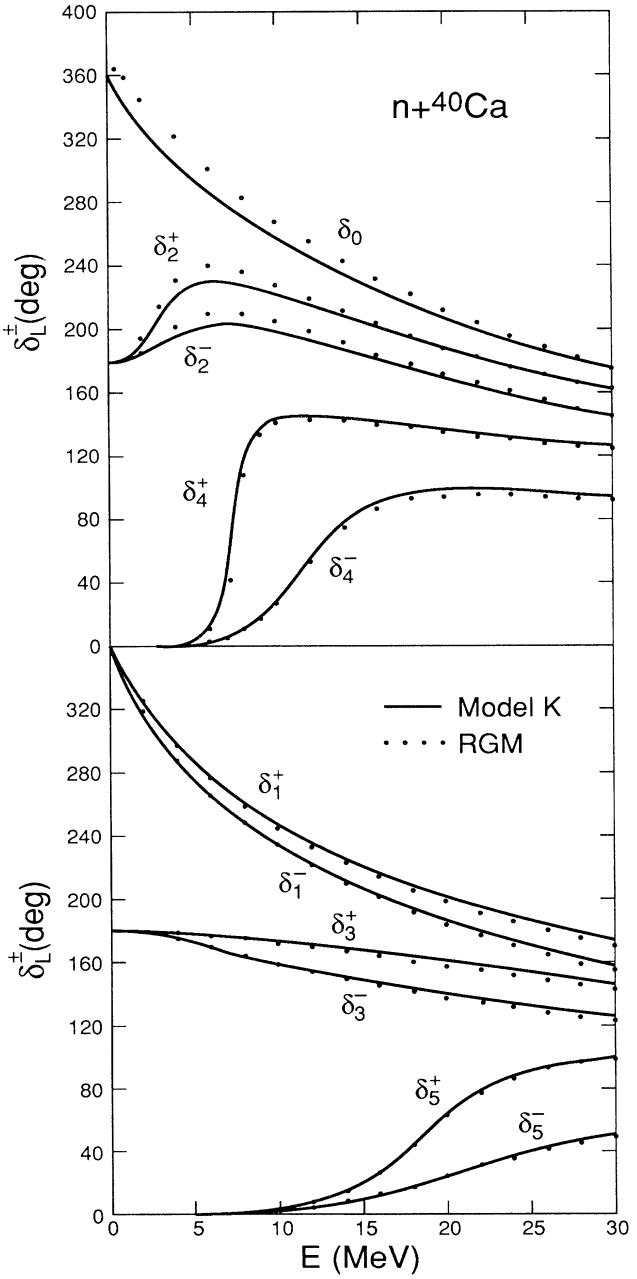

FIG. 6. Comparison of RGM and model-K results for phase shifts in the $n+{}^{40}$Ca system.

TABLE I. Bound-state energies $\tilde{E}$, in MeV, of ^{41}Ca.

Bound state	$\tilde{E}$ (RGM)	$\tilde{E}$ (model K)
$P_{3/2}$	−6.56	−5.03
$P_{1/2}$	−4.26	−2.85
$F_{7/2}$	−6.98	−6.43
$F_{5/2}$	−1.36	−0.51

the MN potential of Eqs. (65)–(67), with the spin-orbit part taken this time into account. The values of the exchange-mixture parameter u and the spin-orbit parameter J_λ are equal to 0.856 and 50 MeV fm^5, respectively, as determined from a previous RGM investigation [11].

Energies of bound states with $L=1$ and 3, measured with respect to the $n+^{40}$Ca threshold, are listed in Table I. Here it is seen that the model-K values are larger than the corresponding RGM values by about 1.5 MeV in the $L=1$ states and by a smaller amount in the $L=3$ states. As in the $n+^{16}$O case, this shows again that model K is not too satisfactory for bound-state studies. With the adoption of this model, one has to tolerate an inaccuracy in the bound-state energy of around 1 MeV.

