Extent of simultaneous parity and time violation in ^{182}W

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In order to relate nuclear gamma-ray distributions to the fundamental parity-time- (PT-) and parity- (P-) violating meson-nucleon interaction, we analyze the case of the mixed ($E1,M2,E3$) 1189-keV gamma ray in ^{182}W which is populated in the decay of cryogenically oriented ^{182}Ta. Within the framework of the quasiparticle random-phase approximation we calculate the value of the complex "irregular" mixing ratio $\varepsilon(\overline{E2}/M2)$ for this transition. We estimate that this mixing ratio will have a P-violating real part of $|\varepsilon|\cos\eta \simeq 5\times 10^{-5}$ which implies an observable forward-backward asymmetry ($\langle \mathbf{J}\rangle\cdot\mathbf{k}$) in the 1189-keV gamma-ray directional distribution of $O_P \simeq 2\times 10^{-5}$ at 10 mK. For the PT-violating imaginary part we find $|\varepsilon|\sin\eta \simeq 200\bar{g}_{\pi NN}^{(I=1)}$, where $\bar{g}_{\pi NN}^{(I=1)}$ is the strength of the isovector PT-violating pion-nucleon coupling. An upper limit to this constant of $\lesssim 3\times 10^{-10}$ may be obtained from the electric dipole moment of the neutron. Whence we conclude that at 10 mK one needs to measure the PT-violating correlation $(\langle \mathbf{J}\rangle\cdot\mathbf{k}_2)(\langle \mathbf{J}\rangle\cdot\mathbf{k}_1\times\mathbf{k}_2)$ to an accuracy of $O_{PT} \lesssim 2\times 10^{-8}$ in order to improve the limit on $\bar{g}_{\pi NN}^{(I=1)}$ set by the neutron electric dipole moment.

I. INTRODUCTION

Following the discovery of CP violation in neutral kaon decay, numerous experimental attempts have been made to probe the extent of time (T) and simultaneous parity and time (PT) symmetry violations in other systems [1]. To date, none of these have shown any significant symmetry noninvariance. The most stringent constraint on such effects is provided by the upper limit on the PT-violating neutron electric dipole moment $d_n < 1.2\times 10^{-25}e$ cm [2]. For a PT-violating pion-nucleon interaction of given isospin structure ($I \leq 2$), this observable may be related to the associated coupling constant $\bar{g}_{\pi NN}^{(I)}$ [3–5]. The possible isoscalar and isotensor interaction constants are thus restricted to $\bar{g}_{\pi NN}^{(I=0,2)} \lesssim 3\times 10^{-11}$. However, because of a reduced sensitivity of the neutron electric dipole moment to the isovector interaction, the limit on the associated coupling constant $\bar{g}_{\pi NN}^{(I=1)}$ is an order of magnitude weaker. By contrast, measurements involving the atomic nucleus are more sensitive to the isovector interaction since the isoscalar and isotensor terms are both hindered by a factor $(N-Z)/A$ [5]. It is therefore of interest to ascertain the competitiveness of symmetry experiments on the atomic nucleus as compared to the neutron electric dipole moment, especially with regard to PT-violating isovector pion exchange.

Previously, Murdoch *et al.* [6] searched for a gamma-ray distribution arising from the PT-violating correlation term $(\langle \mathbf{J}\rangle\cdot\mathbf{k}_2)(\langle \mathbf{J}\rangle\cdot\mathbf{k}_1\times\mathbf{k}_2)$ in ^{180}Hf, where $\langle \mathbf{J}\rangle$ is the expectation value of the cryogenically oriented nuclear spin and $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of two subsequent cascading gamma rays. (For a review of this and alternative distributions, see Ref. [1].) From the resulting limit set on this observable, Herczeg [5] has estimated that the isovector coupling constant be confined to the range $\bar{g}_{\pi NN}^{(I=1)} \lesssim 5\times 10^{-8}$, two orders of magnitude larger than that provided by the neutron electric dipole moment. Nevertheless, similar experiments in more favorable cases may be more competitive in this respect.

In a previous paper [7], henceforth referred to as I, we discussed the requirements that candidates for such experiments should satisfy. In the present work we wish to consider theoretically the merits of one such nucleus, ^{182}W, which we consider to be a generic example of a good experimental case. Therefore, in what follows we perform a detailed theoretical analysis in order to relate directly the observable PT-violating experimental distribution given above to the coupling constant $\bar{g}_{\pi NN}^{(I=1)}$. We will also discuss some general features of our results which may be pertinent to the analysis of other nuclei.

II. GENERAL CONSIDERATIONS

Consider a nuclear system consisting of two states, an initial state $|a_0\rangle$ and a final state $|b\rangle$, both of which are eigenstates of the P- and T-invariant strong-interaction Hamiltonian. These will be linked by an electromagnetic interaction which induces transitions of one or more multipolarities, characterized by the amplitudes $\gamma(\pi L, a_0 \to b)$ [8]. The T invariance of the electromagnetic interaction (which we assume to hold good) requires that these be entirely real. However, because of the atomic final-state effects [9], which involve the virtual interaction of the emitted photons with atomic electrons, these matrix elements acquire a small imaginary term. This phenomenon, parametrized by the phase angle $\xi(\pi L)$, mimics T noninvariance.

