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Successive energy ratios in medium- and heavy-mass nuclei as indicators of different kinds of collectivity

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The systematics of energy ratios of successive levels of collective bands in medium- and heavy-mass even-even nuclei are studied. A measure of their deviation from the vibrational and rotational limiting values is found to have different magnitude and angular momentum dependence in the vibrational, γ-unstable and rotational regions. The usefulness of this new criterion for distinguishing between different kinds of collective behavior is demonstrated in the case of intruder and octupole bands.

Energy ratios have been a useful tool in the study of medium- and heavy-nuclei since their introduction by Mallmann.[1] Knowing the excitation energies $E(J)$ of levels with angular momentum J, one can form the ratios $R(J/2)=E(J)/E(2)$. It was shown by Mallmann that $R(6/2)$ and $R(8/2)$ plotted against $R(4/2)$ show a universal behavior for collective nuclei ranging from the vibrational to the deformed limit. These ratios have been studied in the framework of the variable moment of inertia (VMI) model[2] and its extended versions, like the variable anharmonic vibrator model (VAVM).[3,4] In particular the ratio $R(4/2)$ has been widely used as an indicator of collectivity, having the value 2 in the vibrational limit, $\frac{10}{3}$ in the rotational limit, and values around 2.5 for γ-unstable nuclei. Recently it has been suggested[5] that plots of $R(6/4)$ versus $R(4/2)$ show a smooth systematic behavior, deviations from which indicate coexistence of collective and noncollective configurations.

In the present work we study the series of ratios $R(J+2/J)$, $J=2,4,6,\ldots$. Using this series we construct a quantity showing distinctly different behavior in the vibrational, rotational, and γ-unstable limits. Therefore this series can be used for the safe determination of the character of a collective band, especially in nuclei where mixing of different bands occurs, in which case the $R(4/2)$ ratio might be seriously affected (for an example, see Ref. 6). The applicability and usefulness of this new criterion in the case of octupole[7] and intruder[8] bands is demonstrated.

We start with the study of ground-state bands. In the rotational limit the members of these bands have excitation energies

$$E(J)=AJ(J+1), \tag{1}$$

where A is the rotational constant. Then in this limit we find

$$R(J+2/J)_{\rm rot}=\frac{(J+2)(J+3)}{J(J+1)}. \tag{2}$$

In the vibrational limit the members of the band are

$$E(J)=BJ, \tag{3}$$

so that the relevant ratio is

$$R(J+2/J)_{\rm vib}=\frac{J+2}{J}. \tag{4}$$

One can easily see that in both limits the ratio is decreasing with increasing J. The same is true for the difference

$$R(J+2/J)_{\rm rot}-R(J+2/J)_{\rm vib}=\frac{2(J+2)}{J(J+1)}, \tag{5}$$

which is always positive.

For a given band we construct for each J the quantity

$$r(J+2/J)=\frac{R(J+2/J)_{\rm exp}-R(J+2/J)_{\rm vib}}{R(J+2/J)_{\rm rot}-R(J+2/J)_{\rm vib}}, \tag{6}$$

where $R(J+2/J)_{\rm exp}$ is the experimental value of the ratio. It is clear that this ratio should be close to one for a rotational nucleus and close to zero for a vibrational nucleus, while it should have intermediate values for γ-unstable nuclei.

These systematics should hold for ground-state bands up to the point of backbending,[9] which can be read from the tables of Ref. 4. For other kinds of bands, as octupole[7] and intruder[8] bands, a band head energy must be taken away (for detailed examples, see below). In this case Eqs. (2) and (4) must be modified as follows

$$R(J+2/J)_{\rm rot}=\frac{(J+2)(J+3)-J_{\rm bh}(J_{\rm bh}+1)}{J(J+1)-J_{\rm bh}(J_{\rm bh}+1)}, \tag{7}$$

$$R(J+2/J)_{\rm vib}=\frac{J+2-J_{\rm bh}}{J-J_{\rm bh}}, \tag{8}$$

where $J_{\rm bh}$ is the angular momentum of the band head. In addition

$$R(J+2/J)_{\rm exp}=\frac{E(J+2)-E(J_{\rm bh})}{E(J)-E(J_{\rm bh})}. \tag{9}$$

Using Eqs. (7)–(9) in (6) we get results for these kinds of bands.

