Relativistic σ-ω mean-field theory for hyperons from a quark model

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We suggest a phenomenological description of hypernuclear σ-ω Dirac mean-field theory, where the meson-baryon vertices are based on the quark model with relativistic corrections. While the model is shown to be in agreement with the σ-ω model for regular nuclei, large tensor ωYY couplings are derived for ω-hyperon vertices. This coupling, which is negligible for nucleons, is capable of resolving the problem of the small spin-orbit interaction in Λ hypernuclei within the Dirac approach, as was recently shown independently by Jennings. Our model explains the available data. It can also be the basis for more systematic theoretical studies. Predictions are thus presented for Σ and Ξ hypernuclei. Further improvements of the model are discussed.

I. INTRODUCTION

A potentially fruitful area of research involves the application to hypernuclear physics of the mean-field σ-ω model [1–3], based on the Dirac equation with strong scalar and vector potentials. Hypernuclei, in which a hyperon such as the Λ or the Σ replaces a nucleon, add another dimension to weak, electromagnetic or hadronic probes of nuclear dynamics. Hyperons carry nonzero strangeness that distinguishes them from nucleons. Thus they are hardly or not Pauli excluded from orbitals occupied by nucleons and can penetrate dense nuclear matter inaccessible to other hadronic probes. We can use hypernuclei to explore the role of hyperons in hadronic forces or weak interactions in nuclei. We may also use them to measure changes of electromagnetic properties of hyperons by the nuclear environment. Some of these problems have already been addressed elsewhere. Weak interactions in hypernuclei have been recently reviewed [4], the issue of hypernuclear electromagnetic moments was studied [5], and a review of relativistic aspects in hypernuclear physics is in progress [6], which led to the present work.

Here we deal with the hypernuclear spin-orbit force of the σ-ω model, where we confirm that to explain the small Λ-hypernuclear spin-orbit interaction [6,7] a strong $\omega\Lambda\Lambda$ tensor vertex is required [8,10] and that this feature arises naturally in a quark model developed some time ago [9] without disturbing the successful applications of the σ-ω model to ordinary nuclei [1–3]. Predictions for Σ and Ξ hypernuclei are provided. After this work was completed we learned from A. Gal that similar results for the Λ have been independently obtained by Jennings [10]. The present work goes beyond the results of Ref. [10], however. Since our quark model provides tensor couplings, we are able to make definite predictions for Σ and Ξ hypernuclei in addition to explaining the available data for Λ hypernuclei. The model can also be used for more detailed and refined theoretical studies inasmuch as it predicts also scalar meson couplings. We also present new theoretical insight into the problem of hypernuclear spin-orbit interactions.

II. HYPERON-NUCLEUS POTENTIALS AND THE MEAN-FIELD THEORY (MFT)

In the MFT, the meson-field operators are replaced by their expectation values which are classical fields; thus ϕ_0 is the ground-state (g.s.) expectation value of the scalar field ϕ from the σ meson and $V^\mu=(V_0,\mathbf{V})$ is the vector mean field from the ω meson. The MFT Lagrangian density for nucleon and hyperon (Y) in the presence of these fields is

$$L_{\mathrm{MFT}}=\bar{\psi}_N[\gamma_\mu(i\partial^\mu-g_v^N V^\mu)-(M_N-g_s^N\phi_0)]\psi_N +\bar{\psi}_Y[\gamma_\mu(i\partial^\mu-g_v^Y V^\mu)-(M_Y-g_s^Y\phi_0)]\psi_Y +\text{purely mesonic terms}\ , \tag{1}$$

which provides the simplest hypernuclear σ-ω formalism. While it is satisfactory for the nucleon to include only the vector (γ_μ) ω-baryon couplings [1–3], we will follow Noble [8,11] in arguing that a strong $\omega\Lambda\Lambda$ tensor coupling is required for Λ hypernuclei, although for somewhat different reasons. In fact, a tensor vertex arises naturally in the quark model [9], while leaving undisturbed the satisfactory status of the nucleon. Predictions will be provided for the Σ and Ξ, where no data are available. Note that for most applications in spherical nuclei the mean-field expectation value $V_\mu=\delta_{\mu 0}V_0$. An exception is discussed, e.g., in Ref. [5].