If the full nuclear Hamiltonian contains also a P-violating term, which could be the weak interaction V_P or a new interaction which simultaneously violates T invariance V_{PT}, then the initial and final nuclear states of the electromagnetic transition will contain admixtures of opposite parity states. For simplicity, we will assume that only the initial state $|a_0\rangle$ becomes mixed with a complete set of opposite parity states $|a_z\rangle$. Then to first order we have

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$$|a\rangle = |a_0\rangle + \sum_z \frac{|a_z\rangle}{E_0 - E_z} \langle a_z | V_{PT} + V_P | a_0 \rangle \ . \quad (1)$$

As a result, in addition to the "regular" electromagnetic multipoles (πL) with amplitudes $\gamma(\pi L, a \to b) = \gamma(\pi L, a_0 \to b)$, there will also exist "irregular" multipoles of the opposite parity $(\overline{\pi L})$ with amplitudes

$$\gamma(\overline{\pi L}, a \to b) = \sum_z \gamma(\overline{\pi L}, a_z \to b) \times \frac{1}{E_0 - E_z} \langle a_z | V_{PT} + V_P | a_0 \rangle \ . \quad (2)$$

By focusing on certain aspects of the emitted gamma-ray angular distribution, we may obtain experimental observables which depend upon the irregular mixing ratio $\varepsilon(\overline{\pi L}/\pi L) = \gamma(\overline{\pi L}, a \to b)/\gamma(\pi L, a \to b)$. Specifically, a measurement of the P-violating vector combination $(\langle \mathbf{J} \rangle \cdot \mathbf{k})$, where $\langle \mathbf{J} \rangle$ is the oriented nuclear spin and $\mathbf{k}$ is the momentum of a single emitted photon, yields a quantity which is proportional to the real part of the mixing ratio:

$$|\varepsilon| \cos\eta = \mathrm{Re} \sum_z \frac{\gamma(\overline{\pi L}, a_z \to b)}{\gamma(\pi L, a_0 \to b)} \frac{1}{E_0 - E_z} \langle a_z | V_P | a_0 \rangle \ . \quad (3)$$

Through this relationship the experimental observable may be directly related to the matrix elements of the weak interaction V_P and subsequently to the weak-coupling constants.

If, on the other hand, we measure the PT-violating distribution $(\langle \mathbf{J} \rangle \cdot \mathbf{k}_2)(\langle \mathbf{J} \rangle \cdot \mathbf{k}_1 \times \mathbf{k}_2)$ introduced earlier, then we obtain a quantity which is proportional to the imaginary part of the mixing ratio:

$$|\varepsilon| \sin\eta = \mathrm{Im} \sum_z \frac{\gamma(\overline{\pi L}, a_z \to b)}{\gamma(\pi L, a_0 \to b)} \frac{1}{E_0 - E_z} \left\{ \langle a_z | V_{PT} | a_0 \rangle + i \langle a_z | V_P | a_0 \rangle \left[\xi(\overline{\pi L}) - \xi(\pi L) \right] \right\} . \quad (4)$$

(We note that the matrix elements of V_{PT} and V_P are relatively pure imaginary.) Thus, as before, we may relate the experimental observable to the matrix elements of V_{PT} and ultimately to the as-yet unknown coupling constants of the PT interaction. Unfortunately, there is an added complication here in that the experimental observable cannot distinguish the true PT violation from the pure P violation coupled with the pseudo-T violation arising from the atomic final-state effects. We will not consider these final-state effects further; i.e., we will neglect the second braced term in Eq. (4). However, in a separate publication [21] we calculate these using a previously developed technique [9] and assess their ramifications for our present results.

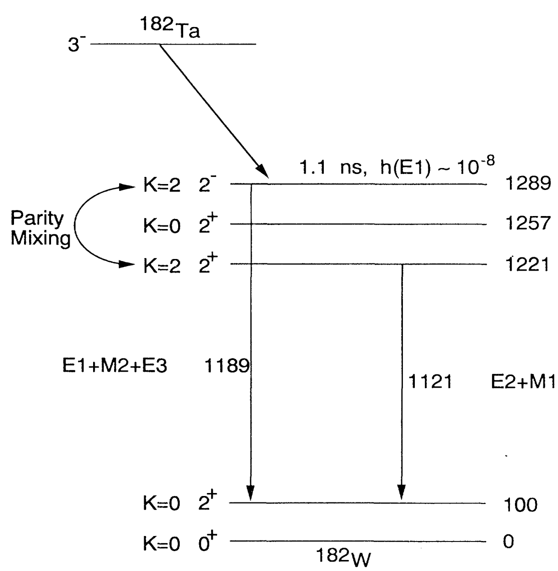

FIG. 1. Simplified decay scheme of ^{182}Ta indicating the levels and gamma rays involved in the parity mixing.

From these relationships it can be seen that in order to obtain optimal sensitivity of the experimentally observed mixing ratio to the matrix elements and coupling constants of the PT- and P-violating potentials, we require that the regular parity state possess a relatively hindered transition [small $\gamma(\pi L, a_0 \to b)$], while simultaneously there exists at least one close-lying state of the opposite parity and suitable quantum numbers which exhibits a relatively enhanced corresponding transition [small $(E_0 - E_z)$ and large $\gamma(\overline{\pi L}, a_z \to b)$]. Such a situation pertains in the deformed nucleus ^{182}W and, to a lesser extent, in several neighboring even-A rare-earth nuclei.

A simplified scheme of levels following the β decay of ^{182}Ta is shown in Fig. 1. Consider the mixed $(E1,M2,E3)$ 1189-keV transition between the first octupole vibrational bandhead $(|J^\pi K\rangle = |2^- 2\rangle)$ at 1289 keV and the first excited member of the ground-state rotational band $(|J^\pi K\rangle = |2^+ 0\rangle)$ at 100 keV. Owing partially to K forbiddedness, the leading order $E1$ component of this transition is hindered by a factor of almost 10^{-8} relative to the single-particle estimate [10].

In the vicinity of the octupole bandhead, there exists a $|2^+ 2\rangle$ gamma vibrational bandhead lying at 1221 keV which also exhibits a transition to the first excited state of the ground-state rotational band. However, in this case it is a strong collective $E2$ (the $M1$ component is K forbidden and therefore much smaller).

