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Strong correlation and saturation of $E2$ and $M1$ transition strengths in even-even rare-earth nuclei

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In heavy deformed even-even nuclei the orbital $M1$ strength below 4 MeV excitation, the so-called "scissors mode," shows a striking correlation with the $E2$ strength to the first excited 2^+ states. A saturation effect, common to both $E2$ and $M1$ transitions, is observed before midshell. The correlation and the saturation effect are discussed.

It has long been recognized that the collective features of deformed nuclei can be considered as a manifestation[1] of residual nucleon-nucleon interactions. In a pioneering work, Federman and Pittel[2] showed that the nuclear deformation is caused by the interplay between the long-range quadrupole force and the short-range pairing force among the valence nucleons. Perhaps the best established examples of low-lying collective excitations are the first 2^+ and 4^+ levels in heavy deformed even-even nuclei. By definition, the square of the mass deformation parameter δ is proportional (see, for example, Ref. 3) to the $E2$ transition strength to the first 2^+ state, i.e., $B(E2,0_1^+\rightarrow 2_1^+)$.

To schematically represent the influence of residual interactions on nuclear collectivity Casten and co-workers[4–6] have employed N_pN_n and $P=N_pN_n/(N_p+N_n)$ schemes, where N_p and N_n are the number of valence protons and neutrons outside closed shells, respectively. The factor P, a normalized form of N_pN_n, can be viewed as giving the averaged numbers of p-n interactions compared to like nucleon interactions. Valence nucleon models such as the interacting boson model in its version two (IBM-2) predict[7,8] that the ground-state transitions from the 2_1^+ level ($E2$) and orbital $M1$ transitions from the low-lying 1^+ levels bear a simple relation to the total number of valence bosons and the P factor, respectively. This observation leads to the conclusion that the experimental data can be used to deduce effective charges, g factors and the number of valence bosons for each nucleus on an individual basis. Alternately, we can use the data from a large number of nuclei to identify the shortcomings of the models.

The purpose of this Rapid Communication is twofold. In recent years, thanks to the concerted efforts of (e,e') and nuclear resonance fluorescence (NRF) groups,[9–11] orbital $M1$ transition strengths [$B(M1)$] for the so-called "scissors mode," originally found[12] in ^{156}Gd, have been reliably determined in rare-earth even-even nuclei up to about 4 MeV excitation energy. Since it was recently shown[11] that $B(M1)\propto\delta^2$ for Samarium isotopes, we investigate to determine if a conspicious correlation persists between the $B(M1)$ and the $B(E2,\ 0_1^+\rightarrow 2_1^+)$ strengths over a wide mass range and in turn for a large range of P values. Second, we address the question of saturation of transition strengths[6,13] near the middle major shells.

As our intention is to examine the systematic trend in a nearly model-independent fashion, we assume only the major shell closures of $Z=50,82$ and $N=82,126$ in counting the number of valence nucleons. Figure 1(a) shows a plot of the $B(E2,\ 0_1^+\rightarrow 2_1^+)$ values[14] in Weisskopf units (W.u.) versus P. As can be seen, these transition strengths increase slowly with P for $0\leq P\leq 3$, steeply rise to a maximum of about 200 W.u. as P changes from 3 to 6, and stay nearly constant for larger P values. This saturation of $B(E2)$ strengths before midshell has been noticed[6,13] earlier. Figure 1(b) shows the orbital $B(M1)$ strengths for the same nuclei as a function of P. The experimental values are taken from (e,e') and NRF results and are also listed in Table I, where the individual references are cited.

A glance at Figs. 1(a) and 1(b) makes clear that a striking correlation persists between the $B(M1)$ and $B(E2)$ values for the entire region of $0\leq P\leq 8$ with one exception. The $B(M1)$ strength in ^{164}Dy is nearly 50% larger than the mean saturation value. We note that Freeman *et al.*[22] concluded that one group of low-lying 1^+ levels are not of the collective type in this nucleus. Though this explanation is not entirely satisfying, the abnormal behavior of ^{164}Dy may be due to the complex structure of these levels. We also stress that the inclusion of the $Z=64$ subshell closure for the nuclei with N less than 90 would not change the common dependence of both transition strengths on P. The only impact would be a smoothing of the steep rise in the range $3\leq P\leq 6$. To quantitatively illustrate the correlation, we fitted the data with an empirical relation for the strength as a function of P as

$$B(E2,M1)=a_1+\frac{a_2}{1+\exp[(c-P)/d]} \qquad (1)$$

where a_1, a_2, c, and d are fit parameters. The fits, shown as solid curves in Figs. 1(a) and 1(b), resulted in $a_1=12.3$ (0.36), $a_2=193.1$ (2.2), $c=4.45$ (4.1), and $d=0.57$ (0.32) for $E2$ ($M1$) transition strengths, respectively. It is interesting to note that, despite larger experimental errors in the $M1$ strength, both are described by the same empirical relation. Also, the parameters c and d which characterize the shape of the curves, are very nearly the same for both multipolarities. Keeping in mind that δ^2 is

