In-medium and core-polarization effects in $^{50}\mathrm{K}(0^-) \xrightarrow{\beta^-} {}^{50}\mathrm{Ca}(0^+)$

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A shell-model calculation in the model space of the $(0d, 1s)$ and $(0f, 1p)$ major shells is made for the first-forbidden β^- transition $^{50}\mathrm{K}(0^-) \rightarrow {}^{50}\mathrm{Ca}(0^+)$. Various truncation schemes were evaluated and the one adopted was to allow at most two neutrons in the $\nu 0f_{5/2}$ orbit. An evaluation of the "final-state" core-polarization effects is made perturbatively for the two rank-zero matrix elements which contribute to $\Delta J = 0$ first-forbidden β decays. The core-polarization effect is also applicable to $\Delta J = 0$ spin-dipole excitations in the $A \approx 40$ region. Comparison of the shell-model (impulse approximation) prediction to experiment indicates the need for an appreciable in-medium contribution to the timelike component of the weak axial current. The enhancement of 52% found in this comparison is in good agreement with the general trend of predictions for one-pion exchange processes in $\Delta J = 0$ first-forbidden decays in $A = 16$ to 96 nuclei.

I. INTRODUCTION

The decay of ^{50}K to the 0^+ ground state of ^{50}Ca has $T_{1/2} = 472 \pm 4$ ms, $Q(\beta^-) = 14.05 \pm 0.30$ MeV and a branching ratio of $60 \pm 10\%$ [1–3], yielding $\log f_0 t = 5.89 \pm 0.09$. The low-lying energy spectrum of ^{50}K can be safely assumed to be a 0^--3^- quartet resulting from a $^{39}\mathrm{K}(\frac{3}{2}^+) \otimes {}^{51}\mathrm{Ca}(\frac{3}{2}^-)$ coupling. A $\log f_0 t$ value of 5.9 is extremely low for a first-forbidden ($\Delta J \leq 2$, $\pi_i \pi_f = -$) beta decay — there are only three known decays for $A < 206$ that are faster [4–6]. Invoking the systematics and theory of first-forbidden decays in the $A \approx 40$ region [7], we conclude that the $\log f_0 t$ value effectively establishes the ^{50}K ground state as $J^\pi = 0^-$.

Such $\Delta J = 0$ first-forbidden transitions are presently of considerable interest for two reasons. The first is the large mesonic enhancement, 40-60 %, predicted [8, 9] for the matrix element M_0^T of the timelike component of the weak axial current γ_5 which is one of the two operators contributing to rank-zero decays. The second reason involves recent speculation — based on in-medium chiral Lagrangians [10–14] — for large density effects on certain operators. The idea is that these effects are manifested via an effective nucleon mass M^*. For the lead region, Rho [14] finds a plausible explanation for the difference between the predicted [15, 16] and extracted [17] values of ϵ_{MEC} of 1.4 and 2.0, respectively, with $M^*/M \approx 0.8$. The other operator contributing to $\Delta J = 0$ first-forbidden beta decay is the spacelike component of the axial current M_0^S which is responsible for $\Delta J = 0$ spin-dipole excitations. This operator is not expected to be affected significantly by meson exchange or by effects due to the nuclear density.

Evaluation of the meson-exchange contribution (MEC) in first-forbidden beta decay — and thus the enhancement factor ϵ_{MEC} by which M_0^T exceeds the impulse approximation — is quite difficult if the same rigor is demanded as can be brought to the impulse approximation calculations via the shell model. Nevertheless, much progress has been made, mainly by Towner and his colleagues [18, 19] and Kirchbach and her colleagues [15, 16]. It has been found that the relative values of the MEC and impulse contributions — and thus the value of ϵ_{MEC} — is insensitive to nuclear structure. This insensitivity is expected [19] because the effect is mainly due to the interaction of the valance nucleons with the core. Kirchbach and Reinhardt [16] have calculated meson enhancements for the matrix element of γ_5 of between 40 and 60% ($\epsilon_{\mathrm{MEC}} = 1.40$–$1.60$) for transitions near $A = 16, 96$, and 206.