Since these two vibrational bandheads have the same spin J and projection K, they may be mixed by the PT- and P-violating scalar potentials in a K-allowed manner. If we focus on the 1189-keV transition from the octupole bandhead, then in addition to the regular $(E1,M2,E3)$ multipoles, we may also expect to observe the presence of

TABLE I. Spectroscopic properties of the 1189- and 1121-keV gamma transitions in ^{182}W. All values are taken from Ref. [10].

Transition (keV)	πL	$B(\pi L)$ (W.u.)	Mixing ratio
1189	$E1$	$1.7(1)\times 10^{-8}$	$\delta(E1/M2)=+2.3(3)$
	$M2$	$1.1(2)\times 10^{-2}$	
	$E3$	9.1(9)	$\delta(E3/M2)=-1.6(2)$
1121	$M1$	$2.7(11)\times 10^{-5}$	
	$E2$	8.0(3)	$\delta(E2/M1)=+30(5)$

irregular ($E2$) components, which we may associate with the 1121-keV transition, due to a small admixture of the gamma bandhead. Since both the $E2$ transition rate is almost 10^4 times faster than the combined regular transition rate and the energy separation of the two levels is only 68 keV, there will be an enhanced sensitivity of the irregular mixing ratio ε to the PT- and P-violating matrix elements. The relevant spectroscopic information for the 1189- and 1121-keV transitions is presented in Table I.

As we have demonstrated in I, the majority of the PT- and P-violating strength is concentrated in the matrix elements of states separated by approximately $1\hbar\omega\sim 7$ MeV. While the energy denominator does much to increase the contribution from close-lying states, they still constitute only a modest fraction of the total-energy-weighted strength. We must therefore consider contributions from the entire spectrum of $|2^+2\rangle$ quadrupole states, not only that of the lowest-lying one. In addition to the gamma bandhead at 1221 keV, there will also be some admixtures into the 1289-keV octupole state from the even closer-lying $|2^+0\rangle$ beta vibrational bandhead at 1257 keV. However, despite this, we expect that it will have a much smaller contribution to ε since both the corresponding $E2$ transition is comparatively weak and the matrix element mixing it into the octupole state is small, being second-order K forbidden. In what follows we consider the angular momentum projection K to be a good quantum number, thereby ignoring beta vibrational states.

Before turning to a theoretical evaluation of the irregular multipole mixing ratio ε in terms of the fundamental PT- and P-violating coupling constants, we will first discuss the nuclear orientation aspects of the ^{182}W system in order to determine how this parameter is related to the experimental observables.

III. NUCLEAR ORIENTATION

Nuclear parity violation may manifest itself as small terms of the form $(\langle \mathbf{J}\rangle\cdot\mathbf{k})$ which are present in the more dominant P- and T-conserving directional distribution of a single photon emitted from an oriented nucleus. In the notation of Steffen and Alder [11], this directional distribution $W(\theta)$ may be generally written as

$$W(\theta)=\sum_{\lambda} B_\lambda A_\lambda U_\lambda P_\lambda(\cos\theta) , \tag{5}$$

where θ is the angle between the axis of nuclear orientation $\langle \mathbf{J}\rangle$ and the direction of observation $\mathbf{k}$. The experimental P-violating observable O_P is then the forward-backward asymmetry defined by

$$O_P=[W(0)-W(\pi)]/[W(0)+W(\pi)] = \sum_{\lambda\ \mathrm{odd}} B_\lambda A_\lambda U_\lambda \Big/ \sum_{\lambda\ \mathrm{even}} B_\lambda A_\lambda U_\lambda . \tag{6}$$

This quantity is now directly proportional to the real part of the irregular mixing ratio, which for the present case we define as $|\varepsilon_{22}|e^{i\eta}=\gamma(\overline{E2})/\gamma(M2)$. This may be seen by writing the odd-rank angular distribution coefficients explicitly [11]:

$$A_{\lambda\ \mathrm{odd}}=\frac{2|\varepsilon_{22}|\cos\eta}{1+\delta_{12}^2+\varepsilon_{22}^2+\delta_{32}^2}[\delta_{12}F_\lambda(12)+F_\lambda(22) +\delta_{32}F_\lambda(23)] , \tag{7}$$

where the regular mixing ratios, which are purely real since we are neglecting atomic final-state effects, are defined to be $\delta_{12}=\gamma(E1)/\gamma(M2)$ and $\delta_{32}=\gamma(E3)/\gamma(M2)$ as given in Table I. Thus, for the decay of ^{182}Ta nuclei cryogenically oriented by the magnetic hyperfine interaction in iron, we find for the 1189-keV transition of interest that $O_P\simeq -0.37|\varepsilon_{22}|\cos\eta$ and $-0.03|\varepsilon_{22}|\cos\eta$ at temperatures of 10 and 20 mK, respectively. The rather poor experimental sensitivity to the P violation, especially at 20 mK, results from phase cancellations and may be somewhat improved by choosing an alternative observation axis θ.

We now consider the PT-violating term $(\langle \mathbf{J}\rangle\cdot\mathbf{k}_2)(\langle \mathbf{J}\rangle\cdot\mathbf{k}_1\times\mathbf{k}_2)$, which requires a measurement of the angular correlation of two successive gamma radiations emitted from an oriented source. The probability distribution for such an occurrence is given in the notation of Krane [12] by

$$W(\theta_1,\theta_2,\Phi)=\sum_{\lambda_0\lambda_1\lambda_2} B_{\lambda_0} A_{\lambda_1}^{\lambda_2\lambda_0} A_{\lambda_2} H_{\lambda_0\lambda_1\lambda_2}(\theta_1,\theta_2,\Phi) , \tag{8}$$

where θ_1 and θ_2 are the angles between the axis of nuclear orientation $\langle \mathbf{J}\rangle$ and the directions of observation of the two photons $\mathbf{k}_1$ and $\mathbf{k}_2$, which are themselves separated by an angle Φ. The experimental observable O_{PT}, defined as

