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Soft-photon analysis of pion-proton bremsstrahlung and the "experimental" magnetic moment of $\Delta^{++}(1232)$

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We present a special two-energy-two-angle amplitude which can be rigorously derived for the pion-proton bremsstrahlung ($\pi^+ p\gamma$) process near the $\Delta^{++}(1232)$ resonance and report the magnetic dipole moment of the Δ^{++}, μ_Δ, extracted from existing data. The extracted values of μ_Δ are 3.7–4.2 $e/2m_p$ from one set of data and 4.6–4.9 $e/2m_p$ from another. Here, m_p is the proton mass. The overall agreement between the theoretical predictions calculated with the extracted μ_Δ and the experimental measurements is excellent.

In this paper, we present a special two-energy-two-angle (TETAS) amplitude which can be rigorously derived for the pion-proton bremsstrahlung ($\pi^+ p\gamma$) process,

$$\pi^+(q_i^\mu)+p(p_i^\mu)\rightarrow \pi^+(q_f^\mu)+p(p_f^\mu)+\gamma(k^\mu)\ ,$$

near the $\Delta^{++}(1232)$ resonance. We discuss briefly how the amplitude is derived by using a useful radiation decomposition identity[1] for bremsstrahlung emission from the internal Δ^{++} line with the anomalous magnetic moment λ_Δ and we discuss some interesting features of the amplitude. Applying the TETAS amplitude to calculate $\pi^+ p\gamma$ cross sections by treating λ_Δ as a free parameter, we have also extracted the value of λ_Δ (which determines the magnetic moment of the Δ^{++}, μ_Δ) from both the University of California at Los Angeles (UCLA) data[2] and the Schweizerisches Institut für Nuklearforschung (SIN) data.[3] Our work represents the first successful attempt to extract μ_Δ by fitting to 85% of the available $\pi^+ p\gamma$ data (45 sets of the UCLA data and 3 sets of the SIN data). Finally, we demonstrate that the overall agreement between the experimental data and the theoretical calculations (based upon the TETAS amplitude with the extracted value of μ_Δ as an input) is excellent. To the best of our knowledge, such an agreement has never before been obtained.

The $\pi^+ p\gamma$ process has attracted much attention, mainly because it can be used to probe the electromagnetic properties of the Δ^{++} resonance. Two experimental groups[2,3] have systematically measured the $\pi^+ p\gamma$ cross sections which can be used to determine μ_Δ. To extract μ_Δ from the $\pi^+ p\gamma$ data, one needs a valid bremsstrahlung amplitude which takes into account photon emission from the internal Δ^{++} line. Such an amplitude can be derived, in principle, from a dynamical model or from a fundamental theorem known as the soft-photon theorem.[4,5] Various soft-photon amplitudes, which are consistent with the soft-photon theorem, have been constructed by using Low's prescription.[4] Low's prescription involves the following steps: (i) Obtain the external amplitude M_μ^E from four external emission diagrams and expand M_μ^E in powers of photon energy k. (ii) Impose the gauge invariant condition, $M_\mu^I k^\mu = -M_\mu^E k^\mu$, to obtain the leading term (order k^0) of the internal amplitude, M_μ^I. (iii) Combine M_μ^E and M_μ^I to obtain the total bremsstrahlung amplitude, M_μ. The first two terms of the expansion of M_μ, which are independent of the off-shell effects, define a soft-photon amplitude. The most important feature of a soft-photon amplitude is that it may be calculated exactly in terms of the corresponding elastic T matrix and the electromagnetic constants of the participating particles. Depending upon how many energies and scattering angles are involved, soft-photon amplitudes have been classified into one-energy-one-angle (OEOA) amplitudes, one-energy-two-angle (OETA) amplitudes, two-energy-one-angle (TEOA) amplitudes, two-energy-two-angle (TETA) amplitudes, and so forth.[6] Recent studies[6–8] show that the combined $\pi^\pm p\gamma$ data and $p\,^{12}\mathrm{C}\gamma$ data can only be described by special two-energy amplitudes [i.e., those amplitudes which depend upon two special energies, the initial energy $(s_i)^{1/2}$ and the final energy $(s_f)^{1/2}$]. Moreover, TETAS amplitudes (those amplitudes which depend upon two special energies and two special scattering angles) are found to give the best fit to the combined data.

