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Unitarity consideration of pion production in relativistic heavy-ion collisions

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Pion production in relativistic heavy-ion collisions is examined with the use of the unitarity relation. We show that the density of states for pion production in an absorptive nuclear medium is not inversely proportional to the in-medium pion group velocity; it depends equally on the imaginary part of the pion self-energy.

In recent years a great deal of experimental effort has been devoted to using relativistic heavy-ion collisions to create highly excited nuclear matter. As pions are the most copiously produced particles, there has been considerable theoretical interest in relating pion multiplicity and pion production cross sections to nuclear temperature and the nuclear equation of state. Because the density of final pion states for calculating the $N_1+N_2\rightarrow N_1'+N_2'+\pi$ cross section in free space is proportional to $\delta(E_{N_1}+E_{N_2}-E'_{N_1}-E'_{N_2}-\omega_\pi)=\delta(k-k_\pi)|d\omega_\pi/dk_\pi|^{-1}$, it has been proposed[1] that, as a result of the interaction [through the pion self-energy $\Pi(\omega_\pi,k_\pi)$] between the pion and the medium, the pion group velocity in the medium will be modified and the density of states available for pion production will be changed by a factor[1,2]

$$F=(|d\omega_\pi/dk_\pi|_{\Pi=0})/(|d\omega_\pi/dk_\pi|_{\Pi\neq0})\,. \tag{1}$$

The presence of the in-medium pion group velocity $|d\omega/dk_\pi|_{\Pi\neq0}$ in Eq. (1) has its origin in the replacement of the free-space pion energy in the above δ function by the in-medium pion energy that satisfies the dispersion relation $\omega_\pi^2=m_\pi^2+\mathbf{k}_\pi^2+\mathrm{Re}\Pi$. A calculation based on this dispersion relation indicated[3] that at high nuclear density ($\rho\gtrsim 3\rho_0$) the in-medium pion group-velocity equals zero at $k_\pi/m_\pi\simeq1.5$ leading to an infinite F. It was further shown that one could expect a large enhancement of the pion-to-nucleon production ratio in relativistic heavy-ion collisions in which very high nuclear densities are created.[1,3] Such theoretical considerations have subsequently been employed in several calculations aimed at using dilepton production as a probe of pion dynamics in heavy-ion collisions.[4,5] Again, it was noted that production cross sections were significantly enhanced in kinematical regions where the in-medium pion group velocity is very small.[4] In a more recent calculation,[6] it has been pointed out that Eq. (1) can lead to singularities that have to be regulated with the use of the imaginary part of the pion propagator. However, the theoretical aspect of Eq. (1) has not been discussed. Furthermore, the effects of the coupling between channels of different pion multiplicities on pion propagation and the contribution to pion self-energies from true pion absorption on two nucleons were not considered.

In this paper, we present a formal analysis of the density of states for pion production and of the reactive content of pion self-energy, using the unitarity relation. Our main purpose is to present a general formulation for the density of states and the pion self-energy for multipion production in heavy-ion collisions.

The unitarity relation that clearly separates the elastic and inelastic contributions to the total cross section has the form[7]

$$T_{el}-T_{el}^\dagger=-2i\pi T_{el}\delta(E-H_0)T_{el}^\dagger + \Omega^\dagger(E)(V_{\mathrm{opt}}-V_{\mathrm{opt}}^\dagger)\Omega(E)\,, \tag{2}$$

or, upon using $T_{el}=-\lambda f_{el}$,

$$\mathrm{Im}[f_{el}(E)]=\lambda f_{el}\{-\mathrm{Im}[G_0^+(E)]\}f_{el}^\dagger + \frac{1}{\lambda}\Omega^\dagger(E)\{-\mathrm{Im}[V_{\mathrm{opt}}(E)]\}\Omega(E)\,, \tag{3}$$

where $G_0^+=(E-H_0+i\epsilon)^{-1}$. f is the elastic-scattering amplitude and $\lambda=(4\pi^2m_{\mathrm{red}})^{-1}$ when the plane-wave states are normalized according to $\langle\mathbf{k}'|\mathbf{k}\rangle=\delta(\mathbf{k}'-\mathbf{k})$, with m_{red} being the reduced mass of the nucleus-nucleus system. Furthermore, V_{opt} is the optical potential, E is the total energy of the interacting system, and $\Omega\equiv\Omega_{\mathrm{opt}}$ is the wave operator. Taking the forward-scattering matrix element of Eq. (3) and multiplying both sides by $4\pi/k$, we obtain the optical theorem with the first and second terms giving rise, respectively, to the total elastic and reaction cross sections. Equation (2) implies, therefore, that the density of states for the elastic and inelastic processes are, respectively, given by $-\mathrm{Im}(G_0^+)/\pi$ and $-\mathrm{Im}(V_{\mathrm{opt}})/\pi$.