The above pure σ-ω (scalar+vector) model works well for regular nuclei [1–3], where one typically finds for medium nuclei that

$$g_v^N V_0(0)\simeq +345\ \mathrm{MeV}\ , \tag{2a}$$

$$-g_s^N\phi_0(0)\simeq -420\ \mathrm{MeV}\ . \tag{2b}$$

These values reproduce the standard empirical low-energy ($E=M_N$) shell-model parameters

$$V_c^N\simeq -53\ \mathrm{MeV}\ , \tag{3a}$$

$$V_{\mathrm{s.o.}}^N\simeq 17\ \mathrm{MeV}\ . \tag{3b}$$

In principle, the vector coupling constant introduced for the Dirac MFT above should be identical with the quark model constants $g_{\omega BB}$ of Secs. III and IV for $B=N$, Λ, Σ, Ξ. (The same applies also to the scalar coupling constants.) However, there is a common practice of assuming total freedom to adjust coupling constants in order to fit a given set of nuclear data, making the model uncomfortably empirical. We believe that the underlying physics demands that all possible constraints imposed by our best present knowledge be applied to the theoretical calculations and predictions. This is admittedly a rather austere program that is not generally adhered to. For this reason we keep separate notation for g_V^B and $g_{\omega BB}$ realizing that a more stringent goal of Dirac nuclear theory would actually require a single coupling constant [12]. Such a refined theory is, of course, outside the scope of the present work.

From the phenomenology of hypernuclear binding energies and level splittings ΛA potential strengths are found that are considerably smaller than the corresponding NA values [5,13].

The naive quark model prediction for the σ-ω meson-Λ coupling constants is

$$g^{\Lambda}\simeq \tfrac{2}{3}g^{N}\,, \tag{4}$$

when the s quark is a spectator. With these relations, the central and spin-orbit potentials are predicted to be much larger than the empirical ones [7], by up to a factor of 4. Thus for consistency with empirics, several authors have assumed in Eq. (4) a ratio in the range 0.2–0.4 [5,13]. We know of no theoretical justification for such a small ratio.

In contrast to the nonrelativistic shell model for ordinary nuclei, the σ-ω model explains the large spin-orbit and shallow central potentials naturally in its nonrelativistic limit. For hypernuclei, however, the nonrelativistic shell model seems to be a perfectly satisfactory starting point with its small Λ-nuclear spin-orbit interaction. In this case, the pure scalar+vector Dirac model yields a much too large spin-orbit interaction. However, the $\omega\Lambda\Lambda$ tensor vertex predicted from a quark model transforms the complicated Λ-nuclear single-particle Dirac equation with scalar, vector, and tensor potentials into a simple nonrelativistic shell-model wave equation with just a shallow central potential in the nonrelativistic limit [10]. In a hypernucleus one deals with nucleons and the hyperon simultaneously, for both of which the Dirac approach provides a natural framework without ad hoc forces. Thus, we consider the Λ case as a success of the Dirac MFT, because such a consistent and natural description for both nucleons and a hyperon does not seem to exist for the nonrelativistic model. Therefore, we shall argue that the Dirac theory can be used to predict results for systems such as Σ and Ξ hypernuclei. Although no good data exist for Σ hypernuclei to provide information on the Σ-hypernuclear spin-orbit coupling, predictions will be given in Sec. V for Σ as well as Ξ hypernuclei. Such predictions can be provided here based on the quark model used. Moreover, our model can serve as a basis for a systematic theoretical study in an attempt to improve upon the purely phenomenological level.

III. QUARK MODEL FOR THE NUCLEON

Effective meson-field theoretical approaches have been successful, but not always consistent and hardly predictive, in describing baryon-baryon and meson-baryon interactions [14]. At the fundamental level, the internal structure of hadrons is described by quantum chromodynamics (QCD), the SU(3) color gauge theory of elementary quarks and gluons as massless gauge bosons [15]. Only at short distances and high energy is the running quark-gluon coupling of QCD small enough for perturbation theory to apply, while at large distances QCD-motivated (quark and other) models are used to understand the nonperturbative aspects of QCD. Such models incorporate color confinement along with the fundamental symmetries in their description of hadron structure and dynamics. We view such models as useful interpolation between QCD and the hadronic picture. We will see that the hypernuclear spin-orbit problem can be conveniently addressed using a quark model.