The TETAS amplitudes have been investigated by Fischer and Minkowski,[9] by Heller,[10] and most recently by us.[7] However, none of the amplitudes obtained by these authors can be used to determine μ_Δ from the $\pi^+ p\gamma$ data. Let us explain this point more precisely. As we know, bremsstrahlung emissions from the internal Δ^{++} line involve two sources: one contribution comes from the charge of the Δ^{++} and another contribution is due to the magnetic moment of the Δ^{++}. Low's prescription can be applied to find the expression for the charge contribution. (The expressions for the charge contribution obtained in Refs. 7, 9, and 10 are all identical even though the expressions are written in different forms.) But it is very difficult to obtain the expression for the magnetic contribution by using Low's prescription. This is because the magnetic contribution involves an important λ_Δ-dependent term which is separately gauge invariant. (If M_λ^μ is the λ_Δ-dependent term which is separately gauge invariant, then we have $M_\lambda^\mu k_\mu = 0$. In that case, M_λ^μ cannot be derived from the external amplitude by imposing the gauge invariant condition. Imposing the gauge invariant condition to determine the leading term of the internal amplitude is the most important step in Low's prescription.) This explains why a soft-photon amplitude which takes into account photon emission from the Δ^{++} (including both the charge contribution and the magnetic contribution) has never before been constructed. Since the amplitudes obtained in Refs. 7, 9, and 10 do not have the λ_Δ-dependent term, these amplitudes cannot be used to extract λ_Δ or μ_Δ from the $\pi^+ p\gamma$ data.

Since Low's original prescription cannot be used to obtain an internal contribution which is separately gauge invariant, we have developed a modified procedure to construct a TETAS amplitude which includes the λ_Δ-dependent term. In this modified procedure, we have added an internal contribution M_μ^Δ, which represents photon emission from the internal Δ^{++} line, to the external amplitude M_μ^E before we impose the gauge invariant condition to obtain the rest of other internal contributions. To obtain the expression for M_μ^Δ which can be incorporated into M_μ^E is the most difficult part in this step. Fortunately, this can be done if we apply a radiation decomposition identity for photon emission from the internal Δ^{++} (an extended Brodsky-Brown identity) to split M_μ^Δ into four quasiexternal amplitudes, which can be easily combined with M_μ^E.

The magnetic moment of the Δ^{++} obtained in this work is based upon the TETAS amplitude [given by Eq. (1)]. In deriving this amplitude, we have ignored the emission from the internal pion-proton loop (or the open pion-proton channel). In a recent study using a nonrelativistic dynamical model, Heller *et al.*[11] have reported that an "effective" (or dressed) magnetic moment of the Δ^{++} can be defined if the contribution from the loop diagrams is involved. This effective moment, which is different from the "bare" moment predicted by the SU(6) or the quark model, is a complex and energy-dependent quantity and its imaginary part, according to their calculation, is not negligible. However, these authors were not able to demonstrate that their model could be used to describe most of the $\pi^+ p\gamma$ data. Thus the problem of defining and calculating the effective magnetic moment for an off-shell unstable particle remains unsolved, warranting further careful studies. Now, it is obvious that the effective moment cannot be rigorously defined in any model independent calculation since it is difficult to take into account the loop contribution in the soft-photon approximation. This is why the magnetic moment of the Δ^{++} extracted from the $\pi^+ p\gamma$ data by using the TETAS amplitude is not exactly the effective moment, and therefore we do not claim that we have solved the effective moment problem in this work. Since it is also difficult to identify our magnetic moment with the "bare" moment, we have used the "experimental" magnetic moment to describe the result obtained by us. Fortunately, our best fit to the data implies that the experimental moment is very close to the effective moment (mainly because the imaginary part is found to be very small, suggesting little contribution from the loop diagrams) and we have found that the experimental moment is in good agreement with the bare moment predicted by a modified SU(6) model.

