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Gamow-Teller decay of $N=50$ nuclei

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The Gamow-Teller decays of $N=50$ nuclei are calculated, making use of an interaction determined by a least-squares fit to the spectra of $N=51$ nuclei. Core polarization is taken into account by evaluating various first-order diagrams, and when these and other corrections are included the calculated strengths are, in most cases, in good agreement with experiment.

I. INTRODUCTION

The almost universal quenching of Gamow-Teller (GT) strength relative to simple shell-model predictions has been a problem of great interest to both theorists and experimentalists in recent years. There have been attempts to account for this quenching by considering core polarization and more exotic corrections such as those arising from meson-exchange currents and isobar excitation [1]. Almost all calculations, however, have been concerned with the total GT strength rather than its distribution in energy.

Experimental groups at GSI Darmstadt and CERN have recently been investigating Gamow-Teller decays of proton-rich nuclei in the region of mass 100, and this has made available much new data concerning nuclei with $N=50$. Quenching of GT strength in these nuclei was examined in a paper by Towner [2], who showed that several processes contribute to the large hindrance factors. These calculations were concerned only with summed strengths, however, and therefore could not determine how much of the strength feeds states which lie at excitation energies above the electron capture Q value.

The present paper examines in detail the GT decay of $N=50$ nuclei. Least-squares fits are performed to determine an interaction which reproduces the spectra of $N=51$ nuclei with very low rms error, and the corresponding wave functions are used in the calculation of GT strengths. Core polarization diagrams are evaluated, and their effect on summed strengths examined in Sec. III. The calculated strengths for masses 93–98 are presented in Sec. IV, and found to be in good agreement with experiment.

II. SPECTRA OF $N=51$ NUCLEI

Spectra of $N=51$ nuclei have been the subject of a number of theoretical papers. Talmi [3], Auerbach and Talmi [4], and Vervier [5] considered just $d_{5/2}$ neutrons, while Gloekner [6] included the $s_{1/2}$ single-neutron level for ^{91}Zr and ^{92}Nb. All these authors deduced effective two-body matrix elements by fits to empirical data. Chuu *et al.* [7] allowed the 51st neutron to occupy the $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ orbitals, using the $N=50$ proton interaction of Ball *et al.* [8] and a surface-delta interaction (SDI) between protons and neutron. A fit of the $T=0$ and $T=1$ SDI strengths to 52 levels in ^{90}Y, ^{91}Zr, ^{92}Nb, and ^{93}Mo gave a rms deviation of 150 keV.

The present calculations were least-squares fits to six ground-state energies and 59 excitation energies in $N=51$ nuclei of mass 90–97. The proton-proton interaction was taken to be the "total energy" set of matrix elements from Gloekner and Serduke [9], and the $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ single-neutron energies relative to the $d_{5/2}$ were taken as free parameters. Various choices of neutron-proton interaction were investigated, with results as summarized below.

(a) With a SDI interaction, as chosen by Chuu *et al.* [7], best-fit $T=0$ and $T=1$ strengths give a rms error of 160 keV. The most obvious deficiency of the interaction is that it is incapable of accounting for the lowest $\frac{7}{2}^+$ states in ^{95}Ru and ^{97}Pd (the lowest calculated $\frac{7}{2}^+$ in ^{97}Pd is 750 keV too high). Experiment clearly shows that the $g_{7/2}$ neutron strength drops steadily with increasing number of $g_{9/2}$ protons [10], but a SDI cannot reproduce this behavior since it gives equal centroid energies for $g_{9/2}(p)g_{7/2}(n)$ and $g_{9/2}(p)d_{5/2}(n)$.

(b) If a volume-delta interaction is used instead of the SDI, the rms errors drops significantly to 105 keV. This is due mainly to the fact that the $g_{9/2}(p)g_{7/2}(n)$ centroid now has a magnitude which (for harmonic-oscillator radial functions) is a factor of 1.83 greater than that of $g_{9/2}(p)d_{5/2}(n)$, causing the effective $g_{7/2}d_{5/2}$ single-neutron gap to decrease by about 300 keV for each additional $g_{9/2}$ proton.