$$O_{PT}=[W(+\theta_2)-W(-\theta_2)]/[W(+\theta_2)+W(-\theta_2)] = \sum_{\substack{\lambda_2\lambda_0\ \mathrm{even}\\ \lambda_1\ \mathrm{odd}}} B_{\lambda_0} A_{\lambda_1}^{\lambda_2\lambda_0} A_{\lambda_2} H_{\lambda_0\lambda_1\lambda_2}(+\theta_2) \Big/ \sum_{\substack{\lambda_2\lambda_0\ \mathrm{even}\\ \lambda_1\ \mathrm{even}}} B_{\lambda_0} A_{\lambda_1}^{\lambda_2\lambda_0} A_{\lambda_2} H_{\lambda_0\lambda_1\lambda_2}(+\theta_2) , \tag{9}$$

where $W(\pm\theta_2)=W(\pi/2,\pm\pi/4,\pi/2)$, is then seen to be directly proportional to the imaginary part of the irregular mixing ratio, since we have

$$A_{\lambda_1 \text{ odd}}^{\lambda_2\lambda_0 \text{ even}} = \frac{2i|\varepsilon_{22}|\sin\eta}{1+\delta_{12}^2+\varepsilon_{22}^2+\delta_{32}^2}[-\delta_{12}F_{\lambda_1}^{\lambda_2\lambda_0}(12) + \delta_{32}F_{\lambda_1}^{\lambda_2\lambda_0}(23)] . \tag{10}$$

In the case of ^{182}W, taking $\mathbf{k}_1$ to be the 1189-keV gamma ray and $\mathbf{k}_2$ to be the subsequent 100-keV ground-state transition, we find that $O_{PT}\simeq +0.25|\varepsilon_{22}|\sin\eta$ and $+0.20|\varepsilon_{22}|\sin\eta$ at temperatures of 10 and 20 mK, respectively. By comparison, the sensitivity of the previous ^{180}Hf experiment [6] was significantly worse, only $-0.03|\varepsilon_{22}|\sin\eta$ at 20 mK. Potentially, this is an important advantage for the ^{182}W system.

IV. SYMMETRY-VIOLATING MATRIX ELEMENTS

Having expressed the experimental observable in terms of the symmetry-violating irregular mixing ratio, we must relate this quantity theoretically to the matrix elements of the symmetry-violating potentials through Eqs. (3) and (4). With reference to the discussion in Sec. II, we do this with $|a_0\rangle$ representing the 1289-keV octupole $|2^-2\rangle$ vibrational state, $|a_z\rangle$ the spectrum of $|2^+2\rangle$ quadrupole states, and $|b\rangle$ the first excited member of the ground-state rotational band at 100 keV.

We model the quadrupole and octupole vibrational states within the framework of the quasiparticle random-phase approximation (QRPA). We restrict the $|2^+2\rangle$ levels to the complete set of one phonon states. Naturally, there exist other multiphonon $|2^+2\rangle$ states. However, the corresponding matrix elements of $V_{PT,P}$, being of higher order in the QRPA, are expected to be smaller than the ones which we include. In addition, as can be seen from Eq. (2), the admixed states $|a_z\rangle$ are required to undergo an electromagnetic transition to the state $|b\rangle$. Since in the present case this latter state is a member of the ground-state rotational band, the one-body electromagnetic operators are expected to connect predominantly to the one-phonon states that we consider here.

The single-particle states, created by the operator

$$a_m^\dagger = \sum_{Nj} A_{Nj}^m b_{Nj\Omega}^\dagger , \tag{11}$$

are derived from an axially symmetric, deformed Nilsson potential, where the creation operators of the spherical harmonic-oscillator states labeled by $(Nj\Omega)$ transform under time reversal according to the relation

$$b_{\widetilde{Nj\Omega}}^\dagger = (-1)^{j+N-\Omega} b_{Nj-\Omega}^\dagger , \tag{12}$$

and A_{Nj}^m are the usual expansion coefficients. The quasiparticle creation operators are then related to the particle operators via the Bogoliubov transformation

$$\alpha_m^\dagger = u_m a_m^\dagger - v_m a_{\tilde{m}} . \tag{13}$$

The single quasiparticle basis in our calculation encompasses all such states lying within 9 MeV ($\simeq 1.2\hbar\omega$) of the Fermi level.

The QRPA phonons

$$Q_{LK}^{a\dagger} = \sum_{m<n} X_{mn}^a \alpha_m^\dagger \alpha_n^\dagger - Y_{mn}^a \alpha_{\tilde{n}} \alpha_{\tilde{m}} , \tag{14}$$

of multipolarity L and angular momentum projection K, are calculated using a separable particle-hole force of the form

$$V_{msnr}^L = -\kappa_{LK}\langle m|r^L Y_K^L|n\rangle\langle r|r^L Y_K^L|s\rangle^* . \tag{15}$$

The coefficients κ_{22} and κ_{32} were deduced by fitting to the experimental collective quadrupole gamma and octupole vibrational bandhead energies in a number of nuclei of the $A=152-190$ mass region. We thus obtained the values $\kappa_{22}=205A^{-7/3}$ MeV fm^{-4} and $\kappa_{32}=121A^{-3}$ MeV fm^{-6}, which are consistent with previous works [13].

