Formal differences in perturbation methods for direct rearrangement collision processes

Sung-Ho Suck Salk
Department of Physics, Pohang Institute of Science and Technology, Pohang 790-600, Korea
(Received 4 September 1990)

Presently there exists a lack of systematic analysis on the differences in perturbation methods for studying direct rearrangement collisions. Emphasis is placed upon elucidating the formal differences in the structures of the T matrix in various perturbation theories by paying attention to differences in perturbation potentials and T operators. Extensive comparison between different perturbation theories and the relationship between them is explicitly shown to further highlight the formal differences among them.

I. INTRODUCTION

Depending on the choice of scattering (or relative) waves for the description of relative motions for colliding particles, formally three different classes of perturbation theory[1–16] are used for studying direct rearrangement collision processes. They are, namely, the plane, distorted (single-channel wave or elastic-scattering wave), and coupled-channel waves (multichannel or inelastic-scattering wave), respectively. Accordingly, the three perturbation methods are the plane-wave (PW), distorted-wave (DW), and coupled-channel wave (CW) Born series expansions of the transition amplitude. As a continuation of our earlier studies,[14] our main objective in the present study is to sharpen their differences and find their relationships by deriving structural differences in the formal expressions of the T matrix (transition amplitude). For completion, self-containment is attempted.

II. FORMAL DIFFERENCES IN PERTURBATION POTENTIALS AND T OPERATORS

To meet a necessity for symbol definitions to be used later, we partially review our earlier studies[14] by paying attention to the formal structures of both the perturbation potential and T operator. By introducing projection operators, explicit distinctions among different perturbation approaches in the structures of both the T operator and perturbation potential are made in a self-evident manner.

A. Differences in perturbation potentials

The Hamiltonian for arrangement α is formally written

$$H=H_\alpha=T_\alpha+V_\alpha+h_\alpha \ . \tag{2.1}$$

Here T_α and V_α are the kinetic- and potential-energy operators between a projectile and a target. h_α is the internal energy operator for both the target and projectile.

The distorted (elastic) wave X_γ^d to describe relative motion in the γ arrangement is obtained from[14(d)]

$$(h_\gamma+T_\gamma+U_\gamma^d)X_\gamma^{d(+)}=EX_\gamma^{d(+)} \ . \tag{2.2}$$

Here U_γ^d is the distorting potential responsible only for elastic collisions, and is defined by[14(d)]

$$U_\gamma^d=P_\gamma V_\gamma P_\gamma \ , \tag{2.3}$$

with the projection operator

$$P_\gamma=|\gamma_0\rangle\langle\gamma_0| \ , \tag{2.4}$$

which projects only onto the entrance channel (e.g., ground state) $|\gamma_0\rangle$.

Now the coupled-channel (inelastic) wave X_γ^c for the relative motion is obtained from[14(d)]

$$(h_\gamma+T_\gamma+U_{\bar{\gamma}}^c)X_\gamma^{c(+)}=EX_\gamma^{c(+)} \ . \tag{2.5}$$

Here U_γ^c is the distorting potential which allows coupling to inelastic channels:

$$U_\gamma^c=(P_\gamma+Q_\gamma)V_\gamma(P_\gamma+Q_\gamma) \ , \tag{2.6}$$

with the projection operator

$$Q_\gamma=\sum_\gamma|\gamma\rangle\langle\gamma| \ , \tag{2.7}$$

which projects onto the Hilbert subspace of the discrete states (e.g., excited states), $|\gamma\rangle$ of the target other than $|\gamma_0\rangle$. For completion we define R_γ as the projection operator onto the remaining Hilbert subspace of the continuum states,

$$R_\gamma=1-P_\gamma-Q_\gamma \ . \tag{2.8}$$

The Schrödinger equation for the total wave function is

$$(h_\gamma+T_\gamma+V_\gamma)\Psi_\gamma^{(+)}=E\Psi_\gamma^{(+)} \ . \tag{2.9}$$

By using (2.3), (2.6), and (2.8), the γ-channel interaction potential V_γ above can be rewritten,

$$V_\gamma=U_\gamma^d+W_\gamma^d \tag{2.10}$$

or

$$V_\gamma = U_\gamma^c + W_\gamma^c \ , \tag{2.11}$$

where, by introducing

$$V_\gamma = (P_\gamma + Q_\gamma + R_\gamma) V_\gamma (P_\gamma + Q_\gamma + R_\gamma) \ , \tag{2.12}$$

the reactive perturbation potentials that cause rearrangement collision are formally written

$$\begin{aligned} W_\gamma^d &= (P_\gamma + Q_\gamma + R_\gamma) V_\gamma (P_\gamma + Q_\gamma + R_\gamma) - P_\gamma V_\gamma P_\gamma \\ &= P_\gamma V_\gamma (Q_\gamma + R_\gamma) + (Q_\gamma + R_\gamma) V_\gamma (P_\gamma + Q_\gamma + R_\gamma) \end{aligned} \tag{2.13}$$