In the following we point out how the hypernuclear spin-orbit interaction problem can benefit from a quark model that goes beyond the broken SU(3) flavor symmetry by extrapolating the chiral-invariant vector coupling of the QCD Lagrangian to long distances starting from a Fierz transformation of its quark action [16]. A bosonization of QCD based on this method is now more generally recognized as a key to the mesonic dynamics of QCD at longer distances and low energy [17]. The model includes relativistic effects of the interacting quark in terms of its small Dirac wave function from some confinement model. Its bag model version [9] is written in the coordinate representation as

$$\bar{q}(\mathbf{r})=\chi^{\dagger}(g(r),i\sigma\cdot\hat{\mathbf{r}}f(r))\,. \tag{5}$$

The following overlap integrals will be useful:

$$F_0^{\pm}(q^2)=4\pi\int_0^R dr\, r^2[g^2(r)\pm f^2(r)]j_0(qr)\,, \tag{6a}$$

$$F_1(q^2)=16\pi\frac{M}{q}\int_0^R dr\, r^2 g(r)f(r)j_1(qr)\,, \tag{6b}$$

where $q^{\mu}=(q_0\cong 0,\mathbf{q})$ $(q=|\mathbf{q}|, q^2\cong -\mathbf{q}^2)$ is the momentum transfer, R the bag radius, and M the baryon mass of the relevant meson-baryon vertex. A constituent quark model version has also been developed in previous applications of the model to a variety of problems [16,18].

In the ensuring discussion, results will be required for nucleons as well as hyperons. Starting with the nucleon, its vector-meson vertex is given by

$$\begin{aligned}\langle\gamma^{\mu}(\tau)^T\rangle_N &=\langle N(p')[3]\tfrac12,\tfrac12|\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\bar{q}(\mathbf{r})\gamma^{\mu}(\tau)^T q(\mathbf{r})|N(p)[3]\tfrac12,\tfrac12\rangle\\ &=(1-q^2/4M_N^2)^{-1}\bar{u}_N(p')[(x_TF_0^+ +q^2y_TF_1/8M_N^2)\gamma^{\mu}-(x_TF_0^+-\tfrac12 y_TF_1)i\sigma^{\mu\nu}q_{\nu}/2M_N](\tau_N)^Tu_N(p)\\ &=\bar{u}_N(p')(F_{1T}\gamma^{\mu}+iF_{2T}\sigma^{\mu\nu}q_{\nu}/2M_N)(\tau_N)^Tu_N(p)\end{aligned} \tag{7}$$

in the bag model [9], and

$$\langle \gamma^{\mu}(\tau)^T \rangle_N = (1-q^2/4M_N^2)^{-1}\bar{u}_N(p')[(\Gamma_{0T}-q^2\Gamma_T/2M_N)\gamma^{\mu}-(\Gamma_{0T}-2M_N\Gamma_T)i\sigma^{\mu\nu}q_\nu/2M_N](\tau_N)^T u_N(p) \quad (8)$$

in the constituent quark model [18]. In Eq. (7)

$$x_T = 3^{1-T}, \quad y_T = -2(5/3)^T , \quad (9)$$

while for Eq. (8)

$$\Gamma_{0T} = (1+\alpha/4m_q^2)^{-1}3^{1-T}[1+\alpha(1+q^2/9\alpha)/4m_q^2] \times \exp(q^2/6\alpha) ,$$
$$\Gamma_T = (1+\alpha/4m_q^2)^{-1}(5/3)^T \exp(q^2/6\alpha)/3m_q . \quad (10)$$

In Eqs. (7) and (8) τ (τ_N) is the quark (nucleon) isospin operator, and $T=0$ or 1; obviously $T=0$ for the isoscalar ω meson. The ω meson couples via $ig\gamma^{\mu}$ to each of the three valence quarks, so its coupling to the nucleon is simply via $ig\langle \gamma^{\mu} \rangle_N$. The ω-nucleon vector coupling constant is

$$g_{\omega NN} = gF_{10}(q^2=0) = gx_0 = 3g , \quad \text{in the bag model} ; \quad (11a)$$

$$g_{\omega NN} = g\Gamma_{00}(q^2=0) = 3g , \quad \text{in the constituent quark model} . \quad (11b)$$

The ω-nucleon tensor coupling is given, respectively, by

$$f_{\omega NN} = gF_{20}(q^2=0) = -gx_0F_0^+(q^2=0) - \tfrac{1}{2}gy_0F_1(q^2=0) , \quad (12a)$$

or

$$f_{\omega NN} = -g[\Gamma_{00}(q^2=0) - 2M_N\Gamma_0(q^2=0)] = g[-3+(1+\alpha/4m_q^2)^{-1}2M_N/3m_q] . \quad (12b)$$

Numerical values for $f_{\omega NN}$ may be obtained by fitting, e.g., m_q to the measured nucleon mass and α to the axial-vector coupling constant $g_A = 1.25$, yielding for Eq. (12b)

$$m_q = M_N/3, \quad \alpha = 12m_q^2/13 ,$$

and consequently

$$f_{\omega NN}/g_{\omega NN} = -0.47 . \quad (13)$$

This is a fairly large tensor-to-vector ratio; however, we show next that a lower ratio is more likely as suggested by a fit of $F_1(0)$ and Γ_T to the nucleon magnetic moment.