In Fig. 1 we show the results of the calculation for the ground-state bands of ten nuclei (data taken from Ref. 10). In 1(a) three vibrational nuclei are shown. The first point in the figures corresponds to $R(4/2)$, which is known to be between 2 and roughly 2.3 for vibrational nuclei. For each nucleus the ratios start with a small value and then increase with J, more rapidly in the beginning and slower at higher J's. In 1(b) we show results for four γ-unstable nuclei. In all cases the ratios obtain values around 0.4–0.6, increasing in the beginning and decreasing later on. It is known that for these nuclei the $R(4/2)$ ratio has values roughly in the region 2.4–2.8. Finally, in 1(c) we show three examples of rotational nuclei. Nuclei in this region have $R(4/2)$ ratios between roughly 2.9 and 3.33. In all cases the ratios start with a value very close to one and then constantly decrease. We remark that in the three regions both the magnitude of the $r(J+2/J)$ ratios and their dependence on J differ drastically. In particular (i) the magnitude is confined in the region 0.1–0.35 in the vibrational limit, takes values between 0.4 and 0.6 for γ-unstable nuclei, and lies in the area between 1.0 and 0.6 in the rotational limit. (ii) More importantly, the ratios as functions of J increase in the vibrational limit, show the opposite behavior, i.e., decrease, in the rotational limit, while they exhibit intermediate behavior (first increasing and then decreasing) in the γ-unstable case.

The same conclusions are drawn from the study of all ground-state bands given in Ref. 10. More examples of vibrational, transitional, and rotational nuclei are given in Tables I–III, respectively.

The clear separation in magnitude [observation (i)] as well as the clearly different functional dependence on J [observation (ii)] can be used for the characterization of a large variety of bands. So far we have examined levels of positive parity. In addition to them, levels of negative parity also exist. In some cases the positive-parity levels of the ground-state band and the negative-parity levels of the lowest $K=1$ band form a band with octupole deformation.[7] Some examples are the Th (Ref. 13) and Ba (Ref. 14) isotopes. Results for the negative-parity levels of some Th isotopes are shown in Fig. 2. In this case $J_{\rm bh}=1$ was used, since the negative-parity levels form a $K^{\pi}=1^-$ band starting with $J=1$. Clearly rotational behavior is seen in all cases. The same is true for the Ba isotopes (Ref. 14), not shown in the figure. For low J the experi-