for the distorted- (single-channel) wave representation of the T matrix, and

$$\begin{aligned} W_\gamma^c &= (P_\gamma + Q_\gamma + R_\gamma) V_\gamma (P_\gamma + Q_\gamma + R_\gamma) \\ &\quad - (P_\gamma + Q_\gamma) V_\gamma (P_\gamma + Q_\gamma) \\ &= (P_\gamma + Q_\gamma) V_\gamma R_\gamma + R_\gamma V_\gamma (P_\gamma + Q_\gamma + R_\gamma) \end{aligned} \tag{2.14}$$

for the coupled-channel (multichannel) wave representation of the T matrix. Using (2.10) and (2.11), we rewrite the Hamiltonian (2.1),

$$H_\gamma = h_\gamma + T_\gamma + U_\gamma^d + W_\gamma^d = h_\gamma + T_\gamma + U_\gamma^c + W_\gamma^c \ . \tag{2.15}$$

The plane wave Φ, for the relative motion in arrangement γ can now be regarded to arise due to the neglect of the elastic, inelastic, and continuum channels, that is, $P_\gamma = Q_\gamma = R_\gamma = 0$ in (2.12). (2.9) simply leads to

$$(h_\gamma + T_\gamma)\Phi_\gamma = E\Phi_\gamma \ . \tag{2.16}$$

B. Three different classes of T operators

The Lippmann-Schwinger equations of interest are, with the use of (2.2), (2.5), (2.15), and (2.16) in (2.9),

$$|\Psi_\gamma^{(+)}\rangle = |\Phi_\gamma\rangle + G_\gamma^{(+)} V_\gamma |\Phi_\gamma\rangle \ , \tag{2.17}$$

$$|\Psi_\gamma^{(+)}\rangle = |X_\gamma^{d(+)}\rangle + G_\gamma^{(+)} W_\gamma^d |X_\gamma^{d(+)}\rangle \ , \tag{2.18}$$

and

$$|\Psi_\gamma^{(+)}\rangle = |X_\gamma^{c(+)}\rangle + G_\gamma^{(+)} W_\gamma^c |X_\gamma^{c(+)}\rangle \ . \tag{2.19}$$

Here the total Green's operator or total wave propagator is given by

$$\begin{aligned} G_\gamma^{(+)} = \frac{1}{E - H_\gamma + i\varepsilon} &= g_\gamma^{0(+)} + g_\gamma^{0(+)} V_\gamma G_\gamma^{(+)} \\ &= g_\gamma^{0(+)} + g_\gamma^{0(+)} V_\gamma g_\gamma^{0(+)} + \cdots \ , \end{aligned} \tag{2.20}$$

where $g_\gamma^{0(+)}$ is the plane-wave propagator,

$$g_\gamma^{0(+)} = \frac{1}{[E - (h_\gamma + T_\gamma) + i\varepsilon]} \ ; \tag{2.21}$$

$$\begin{aligned} G_\gamma^{(+)} &= g_\gamma^{d(+)} + g_\gamma^{d(+)} W_\gamma^d G_\gamma^{(+)} \\ &= g_\gamma^{d(+)} + g_\gamma^{d(+)} W_\gamma^d g_\gamma^{d(+)} + \cdots \ , \end{aligned} \tag{2.22}$$

where $g_\gamma^{d(+)}$ is the distorted- (elastic-scattering) wave propagator,[14(d)]

$$g_\gamma^{d(+)} = \frac{1}{[E - (h_\gamma + T_\gamma + U_\gamma^d) + i\varepsilon]} \ ; \tag{2.23}$$

or

$$\begin{aligned} G_\gamma^{(+)} &= g_\gamma^{c(+)} + g_\gamma^{c(+)} W_\gamma^c G_\gamma^{(+)} \\ &= g_\gamma^{c(+)} + g_\gamma^{c(+)} W_\gamma^c g_\gamma^{c(+)} + \cdots \ , \end{aligned} \tag{2.24}$$

where $g_\gamma^{c(+)}$ is the coupled-channel (inelastic-scattering) wave propagator,

$$g_\gamma^{c(+)} = \frac{1}{[E - (h_\gamma + T_\gamma + U_\gamma^c) + i\varepsilon]} \ . \tag{2.25}$$