The approach used here and in previous studies of the in-medium enhancement of M_0^T is to make as careful an evaluation of the decay rate as possible via the impulse approximation as formulated in the spherical shell model. The extent of enhancement due to effects of the nuclear medium is then ascertained by a comparison to experiment [4, 5, 7, 17]. Although ϵ_{MEC} will be used to represent this enhancement, it should be emphasized that all in-medium effects are included in this enhancement factor and the motive of this study is not to describe the β^- decay of ^{50}K but rather to use this decay to evaluate, as well as possible, the in-medium effect for this decay. We can then compare this result to the expected MEC enhancement to gain some idea of the other in-medium effects.

II. CALCULATION

A. The interaction

Shell-model calculations were performed with the WBMB (Warburton-Becker-Millener-Brown) interaction [20, 7] using the computer code OXBASH [21]. The WBMB interaction has been tested throughly by the

calculation of numerous observables [20, 7]. Ideally the WBMB interaction operates in the full model space of the $(1s, 0d)$ and $(0f, 1p)$ major shells. In the present instance use of the full model space is not possible: the available computational resources do not allow diagonalization of the ^{50}K 0^- states in a full $\pi(1s, 0d)^{-1}\nu(1p, 0f)^{11}$ model space. Thus, calculations of the initial and final wave functions and the one-body-transition densities connecting them were made with various truncations of the $(0f, 1p)$ part of the model space. The $(0f, 1p)$ part of the WBMB interaction is that of McGrory [22]. The McGrory interaction is essentially that of Kuo and Brown [23] but with the eight $f^2_{7/2}$ two-body matrix elements (TBME) adjusted in a least-squares fit to selected binding energies of $40 \leq A \leq 44$ nuclei. This interaction is designed for use in the $A \leq 44(0f, 1p)$ nuclei. It has obvious deficiencies when applied to $A > 46$. These deficiencies were discussed by McGrory, Wildenthal, and Halbert [24] who pointed out that an adequate remedy for specific A was to adjust the single-particle energies (SPE) of the four orbits. Accordingly, the SPE of the orbits were adjusted so as to reproduce as closely as possible the binding energies and spectroscopic factors of the low-lying levels of ^{49}Ca [26] relative to ^{48}Ca. The adjustment was found to be insensitive to the truncation used and consisted of raising the SPE of the $1p_{3/2}$, $1p_{1/2}$, and $0f_{5/2}$ orbits by 1.67, 2.06, and 1.67 MeV, respectively. As pointed out by McGrory, Wildenthal, and Halbert [24], this adjustment is an approximate correction for the fact that the interaction of these three orbits with the $\sim$8 $0f_{7/2}$ neutrons is too strong. The adjustment is approximately equivalent to adding 210, 260, and 210 keV to all the $\langle f_{7/2}j_2|V|f_{7/2}j_2\rangle$ TBME with $j_2 = p_{3/2}$, $p_{1/2}$, and $f_{5/2}$, respectively, which is similar to the f-p'' interaction. It should be stressed that the adjustment made is not arbitrary but is well documented and understood via the work of McGrory, Wildenthal, and Halbert [24]. Also, since the $f_{7/2}$ shell is not directly involved in the β^- transition, the adjustment has only a small effect on the calculated matrix elements.