In the nucleon case, the electromagnetic current is readily obtained from Eqs. (7) and (8) using the quark charge operator $Q = \frac{1}{2}e(\frac{1}{3}+\tau_3)$ for the u and d quarks:

$$\langle N(p')|j^{\mu}|N(p)\rangle = \frac{e}{2}(1-q^2/4M_N^2)^{-1}\bar{u}_N(p')\{[F_1^S(q^2)+F_1^V(q^2)\tau_3^N]\gamma^{\mu}+[F_2^S(q^2)+F_2^V(q^2)\tau_3^N]i\sigma^{\mu\nu}q_\nu/2M_N\}u_N(p) , \quad (14)$$

where the definition of $F_{1,2}^{S,V}$ differ for the bag and constituent quark models. Rather than repeating the same steps leading to Eq. (13), we note that

$$F_1^S(q^2=0) = F_1^V(q^2=0) = 1 \quad (15)$$

represent the corresponding (isoscalar or isovector) nucleon charge in units of $e/2$, while in the bag model (e.g.)

$$F_2^S(q^2=0) = -[1-\tfrac{1}{3}F_1(q^2=0)] = \kappa_p + \kappa_n = -0.12 ,$$
$$F_2^V(q^2=0) = -[1-\tfrac{5}{3}F_1(q^2=0)] = \kappa_p - \kappa_n = 3.70 , \quad (16)$$

where κ_N is the nucleon anomalous magnetic moment in nuclear magnetons. From Eqs. (16) we find an average value of

$$F_1(q^2=0) = 2.72 ; \quad (17)$$

this result is obtained [see Eq. (16)] from a fit to the nucleon magnetic moment. From Eq. (12a) we now obtain the bag model result

$$F_{20}(q^2=0) = -0.28, \quad f_{\omega NN}/g_{\omega NN} = -0.09 \quad (18)$$

for the ratio of the ω-nucleon tensor-to-vector coupling constants. Likewise in the constituent quark model,

$$(\tfrac{1}{3}\Gamma_{00} - \tfrac{2}{3}M_N\Gamma_0)|_{q^2=0} = 0.12 ,$$
$$(\Gamma_{01} - 2M_N\Gamma_1)|_{q^2=0} = 3.70 \quad (19)$$

[cf. Eq. (16)], again yielding $f_{\omega NN}/g_{\omega NN} = -0.09$ as in Eq. (18). The small tensor-to-vector ratio obtained here is in good agreement with the hadronic phenomenology [14,19] (See, however, Ref. [20] for the ω and [21] for the ρ ratio.) In Sec. IV, we will see that a large $\omega\Lambda\Lambda$ tensor coupling is required, and that the magnetic moment provides no input in determining $f_{\omega\Lambda\Lambda}$ (unlike the nucleon case).

IV. QUARK MODEL FOR HYPERONS

The quark model results of Sec. III have been shown to be consistent with the nuclear σ-ω MFT in the nucleon sector. In this section we apply the model to hypernuclei, where a small Λ-hypernuclear spin-orbit splitting is measured (see Sec. II); predictions for Σ and Ξ hypernuclei [22] will also be given.

The underlying quark structure of the Λ in the uds basis, where quarks are treated as distinguishable and the

u, d quarks are antisymmetrized explicitly, is given by

$$|\Lambda(S=\tfrac{1}{2},T=0)\rangle = |(ud)S=0,T=0\rangle |s\rangle_{S_\Lambda=1/2,T_\Lambda=0} , \tag{20}$$

where the u and d quarks are coupled to spin and isospin zero. For the $\Sigma^\pm$,

$$|\Sigma^\pm(S=1/2,T=1)\rangle = [|(qq)S=1,T=1\rangle |s\rangle]_{S_\Sigma=1/2,T_\Sigma=1} , \tag{21}$$

with $q=u(d)$ for $\Sigma^+(\Sigma^-)$. Finally, we also consider here the $S=-2$ Ξ hyperon, where

$$|\Xi^{0,-}(S=1/2,T=1/2)\rangle = [|q>|(ss)S=1\rangle]_{S_\Xi=1/2,T_\Xi=1/2} . \tag{22}$$

