Implications of various spin-one relativistic wave equations for intermediate-energy deuteron-nucleus scattering

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(Received 6 August 1990)

Various relativistic treatments are applied to deuteron-nucleus elastic scattering at intermediate energies. There are various possibilities for spin-one wave equations. Here we have considered three of them, the Kemmer-Duffin-Petiau, Proca, and Weinberg equations. Second-order equations are obtained from each using a similar set of approximations. Elastic-scattering observables including the differential cross section, and the vector and tensor analyzing powers are calculated using all three equations. The different predictions are compared with each other and with experimental data for 400 MeV ^{58}Ni$(d,d)^{58}$Ni and 700 MeV ^{40}Ca$(d,d)^{40}$Ca. We find that within the approximations made and for the assumed interactions there are no significant differences between the three sets of predicted observables.

I. INTRODUCTION

Recently, Dirac-equation-based relativistic approaches to proton-nucleus elastic scattering at medium energies[1–7] were shown to be very successful in representing the experimental data, especially the spin observables. In these approaches the proton is treated as a point particle obeying a Dirac equation which contains a nuclear optical potential consisting of two terms, one a Lorentz scalar and another the timelike component of a Lorentz four-vector. This scalar-vector combination plays a critical role in the representation of the elastic-scattering observables and makes it possible to account for a large body of elastic-scattering data over a wide range of energy and mass number.

In this paper we consider the theoretical description of intermediate-energy deuteron-nucleus scattering. The success of the Dirac-equation approach in describing the propagation of a composite spin-one-half projectile in *proton*-nucleus scattering warrants an analogous study of a Lorentz-invariant description of *deuteron*-nucleus scattering in which the projectile is represented by a relativistic pointlike spin-one particle. Effects due to the finite size and internal structure of the deuteron can, to some extent, be accounted for by the choice of interaction form and by including breakup corrections.

Unlike the case for spin one-half, there are a number of spin-one relativistic wave equations (RWE). Previous applications to deuteron-nucleus scattering of the Weinberg[8–10] and Kemmer-Duffin-Petiau[11–13] (KDP) equations have been reported by different authors using different assumptions. The Breit equation[14] has also been used in this regard but it is not a fundamental-invariance-based spin-one RWE.

Our principal aim in this work is to determine the impact of the nonuniqueness of the spin-one RWE's on the deuteron-nucleus elastic-scattering predictions by evaluating the Weinberg, Proca, and KDP equations using a similar set of approximations and with the same interactions. In the absence of any strong theoretical argument for preferring one RWE over another, phenomenological results may tell us which one is best suited for deuteron-nucleus scattering or whether the choice between different spin-one RWE's is even a relevant issue.

In the next section we discuss the various spin-one equations. The common set of approximations and the reduction to second-order form are treated in Sec. III. In Sec. IV, we present the results of our calculations. Final comments and the conclusions are given in Sec. V. Some of the technical details are provided in the Appendix.

II. SPIN-ONE RELATIVISTIC WAVE EQUATIONS

Following the introduction of the Dirac equation,[15] the search began for similar equations for higher spins. It was soon discovered that, apart from spin one-half, none of the other spins obeys a single relativistic wave equation. For example, it was generally believed that for spins zero and one, the Klein-Gordon[16,17] and Proca[18] equations were unique. However, more than 50 years ago it was found that the Kemmer-Duffin-Petiau[19–21] equations can describe both spin-zero and spin-one objects. Since then, many more systems of equations for arbitrary spins, which originate under different assumptions made regarding their invariance under the Lorentz group, have been found.

Historically, the first equation describing massive particles with arbitrary spins was the Dirac-Fierz-Pauli[22–24] equation. The state vector in this case contains extra components which have to be eliminated by imposing constraints to yield the required physical degrees of freedom. For spin one they lead to the Proca equation which can be written as

$$\partial_\nu F^{\nu\mu}(x)+m^2\psi^\mu(x)=0 \;, \tag{1}$$

where $\psi^\mu(x)$ is the four-vector field for the spin-one particle, m is the mass, and $F^{\mu\nu}$ is the field strength tensor given by

$$F^{\mu\nu}(x)=\partial^\mu\psi^\nu(x)-\partial^\nu\psi^\mu(x) \;. \tag{2}$$

Here $\hbar=c=1$ and Bjorken and Drell[25] conventions have been used throughout. To eliminate the extra component in ψ^μ, the constraint

$$\partial_\mu\psi^\mu=0 \tag{3}$$

is imposed so that Eqs. (1)–(3) taken together describe a spin-one particle. Each component of the field obeys the Klein-Gordon condition

$$(\partial_\nu\partial^\nu+m^2)\psi^\mu(x)=0 \;. \tag{4}$$

The field $\psi^\mu(x)$ transforms as the $D(1,0)\oplus D(0,0)$ representation of the homogeneous Lorentz group.