Calculated RGM and model-K phase shifts for $n+^{40}$Ca scattering in the energy region up to 30 MeV are shown in Fig. 6. From this figure, one easily concludes that model K works rather well, especially at higher energies. In the lower-energy region with $E \lesssim 10$ MeV, there exists a rotational band of broad levels with $L=0$, 2, and 4, and model K is somewhat less satisfactory. But even here, one can see from Fig. 6 that the degree of inaccura-

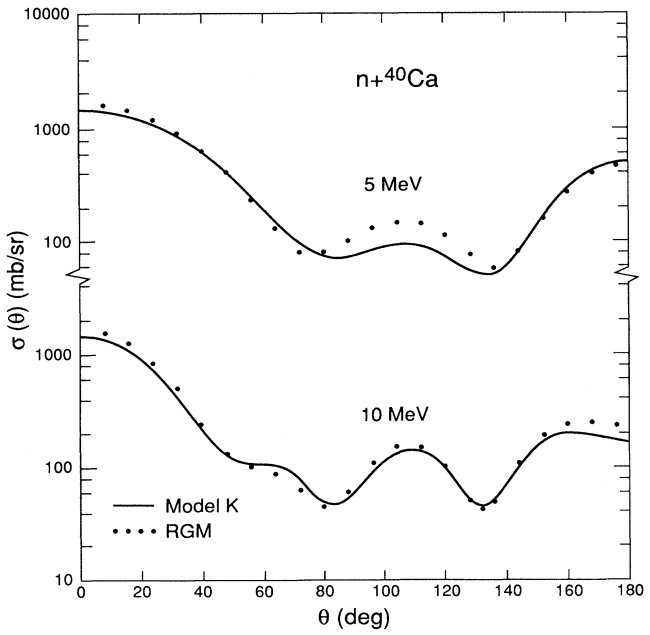

FIG. 7. Comparison of RGM and model-K results for differential scattering cross sections at 5 and 10 MeV in the $n+^{40}$Ca system.

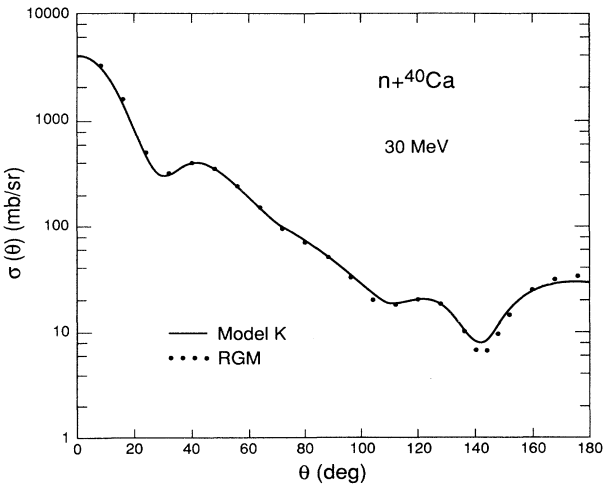

FIG. 8. Comparison of RGM and model-K results for differential scattering cross sections at 30 MeV in the $n+^{40}$Ca system.