Λ hypernuclei can be analyzed on the basis of Eq. (20). Following the approach of Sec. III, we now proceed to write down the ω-Λ coupling. Assuming no admixture of strange quark content in the ω (or the σ), the Okubo-Zweig-Iizuka (OZI) rule tells us that the mesons will only couple to the u and d quarks. [The same assumption leads to Eq. (4).] The $\omega\Lambda\Lambda$ vertex is, therefore, given by [9]

$$\langle\Lambda|g\gamma^\mu|\Lambda\rangle = 2g(1-q^2/4M_\Lambda^2)^{-1}F_0^+\bar{u}_\Lambda(p') \times(\gamma^\mu - i\sigma^{\mu\nu}q_\nu/2M_\Lambda)u_\Lambda(p) , \tag{23}$$

yielding the tensor-to-vector coupling ratio

$$f_{\omega\Lambda\Lambda}/g_{\omega\Lambda\Lambda} = -1 . \tag{24}$$

This ratio does not depend on fitting any particular set of data, in contrast to the situation encountered for the nucleon in Sec. III. This feature was crucial for Ref. [10] to show that the small spin-orbit interaction of the Λ hyperon is consistent with the σ-ω Dirac model. Indeed, it is unrelated to the magnetic moment of the Λ, which comes from the $\boldsymbol{\sigma}$ matrix element of the s quark and does not tell us anything about the value of $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda\Lambda}$; there is no F_1 contribution in Eq. (23) because the $\boldsymbol{\gamma}$ matrix element is zero for the ud pair coupled to spin 0. Furthermore, since the Λ hyperon is neutral, it has only an anomalous magnetic moment. The Λ-electromagnetic current is

$$\langle\gamma^\mu e_s\rangle_\Lambda = -\tfrac{1}{3}(1-q^2/4M_\Lambda^2)^{-1}\bar{u}'_\Lambda \times(-q^2\gamma^\mu\Gamma_1/2M_\Lambda + \Gamma_1 i\sigma^{\mu\nu}q_\nu)u_\Lambda \tag{25}$$

for the constituent quark model (with a similar result for the bag model). As $q^2\to 0$, Eq. (25) does not contain a vector coupling and does not contain information relevant for Eq. (23).

We now turn to the Σ and Ξ hyperons [cf. Eqs. (21) and (22)]. As was the case for the Λ hyperon, the ω meson couples only to the u and d quarks, not to the s quark. We shall use the measured baryon masses in the ensuing discussion. The quark model [9,18] predicts for the convection current of Eq. (37) an octet $F/(F+D)$ ratio $\alpha=1$, while $\alpha=2/5$ for the Pauli current (tensor matrix element) at $q^2=0$. The same α values are obtained from magnetic couplings in the SU(6) symmetry [20] which is broken in quark models. Our quark model goes beyond SU(6) in predicting scalar meson couplings ($\alpha_S=1$) from chiral properties built into the model. The model yields the following $\omega\Sigma\Sigma$ and $\omega\Xi\Xi$ vertices, corresponding to Eq. (7) for the nucleon and Eq. (23) for the Λ hyperon with F_1 of Eq. (6b); similar results hold for the constituent quark model version:

$$\langle\Sigma|g\gamma^\mu|\Sigma\rangle = g(1-q^2/4M_\Sigma^2)^{-1}\bar{u}_\Sigma(p') \times[(2F_0^+ - q^2F_1/3M_\Sigma^2)\gamma^\mu - (2F_0^+ - 4F_1/3)i\sigma^{\mu\nu}q_\nu/2M_\Sigma]u_\Sigma(p) , \tag{26}$$

$$\langle\Xi|g\gamma^\mu|\Xi\rangle = g(1-q^2/4M_\Xi^2)^{-1}\bar{u}_\Xi(p') \times[(F_0^+ + q^2F_1/12M_\Xi^2)\gamma^\mu - (F_0^+ + F_1/3)i\sigma^{\mu\nu}q_\nu/2M_\Xi]u_\Xi(p) , \tag{27}$$

where $F_0^+=1$ at $q^2=0$ and

$$F_1 = \begin{cases} 2.41, & \Sigma \text{ bag version} \\ 2.66, & \Xi \text{ bag version} \end{cases} , \quad F_1 = \begin{cases} 3.46, & \Sigma \text{ Eq. (19) fit} \\ 3.82, & \Xi \text{ Eq. (19) fit} \end{cases} . \tag{28}$$

From Eqs. (26) and (27) we obtain the tensor-to-vector coupling constant ratios (at $q^2=0$)

$$f_{\omega\Sigma\Sigma}/g_{\omega\Sigma\Sigma} = -1+2F_1/3 = \begin{cases} 0.61, & \text{bag version} \\ 1.31, & \text{Eq. (19) fit} \end{cases} \tag{29}$$

$$f_{\omega\Xi\Xi}/g_{\omega\Xi\Xi} = -1-F_1/3 = \begin{cases} -1.89, & \text{bag version} \\ -2.27, & \text{Eq. (19) fit} \end{cases} . \tag{30}$$

A comparison of Eq. (23) for the Λ hyperon with Eqs. (26) and (27) shows that the strong vector and tensor ω-hyperon couplings contain detailed dynamics of the quark model for the Σ and Ξ hyperons (as well as for the nucleon), in contrast to the Λ hyperon.