The use of the Lippmann-Schwinger equations (2.17), (2.18), and (2.19) for the prior interaction form of the transition amplitude,

$$T_{\beta\alpha} = \langle \Psi_\beta^{(-)} | V_\alpha | \Phi_\alpha \rangle \ , \tag{2.26}$$

leads to the following three classes of T operators: the plane-wave operator $T^{\rm PW}$,

$$\begin{aligned} T^{\rm PW} &= V_\alpha + V_\beta G_\beta^{(+)} V_\alpha \\ &= V_\alpha + V_\beta g_\beta^{0(+)} V_\alpha + V_\beta g_\beta^{0(+)} V_\beta g_\beta^{0(+)} V_\alpha + \cdots \ , \end{aligned} \tag{2.27}$$

associated with

$$T_{\beta\alpha} = \langle \Phi_\beta | T^{\rm PW} | \Phi_\alpha \rangle \ ; \tag{2.28}$$

the distorted-wave operator $T^{\rm DW}$,[14(d)]

$$\begin{aligned} T^{\rm DW} &= W_\alpha^d + W_\beta^d G_\beta^{(+)} W_\alpha^d \\ &= W_\alpha^d + W_\beta^d g_\beta^{d(+)} W_\alpha^d \\ &\quad + W_\beta^d g_\beta^{d(+)} W_\beta^d g_\beta^{d(+)} W_\alpha^d + \cdots \ , \end{aligned} \tag{2.29}$$

associated with

$$T_{\beta\alpha} = \langle X_\beta^{d(-)} | T^{\rm DW} | X_\alpha^{d(+)} \rangle \ ; \tag{2.30}$$

and the coupled-channel wave operator $T^{\rm CW}$,[14(d)]

$$\begin{aligned} T^{\rm CW} &= W_\alpha^c + W_\beta^c G_\beta^{(+)} W_\alpha^c \\ &= W_\alpha^c + W_\beta^c g_\beta^{c(+)} W_\alpha^c \\ &\quad + W_\beta^c g_\beta^{c(+)} W_\beta^c g_\beta^{c(+)} W_\alpha^c + \cdots \ , \end{aligned} \tag{2.31}$$

associated with

$$T_{\beta\alpha} = \langle X_\beta^{c(-)} | T^{\rm CW} | X_\alpha^{c(+)} \rangle \ . \tag{2.32}$$

The T operators (2.29) and (2.31) corresponding to the DW and CW representations above are now seen to have distinctively different structures. Obviously this is owing to the manifest differences in the distorting potentials and perturbation potentials that are shown in (2.3), (2.6), (2.13), and (2.14) above.

III. TRANSFORMATION OF DISTORTED-WAVE TRANSITION AMPLITUDE INTO PLANE-WAVE REPRESENTATION

Using (2.29), the DW transition amplitude (2.30) is explicitly

$$T_{\beta\alpha}=\langle X_\beta^{d(-)}|W_\alpha^d+W_\beta^d g_\beta^{d(+)}W_\alpha^d + W_\beta^d g_\beta^{d(+)}W_\beta^d g_\beta^{d(+)}W_\alpha^d+\cdots|X_\alpha^{d(+)}\rangle\ , \tag{3.1}$$

Considering (2.13) and the orthogonality between different projection operators for the same arrangement,

$$P_\gamma Q_\gamma=Q_\gamma R_\gamma=P_\gamma R_\gamma=0\ , \tag{3.2}$$

with commutability, we rewrite (3.1),

$$T_{\beta\alpha}=\langle X_\beta^{d(-)}|(Q_\alpha+R_\alpha)V_\alpha P_\alpha+P_\beta V_\beta(Q_\beta+R_\beta)g_\beta^{d(+)}(Q_\alpha+R_\alpha)V_\alpha P_\alpha + P_\beta V_\beta(Q_\beta+R_\beta)g_\beta^{d(+)}(Q_\beta+R_\beta)V_\beta(P_\beta+Q_\beta+R_\beta)g_\beta^{d(+)}(Q_\alpha+R_\alpha)V_\alpha P_\alpha+\cdots|X_\alpha^{d(+)}\rangle\ . \tag{3.3}$$

In (3.3) above, the projection operators between different arrangements are nonorthogonal due to nonvanishing overlap between different arrangement channels.

Using (2.2) and (2.16), the Lippmann-Schwinger equation for the distorted waves for arrangement γ is written

$$|X_\gamma^{d(+)}\rangle=|\Phi_\gamma\rangle+g_\gamma^{d(+)}U_\gamma^d|\Phi_\gamma\rangle\ , \tag{3.4a}$$

or

$$|X_\gamma^{d(+)}\rangle=|\Phi_\gamma\rangle+(g_\gamma^{0(+)}+g_\gamma^{0(+)}U_\gamma^d g_\gamma^{0(+)}+\cdots)U_\gamma^d|\Phi_\gamma\rangle \tag{3.4b}$$

by introducing the distorted-wave propagator

$$g_\gamma^{d(+)}=g_\gamma^{0(+)}+g_\gamma^{0(+)}U_\gamma^d g_\gamma^{d(+)} =g_\gamma^{0(+)}+g_\gamma^{0(+)}U_\gamma^d g_\gamma^{0(+)} +g_\gamma^{0(+)}U_\gamma^d g_\gamma^{0(+)}U_\gamma^d g_\gamma^{0(+)}+\cdots \tag{3.5}$$

into (3.4a).