Thus, the unitarity relation connects the discontinuity (or the imaginary part) of an optical potential to cross sections for inelastic processes.[7,8] Because the pion self-energy employed in nuclear physics is an effective interaction between the pion and the medium in the sense of a pion optical potential, we can anticipate that the imaginary part (or the reactive content) of the pion self-energy will affect the density of states for pion production. In the following, we will relate $\mathrm{Im}(V_{\mathrm{opt}})$ to the pion self-energy.

A well-known decomposition of the optical potential is[9]

$$V_{\mathrm{opt}}(E)=PVP+PVQ(E-QHQ+i\epsilon)^{-1}QVP\,, \tag{4}$$

where P and Q are the projectors that select, respectively, the elastic and inelastic channel spaces, and V is Hermitian. However, in order to analyze in detail the pion production, we will generalize Eq. (4) by making explicit various inelastic channel subspaces. We first consider the simpler case where only one pion is produced. Because pion production proceeds primarily through the elementary $NN\rightarrow NN\pi$ process, we divide the reaction space into

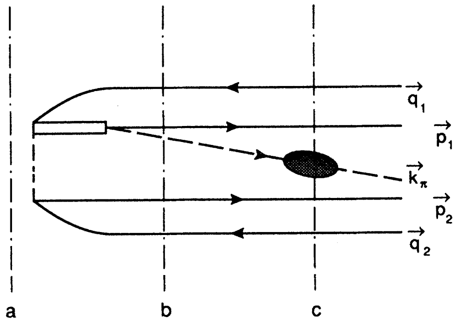

FIG. 1. Schematic diagram for the model space. The dashed, solid, and vertical lines denote, respectively, the pion, the nucleon (or hole), and the channel space. The oval represents the self-energy insertion. The unfilled box represents the bare isobar.

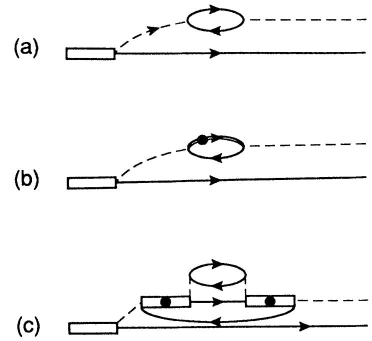

FIG. 2. Born-term insertion to pion propagator. Same caption as for Fig. 1, except that the box containing the dot represents the physical Δ. Diagrams having the pion lines crossed are not shown but are included in the calculations.

three subspaces: (a) the elastic channel space; (b) the doorway-state channel containing one pion and two nucleons in the continuum; and (c) the subspace containing all the other states; i.e., states having more than two nucleons in the continuum. For the sake of having a simple notation, we will not write explicitly the nucleon-nucleon interactions or nucleon self-energies. Upon introducing the projectors P_a, P_b, P_c with $P_a+P_b+P_c\equiv 1$, we can rewrite Eq. (4) as[10]

$$V_{\rm opt}(E)=U_{aa}+U_{ab}(E^+-K_b-U_{bb})^{-1}U_{ba}\,, \qquad (5)$$

where $E^+\equiv E+i\epsilon$ and the interactions U are defined by

$$U_{ij}=V_{ij}+V_{ic}(E^+-K_c-V_{cc})^{-1}V_{cj}\quad (i,j=a,b)\,, \qquad (6)$$

with $U_{ij}=P_iUP_j$, $V_{ij}\equiv P_iVP_j$, and $K_i\equiv P_iKP_i$ being the kinetic energy in the space i.