(c) The final fit was made having as free parameters the six $g_{9/2}(p)d_{5/2}(n)$ two-body matrix elements, the three single-neutron energy gaps, and the $T=0$ and $T=1$ strengths of the volume-delta force which was used for the remainder of the interaction between $g_{9/2}$ protons and the 51st neutron. The centroid energy for a $p_{1/2}$ proton and a neutron in any orbital j was set equal to -374 keV, this being the value for $j=d_{5/2}$ which is deduced from the spectrum of ^{90}Y. This fit gave a rms error of only 67 keV, and very reasonable best-fit parameters. The $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ energies relative to the $d_{5/2}$ are 1.17, 2.15, and 2.98 MeV, which can be compared with the energies of the strong $l=0$, $l=2$, and $l=4$ levels in ^{89}Sr at 1.03, 2.01, and 2.67 MeV. The $g_{9/2}(p)d_{5/2}(n)$

$J=2$–7 energies of -594, -340, -120, -226, -169, and -732 keV have an energy centroid of -368 keV, and the delta force

$$V=V_0(1+xP^{\sigma})\delta(\mathbf{r})$$

has $V_0=665$ MeV fm^3, $x=0.13$.

As might be expected from the low rms error, the fit reproduces the known spectra of $N=51$ nuclei very well. Because of this, a number of predictions can be made with some confidence.

(i) The yrast $\frac{15}{2}^+$ and $\frac{19}{2}^+$ states in ^{91}Zr are predicted to lie at 3.3 and 3.55 MeV.

(ii) The yrast 8^+ in ^{92}Nb is expected very close to the known yrast 9^+ at 2.33 MeV, and the yrast 10^+ should lie at about 3.2 MeV, between the yrast 11^+ and 13^+ states.

(iii) The yrast $\frac{19}{2}^+$ in ^{93}Mo is calculated to lie at 2.8 MeV.

(iv) Second 5^+ and 6^+ states are predicted to lie close to 800 keV in ^{94}Tc, just below the known second states of spins 1–4.

(v) Yrast levels of spins $\frac{11}{2}^+$, $\frac{15}{2}^+$, $\frac{19}{2}^+$, and $\frac{21}{2}^+$ are calculated to lie at 2.1, 2.45, 2.6, and 2.5 MeV in ^{95}Ru.

(vi) ^{96}Rh is predicted to have a 6^+ ground state, with the first excited state being a 3^+ at 65 keV. Gurjrathi *et al.* [11] have suggested a 5^+ ground state with the isomer at 52 keV a 2^+, but the present assignments are supported by Rykaczewski *et al.* [12], who also assign 2^+ to levels at 177 and 775 keV. The calculation gives the two lowest 2^+ states at 230 and 834 keV, and 7^+, 5^+, and 4^+ levels at 110, 142, and 175 keV.

(vii) Yrast $\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{15}{2}^+$, and $\frac{21}{2}^+$ levels are calculated to lie at 0.9, 1.3, 2.3, and 2.65 MeV in ^{97}Pd.

(viii) Huyse *et al.* [13] propose 6^+ or 7^+ for the ground state of ^{98}Ag, and the calculation gives 6^+. The calculated spectrum has a 5^+ at 90 keV, 3^+, 4^+, and 7^+ levels at 150, 155, and 165 keV, and a 2^+ at 430 keV. No long-lived isomer is therefore expected, although the 3^+ might be expected to have an $E2$ lifetime of order 1 μs.