The TETAS amplitude which we have derived for the $\pi^+ p\gamma$ process has the form

$$M_\mu^{\rm TETAS} = \bar{u}(p_f,v_f)\left[\left(\frac{q_{f\mu}}{q_f\cdot k} - \frac{(q_f+p_f-R)_\mu}{(q_f+p_f)\cdot k}\right)T(s_i,t_p) - T(s_f,t_p)\left(\frac{q_{i\mu}}{q_i\cdot k} - \frac{(q_i+p_i-R)_\mu}{(q_i+p_i)\cdot k}\right)\right.$$
$$\left. + \left(\frac{p_{f\mu}-R_{f\mu}}{p_f\cdot k} - \frac{(q_f+p_f-R)_\mu}{(q_f+p_f)\cdot k}\right)T(s_i,t_q) - T(s_f,t_q)\left(\frac{p_{i\mu}-R_{i\mu}}{p_i\cdot k} - \frac{(q_i+p_i-R)_\mu}{(q_i+p_i)\cdot k}\right)\right]u(p_i,v_i)\,, \quad (1)$$

where, $s_i=(q_i+p_i)^2$, $s_f=(q_f+p_f)^2$, $t_p=(p_f-p_i)^2$, $t_q=(q_f-q_i)^2$,

$$\epsilon^\mu R_{\alpha\mu} = \tfrac{1}{4}\,[\not k,\not\epsilon] + \frac{\lambda_p}{8m_p}\{[\not k,\not\epsilon],\not p_\alpha\}, \quad (\alpha=i,f)$$
$$\epsilon^\mu R_\mu = \tfrac{1}{4}\,[\not k,\not\epsilon] + \frac{\lambda_\Delta}{8M_\Delta}\{[\not k,\not\epsilon],\not p\}\,, \quad (2)$$

m_p is the proton mass, $M_\Delta=1232$ MeV, $p^\mu=q_i^\mu+p_i^\mu=q_f^\mu+p_f^\mu+k^\mu$, λ_p is the anomalous magnetic moment of the proton, and ϵ^μ is the photon polarization vector. In Eq. (2), we have used $[A,B]\equiv AB-BA$ and $\{A,B\}\equiv AB+BA$. It is easy to show that $M_\mu^{\rm TETAS}$ is gauge invariant since $R_{i\mu}$, $R_{f\mu}$, and R_μ are separately gauge invariant, $R_{i\mu}k^\mu=R_{f\mu}k^\mu=R_\mu k^\mu=0$. The amplitude $M_\mu^{\rm TETAS}$ has many interesting features: (i) It is relativistic, gauge invariant, and consistent with the soft-photon theorem. (ii) It depends only on the elastic T matrix, evaluated at four different sets of (s,t): (s_i,t_p), (s_i,t_q), (s_f,t_p), and (s_f,t_q), but it is free of any derivative of T with respect to s or t. (iii) It takes into account bremsstrahlung emissions from (a) the incoming pion and the outgoing pion (with charge $+e$), (b) the incoming proton and the outgoing proton (with charge $+e$ and the anomalous magnetic moment λ_p), and (c) the internal Δ^{++} line (with charge $+2e$ and the anomalous magnetic moment λ_Δ). We should point out here that $M_\mu^{\rm TETAS}$ is just an approximate amplitude since we have neglected all off-shell contributions and those terms which cannot be expressed in terms of the complete elastic T matrix in our derivation.

We have used the amplitude $M_\mu^{\rm TETAS}$ to calculate $\pi^+ p\gamma$ cross sections as a function of photon energy k, $d^3\sigma/d\Omega_\pi d\Omega_\gamma dk$, at four bombarding energies, 269, 298, 299, and 324 MeV. λ_Δ has been treated as a free parameter in these calculations and it is to be determined from the UCLA data and the SIN data.

To determine λ_Δ from the UCLA data, we have first calculated the following average experimental cross sections from the UCLA data: $\sigma_{1-10}^{\rm UCLA}(E_i,k_j)$, $\sigma_{11-15}^{\rm UCLA}(E_i,k_j)$, and $\sigma_{1-15}^{\rm UCLA}(E_i,k_j)$. Here $\sigma_{1-10}^{\rm UCLA}(E_i,k_j)$ represents the average experimental cross section over the first ten photon counters, $G1$–$G10$, at the bombarding energy E_i ($E_1=269$ MeV, $E_2=298$ MeV, and $E_3=324$ MeV) and the photon energy k_j ($k_1=22.5$ MeV, $k_2=40$ MeV,