The substitution of (3.4a) into the distorted-wave first Born-approximation term called the DWBA transition amplitude in (3.1) above,

$$T_{\beta\alpha}^{\rm DWBA}=\langle X_\beta^{d(-)}|W_\alpha^d|X_\alpha^{d(+)}\rangle\ , \tag{3.6a}$$

leads to

$$T_{\beta\alpha}^{\rm DWBA}=\langle\Phi_\beta|W_\alpha^d|\Phi_\alpha\rangle+\langle\Phi_\beta|U_\beta^d g_\beta^{d(+)}W_\alpha^d|\Phi_\alpha\rangle +\langle\Phi_\beta|W_\alpha^d g_\alpha^{d(+)}U_\alpha^d|\Phi_\alpha\rangle +\langle\Phi_\beta|U_\beta^d g_\beta^{d(+)}W_\alpha^d g_\alpha^{d(+)}U_\alpha^d|\Phi_\alpha\rangle\ . \tag{3.6b}$$

Using (2.13), we obtain for the first term in (3.6b)

$$\langle\Phi_\beta|W_\alpha^d|\Phi_\alpha\rangle=\langle\Phi_\beta|V_\alpha|\Phi_\alpha\rangle-\langle\Phi_\beta|P_\alpha V_\alpha P_\alpha|\Phi_\alpha\rangle =T_{\beta\alpha}^{\rm PWBA}-\langle\Phi_\beta|U_\alpha^d|\Phi_\alpha\rangle\ . \tag{3.7}$$

The first term in (3.7) above is the familiar plane-wave first Born term called the PWBA transition amplitude.

Now the insertion of (3.5) and (3.7) into (3.6b) leads to

$$T_{\beta\alpha}^{\rm DWBA}=\tau_{\beta\alpha}^{(d)}+T_{\beta\alpha}^{\rm PWBA}-\langle\Phi_\beta|U_\alpha^d|\Phi_\alpha\rangle\ , \tag{3.8}$$

where

$$\tau_{\beta\alpha}^{(d)}=\langle\Phi_\beta|(U_\beta^d+U_\beta^d g_\beta^{0(+)}U_\beta^d+\cdots)g_\beta^{0(+)}W_\alpha^d|\Phi_\alpha\rangle+\langle\Phi_\beta|W_\alpha^d g_\alpha^{0(+)}(U_\alpha^d+U_\alpha^d g_\alpha^{0(+)}U_\alpha^d+\cdots)|\Phi_\alpha\rangle +\langle\Phi_\beta|(U_\beta^d+U_\beta^d g_\beta^{0(+)}U_\beta^d+\cdots)g_\beta^{0(+)}W_\alpha^d g_\alpha^{0(+)}(U_\alpha^d+U_\alpha^d g_\alpha^{0(+)}U_\alpha^d+\cdots)|\Phi_\alpha\rangle\ . \tag{3.9}$$

The expression (3.8) above establishes a formal relation between the DWBA and PWBA transition amplitudes. The DWBA (i.e., the first Born term in the representation of distorted wave) is seen to include the first Born term and infinite higher-order Born series terms in the representation of the plane wave. The higher-order Born series terms (3.9) is seen to consist of the perturbation potential W to the first order only and the distorting potential U to all orders. They represent the contribution of the perturbation interaction to the opening of new arrangement channels and the contributions of multistep elastic-scattering processes due to the appearance of the nonreactive distorting (elastic) potential to all orders.

Now consider the case of negligible influence of the distorting potential in the initial arrangement, i.e., $U_\alpha^d\simeq 0$. We then find $V_\alpha\simeq W_\alpha^d$ in (2.10). Thus (3.6b) leads to

$$T_{\beta\alpha}^{\rm DWBA}\simeq\langle\Phi_\beta|W_\alpha^d|\Phi_\alpha\rangle+\langle\Phi_\beta|U_\beta^d g_\beta^{d(+)}W_\alpha^d|\Phi_\alpha\rangle\ , \tag{3.10}$$

or using (2.13),

$$T_{\beta\alpha}^{\rm DWBA}\simeq\langle\Phi_\beta|(Q_\alpha+R_\alpha)V_\alpha P_\alpha|\Phi_\alpha\rangle +\langle\Phi_\beta|P_\beta V_\beta P_\beta g_\beta^{d(+)}(Q_\alpha+R_\alpha)V_\alpha P_\alpha|\Phi_\alpha\rangle\ . \tag{3.11}$$

Further, for the case of weak reactive potential in the final arrangement, β, that is, $W_\beta^d=0$ for $\gamma=\beta$ in (2.10), we have $V_\beta=U_\beta^d$. Thus the expression (3.10) is reduced to

$$T_{\beta\alpha}^{\rm DWBA}\simeq\langle\Phi_\beta|V_\alpha+V_\beta g_\beta^{d(+)}V_\alpha|\Phi_\alpha\rangle\ , \quad (3.12)$$

and the T operator is then

$$T=V_\alpha+V_\beta g_\beta^{d(+)}V_\alpha\ , \quad (3.13)$$

with

$$g_\beta^{d(+)}=\frac{1}{E-(h_\gamma+T_\gamma+U_\gamma^d)+i\varepsilon}\ , \quad (3.14)$$

as shown in (2.23).

