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Low-energy theorem for neutral pion photoproduction on a proton

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The low-energy theorem (LET) prediction for neutral pion photoproduction on the proton remains valid even if isospin symmetry is broken. All rescattering effects are implicitly taken into account. Therefore, the data should not be corrected for final-state interactions in order to compare with the LET. As a consequence of such a correction, the previously quoted discrepancy between the LET and the experiment is overestimated.

Gauge invariance and the hypothesis of the partial conservation of the axial vector current (PCAC) provide a low-energy theorem (LET) for threshold photoproduction of pions on a nucleon.[1] The latest measurements[2-4] of the process $\gamma p \rightarrow \pi^0 p$ near threshold seem to be in contradiction with the LET prediction. This has been interpreted as a violation of PCAC (Ref. 4) and explanations in terms of explicit chiral-symmetry breaking beyond PCAC have been given (e.g., Refs. 5 and 6).

In this paper we also focus on the reaction $\gamma p \rightarrow \pi^0 p$. It is argued that the usually made assumption of isospin symmetry, implying equal pion masses and equal nucleon masses, is not necessary to arrive at a model independent prediction at π^0 threshold. The theoretical result remains the same except for trivial changes like adjusting the $pp\pi^0$ coupling constant and the masses of the particles in question, causing a small numerical change. An important point discussed below is that *all* rescattering effects are taken into account, including rescattering of a charged pion. Therefore, possible remaining effects directly at the π^0 threshold due to the "cusp" at π^+ threshold, which would vanish for perfect isospin symmetry, are included in the LET. As a consequence, the data should *not* be corrected for these final-state interactions (FSI) but compared immediately with the LET prediction. However, in the recently presented experimental results for the relevant E_{0^+} multipole, $E_{0^+} = (-0.5 \pm 0.3) \times 10^{-3}/m_{\pi^+}$ (Refs. 2 and 3) and $E_{0^+} = (-0.35 \pm 0.1) \times 10^{-3}/m_{\pi^+}$,[4] such FSI have been taken out in a (necessarily) model dependent fashion. Apparently, other authors, for instance Kamal,[7] were unaware of this fact because they treat the presented "experimental" result as if it includes these FSI. As already pointed out by Nozawa, Lee, and Blankleider[8] any theoretical model including all FSI should directly compare to the measured value $E_{0^+} \simeq (-1.5) \times 10^{-3}/m_{\pi^+}$. However, also in this work the LET is interpreted as if it does not contain the FSI mentioned above. In contrast, here we stress that the LET includes all FSI and therefore, the disagreement between data and LET,

$$E_{0^+} = (-2.2) \times 10^{-3}/m_{\pi^+} = (-2.1) \times 10^{-3}/m_{\pi^0} ,$$

is much smaller than previously inferred.

It might be useful to note that a *direct* measurement of the E_{0^+} multipole at threshold is impossible because the cross section vanishes at threshold. Some extrapolation is unavoidable and moreover, LET does not yield a prediction of the cross section above threshold where also other multipoles contribute. However, the extracted E_{0^+} multipole at threshold can immediately be compared with the LET result.

Let us discuss the possible implications of isospin symmetry breaking by following our recent derivation of the LET.[9] The earlier mentioned trivial mass changes will be included from now on. As in the isospin symmetric case, gauge invariance leads to the Ward-Takahashi identities,[10] which relate electromagnetic vertex operators and propagators of the particles involved. One has for a proton with initial momentum p and final momentum p'

$$(p'-p)^\mu\Gamma^p_\mu=e[S^{-1}(p')-S^{-1}(p)]\,. \tag{1}$$

Γ_μ denotes the irreducible electromagnetic vertex operator and S is the full proton propagator. The corresponding relation for a neutral pion with initial momentum q and final momentum q' reads

$$(q'-q)^\mu\Gamma^{\pi^0}_\mu=0\,. \tag{2}$$

The right-hand side of the latter Ward-Takahashi identity, which would contain the full π^0 propagator, vanishes because it is proportional to the charge of the particle in question. The total operator M_μ, describing the process $\gamma(k)+p(p)\rightarrow\pi^0(q)+p(p')$, is related to the π^0pp vertex $\Lambda_5(p',p)$ by the generalized Ward-Takahashi identity[11]

$$\begin{aligned}k^\mu M_\mu(p',q;p,k)=e[&S^{-1}(p')S(p'-k)\Lambda_5(p'-k,p)\\&-\Lambda_5(p',p+k)S(p+k)S^{-1}(p)]\,.\end{aligned} \tag{3}$$

By the use of these identities in the derivation of the LET, gauge invariance has been enforced. Another ingredient of this derivation is the PCAC hypothesis

$$\partial^\mu J^0_{5,\mu}=f_\pi m^2_{\pi^0}\phi^0\,, \tag{4}$$

where $J^0_{5,\mu}$ denotes the neutral axial current, ϕ^0 the neutral pion field, and f_π the pion-decay constant.[12] It already includes chiral symmetry breaking effects due to the non-vanishing pion mass and can, therefore, be maintained as a hypothesis. Including the electromagnetic field does not modify this relation to first order in the charge.[13] The validity of the PCAC hypothesis is supposed to be tested in this reaction $\gamma p\rightarrow\pi^0p$ and the discrepancy between LET and "experiment" has indeed been interpreted as "evidence for PCAC violation."[4] Crossing symmetry relations, previously imposed for the isospin amplitudes,[1,9] also hold for the amplitudes describing neutral pion production separately. Proper behavior under space and time reversal, yielding relations for the form factors, is an isospin independent condition. Therefore, the general assumptions do not cause additional problems in the case of isospin symmetry breaking.

