Coulomb correction to elastic α-α scattering

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The elastic α-α scattering is treated within the framework of a generalized phase-function method (GPFM). This generalization consists in absorbing the effect of Coulomb interaction in the comparison functions for developing the phase equation. Based on values of scattering phase shifts computed by the present method, it is concluded that the GPFM provides an uncomplicated approach to rigorous Coulomb correction in the α-α scattering.

I. INTRODUCTION

The α-α interaction $V_{\alpha\alpha}$ is a combination of the Coulomb potential V_C and some short-range interaction V_S. The short-range interaction is of nuclear origin while the Coulomb potential takes care of the charges. The essential features of V_S have been clarified by the microscopic theories of nucleus-nucleus interaction like the generator coordinate method[1] (GCM) and/or the resonating-group method[2] (RGM). In the RGM there appears an interaction generated from the two nucleon forces which consists of two parts: (i) a direct part and (ii) a fairly complicated exchange part (nonlocal kernel). This nonlocal kernel presents mathematical difficulties for rigorous inclusion of the Coulomb effect in the studies of elastic scattering of α particles.[3]

As an approximation to RGM Saito[4] has introduced an orthogonality condition model (OCM). The OCM constrains the relative wave functions to be orthogonal to redundant states forbidden by the Pauli principle. Neudatchin *et al.*[5] have found that a deep α-α attractive local potential which can simulate the effect of unphysical bound states of the microscopic theories can be used in the conventional Schrödinger equation to describe the experimental α-α phase shift just as well as the full RGM equation or the OCM equation. In this work we shall represent V_S by a deep local potential and compute the α-α elastic-scattering phase shifts by including the Coulomb effect rigorously. Our approach to the problem will be based on a generalization of the phase-function method[6] (PFM) for nonrelativistic potential scattering.

In Sec. II, we present a brief review of the conventional phase method and seek an appropriate generalization of the approach to deal with scattering on V_C+V_S. We devote Sec. III to demonstrate the usefulness of the generalized phase-function method (GPFM). In particular, we work with a Gaussian α-α potential,[7] which is independent of both angular momentum and energy and is accurate enough up to energy of 40 MeV in the center-of-mass system, and examine the effect of Coulomb distortion of the nuclear scattering phases.

II. GENERALIZED PHASE-FUNCTION METHOD

The quantity one deals with in the PFM is a variable phase or phase function $\delta_l(k,\rho)$ which represents the phase shift at an energy k^2 due to the potential $V(r)\Theta(\rho-r)$ [$\Theta(x)$ is the step function which vanishes for $x<0$ and is unity otherwise]. It follows by definition that $\delta_l(k,0)=0$ and $\delta_l(k,\infty)=\delta_l(k)$, the lth partial-wave phase shift. It obeys a first-order nonlinear differential equation called the phase equation. The complete description of a wave-mechanical problem needs an amplitude function $\alpha_l(k,\rho)$ in addition to the phase function. Once the phase function is known, the amplitude function $\alpha_l(k,\rho)$ can be obtained by solving a first-order linear differential equation called the amplitude equation with the initial condition $\alpha_l(k,0)=1$. Newton[8] has shown that $\alpha_l(k,\rho)$ represents the modulus of the Jost function[9] produced by potential truncated at ρ. The phase and amplitude equations which constitute the basic algorithms of the PFM are derived as follows.

Consider a particle whose radial motion in a central field is described by the Schrödinger equation and suppose that its Hamiltonian $H(r)$ can be partitioned into two parts:

$$\begin{aligned} H(r)\psi(k,r)&=[H_0(r)-V(r)]\psi(k,r)\\ &=k^2\psi(k,r)\ . \end{aligned} \tag{1}$$

The partition of $H(r)$ in Eq. (1) is chosen so that $H_0(r)$ has a pair of known independent solutions (u_1,u_2), satisfying

$$\begin{aligned} &H_0(r)u_i(k,r)=k^2u_i(k,r),\quad i=1,2\ ,\\ &u_1(k,0)=0\ . \end{aligned} \tag{2}$$