At higher collision energies, the DWBA transition amplitude (3.8) is approximated[5] as

$$\begin{aligned}T_{\beta\alpha}^{\rm DWBA}\simeq T_{\beta\alpha}^{\rm PWBA}&-\langle\Phi_\beta|U_\alpha^d|\Phi_\alpha\rangle\\&+E^{-1}\langle\Phi_\beta|(U_\beta^d+U_\beta^dE^{-1}U_\beta^d+\cdots)W_\alpha^d|\Phi_\alpha\rangle+E^{-1}\langle\Phi_\beta|W_\alpha^d(U_\alpha^d+U_\alpha^dE^{-1}U_\alpha^d+\cdots)|\Phi_\alpha\rangle\\&+E^{-2}\langle\Phi_\beta|(U_\beta^d+U_\beta^dE^{-1}U_\beta^d+\cdots)W_\alpha^d(U_\alpha^d+U_\alpha^dE^{-1}U_\alpha^d+\cdots)|\Phi_\alpha\rangle\ . \end{aligned}\quad (3.15)$$

As $E\to\infty$, (3.15) above is reduced to

$$T_{\beta\alpha}^{\rm DWBA}=T_{\beta\alpha}^{\rm PWBA}-\langle\Phi_\beta|U_\alpha^d|\Phi_\beta\rangle\ . \quad (3.16)$$

It is now clearly seen from (3.16) above that the DWBA converges to the PWBA transition only if the influence of distorting potential U_α^d is ignored.

The plane-wave representation of the DWBA expression (3.8) with (3.9) is simply written

$$\begin{aligned}T_{\beta\alpha}^{\rm DWBA}=\langle\Phi_\beta|(V_\alpha-U_\alpha^d)+T_\beta^0 g_\beta^{0(+)}W_\alpha^d+W_\alpha^d g_\alpha^{0(+)}T_\alpha^0\\+T_\beta^0 g_\beta^{0(+)}W_\alpha^d g_\alpha^{0(+)}T_\alpha^0|\Phi_\alpha\rangle\ ,\end{aligned}\quad (3.17)$$

where

$$T_\gamma^0=U_\gamma^d+U_\gamma^d g_\gamma^{0(+)}U_\gamma^d+U_\gamma^d g_\gamma^{0(+)}U_\gamma^d g_\gamma^{0(+)}U_\gamma^d+\cdots\ . \quad (3.18)$$

The introduction of (3.18) into (3.9) leads to the simple expression

$$\begin{aligned}\tau_{\beta\alpha}^{(d)}=\langle\Phi_\beta|T_\beta^0 g_\beta^{0(+)}W_\beta^d+W_\alpha^d g_\alpha^{0(+)}T_\alpha^0\\+T_\beta^0 g_\beta^{0(+)}W_\alpha^d g_\alpha^{0(+)}T_\alpha^0|\Phi_\alpha\rangle\ .\end{aligned}\quad (3.19)$$

IV. TRANSFORMATION OF COUPLED-CHANNEL WAVE TRANSITION AMPLITUDE INTO BOTH THE DISTORTED-WAVE AND PLANE-WAVE REPRESENTATIONS

From (2.10) and (2.11), we find the distorting (inelastic interaction) potential,

$$U_\gamma=U_\gamma^c-U_\gamma^d=W_\gamma^d-W_\gamma^c \quad (4.1a)$$

or

$$U_\gamma=Q_\gamma V_\gamma(P_\gamma+Q_\gamma)+P_\gamma V_\gamma Q_\gamma\ . \quad (4.1b)$$

Using (4.1), we rewrite (2.5),

$$(h_\gamma+T_\gamma+U_\gamma^d+U_\gamma)X_\gamma^{c(+)}=EX_\gamma^{c(+)}\ . \quad (4.2)$$

The Lippmann-Schwinger equation for $X_\gamma^{c(+)}$ is then

$$X_\gamma^{c(+)}=X_\gamma^{d(+)}+g_\gamma^{c(+)}U_\gamma X_\gamma^d\ , \quad (4.3)$$

with

$$\begin{aligned}g_\gamma^{c(+)}&=g_\gamma^{d(+)}+g_\gamma^{d(+)}U_\gamma g_\gamma^{c(+)}\\&=g_\gamma^{d(+)}+g_\gamma^{d(+)}U_\gamma g_\gamma^{d(+)}\\&\quad+g_\gamma^{d(+)}U_\gamma g_\gamma^{d(+)}U_\gamma g_\gamma^{d(+)}+\cdots\ .\end{aligned}\quad (4.4)$$

For simplicity, we consider only the coupled-channel wave first Born (CCBA) term in (2.32),

$$T_{\beta\alpha}^{\rm CCBA}=\langle X_\beta^{c(-)}|W_\alpha^c|X_\alpha^{c(+)}\rangle\ . \quad (4.5a)$$