Other approaches have been presented in which Dirac-like equations for arbitrary spins were derived. Generally, they lead to first-order equations of the form

$$(i\beta^\mu\partial_\mu-m)\psi_K(x)=0 \;, \tag{5}$$

where the β^μ are numerical matrices obeying some algebra. One such approach is known as the Bhabha[26] equation which for the spin-one case reduces to the KDP equation. For that case, the β^μ matrices obey the following condition:

$$\beta^\mu\beta^\nu\beta^\lambda+\beta^\lambda\beta^\nu\beta^\mu=g^{\mu\nu}\beta^\lambda+g^{\lambda\nu}\beta^\mu \;. \tag{6}$$

It is interesting to note that these matrices satisfy

$$(\beta^\mu p_\mu)^3=p^2(\beta^\mu p_\mu) \;, \tag{7}$$

which is a necessary and sufficient condition on the β^μ matrices for describing a particle with a unique mass.[27] The β^μ matrices do not have an inverse and so, instead of an algebra, they form a ring.[28] It is interesting to note that one of their representations can be written in terms of Dirac matrices

$$\beta^\mu=\tfrac{1}{2}[I(1)\otimes\gamma_\mu(2)+\gamma_\mu(1)\otimes I(2)] \;, \tag{8}$$

where 1 and 2 refer to two different spaces and I stands for the 4×4 unit matrix. This reduces to three irreducible representations of dimensions 1, 5, and 10 where the last two describe spins zero and one, respectively. For spin one, the 10×10 β^μ matrices can be represented by

$$\beta^0=(\delta_{i2}\delta_{j5}+\delta_{i5}\delta_{j2})+(\delta_{i3}\delta_{j6}+\delta_{i6}\delta_{j3})+(\delta_{i4}\delta_{j7}+\delta_{i7}\delta_{j4}) \;, \tag{9a}$$

$$\beta^1=(\delta_{i1}\delta_{j5}-\delta_{i5}\delta_{j1})-(\delta_{i3}\delta_{j10}-\delta_{i10}\delta_{j3})+(\delta_{i4}\delta_{j9}-\delta_{i9}\delta_{j4}) \;, \tag{9b}$$

$$\beta^2=(\delta_{i1}\delta_{j6}-\delta_{i6}\delta_{j1})+(\delta_{i2}\delta_{j10}-\delta_{i10}\delta_{j2})-(\delta_{i4}\delta_{j8}-\delta_{i8}\delta_{j4}) \;, \tag{9c}$$

$$\beta^3=(\delta_{i1}\delta_{j7}-\delta_{i7}\delta_{j1})-(\delta_{i2}\delta_{j9}-\delta_{i9}\delta_{j2})+(\delta_{i3}\delta_{j8}-\delta_{i8}\delta_{j3}) \;, \tag{9d}$$

where i and j refer to the rows and columns, respectively, and δ_{ij} are Kronecker deltas. The state vector itself is given by

$$\Psi_K(x)=\text{Column}(\phi,\,\mathbf{A},\mathbf{E},\,-\mathbf{B}) \;, \tag{10}$$

in analogy with the electromagnetic field. The field $\Psi_K(x)$ transforms as the

$$D(1,0)\oplus D(0,1)\oplus D(\tfrac{1}{2},\tfrac{1}{2})$$

representation of the homogeneous Lorentz group.