Various truncation schemes were used in order to systematically investigate the effect of using less than the full $\nu(0f, 1p)$ shell. The most ambitious consisted of (a) allowing ≤ 4 neutrons out of the $0f_{7/2}$ orbit, and (b) allowing ≤ 2 neutrons in the $0f_{5/2}$ orbit. In both cases there were no other restrictions. These gave very similar results which can be ascertained from the fact that in truncation (a) the occupancy of the $0f_{5/2}$ orbit by more than two neutrons contributed only 0.3% to the ^{50}K 0^-_1 wave function. The results adopted for the calculation of the matrix elements is that of truncation (b). This calculation gives a ^{50}Ca ground state which is 84% $\nu 0f^8_{7/2}1p^2_{3/2}$ and a ^{50}K ground state which is 80% $\pi 0d^{-1}_{3/2}\nu 0f^8_{7/2}1p^3_{3/2}$. Thus it is clear that the decay will be dominated by a $\nu 1p_{3/2} \rightarrow \pi 0d_{3/2}$ transition. It was found that the main effect on the β^- transition of allowing excitations out of the $0f_{7/2}$ orbit is to allow increased occupancy of the $1p_{1/2}$ orbit. This has a magnified effect on the β^- matrix elements because the contribution of the $\nu 1p_{1/2} \rightarrow \pi 1s_{1/2}$ transition is destructive to the dominant $\nu 1p_{3/2} \rightarrow \pi 0d_{3/2}$ transition. The low-lying energy spectrum of ^{50}K in a truncation to ≤ 5 active $1p_{3/2}$-$1p_{1/2}$-$0f_{7/2}$ neutrons is shown in Fig. 1. This spectrum is quite insensitive to the truncation of the $\nu(0f, 1p)$ model space since it is largely determined by the interaction between the $\pi 0d_{3/2}$ and $\nu 1p_{3/2}$ orbits.

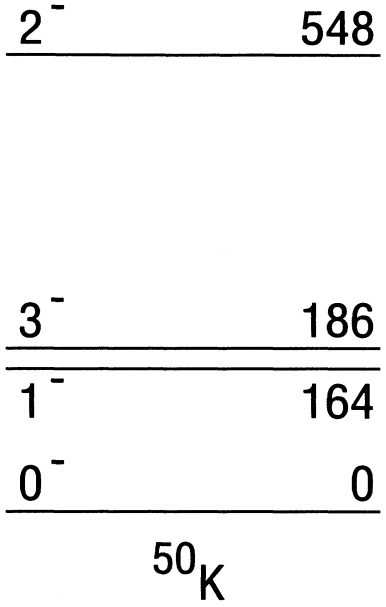

FIG. 1. The $\pi 0d_{3/2}$-$\nu 1p_{3/2}$ quartet of ^{50}K calculated with the WBMB interaction with the adjusted $(0f, 1p)$ SPE given in the text and a truncation such that at most five neutrons occupy the $1p_{3/2}$-$1p_{1/2}$-$0f_{5/2}$ orbits. Level energies are in keV. The J dimensions of the matrices diagonalized in this calculation are 815, 2327, 3508, and 4217 for $J = 0, 1, 2$, and 3, respectively.

When dealing with $\Delta\hbar\omega = 1$ operators it is important to be careful that the wave functions do not contain significant spurious components. In OXBASH, spuriosity is removed by the method of Glockner and Lawson [25]. However, in the present case both the initial and final states have the lowest energies allowed by the Pauli principle and thus their wave functions are free of spuriosity.

B. The matrix elements

For a $0^- \rightarrow 0^+$ β^- transition only operators of zero rank contribute. Thus only single-particle transitions with $j_i = j_f \equiv j$ are allowed. The two rank-zero matrix elements are calculated via

$$M^S_0 = \sum_j \mathcal{M}^S_0(j) = \sum_j D_0(j) M^S_0(j, \text{eff}) = \sum_j D_0(j) q_S(j) M^S_0(j) ,$$
$$M^T_0 = \sum_j \epsilon_{\text{MEC}} \mathcal{M}^T_0(j) = \sum_j D_0(j) \epsilon_{\text{MEC}} M^T_0(j, eff) = \sum_j D_0(j) \epsilon_{\text{MEC}} q_T(j) M^T_0(j). \quad (1)$$

Equation (1) introduces notation to be used in this paper. In Eq. (1) $M_R^\alpha(j)$ is a single-particle matrix element of type α and rank R in the impulse approximation. The quenching factor $q_\alpha(j)$ corrects $M_R^\alpha(j)$ for the finite size of the model space, and $\epsilon_{\rm MEC}$ represents the enhancement of M_0^T over the impulse approximation. The $D_R(j)$ are the one-body-transition densities which are the result of the shell-model calculation. The $M_0^\alpha(j)$ were calculated with Woods-Saxon (WS), Hartree-Fock (HF), and harmonic-oscillator (HO) radial wave functions.