III. ONE- AND TWO-BODY CORE POLARIZATION

For $N=50 \rightarrow N=51$ Gamow-Teller transitions, $(p_{1/2}g_{9/2})^{Z-38}$ and $(p_{1/2}g_{9/2})^{Z-38}(d_{5/2}s_{1/2}d_{3/2}g_{7/2})$ wave functions were used to calculate the bare $(J_f||\sigma||J_i)$ matrix elements, which arise entirely from $g_{9/2}(p) \rightarrow g_{7/2}(n)$ transitions. There are, however, several first-order corrections to these bare matrix elements, which can be represented by the diagrams shown in Fig. 1. Diagrams (a) and (b) are one-body corrections which reduce the effective single-particle matrix element $(g_{7/2}(n)||\sigma||g_{9/2}(p))$ from its bare value of 4.216. Diagrams (c) and (d) are corrections which lead to effective two-body matrix elements

$$(g_{9/2}^2(p){:}J_1||\sigma||g_{9/2}(p)j(n){:}J_2)\ ,$$

and were calculated for $j=s_{1/2}$, $d_{3/2}$, $d_{5/2}$, and $g_{7/2}$. The intermediate state j_1 is equal to j for $j=s_{1/2}$ and $g_{7/2}$, and j_1 is $d_{3/2}$ and $d_{5/2}$ for $j=d_{3/2}$ or $d_{5/2}$.

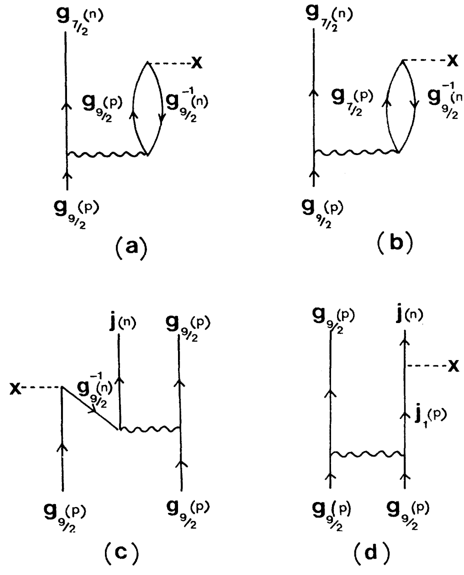

FIG. 1. First-order core polarization diagrams.

These diagrams were evaluated using the volume-delta force given by the fit to $N=51$ spectra. The intermediate-state energies were set equal to single-particle energy gaps deduced from empirical data. The ^{90}Zr (^{3}He,d)^{91}Nb reaction gives $d_{5/2}$, $g_{7/2}$, $s_{1/2}$, and $d_{3/2}$ proton centroid energies of 3.87, 5.56, 5.78, and 5.86 MeV [14], and these were assumed to be the energies of $jg_{9/2}^{-1}$ states. The binding energies of mass 87–89 Sr isotopes suggest that the energy of $d_{5/2}g_{9/2}^{-1}$ is 4.75 MeV for neutrons, and the $N=51$ fits $g_{7/2}d_{5/2}$ gap then gives 7.7 MeV for the neutron $g_{7/2}g_{9/2}^{-1}$ centroid in ^{89}Sr. As discussed above, this centroid should drop by about 300 keV for each $g_{9/2}$ proton, so the $g_{7/2}g_{9/2}^{-1}$ proton energy was set equal to $7.7-0.3n$, where n is the number of $g_{9/2}$ protons. The $s_{1/2}g_{9/2}^{-1}$ and $d_{3/2}g_{9/2}^{-1}$ energies were set equal to 5.9 and 6.9 MeV.

The effect of these core polarization corrections was investigated by a calculation of $g_{9/2}^n \rightarrow g_{9/2}^{n-1}g_{7/2}$ Gamow-Teller strengths, with initial state being of seniority zero for even n, and seniority one for odd n. Table I shows the

TABLE I. Gamow-Teller strengths B_{GT} for $g_{9/2}^n \rightarrow g_{9/2}^{n-1}g_{7/2}$. The 1-body column includes effects of diagrams (a) and (b) of Fig. 1, and the (1+2)-body column also includes effects of diagrams (c) and (d). The last two columns give the corresponding hindrance factors.