$k_3=60$ MeV, $k_4=80$ MeV, $k_5=100$ MeV, $k_6=120$ MeV, and $k_7=140$ MeV). Similarly, $\sigma^{\rm UCLA}_{11\text{-}15}(E_i,k_j)$ represents the average experimental cross section over the next five photon counters, $G11-G15$, and $\sigma^{\rm UCLA}_{1\text{-}15}(E_i,k_j)$ represents the average experimental cross section over all fifteen photon counters, $G1-G15$. The corresponding average theoretical cross sections, calculated using the amplitude $M_\mu^{\rm TETAS}$, will be denoted by $\sigma^{\rm th}_{1\text{-}10}(E_i,k_j)$, $\sigma^{\rm th}_{11\text{-}15}(E_i,k_j)$, and $\sigma^{\rm th}_{1\text{-}15}(E_i,k_j)$. We then use these theoretical and experimental average cross sections to define the following deviation functions:

$$D_x(E_i,\lambda_\Delta)=\sum_j\left[\frac{|\sigma_x^{\rm UCLA}(E_i,k_j)-\sigma_x^{\rm th}(E_i,k_j)|}{\sigma_x^{\rm UCLA}(E_i,k_j)}\right] \quad (3)$$

$$(x=1-10,\ 11-15,\ \text{and}\ 1-15)$$

as a function of λ_Δ. We obtain nine deviation functions since there are three bombarding energies. Varying the value of λ_Δ, we find nine deviation curves. As shown in Figs. 1(a)-1(c), each curve clearly exhibits a minimum point which determines an extracted value of λ_Δ. The average (minimum) values for λ_Δ are 1.4 for photon counters $G1-G10$ [Calculating the χ^2 as a function of λ_Δ, we have also found a minimum point for each χ^2 curve. For photon counters $G1-G10$ at 298 MeV, it gives $\lambda_\Delta=1.4$, in good agreement with the one obtained by using Eq. (3).], 1.6 for photon counters $G1-G15$, and 1.8 for photon counters $G11-G15$. Using these values for λ_Δ, the values of μ_Δ can be calculated. We find, $\mu_\Delta=2(1+\lambda_\Delta)(e/2M_\Delta)$,

$$\mu_\Delta=\begin{cases}3.7\dfrac{e}{2m_p} & \text{for } G1-G10\,,\\[2ex] 4.0\dfrac{e}{2m_p} & \text{for } G1-G15\,,\\[2ex] 4.2\dfrac{e}{2m_p} & \text{for } G11-G15\,.\end{cases}$$

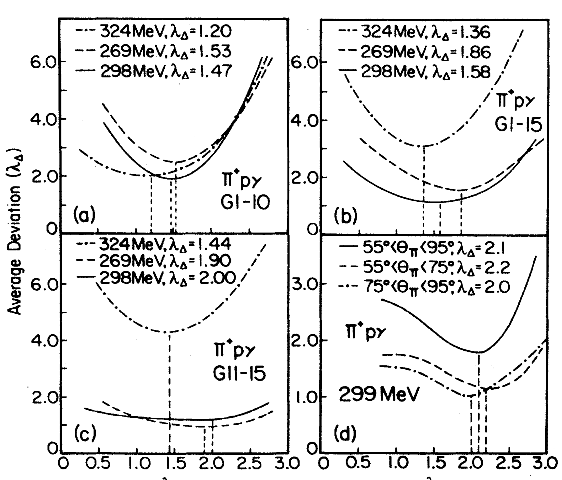

FIG. 1. Average deviation as a function of λ_Δ. The deviation curves shown in (a)-(c) are obtained from the UCLA data while the curves shown in (d) are from the SIN data. The minimum point on each curve determines an extracted value of λ_Δ.

Note that if all the UCLA data sets were analyzed together to yield a single value of μ_Δ, then the extracted value would be $3.8e/2m_p$.