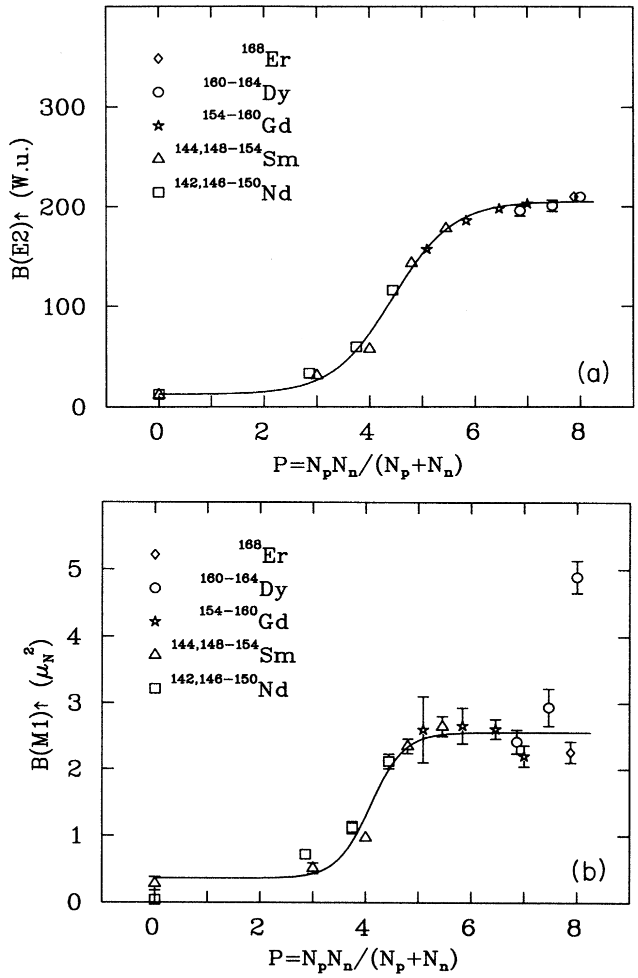

FIG. 1. (a) Plot of $E2$ transition strength of the 2_1^+ states in even-even rare-earth nuclei, indicated in the figure vs P. The solid line corresponds to the fit of Eq. (1). (b) Plot of summed $B(M1)$ strength for the low-lying levels ($E_x \leq 4$ MeV) for the nuclei as in (a) vs P. The transition strengths are taken for (e,e') and NRF experiments (see Table I). The solid line corresponds to the fit of Eq. (1).

proportional to $B(E2)$, we can conclude that there is also a simple δ^2 dependence of the $M1$ strength over a large mass range. This is in agreement with the recent findings of Ziegler et al.,[11] without resorting to the $B(E2)$ data.

It is of interest to get an intuitive physical picture of the behavior of transition strengths as a function of P. Nuclei of small P, with a few valence nucleons, are spherical vibrational and weakly deformed systems and the $E2$ transition strengths are quite small. This persists for $P \lesssim 3$. The rapid increase of transition strength for $4 \leq P \leq 5$ is correlated with the onset of deformation. It can be viewed[2] in terms of increasingly dominant quadrupole interaction strength V_q over the pairing strength V_p as follows: The relative contribution of the V_q with respect to the V_p is given approximately by

$$\frac{V_q}{V_p} \simeq \frac{V_{np}}{V_{NN}} \frac{N_p N_n}{N_p + N_n} = \frac{V_{np}}{V_{NN}} P . \tag{2}$$

TABLE I. Summed $M1$ ground-state transition strength $B(M1)\uparrow(E_x \leq 4$ MeV) for even-even rare-earth nuclei. The values are taken from (e,e') and NRF experiments of the listed references.