Having solved the QRPA equations, we now wish to calculate the $|2^-2\rangle$ to $|2^+2\rangle$ mixing matrix elements. Unless otherwise stated, in what follows we refer explicitly to the case of the isovector PT-violating potential $V_{PT}^{(I=1)}$ [5] as given in I. The P-violating potential V_P may be treated in a similar manner. Thus we require

$$\langle a_0|V_{PT}|a_z\rangle = \langle \text{g.s.}|Q_{32}^{a_0} \sum_{ijkl} \bar{V}_{ijkl} a_i^\dagger a_j^\dagger a_l a_k Q_{22}^{a_z\dagger}|\text{g.s.}\rangle , \tag{16}$$

the mixing matrix elements of the two-body PT- (or P-) violating meson-exchange potential between the lowest $|2^-2\rangle = Q_{32}^{a_0\dagger}|\text{g.s.}\rangle$ octupole state and the spectrum of $|2^+2\rangle = Q_{22}^{a_z\dagger}|\text{g.s.}\rangle$ quadrupole states labeled z, where

$$\bar{V}_{ijkl} = \langle ij|\Gamma V_{PT}^{(I=1)}\Gamma|kl\rangle - \langle ij|\Gamma V_{PT}^{(I=1)}\Gamma|lk\rangle . \tag{17}$$

Here the short-range nucleon-nucleon repulsion is explictly included with the introduction of the function Γ, where

$$\Gamma = 1 - e^{-\gamma_1 r^2}(1-\gamma_2 r^2) , \tag{18}$$

with parameters $\gamma_1 = 1.1$ fm^{-2} and $\gamma_2 = 0.68$ fm^{-2} [14]. The antisymmetrized matrix elements of Eq. (17) were evaluated in a coupled (to total angular momentum J and isospin T) spherical harmonic-oscillator representation $\bar{V}_{1234}^{JT_{12}T_{34}}$, where 1,2,3,4 denote states labeled (Nj), according to

$$\bar{V}_{ijkl} = \sum_{1,2,3,4,J,T_{12},T_{34}} A_1^i A_2^j A_3^k A_4^l (j_1\Omega_1 j_2\Omega_2|J\Omega)(j_3\Omega_3 j_4\Omega_4|J\Omega)(T_1T_{1z}T_2T_{2z}|T_{12}T_z)(T_3T_{3z}T_4T_{4z}|T_{34}T_z)\bar{V}_{1234}^{JT_{12}T_{34}} . \tag{19}$$

Computational restrictions required that this spherical basis be confined to the $N=4,5,6$ oscillator shells.

In evaluating Eq. (16), one may separate the interaction into two more or less distinct parts. The former is a one-body mean-field interaction of the form

$$\sum_{mrpq} \left[X_{mq}^{a_z} X_{rq}^{a_0} + c Y_{mq}^{a_z} Y_{rq}^{a_0} \right] \left\{ \overline{V}_{mprp}(v_p v_p)(u_m u_r + c v_m v_r) + \tfrac{1}{2} \overline{V}_{\bar{m} r p \bar{p}}(u_p v_p)(v_m u_r - c u_m v_r) \right\} , \tag{20}$$

where the dichotomous quantum number c is related to the time-reversal properties of the interaction ($c=+1$ and -1 for PT and P, respectively). The first term in the braces corresponds to the usual effective one-body interaction, which becomes clear if we make the substitution

$$U_{mr} = \sum_p \overline{V}_{mprp}(v_p v_p) . \tag{21}$$

An explicit evaluation of U_{mr} would require a summation over all occupied, deformed core states p. However, in the spherical limit the summation requires only terms of the form $\overline{V}_{1232}^{JT_{12}T_{32}}$. The reduced number of matrix elements required in the presence of the dummy index allows an extension of the basis to include the summation over all core states. Such a procedure has already been carried out in I with the result that, to a very good approximation, we can replace the reduced two-body interaction $\sum_2 \overline{V}_{1232}$ with the effective one-body potential $F(m_\pi R) G(m_\pi, \gamma_1, \gamma_2) U_{PT,\mathrm{asymp}}^{(I=1)}$. (An analogous expression for the P-violating potential has been considered by Adelberger and Haxton [15].) In the deformed case we may therefore make the convenient substitution

$$U_{mr} \simeq \sum_{12} A_1^m A_2^r \langle 1 | F(m_\pi R) G(m_\pi, \gamma_1, \gamma_2) U_{PT,\mathrm{asymp}}^{(I=1)\,'} | 2 \rangle , \tag{22}$$

where the spherical states 1,2 and therefore the deformed single-particle states m,r are now no longer limited to any given shell. By doing so we make two approximations regarding the nature of the core. First, we make an equal-weight core summation up to a sharp Fermi surface. Second, we assume that the core consists of filled spherical subshells. In particular, we model the core of $^{182}_{74}$W by summing protons and neutrons up to the top of the fourth ($Z=70$) and fifth ($N=112$) oscillator shells, respectively.

The second term in braces in the one-body interaction (20) also receives contributions in the core summation from elements outside the present basis. Being a pure pairing term ($u_p v_p \to 0$ in the no-pairing limit), it is not calculable by an analytic reduction of the form used above. However, since the pairing factor increasingly suppresses the effect of states lying farther from the Fermi level, the largest contributions to the summation lie within the $N=4,5,6$ oscillator shells ($u_p v_p \simeq 0.05$ at 9 MeV from the Fermi level). With this truncation we may evaluate this term explicitly, finding that it is typically 10 times smaller than the previous term.

Although the one-body mean-field interaction is usually the most important component of a two-body mixing matrix element, there are two factors which serve to reduce its contribution in the present case. First, a one-body interaction can only scatter one out of the two quasiparticles in the QRPA bosons. Therefore, we require that the quadrupole and octupole phonons have a single-particle label in common, q in Eq. (20). In the case of ^{182}W, we find that the overlap between the lowest quadrupole and octupole vibrational states with a common label is as little as 6%. Second, as illustrated in I, the j-conserving nature of U_{mr} implies that the majority of the PT-, and also P-, violating strength occurs for states separated by $\sim 1\hbar\omega$. For small energy separations one finds only a tiny proportion of the total strength due solely to j mixing in the spin-orbit split intruder subshell. On the other hand, a true two-body interaction may scatter both quasiparticles so that all the components of the QRPA bosons may contribute to the mixing matrix element. Also, while the scalar nature of the interaction still demands angular momentum conservation, the condition is now much less restrictive since it refers now only to the total angular momentum to which two states are coupled [cf. Eq. (19)]. Thus it is possible that the residual two-body component of Eq. (16) competes with and perhaps even dominates over the one-body mean field. Therefore, we calculate also the irreducible two-body matrix element:

$$\begin{aligned} \sum_{mnrs} (X_{mn}^{a_z} X_{rs}^{a_0} - c Y_{mn}^{a_z} Y_{rs}^{a_0}) [\overline{V}_{m\bar{r}s\bar{n}}(u_m v_n v_r u_s - c v_m u_n u_r v_s) - \overline{V}_{m\bar{s}r\bar{n}}(u_m v_n u_r v_s - c v_m u_n v_r u_s) \\ + \overline{V}_{mnrs}(u_m u_n u_r u_s - c v_m v_n v_r v_s)] \\ -(X_{mn}^{a_z} Y_{rs}^{a_0} - c Y_{mn}^{a_z} X_{rs}^{a_0}) [\overline{V}_{m\bar{s}r\bar{n}}(u_m v_n v_r u_s - c v_m u_n u_r v_s) - \overline{V}_{m\bar{r}s\bar{n}}(u_m v_n u_r v_s - c v_m u_n v_r u_s) \\ + \overline{V}_{mnrs}(u_m u_n v_r v_s - c v_m v_n u_r u_s)] . \end{aligned} \tag{23}$$

As before, we restrict the $\overline{V}_{ijkl}$ to the $N=4,5,6$ oscillator shells. Further, to reduce the number of summations, we confined the indices r,s of the octupole phonon to those for which either of the RPA amplitudes $X_{rs}^{a_0}, Y_{rs}^{a_0} \rangle \pm 0.04$. This reduced the basis space of the octupole phonon by a factor of 7, while still retaining 96% by intensity of the wave function.

Finally, an important test of the validity of our results is the prediction of the degree of P violation. The factors which led us to consider both the one-body mean field and the irreducible two-body interaction also apply here. The appropriate two-body P-violating potential is given in Ref. [15]. However, we did not attempt a full calculation of the two-body matrix elements. Instead, we consider only the one-body mean-field interaction analogous to Eq. (22). The numerical results will be shown and discussed in the next section.

V. NUMERICAL RESULTS

Before proceeding to the calculation of the irregular mixing ratio in ^{182}W, we first discuss the relative magnitudes of the matrix elements which are involved. Since one would expect the matrix elements $\langle a_0|V_{PT}+V_P|a_1\rangle$ between the lowest octupole and gamma vibrational states to be the most important in the perturbative summation, we present them in Table II for several rare-earth nuclei. In column 2 we show the matrix elements of the one-body PT-violating mean field, corresponding to the first braced term in Eq. (20). As stated above, the "pure pairing term," corresponding to the second braced term in Eq. (20), is considerably smaller, being about 10% of the value given in column 2. In column 3 we show the irreducible two-body matrix element of Eq. (23). (Because of the sizable computer time involved, we evaluated this matrix element only for ^{182}W.) The entries in columns 2 and 3 are given for $\bar{g}_{\pi NN}^{(I=1)}=3\times10^{-10}$, the approximate upper limit set by the neutron electric dipole moment for the isovector PT-violating interaction. Next, for comparison, we show in columns 4 and 5 the matrix elements of the P-violating one-body potential. The values of the associated coupling constants remain uncertain, and thus we use two different sets of values, those of Desplanques, Donoghue, and Holstein [16] and those of Adelberger [17] (as quoted by Herczeg [5], who considers only the more important ρ-meson-exchange component). In both cases the effects of the short-range nucleon-nucleon correlations were included by reducing the pion- and ρ-meson-exchange terms by 30% and 70%, respectively [15].

It is of interest to compare the matrix elements of the PT- and P-violating potentials. If their ratio for different nuclei and different states is approximately constant, then, as suggested by Herczeg [5], it would be possible to use the experimentally determined degree of P violation to estimate how large the PT violation will be for a given PT coupling constant. To test this hypothesis we show in column 6 the dimensionless parameter $\kappa^{(1)}$ defined as

$$\kappa^{(1)}=\left|\frac{\langle i|U_{PT}|j\rangle/\bar{g}_{\pi NN}^{(I=1)}}{\langle i|U_P|j\rangle/\bar{g}_{\rho NN}}\right| . \quad (24)$$

This is simply the ratio of the one-body PT and P matrix elements of columns 2 and 5 with the symmetry-violating coupling constants removed.

The entries in Table II describe the mixing of the lowest octupole and gamma vibrational states. They can be compared with the average matrix element $\overline{\langle a_0|V_{PT}|a_z\rangle}$, where the averaging is then over all 506 $|a_z\rangle=|2^+2\rangle$ states present in our calculation. The corresponding mean one-body matrix element is 17 μeV with a variance of (51 μeV)^2. The pure pairing one-body term mentioned earlier has a mean value of 1 μeV and variance 2 (μeV)^2, and so it remains small throughout. The irreducible two-body matrix element in Table II represents about 30% of the total PT-violating mixing between the lowest octupole and gamma vibrational states. More generally, we find that the mean two-body matrix element is 5 μeV with a variance of 9 (μeV)^2, continuing the same trend. Despite this smaller average value, the two-body term exceeded the one-body term 25% of the time. Since the relative sign of these two competing terms is random, the two-body cannot be neglected if one is to correctly account for the possibility of phase cancellations.

To illustrate the distribution of the total (one-body mean-field plus irreducible two-body) PT-violating matrix elements, we plot in Fig. 2 the frequency of such terms occurring with a given value. The mean matrix element is 19 μeV with variance 53 (μeV)^2. The large variance is due to a small number of one-body matrix elements with extremely large values (up to 950 μeV). As expected from the one-body angular momentum selection rules, these all lie at excitation energies approaching $1\hbar\omega$.