n	Bare	1-body	(1+2)-body	h_1	h_{1+2}
2	3.55	1.64	0.92	2.16	3.86
3	5.33	2.38	2.02	2.24	2.64
4	7.11	3.06	2.21	2.32	3.22
5	8.89	3.69	3.74	2.41	2.38
6	10.66	4.23	4.08	2.52	2.61
7	12.44	4.71	6.18	2.64	2.01
8	14.22	5.11	6.85	2.78	2.08
9	16.00	5.41	9.63	2.96	1.66
10	17.77	5.61	11.11	3.17	1.60

calculated strengths $B_{\rm GT}$, and the associated hindrance factors h. These can be compared with the results of Towner [2] shown in his Table 5, which were calculated with somewhat different single-particle gaps and various interactions. The present results are similar to his, although the one-body hindrance factor h_1 is now n dependent because of the mass dependence of the neutron $g_{7/2}g_{9/2}^{-1}$ energy gap.

IV. GAMOW-TELLER DECAYS OF $N=50$ NUCLEI

The distribution of Gamow-Teller strength in the decay of $N=50$ nuclei was investigated using ground-state wave functions calculated using the Gloekner and Serduke [9] proton interaction, and $N=51$ wave functions calculated with the interaction given by the fit to $N=51$ spectra. The one- and two-body core polarization corrections discussed in Sec. III were taken into account, and in addition an estimate of the effects of pairing correlations was included by making use of the n-dependent pairing hindrance factors $h_{\rm pair}$ calculated by Towner [2]. It was assumed that the effect of these was to scale the bare single-particle matrix element $(g_{7/2}||\sigma||g_{9/2})$ by a factor of $h_{\rm pair}^{-1/2}$. To represent effects of higher-order correlations discussed by Towner, the calculated values of $B_{\rm GT}$ were reduced by a factor of $|g_A/g_A^{\rm eff}|^2=1.6$.

Figures 2–4 give results for decays of the even-mass nuclei ^{94}Ru, ^{96}Pd, and ^{98}Cd, for which calculated electron capture Q values are 1.62, 3.48, and 5.25 MeV. The first two Q values can be compared with empirical values of 1.589±0.014 and 3.45±0.15 MeV, while the mass-98 value is currently being measured at GSI Darmstadt.

The calculation for $^{94}{\rm Ru}\rightarrow{}^{94}{\rm Tc}$ agrees with experiment in giving two strongly excited 1^+ states below 1 MeV in ^{94}Tc. Their summed Gamow-Teller strength is 1.08, while experiment [15] gives 1.08±0.07. About 28% of the total strength lies above $Q_{\rm EC}$.

Four strongly excited 1^+ states have been isolated below 2 MeV in ^{96}Rh in the decay of ^{96}Pd [12], with a summed strength of $2.32^{+0.86}_{-0.63}$. The calculation gives six 1^+ states below 2 MeV, three of them with significant Gamow-Teller strength, and a summed strength of 2.10. Additional strength of 0.3 is predicted to lie below $Q_{\rm EC}$,

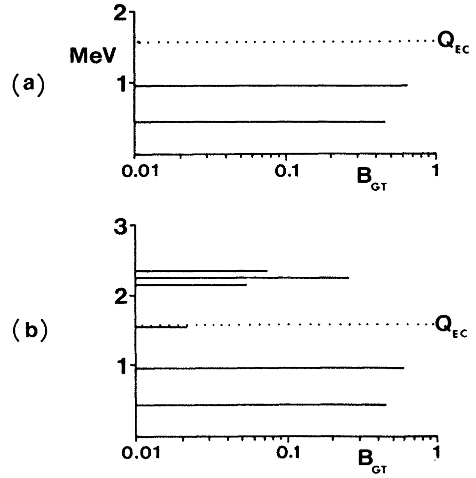

FIG. 2. Gamow-Teller strength in the decay $^{94}{\rm Ru}\rightarrow{}^{94}{\rm Tc}$. (a) Experiment (Ref. [15]). (b) Calculated strength below 3 MeV in ^{94}Tc.