The Woods-Saxon radial wave functions utilized the parameters of Streets, Brown, and Hodgson [27]. With the shell-model occupancies given in Table 2 of Ref. [21], these parameters give $\langle r_p\rangle_{\rm rms} = 3.463$ fm and $\Delta r_{pn} = \langle r_p\rangle_{\rm rms} - \langle r_n\rangle_{\rm rms} = -0.292$ fm as opposed to the experimental values [27] of 3.482(3) and $-0.11(4)$ fm, respectively. The Hartree-Fock wave functions were calculated using the Skyrme SK11 interaction of van Giai and Sagawa [28] as described by Brown, Bronk, and Hodgson [29]. With the same orbit occupancies as in the WS calculation, the HF results for ^{48}Ca are $\langle r_p\rangle_{\rm rms} = 3.484$ fm and $\Delta r_{pn} = -0.154$ fm. The agreement with experiment is considerably better for the HF than WS calculation, especially for Δr_{pn}. For this reason the results for the β^- matrix elements will be based on the HF calculation; the HO and WS calculations provide a measure of the sensitivity of the $M_0^\alpha(j)$ to the radial form assumed.

The WS and HF results depend on the separation energies $S(n)$ for ^{50}K and $S(p)$ for ^{50}Ca. These are related by

$$\begin{aligned} S(p) - S(n) &= Q(\beta^-) - 0.782 \text{ MeV}, \\ S(n) &= 3.270 \text{ MeV} + E_x(\text{MeV}), \\ S(p) &= 16.540 \text{ MeV} + E_x(\text{MeV}), \end{aligned} \tag{2}$$

where E_x is the excitation energy of the parent state in the ^{49}K core and $Q(\beta^-) = 14.05 \pm 0.30$ MeV. It is seen that the large $Q(\beta^-)$ value for ^{50}K $\rightarrow$ ^{50}Ca is associated with quite different values for the proton and neutron separation energies. Because of this difference and, in particular, the loose binding of the neutron in ^{50}K, the matrix elements are relatively sensitive to the value of E_x used in Eq. (2). The method described in a similar calculation [17] for ^{206}Tl $\rightarrow$ ^{206}Pb was used to estimate the effective value of E_x, $\langle E_x(j)\rangle$, to use in Eq. (2) for each value of j. The results were 0.0, 1.0, and 5.2 MeV for $j = \frac{3}{2}, \frac{1}{2}$, and $\frac{5}{2}$, respectively. The HO calculation was made with a value of $\hbar\omega$ equal to the average of the values corresponding to $\langle r_p\rangle_{\rm rms}$ for ^{50}Ca and $\langle r_n\rangle_{\rm rms}$ for ^{50}K (9.44 MeV for WS and 9.68 MeV for HF). With this prescription, the HO $M_0^S(M_0^T)$ were the largest (smallest) and the WS values were the smallest (largest). The spread between these two extremes is ± 12, 3.6, and 3.6% for $j = \frac{3}{2}, \frac{1}{2}$, and $\frac{5}{2}$, respectively. The HF differ from the WS values by 14, 4, and 0.4%, respectively. The greater sensitivity of the $j = \frac{3}{2}$ result is due to the fact that the $\nu 1p_{3/2} \rightarrow \pi 0d_{3/2}$ transition involves a change in the principal quantum number (number of nodes in the radial wave functions) while the other two transitions do not.