We illustrate the different channel spaces in Fig. 1, where the shaded oval represents the pion self-energy. In this doorway model, the reaction proceeds through channel b, i.e., $V_{ac}=0$. This model should be a good description of heavy-ion collisions at energies above the pion production threshold. Consequently, Eq. (6) leads to $U_{aa}=V_{aa}$, $U_{ab}=V_{ab}$, and $U_{ba}=V_{ba}$. In Eq. (5), $U_{bb}=V_{bb}+V_{bc}(E^+-K_b-V_{cc})^{-1}V_{cb}$. We further write $V_{bb}\equiv V_{bb}^{(1)}+V_{bb}^{(2)}$, with $V_{bb}^{(1)}$ and $V_{bb}^{(2)}$ being, respectively, the interactions between the pion and the nucleons having the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ in Fig. 1. The second term of U_{bb} represents, therefore, the interaction between the pion and the rest of the nuclear system; this gives rise to the pion self-energy.

The insertions into the pion line are shown in Fig. 2. The particle-hole excitation diagram in the first row represents true pion absorption on one nucleon. The Δ-hole excitation diagram in the second row is related to quasifree nucleon knockout by the pion. Finally, the diagram in the third row depicts the contribution to pion self-energy from two-nucleon true pion absorption through the formation of a physical Δ having the experimentally observed mass and width. Upon introducing into Eq. (6) a complete set of states χ_b^+, which are the solutions of the equation $(E-K_b-V_{bb}^{(1)})\chi_b^+=0$, we obtain

$$V_{\rm opt}(E)=V_{aa}+\int d\alpha d\beta V_{ab}|\chi_{b,\alpha}^+\rangle\langle\chi_{b,\alpha}^+|\left(E^+-K_b-V_{bb}-V_{bc}\frac{1}{E^+-K_c-V_{cc}}V_{cb}\right)^{-1}|\chi_{b,\beta}^+\rangle\langle\chi_{b,\beta}^+|V_{ba}\,. \qquad (7)$$

Here, α and β denote discrete as well as continuous variables needed to specify the state χ_b^+. One can show that the effect of the interaction $V_{bb}^{(1)}$ is to dress the bare isobar in Fig. 2 to generate the physical mass and width of the isobar. Thus, we have the relations $V_{ab}|\chi_b^+\rangle=t_{ab}|\phi_b\rangle$ and $\langle\chi_b^+|V_{ba}=\langle\phi_b|t_{ba}^\dagger$, where ϕ denotes a plane-wave state and t the pion production operator involving the physical Δ isobar.

Without loss of generality, we will evaluate Eq. (7) using the nuclear-matter approximation. We can further cast the effective one-body propagator in the relativistic form. The forward-scattering matrix element of Eq. (7) is

$$\langle\mathbf{k}|\Omega^\dagger V_{\rm opt}\Omega|\mathbf{k}\rangle=\langle\mathbf{k}|\Omega^\dagger V_{aa}\Omega|\mathbf{k}\rangle+\int d\mathbf{k}_\pi d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{q}_1 d\mathbf{q}_2\delta(\mathbf{k}_\pi+\mathbf{p}_1+\mathbf{p}_2-\mathbf{q}_1-\mathbf{q}_2)\langle\mathbf{k}|\Omega^\dagger t_{ab}^\dagger|\phi_b\rangle$$
$$\times\langle\chi_b^+|\mathbf{k}_\pi,\mathbf{p}_1,\mathbf{p}_2,\mathbf{q}_1,\mathbf{q}_2\rangle\frac{\langle\mathbf{k}_\pi,\mathbf{p}_1,\mathbf{p}_2,\mathbf{q}_1,\mathbf{q}_2|\chi_b^+\rangle\langle\phi_b|t_{ba}\Omega|\mathbf{k}\rangle}{W^2-[m_\pi^2+\mathbf{k}_\pi^2+\Pi(W,\mathbf{k}_\pi)]}\,, \qquad (8)$$

where Π is the pion self-energy. Further,

$$W=E-m_\Delta-\langle V_{bb}^{(2)}\rangle-E_N(\mathbf{p}_1)-E_N(\mathbf{p}_2)+h_N(\mathbf{q}_1)+h_N(\mathbf{q}_2)\equiv W'-E_N(\mathbf{p}_1)-E_N(\mathbf{p}_2)\,,$$

