Multiconfiguration resonating-group study of the six-nucleon system with cluster-rearrangement and pseudo-inelastic configurations

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The six-nucleon system is studied with a multiconfiguration resonating-group calculation which consists of the dominant configuration $d+\alpha$, the cluster-rearrangement configurations $p+{}^5$He and $n+{}^5$Li, and the pseudo-inelastic configuration $d^*+\alpha$. The result shows that, because the deuteron cluster is easily distortable, a $d+\alpha$ single-configuration study is inadequate. With the addition of the other cluster configurations, the ^{6}Li ground-state energy is improved by a large amount equal to 1.76 MeV. From an overall viewpoint, the $p+{}^5$He and $n+{}^5$Li configurations are found to contribute more significantly than the $d^*+\alpha$ configuration, thus confirming the findings from previous seven- and eight-nucleon investigations that single-nucleon transfer processes generally make important contributions and sequential-decay processes are more important than direct-breakup processes in the relatively low-energy region. The $d+\alpha$ total reaction cross sections have also been computed and found to agree rather well with empirical results. At 10 MeV, for example, the calculated total reaction cross section is equal to 85% of the empirical value, which is the highest percentage obtained in our six- to eight-nucleon microscopic resonating-group calculations containing no phenomenological imaginary potentials.

I. INTRODUCTION

Recently, we have carried out a series of multiconfiguration resonating-group investigations[1-5] in light nuclear systems with $A=4-8$. In these investigations, the purposes were to study not only the phase-shift and transmission-coefficient behavior, but also the characteristics of scattering and reaction cross sections. The results from these studies as a whole indicated that some general and important findings concerning reaction mechanisms in light systems can be obtained, and these findings are summarized in the following.

(i) The process of inelastic excitation to a rotational excited state of the participating cluster makes important contributions. For example, in the eight-nucleon case,[1] it was found that the $n+{}^7$Li* cluster configuration, with ^{7}Li* being the total orbital-angular-momentum $L=3$ rotational partner of ^{7}Li in its $L=1$ ground state, must be properly taken into consideration.

(ii) Among rearrangement collisions to two-cluster final states, where one or both of the outgoing clusters may subsequently decay into binary fragments, the one-nucleon transfer process dominates, particularly when no tightly bound clusters need to be broken up. As an example, we note that, for a satisfactory description of the properties of the seven-nucleon system,[2] the reaction process ^{6}Li$(n,d)^5$He, with ^{5}He decaying sequentially into $n+\alpha$, is very important.

(iii) At relatively low energies, sequential-decay processes are considerably more important than direct-breakup processes. In our investigations, direct-breakup processes are taken into account by introducing pseudo-inelastic configurations into the formulation. Somewhat to our surprise, however, it was discovered that the addition of such configurations alone does not generally result in a sufficiently extended model space for satisfactory conclusions. For example, in the six-nucleon system,[3] a calculation employing $d+\alpha$ and a large number of deuteron pseudo-inelastic configurations was found to yield only about 20% of the experimental $d+\alpha$ total reaction cross section, thus strongly indicating that, for this particular system, some other more important cluster configurations must still be incorporated into the formulation.

In this investigation, we continue our series of microscopic studies by considering again the six-nucleon system from the viewpoint of the $d+\alpha$ incident channel, but with the model space expanded to include also cluster-rearrangement configurations. Based on the findings discussed above, we shall perform the calculation with $d+\alpha$, $p+{}^5$He, $n+{}^5$Li, and deuteron pseudo-inelastic configurations. The inclusion of the $p+{}^5$He and $n+{}^5$Li configurations (collectively referred to as the $1N+5N$ configuration) is deemed to be important, because these configurations can be reached from the dominant $d+\alpha$ configuration by one-nucleon transfer processes with only the breakup of the loosely bound deuteron cluster. On the other hand, there are other two-cluster configurations which can be safely omitted without appreciably affecting the objectives of the present investigation. For example,

the $^3H+^3He$ cluster configuration is not likely to influence much the behavior in the $d+\alpha$ incident channel. Although this particular configuration can also be reached from the $d+\alpha$ configuration by a one-nucleon transfer process, there exists the unfavorable factor that a tightly bound α cluster has to be broken up. Experimentally, this is indeed manifested by the finding that the $\alpha(d,^3H)^3He$ partial reaction cross section[6] is only a very small part of the $d+\alpha$ total reaction cross section. Also, inelastic excitations to excited states of the α, ^5He, and ^5Li clusters will not be considered. This is a reasonable approximation to make, since such excitations have not only high-energy thresholds, but also are hindered by the fact that these excited states are not rotational partners of the corresponding nuclei in their ground states. In addition, it is interesting to point out that the $d'+\alpha$ configuration, with d' representing the $T=1$ singlet S state of the deuteron, can be left out of the calculation, because of isospin considerations.

Our next task is to decide the number of deuteron pseudo-inelastic configurations to be used in the calculation. For this decision, there are two contributing factors. First, it is our general understanding[7] that the Pauli principle has the effect of greatly reducing the differences between seemingly distinct cluster configurations when the nucleons are close to one another. This suggests that, since the model space is already spanned by $d+\alpha$ and $1N+5N$ cluster configurations, the number of pseudo-inelastic configurations does not need to be large. The second factor is a more practical one. Because the deuteron cluster in its pseudo-excited state and the $5N$ clusters have large spatial extensions, it is found that we have to deal with severe computational-time and numerical-accuracy problems. Thus, this factor also dictates that only a small number of pseudo-inelastic configurations should be included. Our final decision is to adopt just one such configuration, to be denoted as the $d^*+\alpha$ configuration. Although this number seems rather small, our experience in similar situations[8] does indicate that our aims in this investigation will not be drastically compromised.