The insertion of (4.3) into (4.5a) yields

$$\begin{aligned}T_{\beta\alpha}^{\rm CCBA}=\langle X_\beta^{d(-)}|W_\alpha^c|X_\alpha^{d(+)}\rangle+\langle X_\beta^{d(-)}|U_\beta g_\beta^{c(+)}W_\alpha^c|X_\alpha^{d(+)}\rangle+\langle X_\beta^{d(-)}|W_\alpha^c g_\alpha^{c(+)}U_\alpha|X_\alpha^{d(+)}\rangle\\+\langle X_\beta^{d(-)}|U_\beta g_\beta^{c(+)}W_\alpha^c g_\alpha^{c(+)}U_\alpha|X_\alpha^{d(+)}\rangle\ .\end{aligned}\quad (4.5b)$$

The substitution of (4.1a) into the first term in (4.5b) leads to

$$\begin{aligned}&\langle X_\beta^{d(-)}|W_\alpha^c|X_\alpha^{d(+)}\rangle\\&\quad=\langle X_\beta^{d(-)}|W_\alpha^d|X_\alpha^{d(+)}\rangle-\langle X_\beta^{d(-)}|U_\alpha|X_\alpha^{d(+)}\rangle\ ,\end{aligned}\quad (4.6a)$$

or

$$\langle X_\beta^{d(-)}|W_\alpha^c|X_\alpha^{d(+)}\rangle=T_{\beta\alpha}^{\rm DWBA}-\langle X_\beta^{d(-)}|U_\alpha|X_\alpha^{d(+)}\rangle\ . \quad (4.6b)$$

Using (4.5b) and (4.6b) above, we obtain the following formal relation between the first Born terms in the CW and DW representations,

$$T^{\rm CCBA}_{\beta\alpha}=\tau^{(c)}_{\beta\alpha}+T^{\rm DWBA}_{\beta\alpha}-\langle X^{d(-)}_{\beta}|U_{\alpha}|X^{d(+)}_{\alpha}\rangle\ , \tag{4.7}$$

where

$$\begin{aligned}\tau^{(c)}_{\beta\alpha}=&\langle X^{d(-)}_{\beta}|U_{\beta}g^{c(+)}_{\beta}W^{c}_{\alpha}|X^{d(+)}_{\alpha}\rangle\\&+\langle X^{d(-)}_{\beta}|W^{c}_{\alpha}g^{c(+)}_{\alpha}U_{\alpha}|X^{d(+)}_{\alpha}\rangle\\&+\langle X^{d(-)}_{\beta}|U_{\beta}g^{c(+)}_{\beta}W^{c}_{\alpha}g^{c(+)}_{\alpha}U_{\alpha}|X^{d(+)}_{\alpha}\rangle\ ,\end{aligned}\tag{4.8a}$$

or introducing (4.4),

$$\begin{aligned}\tau^{(c)}_{\beta\alpha}=&\langle X^{d(-)}_{\beta}|(U_{\beta}+U_{\beta}g^{d(+)}_{\beta}U_{\beta}+\cdots)g^{d(+)}_{\beta}W^{c}_{\alpha}|X^{d(+)}_{\alpha}\rangle+\langle X^{d(-)}_{\beta}|W^{c}_{\alpha}g^{d(+)}_{\alpha}(U_{\alpha}+U_{\alpha}g^{d(+)}_{\alpha}U_{\alpha}+\cdots)|X^{d(+)}_{\alpha}\rangle\\&+\langle X^{d(-)}_{\beta}|(U_{\beta}+U_{\beta}g^{d(+)}_{\beta}U_{\beta}+\cdots)g^{d(+)}_{\beta}W^{c}_{\alpha}g^{d(+)}_{\alpha}(U_{\alpha}+U_{\alpha}g^{d(+)}_{\alpha}U_{\alpha}+\cdots)|X^{d(+)}_{\alpha}\rangle\ .\end{aligned}\tag{4.8b}$$

Using (2.14) and (4.1b), we rewrite (4.8a)

$$\begin{aligned}\tau^{(c)}_{\beta\alpha}=&\langle X^{d(-)}_{\beta}|(P_{\beta}V_{\beta}Q_{\beta}+P_{\beta}V_{\beta}Q_{\beta}g^{d(+)}_{\beta}Q_{\beta}V_{\beta}P_{\beta}+\cdots)g^{d(+)}_{\beta}R_{\alpha}V_{\alpha}P_{\alpha}|X^{d(+)}_{\alpha}\rangle\\&+\langle X^{d(-)}_{\beta}|R_{\alpha}V_{\alpha}Q_{\alpha}g^{d(+)}_{\alpha}(Q_{\alpha}V_{\alpha}P_{\alpha}+Q_{\alpha}V_{\alpha}Q_{\alpha}g^{d(+)}_{\alpha}Q_{\alpha}V_{\alpha}P_{\alpha}+\cdots)|X^{d(+)}_{\alpha}\rangle\\&+\langle X^{d(-)}_{\beta}|(P_{\beta}V_{\beta}Q_{\beta}+P_{\beta}V_{\beta}Q_{\beta}g^{d(+)}_{\beta}Q_{\beta}V_{\beta}P_{\beta}+\cdots)g^{d(+)}_{\beta}R_{\alpha}V_{\alpha}Q_{\alpha}g^{d(+)}_{\alpha}\\&\quad\times(Q_{\alpha}V_{\alpha}P_{\alpha}+Q_{\alpha}V_{\alpha}Q_{\alpha}g^{d(+)}_{\alpha}Q_{\alpha}V_{\alpha}P_{\alpha}+\cdots)|X^{d(+)}_{\alpha}\rangle\ .\end{aligned}\tag{4.8c}$$