V. APPLICATION TO HYPERNUCLEAR SPIN-ORBIT INTERACTION

When applied to hypernuclei the results of Sec. IV bring about considerable changes compared with the pure scalar+vector model. They result from the large $f_{\omega YY}/g_{\omega YY}$ ratios, Eqs. (24), (29), (30) relative to the small value of $f_{\omega NN}/g_{\omega NN}$ in Eq. (18).

Starting with the Λ hyperon [see Eq. (24)] we seek an explanation of the small measured spin-orbit interaction [7] of Λ hypernuclei. Toward this end, we discuss two alternative formalisms and then give an intuitive physical explanation which, we believe, is new and helps to clarify the pertinent physical picture beyond the formalism. Predictions will then be discussed for Σ and Ξ hypernuclei.

The (strong) ωYY tensor coupling adds an extra term to the MFT Lagrangian,

$$-f_{\omega YY}\bar{\psi}\sigma^{\mu\nu}F_{\mu\nu}\psi/4M_Y \ , \tag{31}$$

where $F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$. In this work we have derived this tensor part from a quark model. In a purely hadronic theory such terms are sometimes put by hand into the Lagrangian, in analogy with magnetic-moment contributions of Pauli type to represent the interaction of the electromagnetic field with the anomalous magnetic moment of the baryon. It has long been realized that such contributions should emerge (in terms of hadronic degrees of freedom) as higher-order perturbative corrections in the field-theoretical calculations [23]. However, a complete theory along these lines has never been worked out; indeed very similar words were written 40 years ago [24] in very similar circumstances. It is important to keep in mind, however, that the magnetic moments and tensor couplings should not be treated as tree-level contributions in the strict hadronic theory sense.

The ωYY tensor coupling affects the phenomenological Dirac equation as discussed in Ref. [11] (this result is obtained by the usual [2] methods starting from an MFT Lagrangian, Eq. (1), modified to include the term in (31))

$$\left[i\gamma^\mu\partial_\mu-g_v^Y\gamma^0V_0(\mathbf{r})-[M_Y-g_s^Y\phi_0(\mathbf{r})] + \frac{1}{2M_Y}\frac{f_{\omega YY}}{g_{\omega YY}}i\gamma^0\boldsymbol{\gamma}\cdot\nabla g_v^YV_0(\mathbf{r})\right]\psi(\mathbf{r})=0 \ , \tag{32}$$

where we assume that the ratio $f_{\omega YY}/g_{\omega YY}$ obtained in the quark model is directly applicable to the phenomenological Dirac equation. Note that the tensor coupling term in Eq. (31) is important only when the potentials depend explicitly on $\mathbf{r}$, particularly in the nuclear surface, where the r dependence is strongest. For spherically symmetric potentials, $V_0(\mathbf{r})=V_0(r)$, the tensor term becomes

$$\frac{1}{2M_Y}\frac{f_{\omega YY}}{g_{\omega YY}}i\gamma^0\boldsymbol{\gamma}\cdot\hat{\mathbf{r}}\frac{d}{dr}[g_v^YV_0(r)] \ . \tag{33}$$

Using Appendix I of Clark, Hama, and Mercer [3] (where an equivalent Schrödinger potential is obtained from the Dirac equation by eliminating the lower component of ψ and solving for the upper), the tensor coupling adds to the usual spin-orbit interaction the term

$$\frac{1}{rM_Y}\frac{f_{\omega YY}}{g_{\omega YY}}\frac{d}{dr}[g_v^YV_0(r)] \ , \tag{34}$$

implying a large effect on the spin-orbit interaction. The central nonrelativistic potential remains largely unchanged.