A third approach to these arbitrary spin RWE is to find an equation for the physical $2(2s+1)$ components only. Such general spin fields can be built by considering the irreducible representations of the rotation matrices. They are known as the Joos-Weinberg[29,30] fields whose equations of motion are known as Weinberg equations. For spin one they can be written as

$$(p_\mu p_\nu\gamma^{\mu\nu}-m^2)\Psi_W(x)=0 \;, \tag{11}$$

where $\Psi_W(x)$ has six components and the $\gamma^{\mu\nu}$ are 6×6 matrices given explicitly by

$$\gamma^{00}=\begin{bmatrix} I_3 & 0 \\ 0 & -I_3 \end{bmatrix} , \tag{12a}$$

$$\gamma^{0i}=\begin{bmatrix} 0 & S_i \\ -S_i & 0 \end{bmatrix} , \tag{12b}$$

and

$$\gamma^{ij}=\begin{bmatrix} S_iS_j+S_jS_i-\delta_{ij} & 0 \\ 0 & -(S_iS_j+S_jS_i-\delta_{ij}) \end{bmatrix} . \tag{12c}$$

Here I_k are $k\times k$ unit matrices and S_i are spin-one matrices given by

$$(S_i)_{jk}=-i\epsilon_{ijk} \;, \tag{13}$$

where ϵ_{ijk} is the three-dimensional antisymmetric tensor. The field $\Psi_W(x)$ transforms as $D(1,0)\oplus D(0,1)$ under Lorentz transformation so it clearly describes a spin-one particle. Using the above matrix representations, the free spin-one Weinberg equation can be written as

$$\begin{bmatrix} -(\mathbf{S}\cdot\mathbf{p})^2 & (\mathbf{S}\cdot\mathbf{p})i\dfrac{\partial}{\partial t} \\ (\mathbf{S}\cdot\mathbf{p})i\dfrac{\partial}{\partial t} & -[m^2+(\mathbf{S}\cdot\mathbf{p})^2] \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} =0 \;, \tag{14}$$

where $\mathbf{A}$ and $\mathbf{B}$ are three component vectors analogous to the Dirac upper and lower components. For spins greater than one, the Weinberg equations become very complicated equations of third and higher order.[29,30]

All these equations are equivalent for the free-field case but are, in general, not equivalent when interactions are included. For the Coulomb interaction, it has been shown that the Proca and the KDP equations are equivalent.[11–13] When other interactions like the Lorentz scalar and Lorentz four-vector interactions are introduced, they are very different. It has been known for a long time that higher-spin RWE can become pathological in the presence of interactions.[31,32] One finds complex energy eigenvalues, superluminal propagation of waves, loss of constraints, and many other undesirable features. Such behavior is exhibited by the spin-zero and spin-one KDP (Ref. 33) and spin-one Proca[34] equations. In fact, it was shown[35] that the spin three-halves Rarita-Schwinger equation becomes unstable when minimally coupled to the electromagnetic fields. At the present time there are no known systems of arbitrary spin RWE which are free of these problems.

III. SECOND-ORDER EQUATIONS WITH INTERACTIONS

We assume that the point deuteron-nucleus interaction can be represented in terms of a Lorentz scalar interaction S, and a timelike vector interaction V in analogy with Dirac phenomenology for proton-nucleus scattering. We introduce them in Eqs. (1), (5), and (11) with the following substitutions:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + iV\delta_{\mu 0} \ , \tag{15a}$$

$$m \rightarrow \tilde{m} = m + S \ . \tag{15b}$$

The three spin-one equations then become

Proca:

$$D_\mu (D^\mu \psi^\nu - D^\nu \psi^\mu) + \tilde{m}^{\,2} \psi^\nu = 0 \ , \tag{16a}$$

KDP:

$$(i\beta^\mu \partial_\mu - \tilde{m} - \beta^0 V)\Psi_K = 0 \ , \tag{16b}$$

Weinberg:

$$[(i\partial_\mu - V\delta_{\mu 0})(i\partial_\nu - V\delta_{\nu 0})\gamma^{\mu\nu} - \tilde{m}^{\,2}]\Psi_W = 0 \ . \tag{16c}$$

In the case of the KDP equation both S and V can be introduced in a variety of ways.[11–13] However, in order to minimize the differences between the equations we will use the above prescription.

In order to be able to compare the results from the three equations, one must either solve them exactly or make approximations which are similar. The ability to solve them exactly when interactions are present does not currently exist, thus one is forced to formulate the problem in a manner which does allow similar approximations to be made in each case. In our previous work with the KDP formulation for deuteron scattering, Refs. 11–13, we started with an exact equation for the wave function, given by Eq. (3.6) in Ref. 12 and then made simplifying approximations. The equations for the wave function obtained in the Proca and Weinberg cases have a particular structure, different from the KDP equation. It is necessary to reformulate the KDP expression for the wave function to exhibit this structure so that similar approximations can be made.