We now turn to the calculation of the irregular mixing ratio ε_{22} of the 1189-keV transition in ^{182}W using Eqs. (3) and (4). The regular $M2$ transition amplitude $\gamma(M2,a_0\rightarrow b)$ is taken from experiment. Also, for the term involving the lowest gamma vibrational state, we use the experimental irregular transition amplitude $\gamma(\overline{E2},a_1\rightarrow b)$ and energy splitting. All other values are taken from the QRPA calculations. For the PT-violating term, we find

$$|\varepsilon_{22}|\sin\eta=200\bar{g}_{\pi NN}^{(I=1)} . \quad (25)$$

TABLE II. PT- and P-violating mixing matrix elements between the lowest octupole and gamma vibrational states in several rare-earth nuclei.

	U_{PT} (μeV)	V_{PT} (μeV)	U_P (meV)[a]	U_P (meV)[b]	$\kappa^{(1)}$
^{156}Gd	−7.0		+29.7	+59.3	0.98
^{162}Dy	+3.9		+22.0	+47.8	0.68
^{174}Yb	−11.9		−14.9	−40.5	2.44
^{176}Hf	−5.1		−4.9	−31.4	1.35
^{182}W	−10.7	−4.3	−10.0	−13.6	6.56

[a]Calculated with the interaction constants of Ref. [16].
[b]Calculated with the interaction constants of Ref. [17].

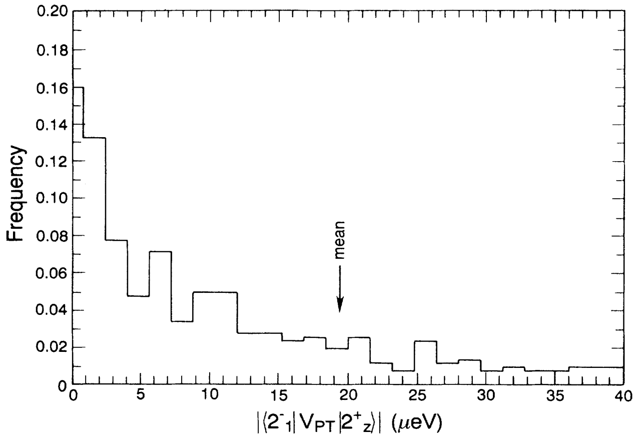

FIG. 2. Frequency of the total PT-violating matrix elements $\langle a_0|V_{PT}|a_z\rangle$ for ^{182}W occurring with a given value. The mean and variance of the distribution are 19 μeV and 53 $(\mu\text{eV})^2$, respectively.

The limit on the neutron electric dipole moment therefore implies that $|\varepsilon_{22}|\sin\eta \lesssim 6\times10^{-8}$. The factor 200 in Eq. (25) represents "nuclear enhancement" in the present case.

For the (one-body mean-field contribution to the) P-violating mixing ratio, we find for the two sets of coupling constants that

$$|\varepsilon_{22}|\cos\eta = \begin{cases} 4\times10^{-5} & (\text{Ref. [16]}) , \\ 5\times10^{-5} & (\text{Ref. [17]}) . \end{cases} \qquad (26)$$

In I we discussed the energy distribution of the PT-violating strength. We concluded that while the perturbative energy denominator did much to enhance that part of the strength at low excitation energies, the majority of the energy-weighted strength lay at energies in the region of $1\hbar\omega$. In the present case there is an additional enhancement of the low-energy strength due to the collectivity of their vibrational transitions. As a result, we find that the admixture with the lowest gamma vibrational state accounts for 80% of the maximum possible contribution to the irregular mixing ratio (which we define by summing absolute values). If we perform the summations of Eq. (3) and (4) with the $E2$ transition amplitudes removed, we find that the lowest gamma vibrational state constitutes only 11% to the maximum possible value of the irregular mixing ratio. Moreover, there is now a significant cancellation between the various terms in the summation. We therefore conclude that when selecting possible cases for studying PT and P violations, one should pay as much attention to the comparative strengths of the irregular and regular transitions as to the size of the energy denominator, since the predominance of a single perturbative admixture allows not only a simple two-level mixing calculation, but also avoids cancellations.

In passing, we compare this situation with the one which may pertain in the case of P-conserving T violation. Here the most general one-body potential can be written as $U_T=(\mathbf{p}\cdot\hat{\mathbf{r}})[f(r)+g(r)\tau_z]+\text{H.c.}$ [18]. If the radial functions $f(r),g(r)$ are at most linear, as assumed in Refs. [19] and [20], then this may be recast in the form $U_T=[H_0,h(r,\tau_z)]$, where H_0 is the strong nuclear Hamiltonian which is assumed to contain no velocity-dependent terms other than a possible spin-orbit interaction. This form of U_T implies that the matrix element between any two states scales as their energy separation. This dependence exactly compensates the energy denominators in the perturbative expansion with the result that we would expect no enhancement in the observed T-violating effect due to close-lying levels. Since the selection rules of U_T ($\Delta j=0$, no parity change) put the vast majority of the T-violating strength within a single oscillator shell, the lack of energy weighting may not be too critical, although it may lead to convergence problems in a limited basis space. Experimentally, if such a potential were valid, this would put the onus entirely on the relative irregular and regular transition rates when selecting favorable candidate nuclei for study.