C. Core polarization

The allowable first-forbidden transitions are customarily restricted to $\Delta Q = 1$, where Q designates a major oscillator shell and is given by $2l + n$, with l the orbital angular momentum and n (= 0,1,...) the principal quantum number of an orbit. This selection rule is exact for HO wave functions. The $q_\alpha(j)$ are used here to represent first-order impulse-approximation effects not included in the model space as expressed formally in Eq. (1). By first order is meant additional terms in the initial (final) state which connect to those terms included in the model space for the final (initial) state. All possible initial-state admixtures of this type are already included; however, $2\hbar\omega$ "final-state correlations" are not. The situation is illustrated schematically in Fig. 2. The missing first-order terms are in the classes $\nu(Q = 2) \rightarrow \pi(Q = 3)$ and $\nu(Q = 3) \rightarrow \pi(Q = 4)$. These contributions were evaluated perturbatively as described by Towner, Warburton, and Garvey [30] in a study of $\Delta J = 2$ β^- transitions ($\Delta J = 2$ spin-dipole excitations) in $A \sim 40$ nuclei and by Warburton [31] in a study of "final-state correlations" in spin-dipole excitations and first-forbidden β^- decay in the lead region. The same interaction(H7B) [32] was used as in the application to $A \sim 208$ [31]. The SPE of Towner, Warburton, and Garvey [30] were used in the present study. In calculating the transition matrix elements a difficult problem arises when the final orbit is unbound as is true for the $Q = 4$ proton orbits. In order

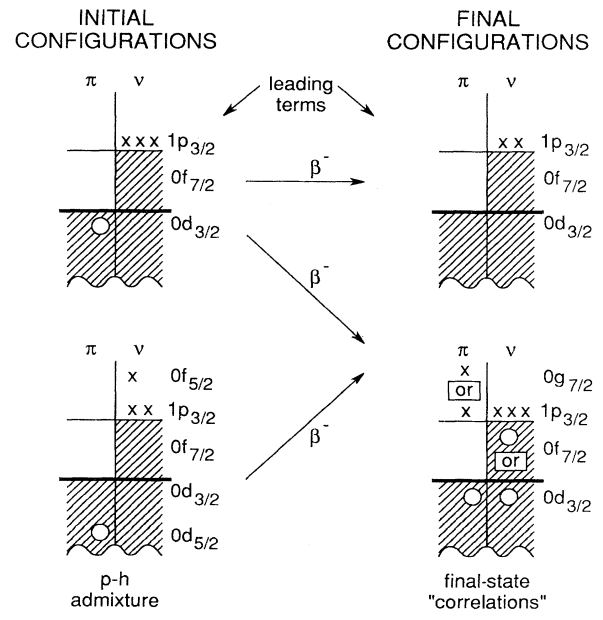

FIG. 2. Schematic illustrating the role in first-forbidden β^- decay and spin-dipole excitations of 1p-1h (one-particle–one-hole) admixtures in the initial state and 2p-2h "final-state correlations." Arrows indicate the configurations linked by the β^- decay. The example shown is specific to ^{50}K$(0^-) \rightarrow$ ^{50}Ca(0^+). Alternative examples are shown of 2p-2h "final-state" admixtures corresponding to $\nu(Q = 3) \rightarrow \pi(Q = 4)$ or $\nu(Q = 2) \rightarrow \pi(Q = 3)$ transitions.

TABLE I. The first-order perturbative quenching factor $1 - q_\alpha(j_i j_f)$ calculated for $\nu(Q = 2) \rightarrow \pi(Q = 3)$ effects on the matrix elements of the three $\Delta j = 0$ $\nu(Q = 3) \rightarrow \pi(Q = 2)$ transitions with HF radial matrix elements and the H7B residual interaction. The decomposition into central and tensor contributions is given, the LS contribution is negligible.