The algebra and the approximations made in obtaining the second-order equations will be illustrated below for the KDP equation. The other two equations are dealt with in the Appendix. Using the representation of the β^μ [Eq. (9)], the KDP equation can be rewritten as a set of four coupled differential equations

$$i\nabla\cdot\mathbf{E} = \tilde{m}\phi \ , \tag{17a}$$

$$\omega\mathbf{E} - i\nabla\times\mathbf{B} = \tilde{m}\,\mathbf{A} \ , \tag{17b}$$

$$\omega\,\mathbf{A} - i\nabla\phi = \tilde{m}\mathbf{E} \ , \tag{17c}$$

and

$$-i\nabla\times\mathbf{A} = \tilde{m}\mathbf{B} \ , \tag{17d}$$

where $\omega = E - V$. Substitution of Eq. (17c) into (17a) and Eqs. (17c) and (17d) into Eq. (17b) and some algebra leads to two equations,

$$(\nabla^2 - \tilde{m}^{\,2} - \Sigma\hat{\mathbf{r}}\cdot\nabla)\phi = -i\omega\nabla\cdot\mathbf{A} + i\omega(\Sigma+\Theta)\hat{\mathbf{r}}\cdot\mathbf{A} \ , \tag{18a}$$

$$(\nabla^2 + \omega^2 - \tilde{m}^{\,2})\mathbf{A} = i\omega\nabla\phi + \nabla(\nabla\cdot\mathbf{A}) - \Sigma\hat{\mathbf{r}}\times(\nabla\times\mathbf{A}) \ , \tag{18b}$$

where

$$\Sigma = \frac{1}{\tilde{m}}\frac{dS}{dr} \ , \tag{19a}$$

and

$$\Theta = \frac{1}{\omega}\frac{dV}{dr} \ . \tag{19b}$$

Here both Σ and Θ have dimensions of energy and $\hat{\mathbf{r}}$ stands for the radial unit vector. These are related to similar quantities defined in Ref. 12 by

$$\Sigma = r\Lambda \tag{19c}$$

and

$$\Theta = -r\Omega \ . \tag{19d}$$

The ϕ equation contains $\hat{\mathbf{r}}\cdot\nabla$ and $\hat{\mathbf{r}}\cdot\mathbf{A}$ terms which can be removed by transforming both ϕ and $\mathbf{A}$ according to

$$\phi = \left[\frac{\tilde{m}}{m}\right]^{1/2} \phi_1 \ , \tag{20a}$$

$$\mathbf{A} = \left[\frac{\tilde{m}E}{m\omega}\right] \mathbf{A}_1 \ . \tag{20b}$$

These transformations preserve the elastic-scattering observables because ϕ_1 and $\mathbf{A}_1$ asymptotically approach ϕ and $\mathbf{A}$, respectively. Equations (18a) and (18b) then become

$$(\nabla^2-\tilde{m}^{\,2}+g^2)\phi_1=-iE\left[\frac{\tilde{m}}{m}\right]^{1/2}(\nabla\cdot\mathbf{A}_1)\ , \tag{21a}$$

$$\begin{aligned}(\nabla^2+\omega^2-\tilde{m}^{\,2})\mathbf{A}_1=&-\left[\Sigma'+\Theta'+(\Sigma+\Theta)\left[\Theta+\frac{2}{r}\right]\right]\mathbf{A}_1+\left[\frac{\Sigma+\Theta}{r}\right][\mathbf{r}(\nabla\cdot\mathbf{A}_1)+\nabla(\mathbf{r}\cdot\mathbf{A}_1)-2(\mathbf{r}\cdot\nabla)\mathbf{A}_1]\\&-\Sigma\hat{\mathbf{r}}\times(\nabla\times\mathbf{A}_1)+\nabla(\nabla\cdot\mathbf{A}_1)+\left[\Sigma'+\Theta'+(\Sigma+\Theta)\left[\Theta-\frac{1}{r}\right]\right]\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{A}_1)+R\ ,\end{aligned} \tag{21b}$$

where

$$g^2=\frac{1}{2}\frac{d\Sigma}{dr}+\frac{\Sigma}{r}-\frac{1}{4}\Sigma^2\ , \tag{22a}$$

and

$$R=i\frac{\omega^2}{E}\left[\frac{m}{\tilde{m}}\right]^{1/2}\left[\frac{\Sigma}{2}\hat{\mathbf{r}}+\nabla\right]\phi_1\ . \tag{22b}$$