From (4.8) above, the CCBA transition amplitude (4.7) is explicitly demonstrated to contain higher-order distorted-wave Born terms in a unique manner, in addition to the distorted-wave first Born (DWBA) term. That is, the first Born term (CCBA) in the representation of coupled-channel (multichannel) waves contains not only the first Born term (DWBA) in the representation of distorted wave, but also the effects of the perturbation potential W^{c}_{α} to the first order responsible for opening new arrangement channels, and the inelastic interaction U_{γ} to all orders to allow multistep inelastic (Q_{γ}) transitions as intermediate steps before the arrival of a final rearrangement channel.

As $E\rightarrow\infty$, (4.5) or (4.7) is reduced to

$$T^{\rm CCBA}_{\beta\alpha}=T^{\rm DWBA}_{\beta\alpha}-\langle X^{d(-)}_{\beta}|U_{\alpha}|X^{d(+)}_{\alpha}\rangle\ , \tag{4.9}$$

and if the distorting potential in the initial arrangement α is weak, we note

$$T^{\rm CCBA}_{\beta\alpha}\simeq T^{\rm DWBA}_{\beta\alpha}\ . \tag{4.10}$$

We now rewrite (3.4b) using (3.18),

$$|X^{d(+)}_{\gamma}\rangle=|\Phi_{\gamma}\rangle+g^{0(+)}_{\gamma}T^{0}_{\gamma}|\Phi_{\gamma}\rangle\ , \tag{4.11}$$

and insert (4.11) into (4.5b) or (4.7) to show a relationship between the CCBA and PWBA,

$$\begin{aligned}T^{\rm CCBA}_{\beta\alpha}=&T^{\rm PWBA}_{\beta\alpha}-\langle\Phi_{\beta}|U^{d}_{\alpha}|\Phi_{\alpha}\rangle\\&-\langle X^{d(-)}_{\beta}|U_{\alpha}|X^{d(+)}_{\alpha}\rangle+\tau_{\beta\alpha}\ ,\end{aligned}\tag{4.12}$$

and a relationship between the CCBA and DWBA,

$$T^{\rm CCBA}_{\beta\alpha}=T^{\rm DWBA}_{\beta\alpha}-\langle X^{d(-)}_{\beta}|U_{\alpha}|X^{d(+)}_{\alpha}\rangle+\tau^{(c)}_{\beta\alpha}\ , \tag{4.13}$$

where $\tau_{\beta\alpha}$ is

$$\tau_{\beta\alpha}=\tau^{(d)}_{\beta\alpha}+\tau^{(c)}_{\beta\alpha}\ , \tag{4.14}$$

and

$$\begin{aligned}\tau^{(c)}_{\beta\alpha}=&\langle\Phi_{\beta}|U_{\beta}g^{c(+)}_{\beta}W^{c}_{\alpha}+T^{0}_{\beta}g^{0(+)}_{\beta}U_{\beta}g^{c(+)}_{\beta}W^{c}_{\alpha}+U_{\beta}g^{c(+)}_{\beta}W^{c}_{\alpha}g^{0(+)}_{\alpha}T^{0}_{\alpha}+T^{0}_{\beta}g^{0(+)}_{\beta}U_{\beta}g^{c(+)}_{\beta}W^{c}_{\alpha}g^{0(+)}_{\alpha}T^{0}_{\alpha}|\Phi_{\alpha}\rangle\\&+\langle\Phi_{\beta}|W^{c}_{\alpha}g^{c(+)}_{\alpha}U_{\alpha}+T^{0}_{\beta}g^{0(+)}_{\beta}W^{c}_{\alpha}g^{c(+)}_{\alpha}U_{\alpha}+W^{c}_{\alpha}g^{c(+)}_{\alpha}U_{\alpha}g^{0(+)}_{\alpha}T^{0}_{\alpha}+T^{0}_{\beta}g^{0(+)}_{\beta}W^{c}_{\alpha}g^{c(+)}_{\beta}U^{c}_{\alpha}g^{0(+)}_{\alpha}T^{0}_{\alpha}|\Phi_{\alpha}\rangle\\&+\langle\Phi_{\beta}|U_{\beta}g^{c(+)}_{\beta}W^{c}_{\alpha}g^{c(+)}_{\alpha}U_{\alpha}+T^{0}_{\beta}g^{0(+)}_{\beta}U_{\beta}g^{c(+)}_{\beta}W^{c}_{\alpha}g^{c(+)}_{\alpha}U_{\alpha}\\&\qquad+U_{\alpha}g^{c(+)}_{\beta}W^{c}_{\alpha}g^{c(+)}_{\alpha}U_{\alpha}g^{0(+)}_{\alpha}T^{0}_{\alpha}+T^{0}_{\beta}g^{0(+)}_{\beta}U_{\beta}g^{c(+)}_{\beta}W^{c}_{\alpha}g^{c(+)}_{\alpha}U_{\alpha}g^{0(+)}_{\alpha}T^{0}_{\alpha}|\Phi_{\alpha}\rangle\end{aligned}\tag{4.15}$$