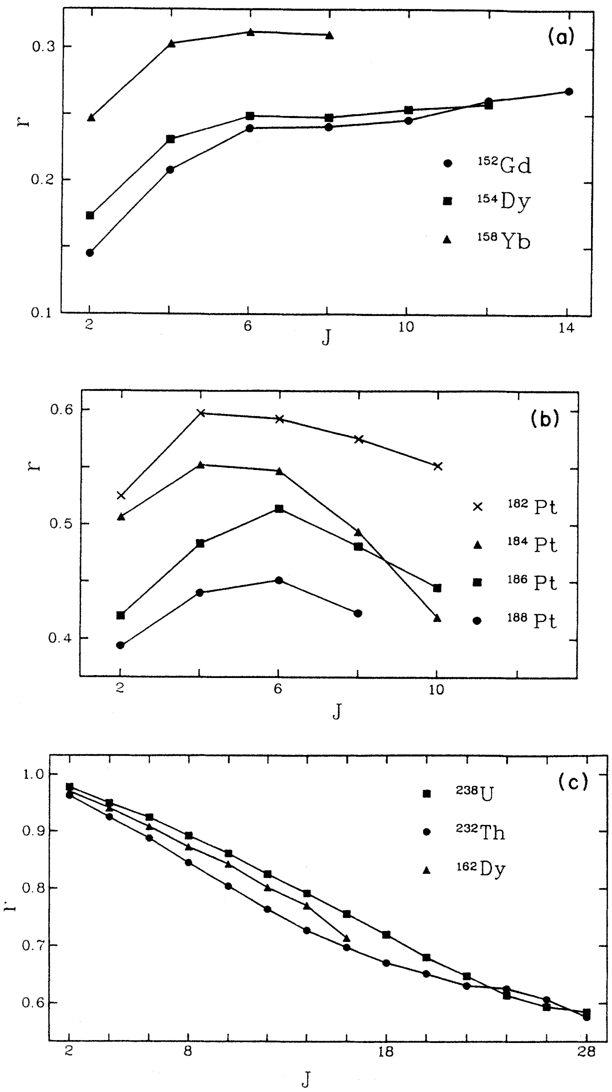

FIG. 1. (a) Examples of $r(J+2/J)$ ratios as functions of J in the vibrational, (b) γ-unstable, and (c) rotational regions. All data are taken from Ref. 10.

TABLE I. $r(J+2/J)$ ratios for ground-state bands of even-even vibrational nuclei. The $R(4/2)$ ratios for these nuclei are between 2.14 and 2.33. All data are taken from Ref. 10 except for ^{176}Pt (Ref. 11).

Nucleus	$r(4/2)$	$r(6/4)$	$r(8/6)$	$r(10/8)$	$r(12/10)$	$r(14/12)$	$r(16/14)$
^{74}Se	0.1106	0.2281	0.2625	0.2904	0.3615	0.3944	0.4099
^{74}Kr	0.1678	0.4294	0.5491	0.5977			
^{82}Sr	0.2371	0.2966	0.3183	0.3294			
^{116}Xe	0.2493	0.2836	0.2850	0.3231			
^{152}Gd	0.1455	0.2078	0.2367	0.2410	0.2459	0.2609	0.2692
^{154}Dy	0.1737	0.2313	0.2491	0.2475	0.2546	0.2586	
^{156}Er	0.2349	0.3032	0.3361	0.3389			
^{158}Yb	0.2472	0.3032	0.3294	0.3255			
^{176}Pt	0.1023	0.1737	0.2839	0.3649	0.4132	0.4321	

TABLE II. $r(J+2/J)$ ratios for ground-state bands of even-even transitional nuclei. The $R(4/2)$ ratios for these nuclei are between 2.38 and 2.84. All data are taken from Ref. 10.

Nucleus	$r(4/2)$	$r(6/4)$	$r(8/6)$	$r(10/8)$	$r(12/10)$	$r(14/12)$
^{76}Se	0.2851	0.3332	0.2937	0.2332		
^{76}Kr	0.3299	0.4936	0.5666	0.5845	0.5237	
^{78}Kr	0.3454	0.4446	0.4727	0.4378		
^{118}Xe	0.3014	0.3736	0.3967	0.3868	0.3443	
^{122}Xe	0.3761	0.4517	0.4689	0.4346		
^{124}Xe	0.3617	0.4365	0.4517	0.3965	0.1115	
^{124}Ba	0.6309	0.6417	0.6081	0.5299		
^{126}Ba	0.5832	0.6227	0.6152	0.5701		
^{128}Ba	0.5156	0.5723	0.5840	0.5698		
^{130}Ba	0.3917	0.4445	0.4483	0.3996		
^{158}Er	0.5575	0.5684	0.5413	0.4964	0.4290	
^{160}Yb	0.4696	0.4947	0.4743	0.4209		
^{162}Hf	0.4225	0.4556	0.4405	0.3907		
^{192}Os	0.6150	0.6266	0.6188	0.5968	0.5876	
^{182}Pt	0.5250	0.5973	0.5924	0.5752	0.5518	
^{184}Pt	0.5065	0.5525	0.5470	0.4939	0.4193	
^{186}Pt	0.4196	0.4830	0.5131	0.4809	0.4454	
^{188}Pt	0.3938	0.4400	0.4512	0.4227		
^{196}Pt	0.3491	0.4025	0.3774	0.2813		
^{222}Th	0.2995	0.3422	0.3276	0.3108	0.3051	0.3036

mental ratios overshoot the rotational value.