To alleviate the severe computational requirements, we have made the simplifications of omitting the Coulomb interaction and by adopting a purely central nucleon-nucleon potential. These simplifications have also been made in our previous studies of the seven- and eight-nucleon systems,[1,2] and are not expected to seriously compromise our main objectives, which are to study the intricate interplay among various cluster configurations in determining the essential behavior of the S-matrix elements and to calculate the total reaction cross sections for the purpose of deciding the important question concerning the adequacy of the chosen model space.

We should mention that there exist other investigations in the six-nucleon system. In these investigations,[9,10] the authors used also the multiconfiguration resonating-group method, with the model spaces spanned by a number of two-cluster configurations. However, they have not included deuteron pseudo-inelastic configurations in their calculations, thus omitting from consideration the effects of deuteron direct breakup. Additionally, it should be pointed out that the emphases of the various existing studies are rather different. In these previous investigations,[9,10] the purpose was to examine charge-asymmetry effects in the $\alpha(d,^3H)^3He$ reaction, while in our present investigation the aim is to study the use of the calculated total reaction cross section as a gauge in determining whether or not the adopted model space is extensive enough. Our opinion is that all these investigations complement one another, and they yield collectively a good understanding regarding the essential properties of the six-nucleon system.

The outline of this paper is as follows. In Sec. II, we present a brief discussion of the formulation of the six-nucleon problem in terms of the $d+\alpha$, $p+^5He$, $n+^5Li$, and $d^*+\alpha$ cluster configurations. Results for bound-state energies and S-matrix elements are shown in Sec. III. In Sec. IV, we discuss the features of the calculated differential scattering and total reaction cross sections in the $d+\alpha$ channel. Concluding remarks are given in Sec. V, where a discussion of the essential findings of this investigation is also made.

II. BRIEF DISCUSSION OF THE FORMULATION

A. Model spaces and nucleon-nucleon potential

The six-nucleon system will first be formulated as a three-cluster $\alpha+n+p$ system, with the α cluster described by a translationally invariant shell-model function of the lowest-configuration in a harmonic-oscillator well of width parameter $\alpha=0.514$ fm^{-2}. From the three-cluster kernel derived, coupled-channel equations involving two-cluster $(\alpha+n)+p$, $(\alpha+p)+n$, and $\alpha+(n+p)$ configurations are then obtained by choosing appropriate relative-motion functions for the $\alpha+n$, $\alpha+p$, and $n+p$ subsystems. These coupled equations are eventually solved by using a variational technique employing Gaussian-type trial functions.[11] For all these steps, thorough discussions have already been given in a previous report[12] and, hence, will not be further described here.

The nucleon-nucleon potential employed in this investigation is the Minnesota (MN) potential given by Eqs. (9)–(11) of Ref. 13, with the Coulomb interaction omitted for simplicity in calculation. The exchange-mixture parameter u contained in this potential will be determined by the requirement that the calculated $d+\alpha$ cluster separation energy in the ground state of ^6Li be close to the empirical value of 2.32 MeV, obtained by making a Coulomb correction of 0.84 MeV to the experimental result of 1.48 MeV.[14] As will be seen below, this procedure results in a value of $u=0.98$, which is quite similar to the u value used in our recent seven- and eight-nucleon investigations.[1,2]

The cluster configurations $d+\alpha$, $1N+5N$ (i.e., $p+^5He$ and $n+^5Li$), and $d^*+\alpha$ will be referred to as configurations a, b, and c, respectively. In this investigation, because one of our main purposes is to see the effects of enlarging the model space, we shall perform the calculations in a number of model spaces, defined according to the cluster configurations included. In Table I, we

TABLE I. Model spaces considered, and the ground-state energies $\tilde{E}_0$ of ^{6}Li, calculated with respect to the $d+\alpha$ threshold.

Model space	Cluster configurations	$\tilde{E}_0$ (MeV)
Single configuration (SC)	$d+\alpha$	−0.54
Double configuration 1 (DC1)	$d+\alpha$, $1N+5N$	−1.43
Double configuration 2 (DC2)	$d+\alpha$, $d^*+\alpha$	−1.96
Triple configuration (TC)	$d+\alpha$, $1N+5N$, $d^*+\alpha$	−2.30

list the various model spaces considered, with SC, DC, and TC denoting single-configuration, double-configuration, and triple-configuration calculations, respectively.

The S-matrix element, obtained by solving the coupled-channel equations, will be denoted as S^L_{fi}, with L being the total orbital angular momentum obtained by coupling the relative orbital angular momentum l between the constituent clusters and the cluster internal orbital angular momentum I ($I=0$, 0, and 1 for clusters d, d^*, and $5N$, respectively). As in our seven- and eight-nucleon studies,[1,2] the initial channel i and the final channel f will be labeled by the value of l and the type of cluster configuration j ($j=a$, b, or c); i.e., they will be specified by a pair of indices (lj). For example, in the $L=2$ state, $S^2_{1b,2a}$ denotes an off-diagonal element describing the coupling between the $d+\alpha$ configuration with $l=2$ and the $1N+5N$ configuration with $l=1$. The parity of this state does not need to be further specified, since it is uniquely determined by the values of l and I. In the example just mentioned, the parity is easily seen to be equal to $+1$.