Finally, we discuss the behavior of the parameter $\kappa^{(1)}$ as defined in Eq. (24). This has two important characteristics. First, the magnitude of $\kappa^{(1)}$ determines the relative sensitivity of the PT- and P-violating matrix elements to their respective coupling constants. On the basis of simple argument, Herczeg has indicated that a value of $\kappa^{(1)}\simeq31$ may be expected [5]. [We included corrections for a numerical factor ($\times0.5$) in his PT-violating potential and the effects of finite pion-range ($\times0.7$) and short-range correlations ($\alpha\simeq3.4$) as discussed in I.] Since this is considerably larger than unity, the PT-violating matrix elements may therefore be expected to exhibit a relatively enhanced sensitivity to the coupling constant $\bar{g}^{(I=1)}_{\pi NN}$ as compared to the P-violating case. Second, if $\kappa^{(1)}$ is fairly well defined within a given range of values, then a degree of proportionality may be established between the matrix elements of the PT- and P-violating potentials. As indicated earlier, in such a case the real and imaginary parts of the irregular mixing ratio [Eqs. (3) and (4)] are related. Since the experimental PT- and P-violating observables O_{PT} and O_P are directly proportional to $|\varepsilon|\sin\eta$ and $|\varepsilon|\cos\eta$, we may write

$$\frac{O_{PT}}{O_P}\sim\tan\eta\simeq\kappa^{(1)}\frac{\bar{g}^{(I=1)}_{\pi NN}}{\bar{g}_{\rho NN}}+[\xi(\overline{\pi L})-\xi(\pi L)] , \qquad (27)$$

where $\bar{g}_{\rho NN}\simeq2.5\times10^{-6}$ is the weak ρ-nucleon coupling [17]. (We follow Herczeg and assume that the most important contribution to the P-violating interaction comes from ρ-meson exchange.) To judge the constancy of $\kappa^{(1)}$, we evaluate its frequency distribution using the set of one-body mean-field matrix elements $\langle a_0|U_{PT,P}|a_z\rangle$ of all the nuclei shown in Table II. In fact since we are dealing with ratios, we consider logarithmic values which are plotted in Fig. 3. From this distribution we find a mean and variance for $\log_{10}|\kappa^{(1)}|$ of $+0.8$ and $(0.7)^2$, respectively. Typically therefore $\kappa^{(1)}$ is notably lower than the earlier estimate.

In general, the variation in $\kappa^{(1)}$ would be too great to allow any form of equality between the perturbative summations of Eqs. (3) and (4). However, if one term in the summation is expected to dominate, such as in the

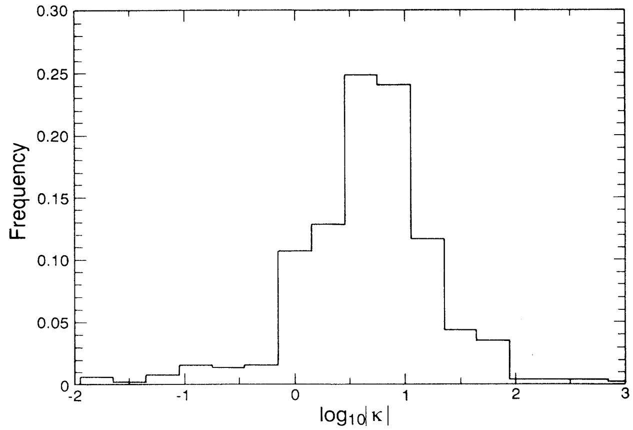

FIG. 3. Frequency of the parameter $\log_{10}|\kappa^{(1)}|$ as defined in Eq. (27) occurring with a given value. The mean and variance of $\log_{10}|\kappa^{(1)}|$ are $+0.8$ and $(0.7)^2$, respectively.

present case, then Eq. (27) may be used to obtain a rough estimate of the PT-violating effects of a given coupling constant. In particular, we find that in order to compete with the limit on $\bar{g}_{\pi NN}^{(I=1)}$ set by the neutron electric dipole moment, then we must have that $\tan\eta \sim (1-40)\times 10^{-4}$. That is, one must measure the PT-violating mixing ratio at least three orders of magnitude more accurately than the value determined for the P-violating part. For comparison, the limit set by the ^{180}Hf experiment was only $\tan\eta < 1.3$ [6].

When applying the quantity $\kappa^{(1)}$ to a given case, one should bear in mind its limitations. For example, the one-body potentials U_{PT} and U_P exhibit different isospin properties. Thus, when taking matrix elements between states whose wave functions receive contributions from both protons and neutrons, there may be additional variations of $\kappa^{(1)}$. (However, the values of $\kappa^{(1)}$ given in Table II pertain to just such states and we find no great deviations from the expected range.)

CONCLUSIONS

To conclude, we have calculated the real and imaginary parts of the $(\overline{E2},M2)$ mixing ratio of the 1189-keV transition in ^{182}W and find that $|\varepsilon_{22}|\cos\eta \simeq 5\times 10^{-5}$ and $|\varepsilon_{22}|\sin\eta \simeq 200\bar{g}_{\pi NN}^{(I=1)}$, respectively. For the case of the low-temperature nuclear orientation experiments outlined in Sec. III, one would therefore expect a P-violating forward-backward gamma-ray asymmetry of $O_P \simeq 2\times 10^{-5}$ and a PT-violating gamma-gamma correlation effect of $O_{PT} \simeq 50\bar{g}_{\pi NN}^{(I=1)}$ at 10 mK. In order to compete with the limit on the isovector PT-violating coupling constant set by measurements of the neutron electric dipole moment, one therefore needs to measure a PT-violating asymmetry to the level of $\sim 10^{-8}$.

We found that the contributions of the effective one-body PT- and P-violating mean fields to these mixing ratios were significantly suppressed due to a poor overlap between the lowest quadrupole and octupole phonons. This reduction, by a factor of more than 10, requires that the irreducible two-body contribution also be considered. While these are somewhat smaller than the one-body matrix elements, both are necessary for a reliable calculation. Provided that the one-body suppression is large enough, this will be generally true of all calculations in even-even A and odd-odd A nuclei, which involve the mixing of two-quasiparticle states. By contrast, in principle, a one-body operator may always link the one-quasiparticle states of odd-A nuclei. The lack of suppression means that not only can the one-body terms be considered exclusively, but also one can expect larger PT- and P-violating matrix elements. Odd-A nuclei therefore seem to offer the best opportunity for studying symmetry-violating effects in nuclei.

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