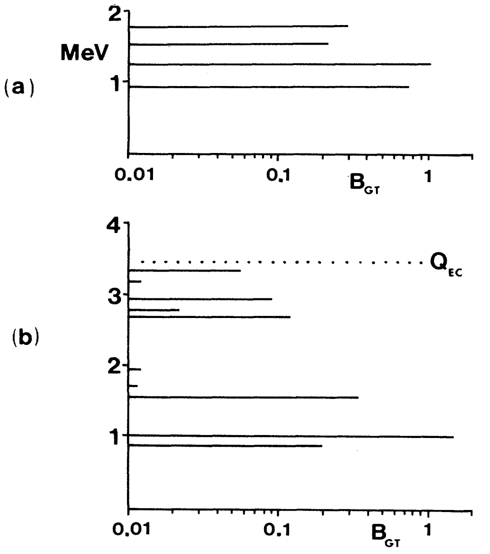

FIG. 3. Gamow-Teller strength in the decay $^{96}{\rm Pd}\rightarrow{}^{96}{\rm Rh}$. (a) Experiment (Ref. [12]). (b) Calculated strength below 4 MeV in ^{96}Rh.

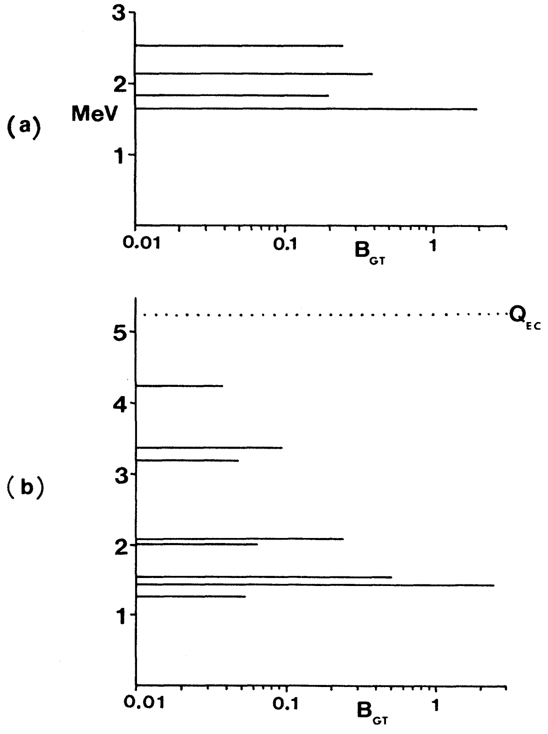

FIG. 4. Gamow-Teller strength in the decay $^{98}{\rm Cd}\rightarrow{}^{98}{\rm Ag}$. (a) Experiment (Ref. [16]). (b) Calculated strength below 5 MeV in ^{98}Ag.

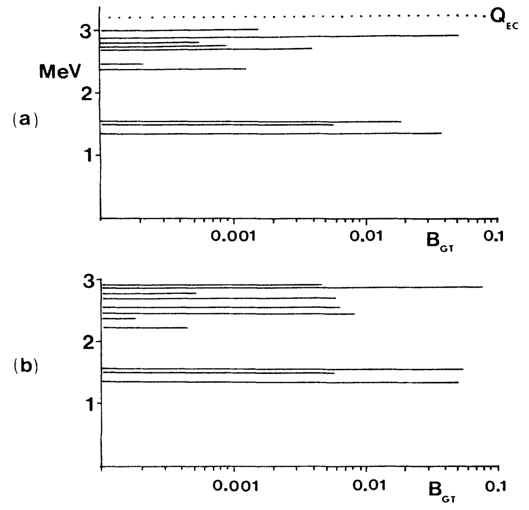

FIG. 5. Gamow-Teller strength in the decay $^{93}\mathrm{Tc}\rightarrow{}^{93}\mathrm{Mo}$. (a) Experiment (Ref. [17]). (b) Calculated strength below 3 MeV in $^{93}\mathrm{Mo}$.

and only 0.014 lies above this energy.