Transition $\nu \rightarrow \pi$	2p-2h (%)	$1 - q_\alpha(j)$ Total	Central	Tensor
			M_0^S	
$0f_{5/2} \rightarrow 0d_{5/2}$	0.03	+0.0031	+0.1089	−0.1058
$1p_{3/2} \rightarrow 0d_{3/2}$	0.34	−0.0907	+0.0831	−0.1738
$1p_{1/2} \rightarrow 1s_{1/2}$	0.15	−0.0215	+0.0925	−0.1141
			M_0^T	
$0f_{5/2} \rightarrow df_{5/2}$	0.03	−0.0023	−0.1143	+0.1119
$1p_{3/2} \rightarrow 0d_{3/2}$	0.34	+0.0986	−0.0886	+0.1872
$1p_{1/2} \rightarrow 1s_{1/2}$	0.15	+0.0246	−0.1075	+0.1321

to calculate these matrix elements with HF wave functions it was assumed the proton $Q = 4$ orbits were bound by 0.2 MeV. This is a crude assumption which will cause an unknown but potentially large percentage error. However, it turns out that the effect of the $Q = 3 \rightarrow Q = 4$ transitions is small and so this assumption is justified. Three Q=3 → Q=4 transitions are not allowed by the Hartree-Fock condition which forbids purely $\Delta n = 2$ excitations such as $\pi 0d_{3/2}^{-1} 1d_{3/2}$. The results presented in Table I represent the effect for a full $\nu(1s, 0d)$ shell and an empty $\pi(0f, 1p)$ shell. The results of Table II assume a full subshell for the indicated ν orbit and an empty π subshell for the π orbit. The reader is cautioned that the first column in Table I contains the three $\Delta J = 0$ transitions possible in the model space assumed while the first column of Table II contains the perturbing transitions allowed by "final-state correlations."

In Table I, the contributions to the $1 - q_\alpha(j)$ are decomposed into central and tensor pieces into order to explicitly display the large role of the tensor force on these quenching factors. The large tensor contribution has been extensively discussed before for $A = 16$ (Ref. [11]), $A = 12$ (Ref. [12]), and $A = 208$ (Ref. [10]). It is seen to be large at $A \sim 40$ also. Because of this

TABLE II. The first-order perturbative quenching factor $1 - q_\alpha(j)$ calculated for $\nu(Q = 2) \rightarrow \pi(Q = 3)$ and $\nu(Q = 3) \rightarrow \pi(Q = 4)$ effects on the matrix elements of the three $\Delta j = 0$ $\nu(Q = 3) \rightarrow \pi(Q = 2)$ $(j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2})$ transitions with HF radial matrix elements and the H7B residual interaction. Results for individual orbits are given so that the results can be applied to nuclei with different N, Z values (different orbit occupancies) via Eq. (3). The result labeled $^{50}\mathrm{K} \rightarrow {}^{50}\mathrm{Ca}$ assumes for $^{50}\mathrm{Ca}$ a $^{40}\mathrm{Ca}$ core plus $\nu 0f_{7/2}^8 1p_{3/2}^2$.

Admixing transition $\nu \rightarrow \pi$ \ j	$1 - q_\alpha(j)$ M_0^S $\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	M_0^T $\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
$0d_{5/2} \rightarrow 0f_{5/2}$	+0.0190	−0.0300	+0.0034	−0.0222	+0.0300	−0.0036
$0d_{3/2} \rightarrow 1p_{3/2}$	−0.0044	−0.0541	−0.0051	+0.0060	+0.0622	+0.0061
$1s_{1/2} \rightarrow 1p_{1/2}$	−0.0361	−0.0066	+0.0048	+0.0409	+0.0064	−0.0048
Sum $(Q = 2) \rightarrow (Q = 3)$	−0.0215	−0.0907	+0.0031	+0.0246	+0.0986	−0.0023
$0f_{7/2} \rightarrow 0g_{7/2}$	+0.0364	+0.0070	+0.0174	−0.0425	−0.0070	−0.0182
$0f_{5/2} \rightarrow 1d_{5/2}$	−0.0170	−0.0875	a	+0.0180	+0.0795	a
$1p_{3/2} \rightarrow 1d_{3/2}$	−0.0239	a	+0.0122	+0.0202	a	−0.0092
$1p_{1/2} \rightarrow 2s_{1/2}$	a	−0.0153	−0.0024	a	+0.0108	+0.0018
$^{50}\mathrm{K} \rightarrow {}^{50}\mathrm{Ca}$	+0.0030	−0.0837	+0.0266	−0.0078	+0.0916	−0.0159

[a] Not allowed by the Hartree-Fock condition (see text).