The solutions $u_1(k,r)$ and $u_2(k,r)$ are 90° out of phase. Thus they can be viewed[10] as a pair of Cartesian axes in the two-dimensional space of solutions of the operator $H(r)$. The PFM proceeds by an ansatz

$$\psi(k,r)=\alpha(k,r)[u_1(kr)\cos\delta(k,r)-u_2(k,r)\sin\delta(k,r)] \tag{3}$$

and the constraint

$$\begin{aligned} \psi'(k,r)=\alpha(k,r)[&u_1'(k,r)\cos\delta(k,r)\\ &-u_2'(k,r)\sin\delta(k,r)]\ . \end{aligned} \tag{4}$$

Here the prime denotes the differentiation with respect to

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r. It is important to note that the particular solution (3) of Eq. (1) can be constructed by using Lagrange's method with u_i's as comparison functions, and the constraint (4) follows naturally from the freedom implied by the method of variation of parameters.[11] Equations (1)–(4) can be combined to write the phase and amplitude equations as

$$\delta'(k,r) = -V(r)[W(u_1(k,r),u_2(k,r))]^{-1} \times [u_1(k,r)\cos\delta(k,r) - u_2(k,r)\sin\delta(k,r)]^2 \quad (5)$$

and

$$\alpha'(k,r) = -[V(r)\alpha(k,r)/W(u_1(k,r),u_2(k,r))] \times [u_1(k,r)\cos\delta(k,r) - u_2(k,r)\sin\delta(k,r)] \times [u_1(k,r)\sin\delta(k,r) + u_2(k,r)\cos\delta(k,r)] , \quad (6)$$

where $W(u_1(k,r),u_2(k,r))$ stands for the Wronskian determitant for $u_1(k,r)$ and $u_2(k,r)$.

In the conventional phase method $H_0(r)$ refers to a free-particle Hamiltonian so that $u_1(k,r)$ and $u_2(k,r)$ stand for the Riccati Bessel and Riccati Neumann functions of order l. But here we are interested in a short-range potential superimposed over the Coulomb potential. We thus look for a generalized approach in which we absorb the Coulomb part of the potential in $H_0(r)$. Admittedly, the comparison functions become the regular and irregular Coulomb functions. If we now set up a phase equation that contains explicitly only the short-range potential, it will yield directly the additional phase shift due to this potential. Note that this formulation of the problem will bypass the characteristic difficulties associated with the long-range nature of the Coulomb interaction. We have chosen to work with the following regular and irregular Coulomb functions:[12]

$$F_l(\rho) = u_1(k,r) = c_l(\eta)\rho^{l+1}\exp(-i\rho)\,{}_1F_1(l+1+i\eta;2l+2;2i\rho) \quad (7)$$

and

$$G_l(\rho) = u_2(k,r) = [2\eta/c_0^2(\eta)]F_l(\rho)[\ln 2\rho + q_l(\eta)/p_l(\eta)] + \Theta_l(\rho) , \quad (8)$$

where $\rho = kr$ and

$$c_l(\eta) = 2^l\exp(-\pi\eta/2)|\Gamma(l+1+i\eta)|/\Gamma(2l+2) . \quad (9)$$

The quantities $q_l(\eta)/p_l(\eta)$ and $\Theta_l(\rho)$ have quite complicated mathematical structures and are given explicitly in Ref. 12. Here η represents the so-called Sommerfeld parameter. The Wronskian $W(F_l,G_l) = k$. In terms of Eqs. (7) and (9) the phase equation for Coulomb distorted nuclear scattering on $V(r)$ is obtained as

$$\delta_l'(\rho) = -k^{-2}V(\rho)[F_l(\rho)\cos\delta + G_l(\rho)\sin\delta]^2 . \quad (10)$$

Here the prime denotes the differentiation with respect to ρ and $V(\rho)$ is the reduced potential in units of $\hbar^2/2m$. In the next section, we shall make use of Eq. (10) to compute α-α elastic-scattering phases for the partial waves $l=0$, 2, 4, and 6 up to center-of-mass energy $E_{\text{c.m.}} = 40$ MeV. Note that in no Coulomb limit Eq. (10) gives the phase equation for scattering only by the nuclear interaction. Thus our approach to the problem provides a natural basis for examining the importance of Coulomb effects in elastic α-α scattering. Wherever necessary we shall use $\delta_l^{cs}(k)$ for the Coulomb distorted nuclear phase shift and $\delta_l^{s}(k)$ for the phase shift induced by the nuclear potential alone.

