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Enhancement in interactions involving two steps

W. Von Oertzen
Centre de Recherches Nucléaires, 67037 Strasbourg CEDEX, France
and Hahn Meitner Institut, D-1000 Berlin 39, Germany

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The enhancement of two-step processes is analyzed for cases where the intermediate states and final states are influenced by a residual interaction which splits the intrinsic configurations without changing total strength.

In nuclear and atomic collisions, a variety of two-step processes are observed, which are enhanced relative to the expectations of two independent sequential steps or are underestimated by a "calibrated" second-order Born approximation. If the probability of a single step is P_1, the enhancement factor (EF) of the two-step process P_2 can be defined by the simple equation

$$P_2 = \mathrm{EF}(P_1)^2 . \tag{1}$$

Examples of such observations are the enhanced two-nucleon transfer between nuclei[1,2] and the removal of two electrons (double ionization) in ion-atom collisions.[3,4]

Using the nomenclature of atomic physics[3] (ion-atom collisions) there are two distinct mechanisms of enhancement, the static correlations leading to configuration mixing and the dynamic correlations. The first is well known in nuclear physics and virtually absent in atomic physics. The second is a "newly discovered" field of research in atomic physics; it is not completely unknown in nuclear physics. Dynamically induced correlations are particularly strong in the case of mixing of configurations of different parity, known as hybridization.[5,6] It leads to enhancement because of strong distortions in the geometrical shapes of the configurations. The direct one-step population of the final state, caused by a field which acts on a collective ensemble of particles, will give an additional contribution. It is an independent process which contributes to the cross section and causes additional enhancement. The origin of the enhancement, the interaction which causes correlations among the particles (or excitations), can be directly the interaction between the constituents (short-range pairing interaction, Coulomb interaction) or their participation in the formation of a collective state.

The discussion of enhancement in the cases presented here will be simplified by two assumptions: (i) the probabilities for one step are ≤ 0.1, in which case essentially first-order approximations are used in each step; and (ii) final states are populated only via unique amplitudes, i.e., without interference effects from different dynamical routes which can be due to variations in dynamical coordinates (impact parameter, etc.) and which can be well treated in semiclassical models.[1,7] We use the semiclassical approximations for scattering problems as they have been used in Coulomb excitation[8] and transfer reactions using heavy ions.[1]

The various situations which will be discussed here are illustrated in Fig. 1. Starting with state 0, the amplitude to reach state 1 is defined by a_{01}, the second step from state 1 to state 2 is reached by amplitude a_{12}. The differential cross sections are obtained from the product of the scattering cross sections $\sigma_{SC}(\theta)$, and a probability $P_i(\theta)$.

For case I illustrated in Fig. 1, we have only one state 1 and 2, the probability for the transition P_1 is given by

$$P_1 = |a_{01}|^2 , \tag{1a}$$

further, we have for the probability to reach state 2

$$P_2 = |a_{02}|^2 = |a_{01} \times a_{12}|^2 \tag{1b}$$

and for $|a_{01}| \simeq |a_{12}|$ we obtain

$$P_2^{\mathrm{I}} = (P_1)^2 \tag{2}$$

for case I.

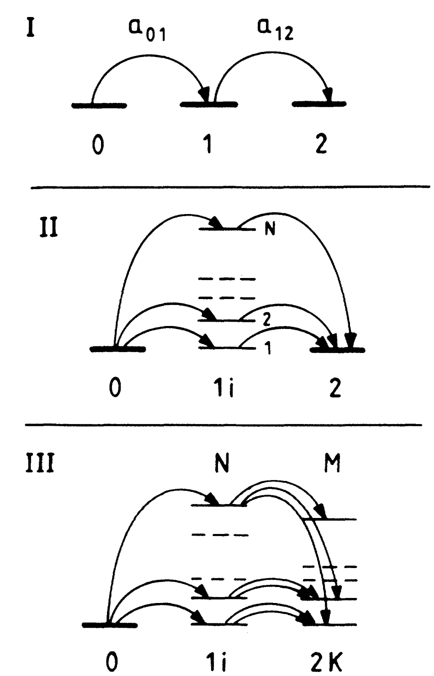

FIG. 1. Schematic illustrations of two-step transitions. Case I: one intermediate state, one final state; Case II: N intermediate states, one final state; Case III: N intermediate states and M final states.