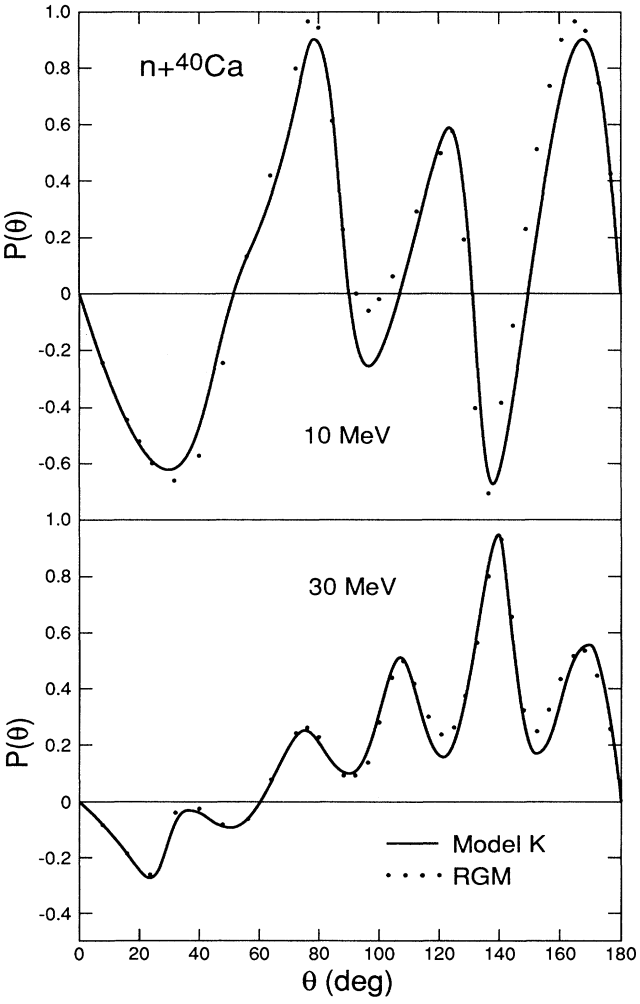

FIG. 9. Comparison of RGM and model-K results for polarizations at 10 and 30 MeV in the $n+^{40}$Ca system.

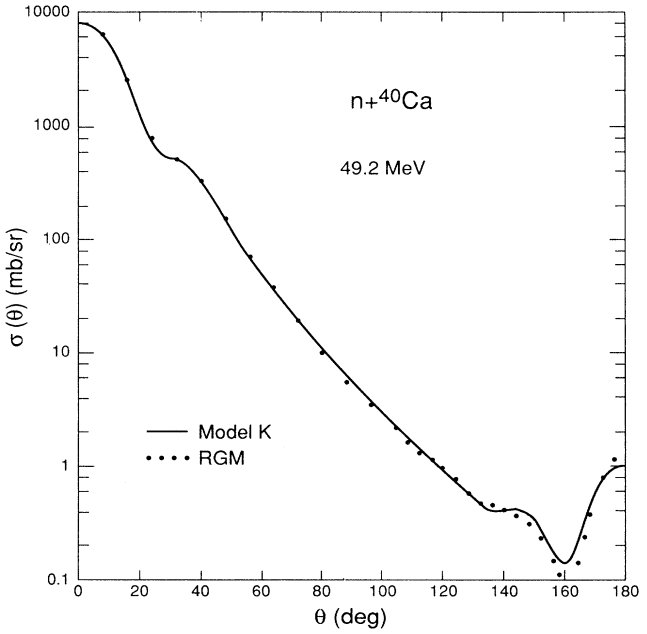

FIG. 10. Comparison of RGM and model-K results for differential scattering cross sections at 49.2 MeV in the $n+{}^{40}$Ca system.

cy associated with model K is still not large enough to be really detrimental.

That model K is less satisfactory at lower energies can be seen from Fig. 7. In this figure, the RGM and model-K differential cross sections at 5 and 10 MeV are compared. At 5 MeV, it is found that model K reproduces correctly the oscillatory features of the RGM result, but not the magnitude at the cross-section peak near 105°. On the other hand, already at a higher energy of 10 MeV, the agreement between RGM and model-K results is seen to be quite reasonable over the whole angular region, thus confirming that one needs to be somewhat cautious about the results of model K only at energies where resonance effects are important.

The expectation that model K works well at relatively high energies is further verified by a comparison at 30 MeV shown in Fig. 8. Here one finds that there is a good agreement between RGM and model-K results in the whole angular region. The discrepancy between the differential cross sections of these two calculations at large backward angles, already rather small in the $n+{}^{16}$O case, becomes smaller still in the $n+{}^{40}$Ca case. The reason for this is clear; it is a consequence of the fact that the nucleon-number difference (N_A-N_B) is even larger here than that in the $n+{}^{16}$O system.

In Fig. 9, polarizations obtained with RGM and model-K calculations are compared at 10 and 30 MeV. Here one finds that, as expected, the agreement is reasonably satisfactory at 10 MeV, but becomes quite good at the higher energy of 30 MeV.

Finally, we have also compared the differential cross sections calculated with the RGM and model K at a rather high energy of 49.2 MeV. For this comparison, the nucleon-nucleon potential used is that of Ref. [7] with the exchange-mixture parameter $u=0.808$ and the spin-orbit interaction omitted [21]. The results are shown in Fig. 10. From this figure, it is seen that, even though the cross section spans 5 orders of magnitude, the agreement between these two calculations is clearly satisfactory.