III. RESULTS AND DISCUSSIONS

We have chosen to work with $V(\rho)$ $[=V_S(\rho)]$ in the form

$$V(\rho) = V_1\exp(-\rho^2/k^2a^2) \quad (11)$$

with

$$V_1 = -11.828\,157\ \text{fm}^{-2} \text{ and } a = 2.132\ \text{fm} .$$

As already noted the two-parameter Gaussian potential (11) is angular momentum and energy independent and is quite accurate up to $E_{\text{c.m.}} = 40$ MeV. For this potential we have integrated Eq. (10) by the Runge-Kutta method with an appropriate stability check to compute the values of $\delta_l^{cs}(k)$, the Coulomb distorted nuclear phase shift. During the course of our integration we have generated the Coulomb functions in terms of the integral representation

$$F_l(\rho)+iG_l(\rho) = [\rho^{l+1}\exp(-\pi\eta)/(2l+1)!c_l(\eta)]\int_0^\infty \{1-\tanh^2 t)^{l+1}\exp[-i(\rho\tanh t - 2\eta t)] + i(1+t^2)^l\exp(-\rho t + 2\eta\arctan t)\}dt . \quad (12)$$

Since Eq. (12) is an infinite integral it is rather crucial to check the convergence of the integrand at a truncated upper limit. Such convergence was ensured by computing known values for $F_l(\rho)$ and $G_l(\rho)$ given either in McCarthy[13] or in Abramowitz and Stegun.[12] We have computed the pure nuclear phase shifts $\delta_l^s(k)$ by using the same set of algorithms but turning off the Coulomb interaction ($\eta=0$) in Eq. (10).

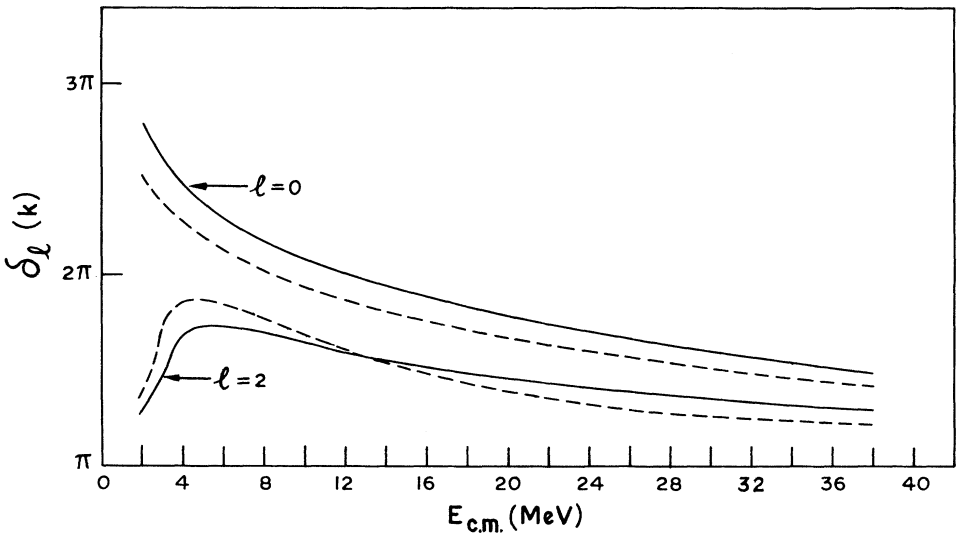

FIG. 1. The S- and D-wave scattering phase shifts $\delta_0(k)$ and $\delta_2(k)$ as a function of $E_{\text{c.m.}}$. The solid and dashed curves give the numbers with and without the Coulomb interaction.