with $\langle V_{bb}^{(2)} \rangle$ denoting the energy shift caused by the pion interaction with the nucleon of momentum $\mathbf{p}_2$, and E_N and h_N the nucleon and hole energies, respectively. We emphasize that the off-shell pion energy W is a variable depending on many-body kinematics. Because V_{aa} is real, the first term of Eq. (8) does not contribute to $\text{Im}(V_{\text{opt}})$. Consequently, the density of states for pion production is given by the imaginary part of the second term in Eq. (8), namely

$$-\frac{1}{\pi}\text{Im}(G_\pi)=\frac{-\text{Im}\Pi(W,k_\pi)/\pi}{[W^2-\mathbf{k}_\pi^2-m_\pi^2-\text{Re}\Pi(W,\mathbf{k}_\pi)]^2+[\text{Im}\Pi(W,\mathbf{k}_\pi)]^2}\,, \tag{9}$$

which only becomes

$$\delta[W^2-m_\pi^2-\mathbf{k}_\pi^2-\text{Re}(\Pi)]=\delta\{W'-E_N(\mathbf{p}_1)-E_N(\mathbf{p}_2)-[m_\pi^2+\mathbf{k}_\pi^2+\text{Re}(\Pi)]^{1/2}\}/2W$$

in the limit that $\text{Im}\Pi\to 0$, a limit that is necessary for obtaining Eq. (1) but does not exist in relativistic heavy-ion collisions. In the limit of a very large imaginary part of the self-energy, Eq. (9) decreases as $(\text{Im}\Pi)^{-1}$. The physical meaning of this result is clear: the greater the absorptiveness of the medium, the less is the probability that the pion can emerge from the collision region. If we do not use the nuclear-matter approximation, the pion self-energy will be nonlocal in pion momenta, in a way similar to a nonlocal momentum-space pion optical potential. One sees easily that this nonlocality will not change the role of the $\text{Im}(G_\pi)$.

In Fig. 3, we show $-\text{Im}(G_\pi)$ at $\rho=3\rho_0$ as a function of k_π and the off-shell pion energy W. Following Refs. 1 and 3, we incorporate the density dependence of the interactions through the use of a density-dependent Fermi-gas model. Clearly, studies of better models for density-dependent interactions are called for. In evaluating the $\Pi(W,\mathbf{k}_\pi)$, we have summed the series generated by the Born diagrams in Fig. 2, using[11] $f_{\pi NN}=1.009$, $f_{\pi\Delta N}$ a$=2.156$, $\Lambda_{\pi NN}=\Lambda_{\pi\Delta N}=1200$ MeV/c, and $g'=0.4$. As we can see, the results depend strongly on the off-shell pion energy W which is a variable of integration in the many-body problem. The fact that the curves are all very different from what one would expect from a δ-function dependence proposed in Ref. 1 confirms numerically that Eq. (1) is not a good approximation to the exact result of Eq. (9). We also note that the contribution to pion self-energy by two-nucleon pion absorption is very important. Hence, they must not be neglected in the calculations.

We now generalize the above analysis to include the

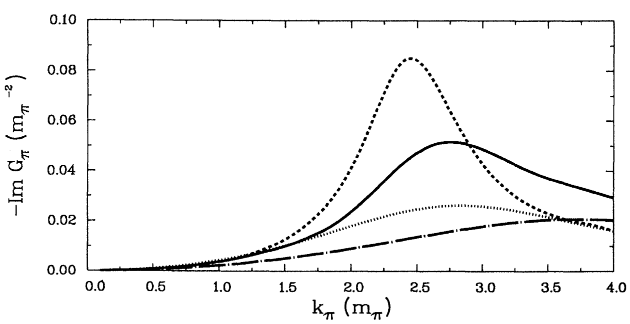

FIG. 3. The $-\text{Im}G_\pi$ as a function of W and k_π, calculated with $\rho=3\rho_0$ and $g'=0.4$. Results obtained with the inclusion of the contribution from two-nucleon pion absorption are given as the solid and dash-dotted curves for $W=5m_\pi$ and $7m_\pi$, respectively. Those without the inclusion of two-nucleon pion absorption are given as the dashed curve ($W=5m_\pi$) and the dotted curve ($W=7m_\pi$).