We have also extracted the value of μ_Δ from the SIN data. Meyer *et al.* have measured the $\pi^+p\gamma$ cross sections at 299 MeV. Depending upon the angular regions for the outgoing pions, three sets of cross sections have been obtained by the group. We shall name the set for $55°<\vartheta_\pi<95°$ as the first set, the set for $55°<\vartheta_\pi<75°$ as the second set, and the set for $75°<\vartheta_\pi<95°$ as the third set. Using the similar method used to obtain the deviation curves for the UCLA data, we have also obtained three deviation curves for the SIN data. As shown in Fig. 1(d), each curve has a minimum point. The values of λ_Δ at these minimum points are 2.1 for the first set ($55°<\vartheta_\pi<95°$), 2.2 for the second set ($55°<\vartheta_\pi<75°$), and 2.0 for the third set ($75°<\vartheta_\pi<95°$). These values of λ_Δ give

$$\mu_\Delta=\begin{cases}4.7\dfrac{e}{2m_p} & \text{for } 55°<\vartheta_\pi<95°\,,\\[2ex] 4.9\dfrac{e}{2m_p} & \text{for } 55°<\vartheta_\pi<75°\,,\\[2ex] 4.6\dfrac{e}{2m_p} & \text{for } 75°<\vartheta_\pi<95°\,.\end{cases}$$

If, on the other hand, all the SIN data sets were analyzed together to yield a single value of μ_Δ, we would find $\mu_\Delta=4.6e/2m_p$.

It is clear that the values of μ_Δ extracted from either the UCLA data or the SIN data are smaller than the value $\mu_\Delta=5.58e/(2m_p)$ predicted by the SU(6) model and the quark model. However, as pointed out by the UCLA group, a modified SU(6) model (with mass corrections) suggested by Beg and Pais[12] predicts $\mu_\Delta=(m_p/M_\Delta)\times5.58e/(2m_p)=4.25e/(2m_p)$. Moreover, Meyer *et al.* have also pointed out that bag-model corrections to the quark model[13] give $\mu_\Delta=4.41-4.89e/(2m_p)$. Thus the values of μ_Δ extracted from the data (the average value of μ_Δ determined from both the UCLA data and the SIN data is $4.2e/2m_p$) are in much better agreement with the

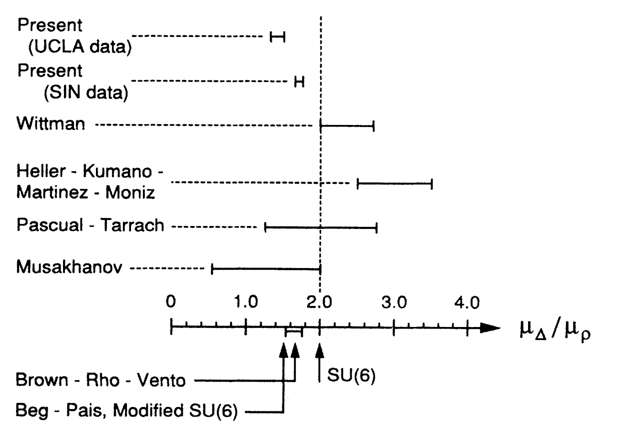

FIG. 2. Compilation of μ_Δ/μ_p results obtained by different groups using various approximations and methods. $\mu_p=2.79e/(2m_p)$.

value predicted by the modified SU(6) model or the quark model with corrections. The values of μ_Δ previously obtained by other authors were 3.6 ± 2.0 by Musakhanov,[14] 5.6 ± 2.1 by Pascual and Tarrach,[15] 7.0–9.8 by Heller *et al.*[11], and 5.58–7.53 by Wittman[16] in the unit of $e/(2m_p)$. These results are summarized in Fig. 2.