Experiment gives four 1^+_1 levels below 3 MeV in $^{98}\mathrm{Ag}$ which are excited with $B_{\mathrm{GT}}>0.1$ in the decay of $^{98}\mathrm{Cd}$ [16]. The calculation gives three, and three others with combined strength of over 0.1. The summed strength of the six is 3.39, while the quoted emprical value is $2.7^{+0.9}_{-0.7}$. The calculated Q value of 5.25 MeV is 250 keV lower than the value estimated by the CERN group, and, if correct, would mean that the experimental value of B_{GT} is underestimated by a factor of about 1.45 [17].

Figures 5 and 6 give results for the $^{93}\mathrm{Tc}\rightarrow{}^{93}\mathrm{Mo}$ and $^{95}\mathrm{Rh}\rightarrow{}^{95}\mathrm{Ru}$ decays. Calculated electron capture Q values are 3.12 and 5.00 MeV, experiment giving 3.20 and 5.11 ± 0.15 MeV. The calculation for mass 93 agrees with experiment in giving three relatively strong levels close to 1.5 MeV, then a gap of several hundred keV before a group of seven or eight levels lying below 3.1 MeV. The strongly excited $\frac{9}{2}^+$ level calculated to lie at 2.89 MeV can be associated with the level observed at 2902 keV. The total calculated strength to levels below 3 MeV is 0.21, while experiment [18] gives about 0.13. Almost 90% of the Gamow-Teller strength is unobserved, because it lies above 3.2 MeV.

In the case of mass 95, it appears that almost all levels between 2 and 3 MeV have been isolated experimentally [19], but the calculation suggests that there are many more strong levels between 3 and 4 MeV than have been observed to date.

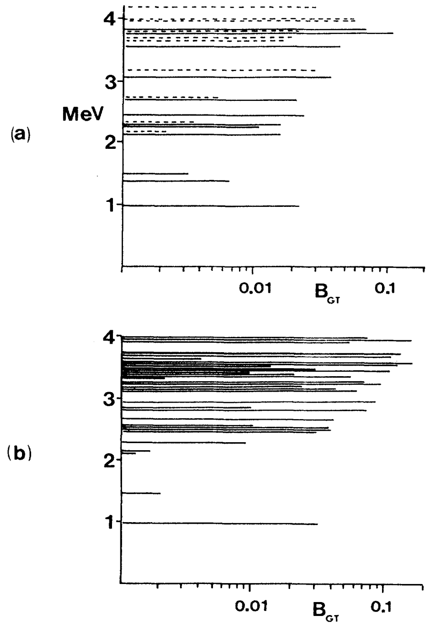

FIG. 6. Gamow-Teller strength in the decay $^{95}\mathrm{Rh}\rightarrow{}^{95}\mathrm{Ru}$. (a) Experiment (Ref. [18]). (b) Calculated strength below 4 MeV in $^{94}\mathrm{Ru}$.

V. CONCLUSIONS

The present calculations are the first which attempt to give a detailed account of the Gamow-Teller decay of $N=50$ nuclei, rather than just accounting for quenching of the total strength. Their success is due largely to the determination of a neutron-proton interaction which accounts remarkably well for the spectra of $N=51$ nuclei. Core polarization is found to be important both in being a major contributor to the quenching and in modifying the distribution of strength. To obtain total strengths in agreement with experiment it is necessary to assume that the effective value of g_A/g_V is close to unity, rather than the free-nucleon value of 1.26; a similar modification of g_A was found necessary in the calculations of Towner and Khanna [1] for closed-shell-plus-one nuclei, and of Brown and Wildenthal [20] for *sd*-shell nuclei, and is believed to be due to the effects of higher-order core polarization, meson-exchange currents, and isobar excitation.

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