The diagonal element of the S matrix will be parametrized in terms of the reflection coefficient η^L_{ii} and the phase shift δ^L_{ii}, i.e.,

$$S^L_{ii}=\eta^L_{ii}\exp(2i\delta^L_{ii}) . \tag{1}$$

For the coupling or off-diagonal element S^L_{fi}, we shall mainly be interested in its absolute value, namely, the transmission coefficient η^L_{fi} given by

$$\eta^L_{fi}=|S^L_{fi}| . \tag{2}$$

To obtain reliable results for the various physical quantities, we have carefully carried out the variational calculation to insure that the unitarity of the S matrix is accurately satisfied.

B. Cluster internal wave functions

1. d and d^* wave functions

The $n+p$ relative-motion wave functions χ_d and χ^*_d in the deuteron ground state d and pseudo-excited state d^* are assumed to be superpositions of two normalized Gaussian-type basis functions, i.e.,

$$\chi_d(\mathbf{r}_{11})=\phi^1_2+c_d\phi^2_2 , \tag{3}$$

where

$$\phi^i_2=A^i_2\exp(-\tfrac{1}{4}\eta^i_2\alpha r^2_{11})Y_{00}(\hat{r}_{11}) , \tag{4}$$

with

$$A^i_2=(4\pi)^{1/2}(\eta^i_2\alpha/2\pi)^{3/4} , \tag{5}$$

and a similar expression for χ^*_d, but with c_d in Eq. (3) replaced by c^*_d. The variational parameters are determined by minimizing the ground-state energy expectation value of the deuteron Hamiltonian and by the condition that the d and d^* wave functions be orthogonal. The results are

$$\begin{aligned}&\eta^1_2=0.245 ,\\ &\eta^2_2=1.714 ,\\ &c_d=0.757 ,\\ &c^*_d=-1.087 .\end{aligned} \tag{6}$$

With these parameter values, the energy expectation values and rms matter radii of d and d^* are given by

$$\begin{aligned}&\tilde{E}_d=-2.02\text{ MeV},\quad \tilde{R}_d=1.79\text{ fm} ,\\ &\tilde{E}^*_d=14.36\text{ MeV},\quad \tilde{R}^*_d=2.27\text{ fm} .\end{aligned} \tag{7}$$

From the values of $\tilde{E}_d$ and $\tilde{E}^*_d$, we find that the $d^*+\alpha$ threshold occurs at 16.38 MeV above the $d+\alpha$ threshold. This is a rather high value and represents an undesirable feature resulting from our use of a single pseudo-inelastic configuration to simplify the calculation. However, since our calculation will cover a large energy range up to 25 MeV, we do expect that the important features of the deuteron direct-breakup process can still be learned in a reasonable manner.

2. $1N+\alpha$ wave function

The relative-motion function χ_5 in the $1N+\alpha$ system ($n+\alpha$ or $p+\alpha$) is also written as the superposition of two normalized Gaussian-type wave functions. It has the form[15]

$$\chi_5(\mathbf{r}_{14})=\phi^1_5+c_5\phi^2_5 , \tag{8}$$

where

$$\phi^i_5=A^i_5\exp(-\tfrac{2}{5}\eta^i_5\alpha r^2_{14})r_{14}Y_{1M}(\hat{r}_{14}) , \tag{9}$$

with

$$A^i_5=(8\pi^2/3)^{1/2}(4\eta^i_5\alpha/5\pi)^{5/4} . \tag{10}$$

As was discussed in Ref. 2, the parameters η_5^1, η_5^2, and c_5 are determined variationally under the conditions that the nucleus ^{5}He should possess strong clustering property, and that the $n+\alpha$ cluster separation energy be close to -2.22 MeV, a value obtained by utilizing experimental information for the $^2P_{3/2}$ and $^2P_{1/2}$ states. The final values used are

$$\eta_5^1=0.20\ ,$$
$$\eta_5^2=0.84\ , \tag{11}$$
$$c_5=0.344\ .$$

With these parameter values and the value of $u=0.98$ to be adopted eventually, the $1N+5N$ threshold is found to be at 4.32 MeV above the $d+\alpha$ threshold.

III. RESULTS FOR BOUND-STATE ENERGIES AND S-MATRIX ELEMENTS

A. Determination of the exchange-mixture parameter u and the ground-state rotational band of ^{6}Li

As was mentioned in Sec. II A, the exchange-mixture parameter u is determined by the condition that, in the TC calculation, the calculated $d+\alpha$ cluster separation energy in the $L=0$ ground state of ^{6}Li be nearly equal to the empirical value of 2.32 MeV, obtained by making a Coulomb correction of the experimental result. The value of u found in this way is 0.98, which yields a separation energy of 2.30 MeV. Therefore, unless otherwise stated, this particular u value will be adopted in all our subsequent calculations.

It is interesting to compare the value of u obtained here with those required in our previous seven- and eight-nucleon calculations.[1,2] Since the model spaces in all these calculations are fairly extensive, one anticipates that the resultant u values should be rather similar. Indeed, this turned out to be the case. In the eight-nucleon calculation, the adopted value of $u=1.0$ yielded a neutron separation energy of 1.23 MeV for the $(L^\pi,S)=(1^+,0)$ level. This latter value is 0.18 MeV larger than the experimental result[14] of 1.05 MeV which can be obtained if we reduce the u value to 0.98. As for the seven-nucleon case, it was found that, with $u=1.0$, the calculated $t+\alpha$ cluster separation energy in the lowest $(L,S)=(1,\frac{1}{2})$ state is 3.50 MeV. Here again, if one wishes to reduce this calculated value to the empirical value of 3.17 MeV, then u must also be reduced to 0.98. Thus, we find that, in all these three systems with $A=6$, 7, and 8, a single value of u equal to 0.98 can reproduce the experimental cluster separation energies in the lowest bound states. This is not only gratifying, but also an indication that the adopted model spaces are reasonably adequate and the corresponding calculations can explain many of the important properties in the low-excitation regions of these nuclear systems.