TABLE III. Predicted values for the one-body-transition densities $D_0(j_i j_f)$ and matrix elements of Eq. (1) for $^{50}\mathrm{K}(0^-) \rightarrow {}^{50}\mathrm{Ca}(0^+)$.

νj_i	πj_f	$D_0(j)$	$M_0^S(j,\mathrm{eff})$	$\mathcal{M}_0^S(j)$	$M_0^T(j,\mathrm{eff})$	$\mathcal{M}_0^T(j)$
$0f_{5/2}$	$0d_{5/2}$	0.0422	+5.2914	+0.223	−115.347	−4.868
$1p_{3/2}$	$0d_{3/2}$	1.3947	−2.4822	−3.463	+44.066	+61.459
$1p_{1/2}$	$1s_{1/2}$	0.1723	+2.7383	+0.472	−51.643	−8.899
		Totals		−2.768		47.692

sensitivity to the composition of the interaction it is important to use as realistic an interaction as possible in these calculations. The one used here is derived from a G-matrix treatment of nucleon-nucleon data [32] and as such is representative of our best current knowledge of the nucleon-nucleon force.

The results of Table I and the $(Q = 2) \rightarrow (Q = 3)$ sums of Table II are applicable to the $N = 20, Z \leq 20$ isotones. They are directly comparable to the results obtained by Warburton *et al.* [7] in a calculation for $^{40}\mathrm{K}(0^-) \rightarrow {}^{40}\mathrm{Ca}(0^+)$ using a full $(0+2)\hbar\omega$ diagonalization in the WBMB model space. The two calculations are in good agreement. The matrix element M_0^S is responsible for $\Delta J = 0$ spin-dipole excitations as well as its role in β decay. The quenching of M_0^S due to 2p-2h correlations in the $^{40}\mathrm{Ca}$ ground state was recently considered by Boucher and Castel [34] whose α in the last column of their Table I is equivalent to $1 - q_S$ in present notation. Unfortunately they used a purely central surface-delta interaction and so, as is clear from Table I, their results are not applicable to real nuclei.

The prescription used to apply the results of Table II to $^{50}\mathrm{K}$ decay is

$$1 - q_\alpha = \sum_j \left[\frac{n_n(j)}{(2j+1)}\right]^{1/2} [1 - q_\alpha(j)] \ , \tag{3}$$

where n_n(j) is the occupancy of the neutron orbit j in the final state. The results are similar to those for $A = 40$ discussed above, the only non-negligible effect is on the $j = \frac{3}{2}$ transition.

What about second-order core-polarization effects? These are quite different for $\Delta\hbar\omega = 1$ operators than for the $\Delta\hbar\omega = 0$ operators ($M1$, $E2$, Gamow-Teller, etc.) with which most of us are most familar. For first-forbidden beta decay the repulsive nature of the particle-hole interaction is the dominating factor in determining the general form of the effect. Going to second order entails an expansion of the initial state from $1\hbar\omega$ to $(1+3)\hbar\omega$ and the final state from $(0+2)\hbar\omega$ to $(0+2+4)\hbar\omega$ where we specify the calculation including the perturbative first-order core-polarization correction just described. Various schematic calculations such as those described by Warburton *et al.* [7] indicate that the added $3\hbar\omega \rightarrow (2+4)\hbar\omega$ contribution will be roughly proportional and in phase with the original $0\hbar\omega \rightarrow (0+2)\hbar\omega$ contribution and will approximately compensate for the renormalization due to these added terms. We test this expectation by comparison to what is know about the rank-one and rank-two components of first-forbidden beta decay. Towner, Warburton, and Garvey [30] found extremely good agreement with experiment for a large collection of first-forbidden unique (rank-two) decays near $A = 40$. The calculation included only first-order core-polarization for which the average effect on the decay rates was a factor of seven. Similarly, the rank-one components of first-forbidden decays in the lead region are extremely well explained with first-order core polarization alone [17]. Note there is no reason to expect the rank-zero components to behave any differently in this respect than the rank-one and rank-two components. Thus higher-order core-polarization effects are expected to be small and are neglected.