case in which the production of two pions becomes energetically possible. Since the one-pion and two-pion channels can couple to each other through pion absorption and creation processes, it is of interest to examine the effects of channel coupling on the production amplitude and on the density of states for pion production. We divide, therefore, the subspace b into two parts, one for the 1π doorway state (denoted b_1) and the other for the doorway state having two pions and either two or three nucleons in the continuum (denoted b_2). The V_{opt} then has the decomposition

$$V_{\text{opt}}(E)=V_{aa}+U_{ab_1}G_{b_1}U_{b_1a}+W_{ab_2}g_{b_2}W_{b_2a}\,, \tag{10}$$

where $G_{b_1}=(E^+-K_{b_1}-U_{b_1b_1})^{-1}$ and $g_{b_2}=(E^+-K_{b_2}-W_{b_2b_2})^{-1}$. Further, $W_{ab_2}\equiv U_{ab_2}+U_{ab_1}G_{b_1}U_{b_1b_2}$ and $W_{b_2b_2}\equiv U_{b_2b_2}+U_{b_2b_1}G_{b_1}U_{b_1b_2}$. (We recall that in the doorway model $U_{ab_1}=V_{ab_1}$, etc.) The first and second terms of W_{ab_2} are, respectively, the amplitudes for one-step 2π production and two successive 1π productions. In $W_{b_2b_2}$, the first and second terms represent the contributions to pion self-energy arising from simultaneous absorption of the two pions and from two sequential one-pion absorptions. This last term prevents us from approximating the propagator in the 2π channel by a product of two single-pion propagators. Had we started our analysis by simply assigning the subspace b in Eq. (5) to the 2π channel and including the 1π channel in the subspace c, the above coupling effects would have been obscured. It is only in the limit of weak coupling (i.e., small $U_{b_2b_1}$ and $U_{b_1b_2}$, we have $W\to U$ and $g\to G$. Equation (10) indicates that the density of states for 1π production is $-\text{Im}(G_{b_1})/\pi$ and that for 2π production is $-\text{Im}(g_{b_2})/\pi$. This result represents an extension of Eq. (9) and can be readily generalized to include higher pion multiplicities. This formal structure of the density of states will also not be affected by the inclusion of temperature dependence of the nuclear state in the analysis.

In summary, the density of states for pion production in an absorptive medium is given by $-\text{Im}(G)/\pi$, with G being the relevant many-body pion propagator. Hence, Eq. (1) should not be used. Realistic evaluation of $\text{Im}(G)$ requires a detailed calculation of pion self-energy, this must include the contributions from one of the most important inelastic channels, the true pion absorption on two nucleons.

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[1]G. E. Brown and V. Koch, in Proceedings of the Eighth High Energy Heavy Ion Study, Berkeley, California, November, 1987 [Lawrence Berkeley Laboratory Report No. LBL-24580, 1987 (unpublished)].

[2]Strictly speaking, the δ function involving ω_π gives more than just the $|d\omega_\pi/dk_\pi|^{-1}$ because the momentum conservation makes the momentum of one of the other nucleons depend also on $\mathbf{k}_\pi$. This detail is, however, not essential to the discussion.

[3]G. E. Brown and V. Koch, in *Proceedings of the Texas A. and M. Symposium on Hot Nuclei, College Station, Texas, December, 1987,* edited by S. Shlomo, R. P. Schmitt, and J. B. Natowitz (World Scientific, Singapore, 1988).

[4]C. Gale and J. Kapusta, Phys. Rev. C **35**, 2107 (1987).

[5]L. H. Xia, C. M. Ko, L. Xiong, and J. Q. Wu, Nucl. Phys. A **485**, 721 (1988).

[6]C. M. Ko, L. H. Xia, and P. J. Siemens, Phys. Lett. B **231**, 16 (1989).

[7]D. J. Ernst, C. M. Shakin, and R. M. Thaler, Phys. Rev. C **9**, 1374 (1974).

[8]L. C. Liu and C. M. Shakin, Phys. Rev. C **20**, 2339 (1979), and the references contained therein.

[9]H. Feshbach, Ann. Phys. (N.Y.) **5**, 357 (1958); *ibid.* **19**, 287 (1962).

[10]L. C. Liu, Phys. Rev. C **23**, 814 (1981).

[11]B. K. Jain, J. T. Londergan, and G. E. Walker, Phys. Rev. C **37**, 1564 (1988). The model for the width of Δ is taken from R. S. Bhalerao and L. C. Liu, Phys. Rev. Lett. **54**, 865 (1985).