However, we need to look into some more detail to the actual derivation of the LET (Ref. 9) before any conclusive statement can be made. A division of all possible contributions to the amplitude is made into a class A and a class B. This was introduced by Gell-Mann and Goldberger for Compton scattering.[14] The class A terms are defined as diagrams where the photon and pion vertex are separated by a single pion or nucleon propagator. Class B is the rest and contains insertions into the pion-nucleon vertex, isobar contributions, etc. We mention that both classes include (charged and neutral) pion rescattering contributions. Class A contains the nucleon and pion pole contributions and as a consequence it is assumed that the invariant Ball amplitudes[15] due to class B diagrams can be expanded in a power series in the kinematical variables ν and ν_I

$$\begin{aligned}\nu&=\frac{1}{2M^2}(p+p')\cdot k\,,\\ \nu_I&=\frac{1}{2M^2}q\cdot k\,.\end{aligned} \tag{5}$$

We will address this assumption below; here we only refer to the caveat made in Ref. 9 in connection to the work of Li and Pagels.[16] Such an expansion is not possible for the class A contribution because it is indeed nonanalytic.[9] In the isospin symmetric case the thresholds for neutral and charged-pion production coincide and their nonanalytic contributions (at least at the relevant values of the kinematical variables) are all contained in class A. If isospin symmetry is broken the thresholds do not coincide because $m_{\pi^0}\neq m_{\pi^+}$ and above the π^0 threshold the π^+ channel opens. Class A terms, which may be nonanalytical, cannot be shifted by this isospin symmetry breaking from class A to class B. In other words, class A diagrams remain class A. Furthermore, we will argue that it is impossible that a class B term is nonanalytic at π^0 threshold for different pion masses, while yielding an analytic contribution for equal pion masses. This justifies the statement that the recent derivation of the LET for neutral pion photoproduction on the proton at threshold[9] remains valid if isospin symmetry is broken. From this derivation it is clear that all FSI are contained in the LET prediction. In other words, this procedure immediately yields the total amplitude.

A violation of the LET, without disobeying the earlier mentioned general principles, necessarily excludes the existence of the assumed Taylor series expansion of the class B contribution. A formal justification of this assumption, made in Refs. 9 and 11, in Ref. 1 for the "non-Born part," and similarly in Ref. 14 in case of Compton scattering, is indeed still lacking. Landau equations[17] in combination with the method of majorization of diagrams[18], which could provide conclusions about the analytical structure of (classes of) diagrams, are not straightforwardly applicable because of the presence of zero-mass photon(s). These techniques nevertheless indicate that if the assumption is true for the neutral pion production on the proton in the isospin symmetric case, then it will remain true in case of isospin symmetry breaking. The reason is that the mass of the charged pion and neutron is greater than the mass of the neutral pion and proton, respectively. A consequence of isospin symmetry breaking may be that the opening of the π^+n channel above the π^0 threshold affects the radius of convergence of the Taylor series in question. Therefore, *extrapolations* of the LET prediction above threshold into the cusp region may be cumbersome. However, as is clear from the derivations (e.g., Refs. 1 and 9), the LET predicts the amplitude only at π^0 threshold.

Finally, we want to point out why it is often (e.g., in Refs. 2-4, 7, and 8) falsely believed that the LET does not contain all FSI in case of isospin symmetry breaking. It arises from the fact that the isospin symmetric LET result happens to be equal to the result which one obtains calculating Born diagrams using pseudovector (PV) pion-nucleon coupling. The underlying strong interaction Lagrangian is isospin symmetric and—apart from the pion mass term—chirally symmetric. The rescattering concept is strongly coupled to Born approaches. Indeed if isospin symmetry is broken one has to correct for rescattering in the PV Born model. Let us illustrate this by writing,[7,8]