An interesting category of bands are the intruder bands.[8] The equivalence between their description in terms of particle-hole excitations across a closed shell in the spherical shell model and a description starting from the Nilsson model has been recently demonstrated.[15] For recent experimental findings and theoretical approaches see Refs. 16 and 17, respectively. The results for some intruder bands in Hg isotopes are also shown in Fig. 2 (Refs. 10 and 18). We used $J_{bh}=0$ in these cases, since these are $K^\pi=0^+$ bands, starting with $J=0$. Clearly rotational behavior is seen in all cases, as expected.[8] Because of strong mixing,[19] the $E(2)$ and $E(4)$ levels are severely influenced, so that the $R(4/2)$ ratio is not a useful indicator in these cases. For the same reason, $r(4/2)$ has been left out from the figure.

We should remark at this point that the ratios proposed in this paper should be used in parallel with the well-known methods of classifying bands and not as a substitute. It is clear, for example, that in order to decide which levels form a band, $B(E2)$ transition probabilities should be checked, requiring that levels of the same band are connected by strong $B(E2)$ transitions. The usefulness of the present method shows up mainly after one has decided which levels form each band. The $r(J+2/J)$ ratios offer a criterion for the characterization of a band as rotational, vibrational, or γ unstable, which is much more general than the usual criterion of the $R(4/2)$ ratio. While the latter criterion cannot be used in cases in which $E(2)$ and/or $E(4)$ are influenced by mixing, as in the case of intruder bands, the criterion based on the $r(J+2/J)$ ratios gives the right answer. In short, the $r(J+2/J)$ ratios offer a criterion for the characterization of a band as rotational, vibrational, or γ unstable, which depends on all the levels of the band and not just on the first two levels of the band, as the $R(4/2)$ ratio.

The different behavior observed in the vibrational and rotational limits can be qualitatively understood as follows. Rotational bands are known to be well described by

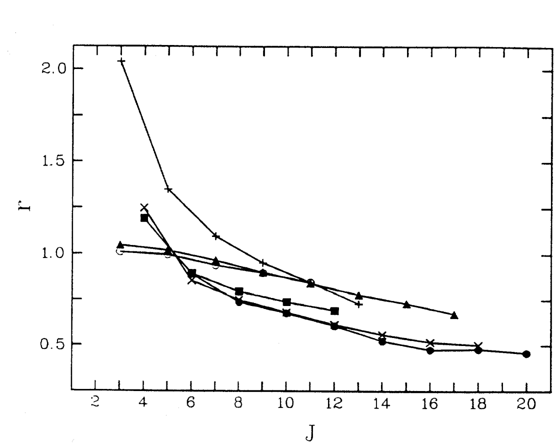

FIG. 2. Examples of $r(J+2/J)$ ratios for odd-spin levels of octupole bands for ^{224}Th (Ref. 13) (plusses), ^{226}Th (Ref. 13) (solid triangles), ^{228}Th (Ref. 13) (open circles), as well as for levels of intruder bands for ^{184}Hg (Ref. 10) (solid squares), ^{186}Hg (Ref. 10) (crosses), ^{188}Hg (Ref. 18) (solid circles).

TABLE III. $r(J+2/J)$ ratios for ground-state bands of even-even rotational nuclei. If for a given nucleus more than seven ratios are known, the extra ratios are given in the next line, in increasing order [i.e., the second line contains the ratios $r(18/16), r(20/18), \ldots$]. The $R(4/2)$ ratios for these nuclei are larger than 2.93. Data for ^{244}Pu are taken from Ref. 12, while for the rest of the nuclei from Ref. 10.