IV. CONCLUSION

Because of analytical complexities in deriving the kernel functions representing the nonlocal part of the internuclear interaction, the resonating-group method has been limited in its application only to very light nuclei and a few selected heavier systems. To enhance its general utility in the domain of medium and heavy nuclei, it seems that one must relax some of the stringent requirements inherent in resonating-group calculations. In this investigation, we take the first step in this direction by introducing a much simpler model, called model K, for nucleon + nucleus interaction, which has the important advantage that the kernel function can be readily derived in an analytical manner, yet still represents a good approximation to the exact RGM treatment.

Model K is a microscopic nonlocal model for the nucleon + nucleus interaction. It is obtained by introducing two simplifying assumptions into the RGM formulation. These assumptions are (i) recoil effects of the target nucleus are not considered, and (ii) core-exchange type-a and type-d terms are not taken into account [22]. With these assumptions, the nucleon + nucleus interaction consists then only of direct and knock-on-exchange terms, and the general expressions for these terms can be analytically derived with very little difficulty.

Comparisons between RGM and model-K results in the $n+\alpha$, $n+{}^{16}$O, and $n+{}^{40}$Ca systems for bound-state energies, phase shifts, differential scattering cross sections, and polarizations yield information concerning the conditions under which model K is valid. These conditions are as follows.

(1) The nucleon-number ratio N_A/N_B and the nucleon-number difference (N_A-N_B), where N_A and N_B denote the nucleon numbers of the target and incident nuclei, respectively ($N_B=1$ in the nucleon + nucleus system), must be both much larger than 1. Based on our present investigation, it can be estimated that, for model K to work well in the nucleon + nucleus case, the value of N_A should be larger than about 10.

(2) Model K is not very accurate for bound-state calculations. In the scattering case, it is a very satisfactory model at energies where resonance effects are not important; in other words, it works well in the c.m. energy region higher than about 10 MeV.

The energy condition (2) above does not seem to be as strict as the condition (1) on the target mass number. For bound states, our investigation shows that model K overestimates the bound-state energy by about 1 MeV. This is not an insignificant amount, but can be tolerated in many situations.

At energies higher than about 10 MeV, model K repro-

duces very well the RGM results. Thus, it can be adopted to make systematic and large-scale investigations of many nucleon + nucleus systems. For this purpose, however, we must first generalize our present formulation to also cover non-closed-subshell target nuclei. This may necessitate the introduction of additional simplifying assumptions, but in our opinion is a rather minor problem.

Model K can also be employed to examine the general properties of the nucleon + nucleus interaction. For example, we can use it to construct equivalent local potentials and then investigate the energy dependence of these potentials. Also, it can be adopted to study the interaction between a nucleon and a nucleus in its excited state. In short, these are the types of projects which we intend to look into in the near future. Meanwhile, we are also examining the α+ nucleus system to see if reasonable assumptions can be introduced to simplify the analytical consideration of this system. Because of the composite nature of the α particle, this will undoubtedly be a more difficult problem; however, in view of the computational complexities which one faces in using the exact RGM approach, we are of the opinion that this is certainly a project worth pursuing.

ACKNOWLEDGMENTS

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[5] See, for example, T. Ando, K. Ikeda, and A. Tohsaki-Suzuki, Prog. Theor. Phys. **61**, 101 (1979); K. Langanke, Ph.D. thesis, Universität Münster, 1980; T. Wada and H. Horiuchi, Phys. Rev. C **38**, 2063 (1988).

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[14] In a general two-cluster system consisting of clusters A and B with nucleon numbers equal to N_A and N_B ($N_A > N_B$), the exchange terms can be conveniently classified by using a pair of indices (xq). The index x represents the number of nucleons interchanged between A and B, and the index q denotes the interaction type. For each value of x from 1 up to (N_B-1), there are five interaction types which are denoted by the index $q=a$, b, c, d, and e (see Refs. [1] and [8] for a careful explanation of the meaning of these interaction types), while for $x=N_B$, there are only three interaction types, namely, $q=a$, c, and d. Thus, in our present study of the nucleon + nucleus interaction, there exist exchange terms with $xq=1a$, $1c$, and $1d$, in addition to the direct or no-exchange term $(x=0)$ which must always be taken into consideration.

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[22] It is interesting to mention that, in many other works on the optical potentials, these assumptions have been routinely made with little or no justification.