$$E_{0^+}^{T}=E_{0^+}^{B}+E_{0^+}^{R}\,, \tag{6}$$

where T, B, and R indicates the total, Born, and rest amplitude, respectively. In calculating Born diagrams one uses renormalized (physical) masses and coupling constants. Consequently, these Born terms include part of the FSI and one can interpret the rest amplitude as "remaining FSI." Therefore, we do not use the somewhat confusing notation (FSI) (Ref. 8) for the rest amplitude. In general, this rest amplitude contains FSI contributions from the π^0p channel and from the π^+n channel. In the isospin symmetric case the FSI contributions of both channels vanish at threshold and the rest amplitude does not contribute to the LET. For broken isospin symmetry the π^+n channel contribution is in general nonzero at π^0 threshold and, consequently, the Born result has to be corrected to obtain the total amplitude. This corrected Born result is then presented as the LET measurement yielding the large discrepancy with theory. In contrast, our preceeding statement is $E_{0^+}^{T}=E_{0^+}^{\mathrm{LET}}$, with the appropriate constants. Let us recall the isospin symmetric result

$$E_{0^+}^{\mathrm{LET},i}=\frac{eg}{16\pi M}[-2\mu+(3+\kappa_p)\mu^2]\,, \tag{7}$$

where g represents the πNN coupling constant, $g^2/4\pi=14.3$, κ_p the anomalous magnetic moment of the proton, $\kappa_p=1.79$, and μ the pion-nucleon mass ratio, $\mu=m_\pi/M$. The PV Born result can easily be obtained from this equation by replacing g/M by f/m_π with $f^2/4\pi=0.08$. The well-known numerical result $E_{0^+}=(-2.3\pm0.3)\times10^{-3}/m_{\pi^+}$ is obtained for a mass ratio $\mu=0.15$. For broken isospin symmetry the LET prediction does not change except for a modified mass ratio $\mu=0.14$ and coupling constant. Recently, the $pp\pi^0$ coupling constant has been redetermined by Bergervoet *et al.*;[19] they found the value $g_0^2/4\pi=13.55$. The earlier quoted number $E_{0^+}=(-2.2)\times10^{-3}/m_{\pi^+}=(-2.1)\times10^{-3}/m_{\pi^0}$, was calculated using this value. The PV Born result does not change numerically. Estimating the remaining FSI contribution in the PV Born approach yields $E_{0^+}^{R}\simeq0.1\times10^{-3}/m_{\pi^+}$. This is even smaller than the result of Nozawa, Lee, and Blankleider,[8] $E_{0^+}^{R}=0.37\times10^{-3}/m_{\pi^+}$, which was obtained using a dynamical model to account for the FSI. Note that these FSI terms in the analyses of the data,[2-4] $E_{0^+}^{R}\simeq(-1.0)\times10^{-3}/m_{\pi^+}$, are even of opposite sign.

We summarize the results: the LET prediction for $E_{0^+}(\gamma p\rightarrow\pi^0p)$ can immediately be compared to the experimental value. No FSI correction due to isospin symmetry breaking should be applied to the data because LET already contains them. As a result the discrepancy between theory and experiment is smaller than previously inferred. This is not in contradiction with the fact that the pure PV Born terms should be corrected for FSI if isospin symmetry is broken.

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[1] P. de Baenst, Nucl. Phys. B **24**, 633 (1970).

[2] E. Mazzucato *et al.*, Phys. Rev. Lett. **57**, 3144 (1986).

[3] P. Argan *et al.*, Phys. Lett. B **206**, 4 (1988).

[4] R. Beck, B. Schoch, F. Kalleicher, G. Koch, V. Metag, H. Stroeher, and R. O. Owens, Panic XII, International Conference on Particles and Nuclei, Cambridge, 1990, Contributed Papers, I-1 (unpublished).

[5] L. M. Nath and S. K. Singh, Phys. Rev. C **39**, 1207 (1989).

[6] T. Schäfer and W. Weise, Panic XII, International Conference on Particles and Nuclei, Cambridge, 1990, Contributed Papers, I-32 (unpublished).

[7] A. N. Kamal, Phys. Rev. Lett. **63**, 2346 (1989).

[8] S. Nozawa, T.-S. H. Lee, and B. Blankleider, Phys. Rev. C **41**, 213 (1990).

[9] H. W. L. Naus, J. H. Koch, and J. L. Friar, Phys. Rev. C **41**, 2852 (1990).

[10] Y. Takahashi, Nuovo Cimento **6**, 371 (1957).

[11] E. Kazes, Nuovo Cimento **13**, 1226 (1959).

[12] Strictly speaking, one should write f_{π^0}. However, the pion-decay constant(s) drop out of the final LET predictions.

[13] S. L. Adler, Phys. Rev. **139**, B1638 (1965).

[14] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1954).

[15] G. S. Ball, Phys. Rev. **124**, 2014 (1961).

[16] L. F. Li and H. Pagels, Phys. Rev. Lett. **26**, 1204 (1971).

[17] R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Cambridge Univ. Press, Cambridge, England, 1966).

[18] I. T. Todorov, *Analytic Properties of Feynman Diagrams in Quantum Field Theory* (Pergamon, Oxford, 1971).

[19] J. R. Bergervoet, P. C. van Campen, R. A. M. Klomp, J.-L. de Kok, T. A. Rijken, V. G. J. Stoks, and J. J. de Swart, Phys. Rev. C **41**, 1435 (1990).