The u value of 0.98 required here is almost exactly the same as that required in a previous calculation[3] in which the model space was spanned by $d+\alpha$ and 14 deuteron pseudo-inelastic configurations. This is an important finding, because it not only demonstrates in a dramatic way the reduction effect of the Pauli principle, but also gives us confidence that both the present and the previous calculations yield an accurate description of the ground-state behavior of ^{6}Li.

With $u=0.98$, the $L=0$ ground-state energies $\tilde{E}_0$ of ^{6}Li, calculated with respect to the $d+\alpha$ threshold, are listed in Table I for the various model spaces. Here one notes the following important features.

(i) The single-configuration (SC) calculation is inadequate. With the addition of $1N+5N$ and $d^*+\alpha$ configurations, $\tilde{E}_0$ is improved by a large amount equal to 1.76 MeV. The reason for this is clear. The deuteron cluster is easily distortable, because of its high compressibility.

(ii) As far as the ground state of ^{6}Li is concerned, the addition of the $d^*+\alpha$ configuration is even more effective than the addition of the $1N+5N$ configuration. This is quite likely a general finding. In other light systems,[8] we have also found that pseudo-inelastic configurations are particularly effective in improving the behavior of the system in low-L states.

We have also made an $L=0$ calculation in which the $1N+5N$ configuration alone is considered. The result for $\tilde{E}_0$ turns out to be equal to 0.82 MeV, which is considerably higher than the value of -0.54 MeV obtained in the SC calculation. This is interesting, because it verifies theoretically the empirical finding[16] that the nucleus ^{6}Li in its ground state has predominantly a $d+\alpha$ cluster structure.

The resonance energies of the $L=2$ state, which belongs to the ground-state rotational band, are obtained in the various model spaces by analyzing the phase-shift results to be discussed below. The result is shown in Fig. 1. From this figure, one notes the interesting feature that, although the $d^*+\alpha$ configuration is more effective than the $1N+5N$ configuration in the $L=0$ state, the opposite is true in the $L=2$ state which has a higher value for the total orbital angular momentum. In the TC calculation, the energy spacing between the $L=0$ and 2 states is equal to 3.70 MeV which agrees very well with the empirical value of 3.60 MeV, obtained by using experimental excitation energies of the $^3D_3$, $^3D_2$, and $^3D_1$ levels, weighted according to the expectation values of $\mathbf{L}\cdot\mathbf{S}$.

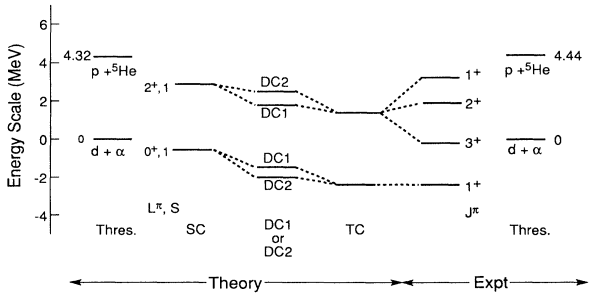

FIG. 1. Energy values of $L^\pi=0^+$ and 2^+ states obtained with SC, DC1, DC2, and TC calculations. An identification between calculated and experimental levels is also shown.

B. Phase shifts and transmission coefficients

Results for the $d+\alpha$ phase shifts and transmission coefficients as a function of E_a, the relative energy of the d and α clusters in the c.m. system, will be graphically presented in Figs. 2–6. In these figures, the calculated phase shifts will be represented by dashed curves in the SC case, open circles in the DC1 case, crosses in the DC2 case, and solid curves in the TC case. For the transmission coefficients, only the results in the TC case (solid curves) will be shown. Also, for clarity in presentation, we shall not show the phase shift in any L^π state where its value is smaller than 5° in the considered energy range of 0–25 MeV for E_a. Similarly, when the transmission coefficient has a value smaller than 0.2 in this energy range, it will also not be presented, simply for the sake of not overcrowding the graph involved.

Because of the use of totally antisymmetric wave functions in microscopic calculations, Pauli resonances (or almost forbidden states) with definite characteristic energies are present. As has been discussed previously,[2] these are real resonances in the calculation, but their characteristics are sensitively dependent upon the extension of the adopted model space. Therefore, their presence does prevent a simple and clear understanding of the level structure and the reaction mechanisms. Indeed, it is just for this reason that we choose to restrict our study to relatively low energies below 25 MeV, in order that the effects of these resonances can be minimized. For example, in the $L=1$ state, the characteristic energies of the Pauli resonances are equal to 30.5, 39.1, 43.1, and 58.0 MeV in the TC case, which are significantly beyond the energy region to be considered in this investigation.

1. $L^\pi=0^+$ state (Fig. 2)

Because the deuteron cluster is easily distortable, the effect of enlarging the model space is, as expected, quite appreciable. As can be seen from Fig. 2, the $d+\alpha$ phase shifts obtained in the SC and TC calculations differ by a large amount of around 20°.

The $d^*+\alpha$ configuration is clearly more effective than the $1N+5N$ configuration in this low orbital-angular-momentum state. In fact, it is seen from this figure that the phase-shift result of the DC2 calculation is already rather similar to that of the TC calculation in the whole energy range considered.