Nucleus	$B(M1)$ (μ_N^2) (e,e')	NRF	Reference
^{142}Nd	···	0.02 ± 0.01	15
^{146}Nd	···	0.72 ± 0.06	15
^{148}Nd	···	1.12 ± 0.09	15
^{150}Nd	1.40 ± 0.23	2.12 ± 0.11	15,16
^{144}Sm	···	0.28 ± 0.10	11
^{148}Sm	···	0.51 ± 0.08	11
^{150}Sm	···	0.97 ± 0.06	11
^{152}Sm	2.09 ± 0.27	2.35 ± 0.11	11,16
^{154}Sm	2.53 ± 0.10	2.65 ± 0.15	11,16
^{154}Gd	2.60 ± 0.50	···	17
^{156}Gd	2.30 ± 0.50	2.66 ± 0.27	17,18
^{158}Gd	2.30 ± 0.50	2.61 ± 0.15	17,18
^{160}Gd	···	2.20 ± 0.16	18
^{160}Dy	···	2.42 ± 0.18	19
^{162}Dy	···	2.94 ± 0.28	19
^{164}Dy	5.17 ± 0.52	4.82 ± 0.24	19,20
^{168}Er	2.50 ± 0.21	2.20 ± 0.16	16,21

The typical p-n interaction strength is $V_{np} \simeq 200$–300 keV, while the pairing strength is $V_{NN} \simeq 1$ MeV, where NN denotes an identical nucleon pair (nn or pp). Thus we get $V_q > V_p$ at $P \simeq 4$ the onset of deformation. However, it is not immediately obvious as to why $E2(M1)$ strengths saturate for $P \geq 6$.

We shall first examine if the IBM-2 can account for the trend of transition strengths. From Fig. 1(a), we can see [quite independent of Fig. 1(b)] that for $P \geq 4$, nuclei are well deformed. In the SU(3) limit, approximately valid for good rotors, the IBM-2 makes distinct predictions[8] for transition strengths, viz.,

$$B(E2) = (e_\nu N_\nu + e_\pi N_\pi)^2 \frac{(2N+3)}{N} \tag{3}$$

and

$$B(M1) = \frac{3}{4\pi}(g_\nu - g_\pi)^2 \frac{8 N_\nu N_\pi}{(2N-1)} , \tag{4}$$

where $N_\nu(=N_n/2)$ and $N_\pi(=N_p/2)$ are valence neutron and proton boson numbers, respectively. For large N (≥ 10), the above relations can be approximated as $B(E2) = e_B^2 N^2$, where e_B is an effective boson charge[8] and $B(M1) = 3/(4\pi) g^2 P$ with $g = g_\nu - g_\pi$. It is thus apparent that the IBM-2 fails to exhibit the simple correlation between the $E2$ and the $M1$ strengths at least under the assumption of SU(3) and F-spin symmetry.

From this we can deduce two important results. First, the number of effective pair interactions saturate in heavy nuclei around $P \simeq 6$, which is generally before the major shells are half filled, especially for the $50 \leq Z \leq 82$ and $82 \leq N \leq 126$ region. The addition of more nucleons does not increase the number of effective interactions. In Ref. 6 the saturation of $B(E2)$ strength was interpreted in terms of a saturation of quadrupole p-n interactions, calculated in a Nilsson scheme. There, a key ingredient was the poor overlap of proton and neutron orbits oriented at very different angles to the symmetry axes. In effect, nucleons in such orbits are on average, quite far apart. The fact that the low-lying $B(M1)$ strength also shows the same feature implies that a common mechanism is operative for these seemingly different phenomena. We are tempted to conclude that orbital $M1$ transitions and $E2$ ground-state transitions have common origins.

Second, the correlation of $B(E2)$ and $B(M1)$ strengths is not obvious in any F-spin symmetric limit of the IBM-2. While the increase of transition strengths for $2 \leq P \leq 4$ could be seen as a transition between symmetric limits, we cannot explain the saturation effects for larger P in a natural way. The correlation and the saturation effect should therefore have a strong impact on the determination of symmetry-breaking terms in the IBM Hamiltonian and should further give important constraints for the determination of effective charges and boson numbers in the $M1$ and $E2$ transition operators. It might also raise the question of whether the p-n interactions responsible for nuclear deformation are fully taken into account in the IBM-2. As Talmi pointed out,[23] one essential reason why the p-n interaction is so important in inducing configuration mixing is that the $T=0$ two-nucleon matrix elements are not subject to the same Pauli restrictions as the $T=1$ matrix elements and result in large matrix elements leading to collectivity. Such configurations do not exist in a two-particle system of like nucleons in identical orbits. It appears that the saturation effect can be, at least intuitively, understood as due to the saturation of effective numbers of nucleon-nucleon interactions.

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