For case II illustrated in Fig. 1, the intermediate state 1 is split into N substates and the total transition probability is kept unchanged by defining the probability for the sum of the N final states

$$P_1=\sum_i^N |a_{01i}|^2 . \tag{3a}$$

P_1 is the same as in Eq. (1a), if we assume that the splitting distributes the strength equally, $|a_{01_i}|\approx(1/\sqrt{N})|a_{01}|$. The population of the final state 2 is obtained by *a coherent sum* where all N configurations responsible for the splitting of state 1 are collected into *one final state* 2; we use the definition

$$a_{01i}\times a_{1i2}=a_{0i2}\approx(a_{01i})^2=\frac{1}{N}|a_{01}|^2 .$$

With these definitions we have

$$P_2^{\rm II}=|\sum_i(a_{01i}\times a_{1i2})|^2=\sum_i^N\left[\frac{1}{N}|a_{01}|^4+\text{intf. terms}\right] . \tag{3b}$$

The interference term contains amplitudes like $a_{01i}a_{1i2}\times a_{01j}a_{1j2}$; the phases of these amplitudes will depend on the mechanism of the splitting. Three extreme situations can be identified: (i) Systematic splitting with a continuous change of the phase, e.g., from $-\delta$ to δ; (ii) random phases: In both cases we expect that the interference term vanishes and $P_2^{\rm II}=(P_1)^2=|a_{01}|^4$, the result of case I is recovered if the total transition strength stays normalized. (iii) Maximum coherence: All phases are the same and will give the maximum possible cross section.

$$P_2^{\rm II}=\sum_i^N\left[\frac{1}{N}|a_{01}|_i^4+\frac{N-1}{N}|a_{01}|_i^4\right] , \tag{3c}$$

and we have an enhancement factor EF given by the number of intermediate states,

$$P_2^{\rm II}=N(P_1)^2,\quad \text{EF}=N;\ \text{for case II} . \tag{3d}$$

The phases may also induce a destructive effect of the interference terms, a cross section which is $1/N$ of the one in case I may be obtained. Further, we may mention that the splitting of the intermediate states can be possible in more than one dimension giving rise to a very large number of states N. Mechanisms to ensure a constructive behavior of all phases are not very common, but will be generally connected to a phase transition.

For completeness we discuss case III (Fig. 1). Here the second step may populate M final states, which collect the total strength in states with different quantum numbers spanning the whole space of the variables characterizing the intermediate states (e.g., spins from the coupling of the quantum numbers of the intermediate steps). We define (total strength kept unchanged) for a transition from intermediate state j, going to M "substates" of channel 2, an incoherent sum of states 2:

$$P_{1j2}=\sum_K^M|a_{1j2K}|^2\simeq|a_{01j}|^2 .$$

If the final states "$2K$" are reached *only by one route* "$1i$," which implies one configuration, the final probability summed over K and N is incoherent, and Eq. (3b) becomes

$$P_2=\sum_i^N\sum_K^M|(a_{01i}\times a_{1i2K})|^2 \tag{4a}$$

and we recover the result of case I because the coherence effect discussed for case II was not included.

$$P_2^{\rm III}\cong(P_1)^2,\quad \text{EF}\equiv 1;\ \text{for case III} . \tag{4b}$$

The reality may often be represented by a case intermediate between cases II and III; some states of class 2, with index K may collect amplitudes via states of class 1 as in case II. This is indicated in Fig. 1 by the additional double lines, some states (e.g., the low lying $0^+,2^+$ states) may thus collect several coherent contributions. In this case parts of the summation with index i in Eq. (4a) will be under the vertical bar. An enhancement larger than 1 is expected in this case, however, smaller than N.

Before discussing two examples, other possibilities for enhancement have to be mentioned, which will originate from the properties of a one-step amplitude a_{02}. In the case of two-particle transfer, the pairing field acting on the center of mass of a correlated pair gives an additional one-step amplitude; this amplitude will depend on the geometry of the two-particle wave function which can be strongly influenced by configuration mixing. Mixing of configurations with different parity may induce, in addition, an extreme localization (hybridization) in configuration space and strongly enhance the one-step amplitude due to geometric effects.[6] Further, coupling of two multipolarities (e.g., in collective two-step excitation) allows the total multipolarity of the a_{02} amplitude to be lower and give dynamically favored conditions for the latter. Finally, the enhancement may in some cases be attributed to a collective state of the total system, which will be enhanced at a particular relative velocity, for example, in the formation of a resonance.[9]

The total enhancement according to Eq. (1) can be related to the strength of a single-particle state in cases where single steps can be measured and defined. Thus an enhancement in single-particle units can be defined.