The effectiveness of pseudo-inelastic configurations in low-L states is a very useful finding. It means that, especially for the description of the ground state of a nuclear system, which has generally a rather low value for the total orbital angular momentum, the introduction of pseudo-inelastic configurations into the formulation will frequently be sufficient.[17] This is quite convenient from the computational viewpoint, since the kernel functions representing the couplings among the various cluster configurations can be analytically derived in a particularly simple manner, if only such configurations (i.e., no cluster-rearrangement configurations) are involved in the calculation.

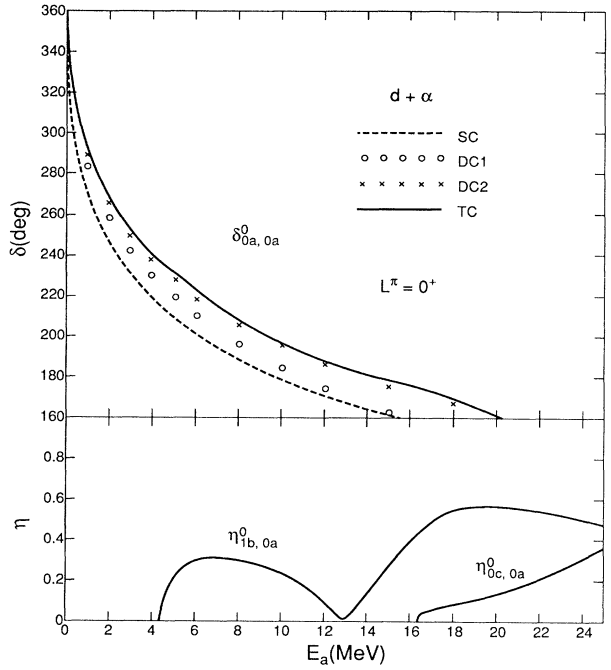

FIG. 2. Calculated phase shifts and transmission coefficients for $L^\pi=0^+$ in the $d+\alpha$ channel. Dashed curve, open circles, crosses, and solid curves represent results obtained with SC, DC1, DC2, and TC calculations, respectively.

2. $L^\pi=1^-$ state (Fig. 3)

The features of the phase shift and the transmission coefficient in this state can be summarized as follows.

(i) The phase-shift points in the DC2 case show the presence of a resonance state at $E_a \cong 18$ MeV. This state has a dispersionlike behavior, indicating that it has

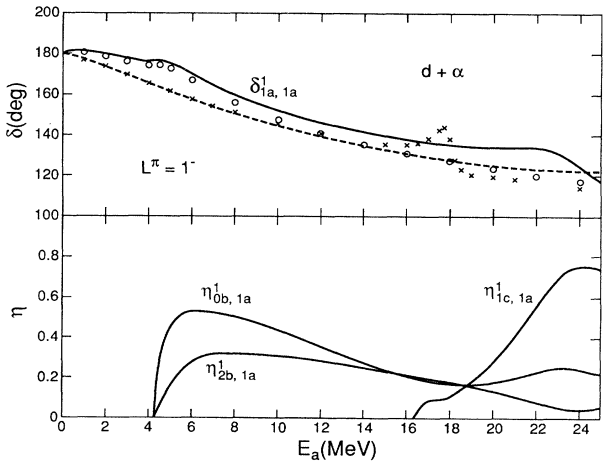

FIG. 3. Same as Fig. 2, except that $L^\pi=1^-$.

predominantly a $d^*+\alpha$ cluster structure. With the further addition of the $1N+5N$ configuration to go into the TC space, it is noted that this resonance state has moved to a higher energy around 24 MeV (see also the behavior of $\eta^1_{1c,1a}$).

(ii) A cusplike behavior occurs in the $d+\alpha$, TC phase-shift curve, due to the coupling between the $d+\alpha$ configuration and the $1N+5N$ configuration with $l=0$. However, this cusp is not too prominent, and its presence is not expected to have much influence on the experimental cross sections.

(iii) At low energies, it is seen that $\eta^1_{0b,1a}$ is appreciably larger than $\eta^1_{2b,1a}$. This shows that, because of centrifugal-barrier effects, the aligned configuration (i.e., $l=L-1$) of the $1N+5N$ structure makes generally the most significant contribution in the low-energy region.

3. $L^\pi=2^+$ state (Fig. 4)

The phase-shift behavior in the resonance region shows clearly that the SC calculation does not lead to a sufficiently accurate result. Upon expanding the model space to go to the TC calculation, the $L=2$ resonance energy is improved by an appreciable amount equal to 1.48 MeV. This latter value is, however, still smaller than the corresponding value of 1.76 MeV found in the $L=0$ ground state (see Table I), indicating that the phenomenon of centrifugal stretching is important in a light system and, hence, the influence of specific distortion decreases with increasing orbital angular momentum.

By comparing the DC1 and DC2 results, it is seen that, in the low-energy resonance region, the $1N+5N$ configuration is appreciably more important than the $d^*+\alpha$ configuration, although the latter does still make a significant contribution. At higher energies, the reverse seems to be true, which is due to the fact that, at $E_a\cong 25$ MeV, there exists in the DC2 calculation a broad resonance structure having predominantly a $d^*+\alpha$ cluster configuration.