For the $Y=\Lambda$, Σ, and Ξ hyperons the spin-orbit interaction is

$$M_Yr_0^2V_{\text{s.o.}}^Yf'(r)/r =r^{-1}(E+M_Y-g_s^Y\phi_0-g_v^YV_0)^{-1}\frac{d}{dr}(g_s^Y\phi_0+g_v^YV_0) +r^{-1}M_Y^{-1}\frac{d}{dr}(g_v^YV_0)f_{\omega YY}/g_{\omega YY} \ . \tag{35}$$

Evidently for the Λ, using $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda\Lambda}=-1$ [Eq. (24)], the two terms in Eq. (35) cancel each other to a large extent. A numerical estimate of Eq. (35) may be obtained by substituting $\phi_0(r)\cong V_0(r)$ and $E\cong M_\Lambda$, yielding

$$M_\Lambda r_0^2v_{\text{s.o.}}^\Lambda f'(r)/r=r^{-1}(1/M_\Lambda^*-1/M_\Lambda)\frac{d}{dr}[g_v^\Lambda\phi_0(r)] \ . \tag{36}$$

The scalar Λ-matrix element contains the mesonic coupling constant $2g/3$ instead of g for the nucleon. Using $g_s^\Lambda/g_s^N=0.6$ [see also Eq. (4) and the discussion following it], we find that

$$1/M_\Lambda^*-1/M_\Lambda=0.27/M_\Lambda \ ,$$

reducing the spin-orbit interaction for Λ hypernuclei by another factor of 4 relative to the prediction of the pure scalar+vector Dirac model. The approximate ratio of vector-to-tensor potentials,

$$\bar{\kappa}_\Lambda=M_\Lambda^*/M_\Lambda=0.79 \ ,$$

is in surprising agreement with Noble's [8] value of $\bar{\kappa}_\Lambda=0.8$ adopted on the basis of the anomalous magnetic moment of the Λ hyperon.

Furthermore, the spin-orbit potential acts mostly in or near the nuclear surface, where V_0 and ϕ_0 are smaller than their central density values. Taking the potentials at half of the latter strength [8] gives a suppression factor $0.12/M_\Lambda$, decreasing the spin-orbit interaction by a factor of about 10 relative to the pure scalar+vector prediction. This reduction occurs over and above what has previously been attained in the literature within the pure scalar+vector model by reducing the meson-baryon couplings g_s and g_v.

For Λ hypernuclei, where Eq. (24) applies, a similar result (at least at a semiquantitative level) may be obtained by using the (on-shell) Gordon decomposition

$$\gamma_\mu-i\sigma_{\mu\nu}q^\nu/2M_\Lambda=(p+p')_\mu/2M_\Lambda \ , \tag{37}$$

where p, p' are the initial and final Λ four-momenta. The importance of this relation in this context was already recognized in Ref. [9] and utilized in Ref. [10].

Since the presence of a tensor coupling results in the modified MFT Dirac equation [cf. Eq. (1)]

$$\left[i\gamma_\mu\gamma^\mu-g_v^Y\left[\gamma_\mu V^\mu+\frac{1}{2M_Y}\frac{f_{\omega YY}}{g_{\omega YY}}\sigma^{\mu\nu}F_{\mu\nu}\right] -(M_Y-g_s^Y\phi_0)\right]\psi=0 \ , \tag{38}$$

we use Eq. (37) for the case of weak overall Λ binding to get for $Y=\Lambda$

$$i\gamma_\mu \partial^\mu \psi - \left[M_\Lambda + g_v^\Lambda \frac{(p+p')^\mu}{2M_\Lambda} V_\mu - g_s^\Lambda \phi_0 \right] \psi = 0 \; . \tag{39}$$

For spherical nuclei and intensive observables, where only the timelike component of a four-vector potential is important, Eq. (39) yields

$$\left[i\gamma_\mu \partial^\mu - \left[M_\Lambda + \frac{E_\Lambda}{M_\Lambda} g_v^\Lambda V_0 - g_s^\Lambda \phi_0 \right] \right] \psi = 0 \; . \tag{40}$$

Interestingly, only a fairly weak scalar potential appears in Eq. (40), of the order

$$V_c \cong \frac{g^\Lambda}{g^N} (75 \text{ MeV}) \; .$$

The net result is a transformation from a complicated Λ-hypernuclear single-particle Dirac equation with scalar, vector, and tensor potentials to a simple Dirac equation virtually equivalent to the nonrelativistic shell model with only a shallow central potential.