Using the values of μ_Δ extracted from the experimental data as input, we have applied the amplitude M_μ^{TETAS} to calculate all $\pi^+ p\gamma$ cross sections which can be compared with the UCLA data and the SIN data. Some of these calculations, compared with the experimental data, are shown in Figs. 3 and 4. From the results shown in Figs. 3 and 4 and other results which are not shown here in this paper, we have found that the agreement between the theoretical predictions and the experimental measurements is excellent in general. This fact can also be seen from the following χ^2 values. We have calculated the χ^2 values for those UCLA cross sections shown in Fig. 3 by using three different values of $\mu_\Delta = (3.8, 4.0, 4.2) e/2m_p$ as input for theoretical predictions. The χ^2 values [corresponding to $\mu_\Delta = (3.8, 4.0, 4.2) e/2m_p$] are (0.4,0.4,0.4), (0.4,0.6, 0.7), (1.7,1.6,1.5), and (0.7,0.7,0.7) for $G2$ [Fig. 3(a)], $G6$ [Fig. 3(b)], $G13$ [Fig. 3(c)], and $G14$ [Fig. 3(d)], respectively. Similarly, for the SIN data, we have also calculated the χ^2 values for those cross sections shown in Fig. 4 by using $\mu_\Delta = (4.2, 4.6, 4.9) e/2m_p$ as input for theoretical predictions. The χ^2 values are (5.4,4.3,3.4), (2.7, 2.6,2.8), and (2.7,2.2,2.1) for Figs. 4(a)–4(c), respectively. Since the calculated $\pi^+ p\gamma$ cross sections are very sensitive to the precise form of the internal amplitude,[7] the overall excellent agreement between theory and experiment shows not only that the extracted values of μ_Δ give the best fit to the experimental data but also that the amplitude M_μ^{TETAS} is valid for the $\pi^+ p\gamma$ process near the $\Delta^{++}(1232)$ resonance.

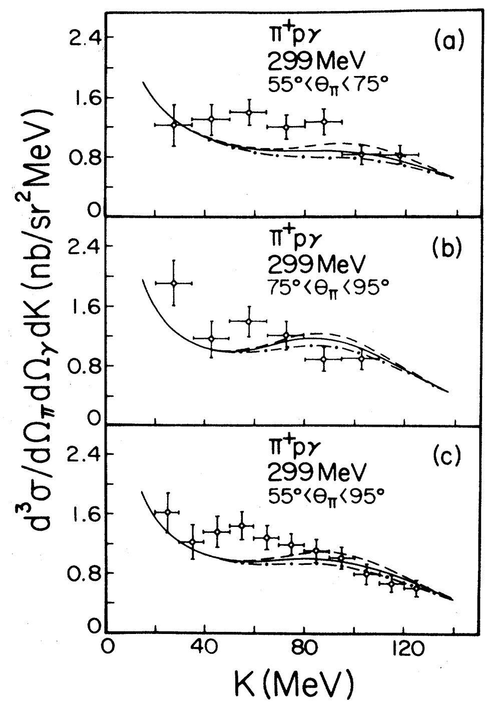

FIG. 4. The $\pi^+ p\gamma$ cross sections as a function of k. The dashed, solid, and dot-dashed curves are calculated with $\mu_\Delta = 4.9$, 4.7, and $4.6 e/(2m_p)$, respectively. The SIN data are from Ref. 3.

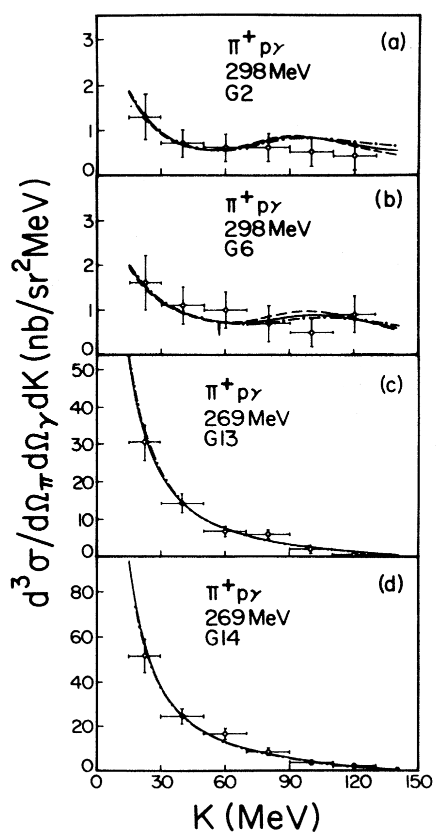

FIG. 3. The $\pi^+ p\gamma$ cross sections as a function of photon energy k. The dashed, solid, and dot-dashed curves are calculated with $\mu_\Delta = 4.2$, 4.0, and $3.7 e/(2m_p)$, respectively. The UCLA data are from Ref. 2.