We come back to the previous discussion and look at two examples. The first is for nuclear collisions, the one- and two-neutron transfer between tin isotopes, ^{112}Sn + ^{120}Sn, below the Coulomb barrier.[1] The relevant reaction routes for one of the nuclei (target ^{112}Sn) is shown in Fig. 2; these routes (amplitudes) have to be multiplied with the corresponding scheme of states for the projectile, ^{120}Sn, which encounters almost the same intermediate states. The first step (one neutron transfer on the 0^+ target) populates single-particle states which are split by the different shell-model quantum numbers and the strength in population is smeared out by the pairing interaction (the sum over all these states was measured in the experiment as P_1). The second step representing P_2 shows dominantly three states ($0^+,2^+,3^-$) among these, the 0^+ can collect all configurations $(lj)^2$, the 2^+ state 70%, and the 3^- is $\sim$15%. The measured ratio, $P_2/(P_1)^2\approx 3=$EF, shows a value intermediate between cases II and III. This value of the enhancement can be attributed to the

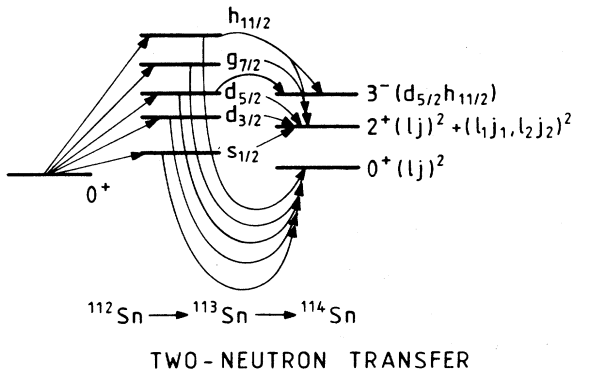

FIG. 2. States contributing as intermediate amplitudes in the two-neutron transfer in the reaction $^{112}Sn(^{120}Sn, ^{118}Sn)^{114}Sn$ at an energy below the Coulomb barrier.

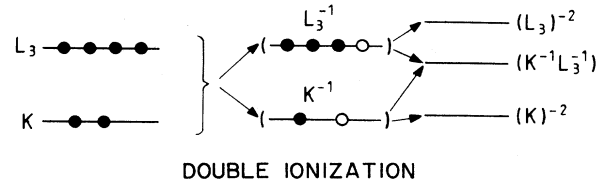

FIG. 3. Scheme of states contributing to the double ionization of ^{58}Ni in a collision with ^{208}Pb at 1.5 MeV/nucleon.

coherent phases introduced by the pairing interaction in the sequential two-step amplitudes. Larger values for EF are found for proton-pair transfer,[2] which can be attributed to a one-step contribution due to a true pair-transfer situation.

The second case concerns double ionization in atomic collisions. The particular case considered is shown in Fig. 3 and discussed in Ref. 4(a); it is $^{58}Ni + ^{208}Pb$ at 1.5 MeV/nucleon. From the figure we can see that a constructive interference in the second final level (K^{-1}, L_3^{-1}), which can be populated by two different routes, should produce an enhancement. In the spirit of the discussion given above we compare the total strength in the first step [denoted by $Q(1)$ in Ref. 4(a)] and the second step [denoted by $P(2)$]. The result of the experiment given in Fig. 3 of Ref. 4(a) for an impact parameter range of 300–400 fm is $Q(1) = P_1' = 2.5 \times 10^{-1}$ and $P(2) = P_2 = 3.0 \times 10^{-2}$. The result would give EF=0.48 instead of unity (expected possibly here). This is due to the treatment in first order which is not applicable for large probabilities (larger than 0.1). We proceed as in Ref. 4(a) and define $P_1 + 2P_2 = Q(1)$ by adding back the flux which has been removed from P_1 due to the two second steps leading to double ionization. Now the result is $P_1 = 1.9 \times 10^{-1}$ and P_1^2 is 3.6×10^{-2}, and the net result is now EF$\simeq$1. This means no enhancement is observed in this case. The more elaborate discussion in Ref. 4(a) produces an enhancement of factor 3.2 by comparing the summed probability of all three final states with the probability of a single step which must be three, as can be deduced from the discussion given above. This result, however, is not an enhancement.

To summarize, the enhancement in two-step processes can suitably be defined for conditions where the total strength in a multiparticle aggregate for a single step can be well determined. The splitting into intermediate states, if causing specific phase relations, can be seen as the origin of enhancement. In the particular case where phase transitions occur (superfluid phase, deformed phase in nuclei) a definition of enhancement in appropriate single-particle units[1,8] can be made and will be in these cases a quantitative measure of the collectivity of the total transition probability, which often will contain an additional one-step contribution.

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