D. Results

The D_0(j), $q_\alpha(j)$, and M_0^α(j) were combined via Eq. (1) as shown in Table III. To a very good approximation the prediction for the decay can be compared to experiment via a rank-zero matrix element $M_1^{(0)}$ which is defined experimentally by [7, 17]

$$B_1^{(0)} = (M_1^{(0)})^2 = \frac{9195 \times 10^5}{f_0 t} \ \mathrm{fm}^2, \tag{4}$$

and theoretically by

$$B_1^{(0)} = (M_1^{(0)})^2 = (\epsilon_{\mathrm{MEC}} M_0^T + a_S M_0^S) \ \mathrm{fm}^2, \tag{5}$$

where $a_S(Z, W_0, r_u)$ is a (largely) kinematical factor fully described previously [7, 17] and evaluated as 13.80 in the present case. From Eq. (4) we obtain an experimental rank-zero matrix element of $M_1^{(0)} = 34.4 \pm 3.4$ fm^2 and equating Eqs. (4) and (5) gives $\epsilon_{\mathrm{MEC}} = 1.52 \pm 0.07$ where the quoted uncertainty is purely experimental.

III. CONCLUSIONS

The main difficulty in the present endeavor is that of isolating the various contributions to the matrix element of M_0^T. This cannot be done experimentally, rather each must be estimated theoretically. Thus the impulse contribution and first-order core polarization were treated in detail and it was argued that higher-order core-polarization corrections are small compared to the first-order terms. The time is certainly ripe for a careful and detailed calculation of the MEC effect for those cases which have been considered carefully in the impulse approximation. However, in lieu of this, the estimates made to date are probably accurate enough to serve the present purpose of searching for other significantly observable in-medium effects. Most estimates of the enhancement due to pion-exchange processes have

been made for transitions near $A = 16$ (Ref. [5]). The only detailed calculations for heavier nuclei appear to be those of Kirchbach and Reinhardt [16] for $s_{1/2} \leftrightarrow p_{1/2}$ transitions for which they obtain enhancement factors as follows: $1.50(A = 16)$, $1.54(A = 18)$, $1.60(A = 96)$, and $1.40(A = 208)$. The present estimate of 1.52 for $A = 50$ is right in line with these predictions as were the previous best estimates of 1.64 at $A = 16$ (Ref. [5]) and 1.70 at $A = 96$ (Ref. [6]). In stark contrast, a recent result [17] of $\epsilon_{\rm MEC} = 2.01 \pm 0.05$ for $A = 205--212$ nuclei is in serious disagreement with the result predicted from mesonic enhancement alone. As was pointed out in that study of the lead region, the two most likely explanations for the large in-medium effect are (a) that the magnitude of the tensor force is seriously overestimated, and (b) that some mass-scaling effect such as that proposed by Rho [14] is at work. However, the situation is quite complex since the same scaling which is proposed to cause $M^*/M < 1.0$ may also weaken the tensor force [35]. In any case, the present result reinforces the possibility that the disagreement in the lead region is connected to the larger A (higher density?).

The decay of ^{50}K is seen to provide further valuable information on in-medium effects in nuclei. In view of this a more accurate and direct determination of the $\log f_0 t$ value would be most welcome.

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