Inverting Eq. (21a) formally and substituting into Eq. (22b) leads to

$$R=-\omega^2\left[\frac{m}{\tilde{m}}\right]^{1/2}\left[\frac{\Sigma}{2}\hat{\mathbf{r}}+\nabla\right]\frac{1}{\xi_K^2-\hat{Z}}\left[\left[\frac{\tilde{m}}{m}\right]^{1/2}\nabla\cdot\mathbf{A}_1\right]\ , \tag{23}$$

where

$$\xi_K^2=\omega^2-g^2 \tag{24a}$$

and

$$\hat{Z}=\nabla^2+\omega^2-\tilde{m}^{\,2}\ . \tag{24b}$$

On expansion, the inverse operator term in Eq. (23) gives

$$\begin{aligned}\frac{1}{\xi_K^2-\hat{Z}}&=\frac{1}{\xi_K^2}\left[1-\frac{\hat{Z}}{\xi_K^2}\right]^{-1}\\&=\frac{1}{\xi_K^2}\left[1+\frac{1}{\xi_K^2}\hat{Z}+\frac{1}{\xi_K^2}\hat{Z}\frac{1}{\xi_K^2}\hat{Z}+\cdots\right]\ ,\end{aligned} \tag{25}$$

which then is used in R. We assume that $\hat{Z}/\xi_K^2$ operating on $(\tilde{m}/m)^{1/2}\nabla\cdot\mathbf{A}_1$ gives a value small compared to unity. In the noninteracting case it is zero. If interactions are present, we assume that we can retain only the first term in the above expansion. Equation (23) can then be written

$$R\approx-\alpha_K[(\Sigma-c)\hat{\mathbf{r}}+\nabla](\nabla\cdot\mathbf{A}_1)\ , \tag{26a}$$

where

$$c=\frac{d}{dr}\ln\xi_K^2 \tag{26b}$$

and

$$\alpha_K=\frac{\omega^2}{\xi_K^2}\ . \tag{26c}$$

Using Eq. (26a), we can rewrite Eq. (21b) as

$$\begin{aligned}(\nabla^2+\omega^2-\tilde{m}^{\,2})\mathbf{A}_1=&-\left[\Sigma'+\Theta'+(\Sigma+\Theta)\left[\Theta+\frac{2}{r}\right]\right]\mathbf{A}_1+\left[\frac{\Sigma+\Theta}{r}\right][\nabla(\mathbf{r}\cdot\mathbf{A}_1)-2(\mathbf{r}\cdot\nabla)\mathbf{A}_1]\\&-\Sigma\hat{\mathbf{r}}\times(\nabla\times\mathbf{A}_1)+(1-\alpha_K)\nabla(\nabla\cdot\mathbf{A}_1)+[\Sigma+\Theta-\alpha_K(\Sigma-c)]\hat{\mathbf{r}}(\nabla\cdot\mathbf{A}_1)\\&+\left[\Sigma'+\Theta'+(\Sigma+\Theta)\left[\Theta-\frac{1}{r}\right]\right]\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{A}_1)\ .\end{aligned} \tag{27}$$

In Eq. (27) we drop the $\nabla(\nabla\cdot\mathbf{A}_1)$ term because its coefficient $(1-\alpha_K)$ is very small compared to unity.

Equation (27) can be completely decomposed into spin-one tensors. They are defined as[36]

$$T_{AB}=(\mathbf{S}\cdot\mathbf{A})(\mathbf{S}\cdot\mathbf{B})-\frac{i}{2}\mathbf{S}\cdot(\mathbf{A}\times\mathbf{B})-\tfrac{1}{3}(\mathbf{A}\cdot\mathbf{B})\mathbf{S}^2\ , \tag{28}$$

which leads to

$$\frac{T_{RR}}{r^2}=(\mathbf{S}\cdot\hat{\mathbf{r}})^2-\tfrac{2}{3} \tag{29a}$$

and

$$T_{RP}=(\mathbf{S}\cdot\mathbf{r})(\mathbf{S}\cdot\mathbf{p})-\frac{i}{2}\mathbf{S}\cdot\mathbf{L}-\tfrac{2}{3}\mathbf{r}\cdot\mathbf{p}\ . \tag{29b}$$