There also exists a simple semiclassical explanation of the above observations [26]. First note that the vector field equation of motion [2] (without a tensor coupling),

$$\partial_\mu F^{\mu\nu} + m_v^2 V^\nu = g_v \bar{\psi} \gamma^\nu \psi \; , \tag{41}$$

looks like massive quantum electrodynamics with the conserved baryon current $B^\mu = \bar{\psi}\gamma^\mu\psi$ (rather than the conserved electromagnetic current of QED) as a source. The origin of the spin dependence of the nonrelativistic baryon-nucleus potential is a result of two possible effects. The first of these arises from an external torque from the interaction between a "magnetic dipole" proportional to the spin and a "magnetic" field. The "magnetic" field is producted by the motion (at velocity **v**) of the baryon in an "electric" field proportional to $-g_v \nabla V_0(r)$. The outcome is an effective spin-orbit interaction Hamiltonian

$$H_1 = -g_v \mathbf{s} \cdot \mathbf{v} \times \nabla V_0(r) / M_B \; . \tag{42}$$

The second effect is the Thomas precession [25],

$$H_2 = \mathbf{s} \cdot \mathbf{v} \times \nabla V_c(r) / 2M_B \; . \tag{43}$$

In conventional nonrelativistic nuclear physics, where only one type of potential exists, a scalar potential gives [based on Eq. (43)] a spin-orbit interaction of the correct sign but a factor of 20 too low, as discussed above. In the Dirac approach with both a strong scalar and vector potential, Eq. (42) gives a large spin-orbit force, hence the success of the model. As shown in Eq. (40), the ω-Λ tensor coupling effectively eliminates the vector potential altogether while reducing the scalar potential to the level of the nonrelativistic shell model, thereby eliminating the spin-orbit interaction as required by hypernuclear data.

The Σ-hypernuclear spin-orbit interaction can be obtained in a similar fashion. For $g_v^\Sigma / g_v^N = 0.6$ and the positive ratio in Eq. (29), the tensor potential of the Σ hyperon increases its spin-orbit potential by about 50% (bag version) to 100% [fit of Eq. (17)] relative to the pure scalar+vector theory. For potentials at half strength in the nuclear surface (as done for Λ hypernuclei above), the ratio of Σ to N spin-orbit interaction is 0.6 (bag) to 1 (fit), which amounts to a sizable spin-orbit interaction somewhat smaller than (but comparable with) that of the nucleon. Unfortunately, no experimental data are available at this time to compare with.

For Ξ hypernuclei we use $g_v^\Xi / g_v^N = 1/3$, as only one nonstrange quark is assumed to interact with the ω meson. The standard spin-orbit potential in Eq. (35) is reduced by the negative ratio in Eq. (30) reversing its sign, so that the Ξ-hypernuclear spin-orbit interaction is −15% to −20% of that of the nucleon.

The one-boson-exchange results within a nonrelativistic hypernuclear dynamics were derived and reviewed by Dover and Gal [20]. They found in model D [27] the Σ hypernuclear spin-orbit potential is about 40% of the nucleon's, the Ξ hypernuclear spin-orbit potential is 15% and *equal in sign* to the nucleon one (smaller and opposite in sign in model F), while the Λ-hypernuclear spin-orbit potential is 25% of the nucleon quantity. The meson-exchange models used by Dover and Gal tend to yield too large a Λ-hypernuclear spin-orbit potential. They could have obtained a vanishing Λ-hypernuclear spin-orbit potential from their Eq. (2.40) upon using $f_{\omega\Lambda\Lambda} / g_{\omega\Lambda\Lambda} = -1$. However, their couplings are based on a different model, viz. the nonrelativistic one-boson-exchange picture, which provides different values for the pertinent coupling constants when fitted to the scarce hyperon-nucleon scattering data.

VI. CONCLUSION

We find, in agreement with Jennings [10], that to explain the small Λ-hypernuclear spin-orbit interaction a strong $\omega\Lambda\Lambda$ tensor vertex is required. A strong ω-hyperon tensor coupling arises naturally in the quark model. This ωYY tensor coupling affects the phenomenological Dirac equation, while the central nonrelativistic potential remains largely unchanged, i.e., without disturbing the successful applications of the σ-ω model to ordinary nuclei. The net result is a transformation from a complicated Λ-hypernuclear single-particle Dirac equation with scalar, vector, and tensor potentials to a simple Dirac equation virtually equivalent to the nonrelativistic shell model with only a shallow central potential. The positive ratio of tensor-to-vector coupling for the Σ hyperon increases its spin-orbit potential comparable to that of the nucleon, while for the Ξ hyperon the standard spin-orbit potential is reduced by the corresponding negative ratio and a reduction factor 1/3 in its scalar and vector coupling constants.

The Λ, Σ, Ξ hyperons have different spin-orbit forces: for the Λ a vanishing spin-orbit interaction, for the Σ it is comparable to the nuclear spin-orbit interaction, for the Ξ it is of opposite sign and an order of magnitude smaller than the nuclear case.

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