and

$$\langle X^{d(-)}_{\beta}|U_{\alpha}|X^{d(+)}_{\alpha}\rangle=\langle\Phi_{\beta}|U_{\alpha}+T^{0}_{\beta}g^{0(+)}_{\beta}U_{\alpha}+U_{\alpha}g^{0(+)}_{\alpha}T^{0}_{\alpha}+T^{0}_{\beta}g^{0(+)}_{\beta}U_{\alpha}g^{0(+)}_{\alpha}T^{0}_{\alpha}|\Phi_{\alpha}\rangle \tag{4.16}$$

and

$$g^{c(+)}_{\beta}=g^{0(+)}_{\beta}+g^{0(+)}_{\beta}U^{c}_{\beta}g^{0(+)}_{\beta}+\cdots\ . \tag{4.17}$$

A formal relation between the CCBA and PWBA transition amplitudes is shown in (4.12). Important to note is that the coupled-channel wave first Born term as shown in (4.7) through (4.8c) and (4.12) through (4.17), respectively, implies the infinite Born series expansion in the representations of both the distorted and plane waves. In analogy to the quantum field theory, the coupled-channel wave first Born term (or CCBA) can now be regarded as a "dressed" or "renormalized" transition amplitude in either the plane-wave or distorted-wave representation. Thus the plane-wave first Born term that appears in the expansion of CCBA simply corresponds to the "bare" term. The fourth term $\tau_{\beta\alpha}$ in the CCBA expression (4.12) represents the "clothing" which encompasses all possibilities of intermediate multistep elastic and inelastic transitions before the arrival of the final arrangement channel.

Finally, at $E \rightarrow \infty$, (4.12) is reduced to

$$T_{\beta\alpha}^{\text{CCBA}} = T_{\beta\alpha}^{\text{PWBA}} - \langle \Phi_\beta | U_\alpha^c | \Phi_\alpha \rangle \ , \tag{4.18}$$

similar to the description made in Sec. III. It is now seen that only if the inelastic distorting potential U_α^c in the initial arrangement α is negligible (i.e., $U_\alpha^c \simeq 0$), the CCBA converges to the PWBA, that is,

$$T_{\beta\alpha}^{\text{CCBA}} = T_{\beta\alpha}^{\text{PWBA}} \ . \tag{4.19}$$

V. SUMMARY

For the sake of completion, an extension of our earlier studies[14(d)] of the T matrix for rearrangement collision was presented here. Using the projection operators, distinctions between formal relations among the three different (PW, DW, and CW) classes of perturbation methods were sharpened. By deriving formally transformations between the first Born terms, PWBA, DWBA, and CCBA, both the distorted-wave and coupled-channel wave first Born terms were seen to be equivalent to the infinite-order plane-wave Born series expansion in a clearly different manner. Differences in structural details between the two were elucidated in Secs. III and IV. A question on the convergence of perturbation theory is often raised in treating the plane-wave Born series expansion[5,17] of the T matrix. The same degree of concern may not occur with the coupled-channel wave Born series expansion for treating direct rearrangement collision. This is because the CCBA, that is, the first Born term alone in the coupled-channel wave representation, already contains the infinite Born series terms in the representation of plane or distorted wave.

As noted from earlier discussions in Sec. IV, the CCBA transition amplitude represents infinite-order Born terms in a unique way, if it is viewed in the representation of either the distorted wave or plane wave. To be more specific, it is shown that the CCBA T matrix given by the distorted-wave representation contains not only the distorted-wave first Born term (DWBA), but also the effect of multistep inelastic transitions as intermediate steps before the arrival of final arrangement channel. Thus, we have explicitly shown that the first Born term alone in the representation of the coupled-channel (multichannel) wave is equivalent to the sum of the infinite plane-wave Born series terms in a unique manner and acts as a "dressed" or "renormalized" transition amplitude.

ACKNOWLEDGMENTS

The author acknowledges partial financial support of the Dohang Institute of Science and Technology and of the Korean Ministry of Education. He is also grateful to Dr. C. K. Lutrus for reading the present manuscript with valuable comments.

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