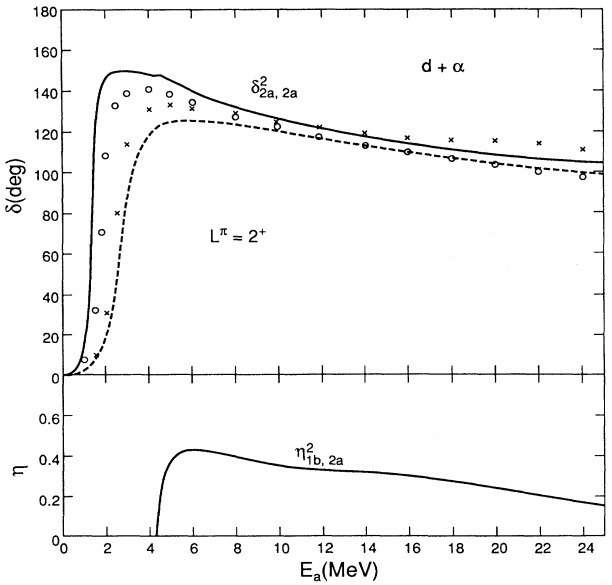

FIG. 4. Same as Fig. 2, except that $L^\pi=2^+$.

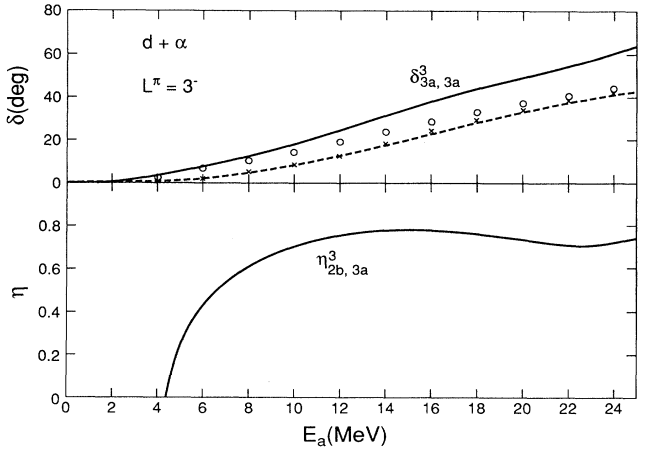

FIG. 5. Same as Fig. 2, except that $L^\pi=3^-$.

4. $L^\pi=3^-$ and 4^+ states (Figs. 5 and 6)

In the $L^\pi=3^-$ state, the effect of enlarging the model space is again quite large. For instance, at $E_a=24$ MeV, the phase shifts in the TC and SC calculations are equal to 60.6° and 41.3°, respectively. In addition, we note from Fig. 5 a very interesting feature which occurs at higher energies. The phase-shift values at 24 MeV are 45.1° in the DC1 case and 42.6° in the DC2 case, indicating that adding either the $1N+5N$ or the $d^*+\alpha$ configuration to the $d+\alpha$ configuration does not greatly improve the SC result in this state. The large improvement obtained in the TC case comes, in fact, from a coherent interplay among all three cluster configurations included in this calculation.

The situation in the $L^\pi=4^+$ state is rather different. From Fig. 6, one finds that, in the whole energy range considered, the DC2 result is similar to the SC result,

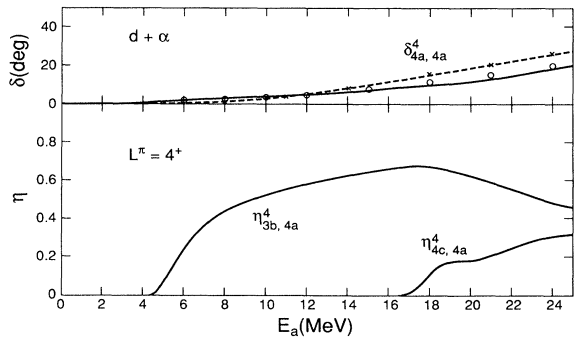

FIG. 6. Same as Fig. 2, except that $L^\pi=4^+$.

while the TC result is similar to the DC1 result. Both of these findings indicate that the contribution of the $d^*+\alpha$ configuration is quite minor in this state. Also, it is noted that, at higher energies, the phase-shift values of the TC calculation are smaller than those of the SC calculation, which is not a frequent occurrence.

Transmission coefficients $\eta^3_{2b,3a}$ and $\eta^4_{3b,4a}$ to the $1N+5N$ aligned configurations have large magnitudes. This suggests that these states are important in calculating the reaction cross sections of the $d+\alpha$ channel.

5. $L^\pi=5^-$ and 6^+ states (not shown)

Phase shifts in $L^\pi=5^-$ and 6^+ states increase slowly with energy in the TC case and, at $E_a=25$ MeV, have values equal to 3.7° and 1.4°, respectively. Similarly, the dominant transmission coefficients, which couple the $d+\alpha$ configuration with the $1N+5N$ aligned configuration, are not large, but still have appreciable magnitudes at higher energies. These results suggest that, for an accurate computation of the scattering and reaction cross sections, these L states must still be included in the calculation.

IV. RESULTS FOR CROSS SECTIONS

A. Total reaction cross section

In Fig. 7, we compare the calculated $d+\alpha$ total reaction cross sections σ_R (solid curve) in the TC case with the empirical values of Jenny *et al.*[18] (solid circles) obtained by a careful analysis of experimental data on differential cross sections, vector and tensor analyzing powers of $d+\alpha$ scattering. As is seen from this figure, the calculation explains quite well the energy-dependent behavior of the empirical result. The main defect is that, in an overall sense, the calculated σ_R is about 70 mb too small. For example, at $E_a=10$ MeV, the calculated value[19] is 431 mb which is equal to 85% of the empirical value of 509 mb. As to the reason for this underestimate, it is our opinion that the adoption of a single deuteron pseudo-inelastic configuration, with a rather high-energy threshold, may be mainly responsible. In addition, it should be noted that, at $E_a \geq 14.3$ MeV, the reaction channel $\alpha(d,{}^3\mathrm{H})^3\mathrm{He}$ also becomes open. However, we do not think that the omission of the ${}^3\mathrm{H}+{}^3\mathrm{He}$ cluster configuration in our formulation is an important factor, because the empirical value[6] of the $\alpha(d,{}^3\mathrm{H})^3\mathrm{He}$ reaction cross section is only about 20 mb, even when E_a is as high as 25 MeV.