In Figs. 1 and 2, we portray the variation of the scattering phase shifts $\delta_l(k)$ [$\delta_l^{cs}(k)$ or $\delta_l^{s}(k)$] as a function of $E_{\text{c.m.}}$. The solid and dashed curves denote the variation of $\delta_l^{cs}(k)$ and $\delta_l^{s}(k)$, respectively. All phase shifts start at the threshold ($E_{\text{c.m.}}=0$) with $n_l\pi$. In each partial wave n_l is the sum of the number of physical bound states and the number of redundant states. Note that, for the α-α system, the number of nonredundant or physical bound states is zero. In plotting the values of $\delta_l^{cs}(k)$ or $\delta_l^{s}(k)$ we have made use of an instructive representation by Neudatchin *et al.*[5] This representation clearly exhibits the requirement of a generalization of the Levinson's theorem[14] in the presence of redundant states that the high-energy limit of all phase shifts is zero. The S- and D-wave phase shifts are plotted in Fig. 1 while the corresponding results for G and H waves are shown in Fig. 2.

The residual phase written as $\delta_l^{r}(k)=\delta_l^{cs}(k)-\delta_l^{s}(k)$ is a critical quantity[15] for examining the role of Coulomb interaction in the phase-shift calculation of charged hadrons and also for comparing different methods for calculating Coulomb effects. Looking closely into our curves

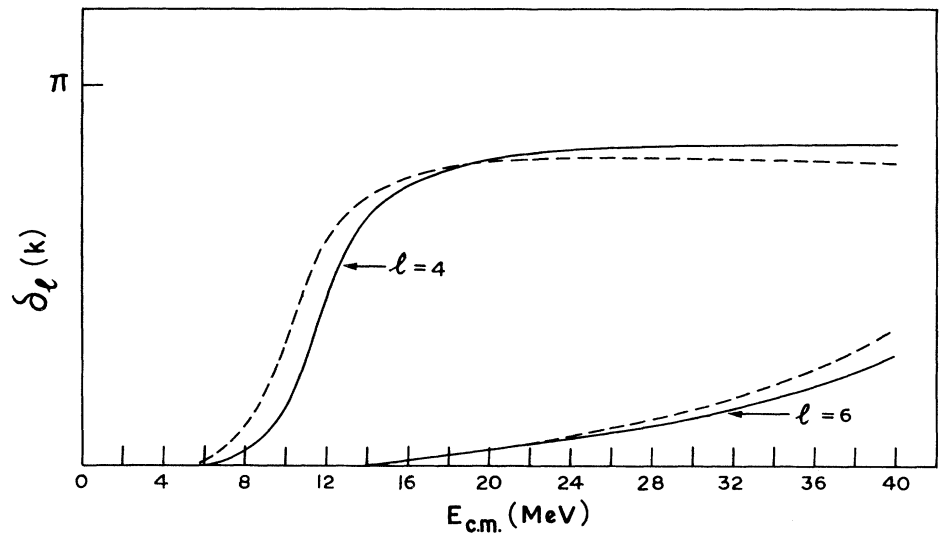

FIG. 2. The G- and H-wave scattering phase shifts $\delta_4(k)$ and $\delta_6(k)$ as a function of $E_{\text{c.m.}}$. The solid and dashed curves have the same meaning as in Fig. 1.

we see that, in general, $\delta_0^r(k) > \delta_l^r(k)$ ($l=2$, 4, and 6). Thus the S-wave phase shifts appear to be more sensitive to Coulomb effects than the corresponding results for higher partial waves. Our results for $\delta_0^{cs}(k)$ may be regarded as somewhat improved when compared with the numbers of Lumbroso[16] obtained by direct solution of the Hill-Wheeler equation presumably because Lumbroso calculated the Coulomb effect only approximately. Further, we note that the magnitude of the Coulomb correction $\delta_l^r(k)$ as obtained by us agrees quite well with those calculated by Baldock[17] and quoted by Barett *et al.*[18] In these studies a microscopic R-matrix method was used to compute the Coulomb matrix elements without approximation.

The phase shifts presented in this work were generated by using an IBM compatible PC/AT with numeric coprocessor (Intel 80287) running at 12 MHz and zero wait state. It took about 15 s CPU time to compute a value for $\delta_l^s(k)$ whereas generation of the corresponding results for $\delta_l^{cs}(k)$ required roughly 55 s. We have verified that these times are only 25% of the time consumed in an R-matrix method calculation.[17,18] We, therefore, conclude by noting that the generalized PFM used by us is an uncomplicated method for rigorous Coulomb correction to α-α elastic scattering.

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