Nucleus	$r(4/2)$	$r(6/4)$	$r(8/6)$	$r(10/8)$	$r(12/10)$	$r(14/12)$	$r(16/14)$
^{152}Sm	0.7568	0.7150	0.6795	0.6478	0.6206	0.5948	
^{154}Gd	0.7610	0.7242	0.6857	0.6493	0.6174	0.5838	0.5439
^{156}Gd	0.9296	0.8816	0.8335	0.7814	0.7295	0.6682	0.6101
^{156}Dy	0.6992	0.6770	0.6434	0.6078	0.5740	0.5392	0.4511
^{162}Dy	0.9703	0.9412	0.9087	0.8728	0.8433	0.8021	0.7706
^{164}Er	0.9576	0.9194	0.8775	0.8325	0.7897	0.7297	
^{166}Er	0.9665	0.9307	0.8851	0.8310	0.7719	0.7096	0.6537
^{168}Yb	0.9497	0.9043	0.8505	0.7840	0.7319	0.6632	0.6026
^{170}Yb	0.9696	0.9457	0.9097	0.8713	0.8238	0.7488	0.6277
^{174}Hf	0.9514	0.9082	0.8562	0.7975	0.7338	0.6628	0.6075
^{176}W	0.9070	0.8352	0.7786	0.7036	0.6357	0.5744	0.5248
^{178}Os	0.7636	0.6840	0.6159	0.5719	0.5497	0.5400	0.5245
^{232}Th	0.9629	0.9254	0.8881	0.8453	0.8043	0.7641	0.7271
	0.6974	0.6708	0.6518	0.6308	0.6259	0.6069	0.5760
^{236}U	0.9779	0.9541	0.9252	0.8953	0.8605	0.8071	0.7887
	0.7477	0.7108	0.6759	0.6484	0.6302	0.6193	0.6104
^{238}U	0.9780	0.9500	0.9243	0.8924	0.8615	0.8255	0.7919
	0.7560	0.7202	0.6803	0.6477	0.6142	0.5938	0.5846
^{242}Pu	0.9804	0.9623	0.9409	0.9107	0.8851	0.8569	0.8282
	0.7995	0.7715	0.7414	0.7054	0.6453		
^{244}Pu	0.9778	0.9439	0.9299	0.9057	0.8798	0.8465	0.8160
	0.7817	0.7440	0.6936	0.5938	0.4745		
^{248}Cm	0.9885	0.9560	0.9437	0.9210	0.8950	0.8632	0.8270
	0.7871	0.7444	0.7038	0.6695	0.6455	0.6341	

an expansion of the form

$$E(J)=AJ(J+1)+B[J(J+1)]^2+C[J(J+1)]^3 + D[J(J+1)]^4+\ldots , \tag{10}$$

in which A is positive, B is negative and roughly 3 orders-of-magnitude smaller than A, C is positive and roughly 6 orders-of-magnitude smaller than A, D is negative and roughly 9 orders-of-magnitude smaller than A.[12] Using typical values of A, B, C, D (as these given in Ref. 12) in Eq. (10) and then using the results in Eq. (6), one easily verifies that $r(J+2/J)$ ratios decreasing with increasing J are obtained. It is well known that rotational stretching is the physical mechanism making the higher-order terms in the expansion necessary. Therefore the decrease of the $r(J+2/J)$ ratios with increasing J is due to stretching. An expansion alternative to Eq. (10) is the Harris expansion,[20] in terms of even powers of the angular velocity ω. The equivalence of the Harris formula to the VMI model has been demonstrated long ago[21] and a fully microscopic foundation has been given.[22]

In the vibrational limit one can use the formula

$$E(J)=AJ+BJ^2+\ldots . \tag{11}$$

In this case one can easily check (using the data of Ref. 10) that B is positive and roughly 1 to 2 orders-of-magnitude smaller than A. Using such values of the parameters in Eq. (11) and then using the results in Eq. (6) one obtains a sequence of ratios increasing with J. Anharmonicities is, therefore, the physical reason behind the increase of the $r(J+2/J)$ ratios with increasing J in the vibrational limit.

In conclusion, we have shown that the energy ratios $R(J+2/J)$ and the quantities $r(J+2/J)$ are useful tools for the characterization of collective bands. $r(J+2/J)$ shows different magnitude and J dependence in the vibrational, γ-unstable, and rotational limits. They are particularly useful in cases in which the first few levels of a band are strongly influenced by mixing, so that the $R(4/2)$ ratio is not a reliable indicator, as in some cases of intruder bands.

The extension of the present method to superdeformed bands,[23,24] as well as to odd-mass and odd-odd nuclei is receiving attention. Similar systematics for $B(E2)$ values can also be instructive.

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