As we have already mentioned, the effective moment which is a complex quantity has been studied by Heller *et al.*[11] We cannot rigorously define this moment in this work since it is difficult to take into account the loop contribution in the soft-photon approximation. Nevertheless, we have done a numerical study by treating λ_Δ in Eq. (1) as a complex number, $\lambda_\Delta = \lambda_R + i\lambda_I$, in order to get some idea about the importance of the imaginary part λ_I. We have chosen λ_Δ to be $1.47 + i\lambda_I$, $1.6 + i\lambda_I$, and $2.4 + i\lambda_I$. By varying λ_I from -1.0 to 1.0 in each case, we have used the UCLA data to calculate average deviations as a function of λ_I at 298 MeV for photon counters $G1$–$G10$. As a result, we have obtained three deviation curves which have the same interesting feature. The value of the average deviation decreases rapidly as λ_I increases from -1.0 to 0 and then it increases rapidly as λ_I increases from 0 to 1.0. Thus the minimum points for all three average deviation curves are around $\lambda_I = 0$, indicating that the best fit to the UCLA data at 298 MeV for counters $G1$–$G10$ can be obtained by choosing λ_Δ to be a real number as we have done in this work.

To understand why the best fit to the UCLA data can be obtained only if λ_Δ is chosen to be a real number, we have performed another study. Our numerical investigation of the amplitude M_μ^{TETAS} reveals that the best agreement between theory and experiment is obtained when the contribution from the R_μ-dependent terms cancels the total contribution from those terms involving $R_{i\mu}$ and $R_{f\mu}$ in Eq. (1). This cancellation occurs when μ_Δ is around $4e/(2m_p)$. However, no cancellation is possible if λ_Δ is chosen to be a complex number with a large imaginary part since the anomalous magnetic moment of proton λ_p is a real number ($\lambda_p=1.79$). This explains why the minimum point is always found around $\lambda_I=0$, independent of the choice of λ_R, if the average deviation is plotted as a function of λ_I. In our numerical study, we have also found that the spectra calculated by using Eq. (1) agree very well with those spectra predicted by using Eq. (16) of Ref. 7 if μ_Δ used in Eq. (1) is about $4e/(2m_p)$. Both results are in excellent agreement with the experimental data.

Now let us discuss what would happen if those terms involving $R_{i\mu}$, $R_{f\mu}$, and R_μ are canceled out precisely. We would obtain an amplitude M_μ^{TETAS} with $R_{i\mu}=R_{f\mu}=R_\mu=0$. Such amplitude was first proposed by Heller[10] and it was discussed in great details in Ref. 7 [Heller's amplitude is identical to Eq. (3) of Ref. 7]. It is a well-known fact that Heller's amplitude can be successfully applied to describe both the $\pi^\pm p\gamma$ data and the $p\,^{12}\mathrm{C}\gamma$ data. This fact may have two possible implications that are consistent with our findings. (i) The cancellation between the contribution from the magnetic moment of the Δ^{++} (including all possible loop corrections) and the contribution from the magnetic moment of proton exists. (ii) The imaginary part of the effective magnetic moment of the Δ^{++} is small and the real part is 3.7–$4.9e/(2m_p)$. In short, the data seem to suggest that dynamical corrections from the loop diagrams are small. In other words, our best fit implies that the experimental magnetic moment of the Δ^{++} extracted from the $\pi^+p\gamma$ data is a good approximation to the effective moment and it is very close to the bare moment given by the modified SU(6) model or the quark model with corrections.

We wish to thank Professor Robert W. Brown for many stimulating discussions. One of us (MKL) is grateful to Dr. Leon Heller for helpful discussions and for making a useful suggestion to study the effective magnetic moment. This work was supported in part by the City University of New York Professional Staff Congress-Board of Higher Education Faculty Research Award Program.

[1]Radiation decomposition identities have been derived for particles with spin ≤ 1 by Brodsky and Brown [S. J. Brodsky and R. W. Brown, Phys. Rev. Lett. **49**, 966 (1982); R. W. Brown, K. L. Kowalski, and S. J. Brodsky, Phys. Rev. D **28**, 624 (1983)]. We have obtained a general method to derive these identities for particles with arbitrary spin. The identity used here is the one for the Δ^{++} with spin $\frac{3}{2}$. The general method for deriving the Brodsky-Brown identities and the application of these identities to modify Low's prescription for constructing soft-photon amplitudes will be discussed in future papers.

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