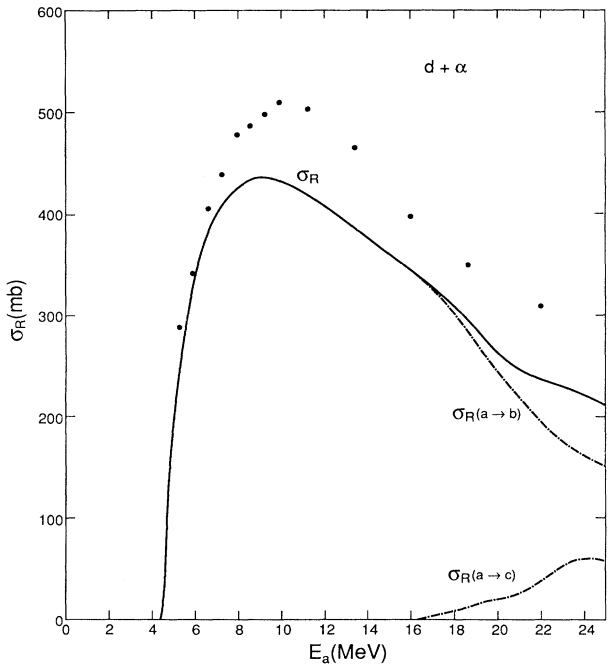

FIG. 7. Calculated $d+\alpha$ total reaction cross section in the TC case (solid curve). Contributions from individual cluster configurations (dot-dashed curves) are also shown. Empirical data (solid circles) are those of Ref. 18.

Partial reaction cross sections $\sigma_R(a\to b)$ and $\sigma_R(a\to c)$ for $d+\alpha$ going into $1N+5N$ and $d^*+\alpha$ (dot-dashed curves) are also shown in Fig. 7. Here one sees that the $\sigma_R(a\to c)$ curve becomes flat at E_a around 24 MeV, with a value in the flat region equal to about 60 mb. This latter value is smaller than the value of 161 mb for $\sigma_R(a\to b)$, and is equal to only 27% of the calculated $d+\alpha$ total reaction cross section at $E_a=24$ MeV.

Both the $a\to b$ and the $a\to c$ reactions lead to the emission of an α particle, a proton, and a neutron. However, the reaction mechanisms involved are quite different. The reaction $a\to b$ represents a sequential-decay process, while the reaction $a\to c$ represents a direct-breakup process. From the discussion given in the preceding paragraph, we can conclude, therefore, that the sequential-decay process is more important than the direct-breakup process in the relatively low-energy region considered here. In this respect, it is interesting to note that, based on indirect but rather convincing evidence, similar conclusions have also been reached in our previous seven- and eight-nucleon investigations.[1,2]

We should emphasize that the transmission coefficients for the process $a\to b$ are especially large in larger-L states, indicating that the one-nucleon transfer process takes place essentially in the peripheral region. On the other hand, the pseudo-inelastic configuration, being responsible for the specific distortion of the deuteron cluster, is found to be more effective in low-L states. This is readily understandable, because cluster distortion is expected to occur mainly in the interior region of the compound nucleus where all nucleons are in the proximity of one another.

The total reaction cross section obtained in the DC1 case follows the same energy-dependent trend as shown in Fig. 7 for the TC case. However, the calculated value of σ_R is, on the average, about 25 mb smaller than the TC value. For example, the results at $E_a=10$ MeV are equal to 408 and 431 mb in the DC1 and TC cases, respectively. As regards the DC2 case, the calculated σ_R curve also becomes quite flat when E_a reaches about 24 MeV, with a value around 70 mb at this energy. This is a

rather small value for σ_R, indicating that an expansion of the model space with only the pseudo-inelastic configuration is grossly inadequate to explain the experimental total reaction cross-section data.[3]

B. Elastic scattering cross section

Differential elastic cross sections for $d+\alpha$ scattering at 8.0 and 21.53 MeV, calculated in the TC case, are shown by solid curves in Fig. 8, where a comparison with experiment[20,21] is also made. Here one finds that the calculation nicely reproduces the oscillatory patterns of the measured cross sections. The disagreement between theory and experiment at $\theta < 20°$ in the 8-MeV case is of no great significance; it results merely from our omission of the Coulomb interaction in the calculation. Similarly, one should not pay too much attention to the fact that the calculated cross-section minimum at about 137° is too deep, because this can be easily remedied by including noncentral components in the nucleon-nucleon potential.