Equation (27) then takes the final form

$$\left[\frac{\mathbf{p}^2}{2E}+U_C^K+U_S^K\mathbf{S}\cdot\mathbf{L}+U_D^K(\mathbf{r}\cdot\nabla)+U_{RR}^K\left[\frac{T_{RR}}{r^2}\right]+iU_{RP}^KT_{RP}\right]\mathbf{A}_1=\frac{k^2}{2E}\mathbf{A}_1\ , \tag{30}$$

where

$$U_C^K=\left[\frac{1}{2E}\right]\left[-\omega^2+\tilde{m}^2+k^2-\tfrac{2}{3}(\Sigma+\Theta)\left[\Theta+\frac{2}{r}\right]-\frac{2}{3}(\Sigma'+\Theta')\right], \tag{31a}$$

$$U_S^K=-\left[\frac{1}{2E}\right]\frac{1}{2r}[(1-\alpha_K)\Sigma+\alpha_K c], \tag{31b}$$

$$U_D^K=-\left[\frac{1}{2E}\right]\frac{1}{3r}[(\alpha_K+2)\Sigma-\alpha_K c+4\Theta], \tag{31c}$$

$$U_{RR}^K=-\left[\frac{1}{2E}\right]\left[\Sigma'+\Theta'+(\Sigma+\Theta)\left[\Theta-\frac{1}{r}\right]\right], \tag{31d}$$

and

$$U_{RP}^K=-\left[\frac{1}{2E}\right]\frac{1}{r}[(1-\alpha_K)\Sigma+\alpha_K c+2\Theta]. \tag{31e}$$

Here the subscripts C, S, and D stand for the central, spin-orbit, and Darwin potentials, respectively, the prime denotes differentiation with respect to r and

$$k^2=E^2-m^2. \tag{32}$$

Expressions similar to Eqs. (30) and (31a)–(31e) have been derived for the Proca and Weinberg equations in the Appendix under similar approximations.

IV. CALCULATIONS AND RESULTS

The main input to the scattering calculations are the nuclear scalar (S_N) and vector (V_N) potentials obtained from fitting nucleon-nucleus scattering observables at half the deuteron kinetic energy. The potentials are written as

$$V_N=V_R f(r,c_{vr},z_{vr})+iV_I f(r,c_{vi},z_{vi}) \tag{33a}$$

and

$$S_N=S_R f(r,c_{sr},z_{sr})+iS_I f(r,c_{si},z_{si}), \tag{33b}$$

where

$$f(r,c,z)=\frac{1}{\{1+\exp[(r-c)/z]\}\{1+\exp[-(r+c)/z]\}}. \tag{34}$$

The effective deuteron-nucleus scalar (S) and vector (V) potentials used here are twice the corresponding nucleon-nucleus potentials evaluated at half the energy in analogy with the Watanabe[37] model assuming a pointlike deuteron (folding over the deuteron wave-function changes this slightly).[12] The Coulomb potential is included in the real part of the vector potential. The effective potentials are written as

$$V(E)=2V_N(E/2)+V_C \tag{34a}$$

and

$$S(E)=2S_N(E/2). \tag{34b}$$

TABLE I. Dirac optical potential parameters used.

Target		Strength (MeV)	Radius/$A^{1/3}$ (fm)	Diffuseness (fm)
^{58}Ni	Vector real	301.96	1.0470	0.6112
	Vector imag	−74.298	1.1226	0.6232
	Scalar real	−406.88	1.0371	0.6369
	Scalar imag	69.266	1.1191	0.5826
^{40}Ca	Vector real	292.27	1.0115	0.6421
	Vector imag	−94.677	1.1253	0.5509
	Scalar real	−414.68	1.0044	0.6690
	Scalar imag	95.739	1.1279	0.5361

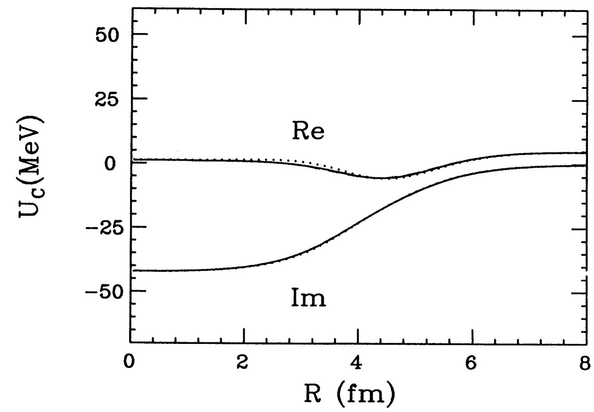

FIG. 1. The effective central potentials for $d+{}^{58}$Ni elastic scattering at a laboratory kinetic energy of 400 MeV. The solid, dotted, and dashed lines present the KDP, Eq. (31a), Proca, Eq. (A8a), and Weinberg, Eq. (A17a), results, respectively.