The overestimate of the magnitudes at the cross-section maxima or shoulders is more serious. For example, at $E_a = 8.0$ MeV, the calculated 100° cross section is equal to 84 mb/sr, which is substantially larger than the measured value of 59 mb/sr. At first, one might believe that this overestimate is entirely correlated with the fact that the calculated $d+\alpha$ total reaction cross section is somewhat too small. However, we tend to think that this is probably not the case. When the SC calculation is improved by going into the TC calculation, the 100° cross section at 8.0 MeV decreases from 106 to 84 mb/sr and, at the same time, the total reaction cross section increases from 0 to 426 mb which is only 52 mb smaller than the empirical value[18] of 478 mb. This suggests that, even if one manages to bring the calculated total reaction cross section up to the empirical value by further expanding the model space, the problem of overestimating the cross sections will likely still persist. At present, we are inclined to believe that the underlying cause is the lack of consideration of noncentral forces (to a lesser extent, also the Coulomb force) in our calculation. As has been shown by Mertelmeier and Hofmann,[22] the tensor force plays an important role in determining the properties of ^{6}Li in its ground state. Thus, if a tensor force were included in our calculation, the strength of the central part of the nucleon-nucleon potential would have to be reduced in order that the cluster separation energy in the ^{6}Li ground state can be reproduced. Together with the reasonable anticipation that the tensor force will become less effective at higher energies, this reduction in the central-force strength will then bring about a significant decrease in the magnitudes at the cross-section peaks. Of course, we realize that this is largely a speculative suggestion; a confirmation of our opinion will certainly require extensive investigations which we intend to carry out in the near future.

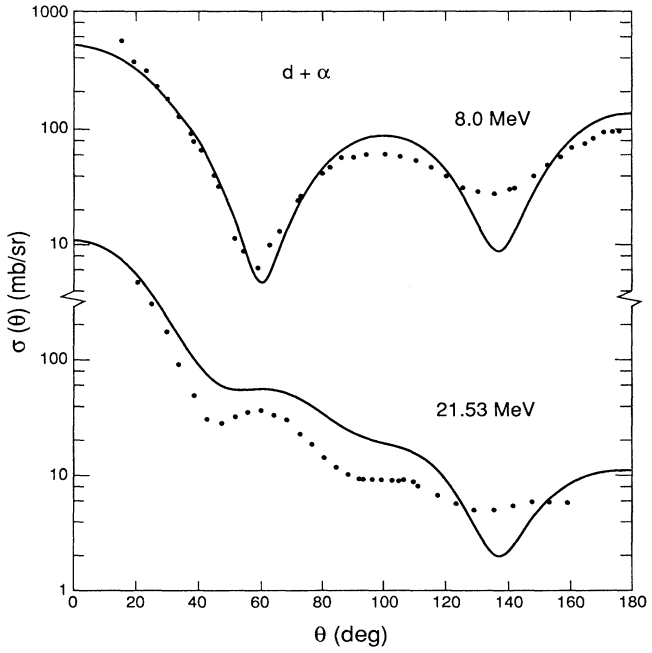

FIG. 8. Comparison of calculated $d+\alpha$ differential scattering cross sections at 8.0 and 21.53 MeV with experiment. Data shown are those of Refs. 20 and 21.

V. CONCLUSION

In this investigation, we study the properties of the six-nucleon system by working in a model space that consists of the dominant configuration $d+\alpha$, the cluster-rearrangement configurations $p+{}^5$He and $n+{}^5$Li (collectively denoted as $1N+5N$), and the pseudo-inelastic configuration $d^*+\alpha$. The main purpose is to see how well the empirical $d+\alpha$ total reaction cross sections can be reproduced by a calculation which takes into account not only the single-nucleon transfer process but also the deuteron direct-breakup process. The result does turn out to be quite satisfactory. At 10 MeV, for example, the calculation yields about 85% of the empirical total reaction cross section, which is the highest percentage obtained in our six- to eight-nucleon microscopic resonating-group calculations containing no phenomenological imaginary potentials. This indicates that, although further improvements can still be made, the present calculation does contain the necessary ingredients required for an adequate description of the behavior of this system.

Because the deuteron cluster is highly distortable, a $d+\alpha$ single-configuration study turns out to be quite inadequate. In the ground state of ^{6}Li, the addition of the $1N+5N$ and $d^*+\alpha$ cluster configurations improves the energy by a large amount equal to 1.76 MeV. For the $L=2$ rotational excited state, the improvement is somewhat smaller, because of centrifugal-stretching effects. But even here, the improvement of 1.48 MeV in the resonance energy is appreciable.

With the exception of the $L=0$ state, the $1N+5N$ configuration is found to contribute more significantly than the $d^*+\alpha$ configuration in an overall sense. This confirms the findings from our previous studies in the seven- and eight-nucleon systems[1,2] that single-nucleon

transfer processes generally make important contributions and sequential-decay processes are more important than direct-breakup processes in the relatively low-energy region. These are far-reaching conclusions, since they can be profitably used to guide our thinking regarding the type of cluster configurations which should be included in future resonating-group investigations of other light nuclear systems.

We should mention that our calculation complements the extensive six-nucleon resonating-group study of Bruno *et al.*[10] In the investigation of these latter authors, a more elaborate nucleon-nucleon potential containing Coulomb and noncentral components was used. On the other hand, our calculation examines the significance of the deuteron direct-breakup process and studies the relative importance of various reaction mechanisms. By combining the information learned from both calculations, we think that there exists now a rather good understanding of the essential characteristics of the six-nucleon system.

Some improvements are still worth making. First, it is desirable to include Coulomb and noncentral forces in the calculation. This will very likely lead to a better agreement between calculated and experimental differential scattering cross sections and enable us to study the features of vector and tensor analyzing powers of $d+\alpha$ scattering. Second, more deuteron pseudo-excited states should be adopted, such that the energy continuum of the deuteron cluster can be represented in a more adequate manner and the problem of high-energy threshold can be avoided. Both of these improvements can, in fact, be readily incorporated into our present resonating-group formulation. The main difficulties are really technical ones associated with computational time and numerical accuracy. We should emphasize, however, that these difficulties are by no means insurmountable. With some further advance in analytical and computational techniques, we should be able to overcome them in the not-too-distant future and, thereby, perform an even more exciting study of the interesting six-nucleon problem.

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