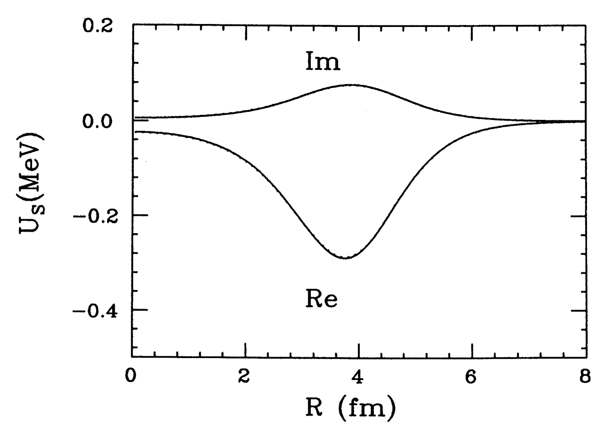

FIG. 2. The effective spin-orbit potentials for d ^{58}Ni elastic scattering at a laboratory kinetic energy of 400 MeV. The solid lines present the KDP, Eq. (31b), results and the dotted lines present the identical Proca, Eq. (A8b), and Weinberg, Eq. (A17b), results, respectively.

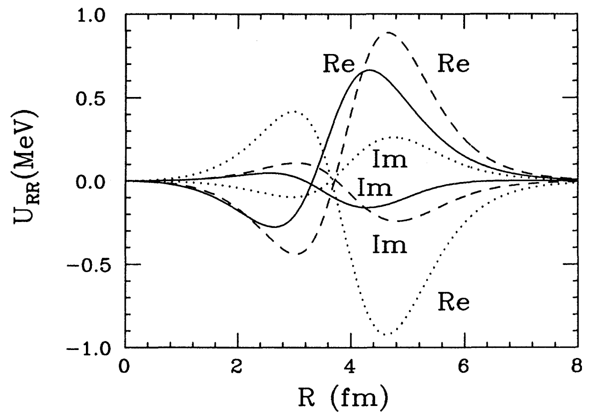

FIG. 3. The effective RR tensor potentials for $d+{}^{58}$Ni elastic scattering at a laboratory kinetic energy of 400 MeV. The solid, dotted, and dashed lines present the KDP, Eq. (31d), Proca, Eq. (A8d), and Weinberg, Eq. (A17d), results, respectively.

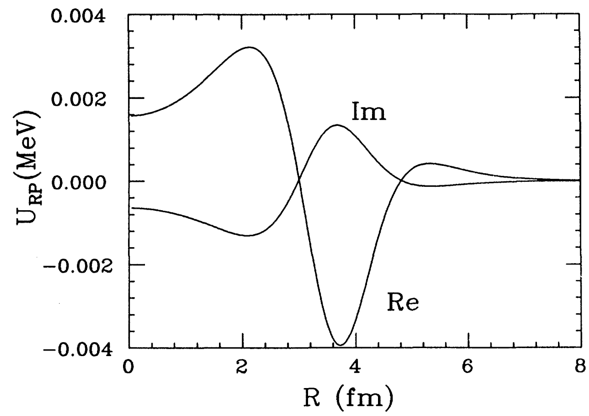

FIG. 5. The effective RP tensor potentials for $d+{}^{58}$Ni elastic scattering at a laboratory kinetic energy of 400 MeV. The solid line shows the results for the KDP results, Eq. (31e). The Proca and Weinberg effective RP tensor terms vanish.

We calculated the results for $d+{}^{40}$Ca at 700 MeV and $d+{}^{58}$Ni at 400 MeV laboratory kinetic energy using the parameters for proton-nucleus scattering at 362 and 200 MeV, respectively. The 362-MeV $p+{}^{40}$Ca data[38] are the closest data set to 350 MeV available. The energy dependence of S_N and V_N in this energy region is modest,[39,40] hence, the 362-MeV parameters should be adequate. For $d+{}^{58}$Ni we used the phenomenological $p+{}^{40}$Ca potentials with radius parameters rescaled by the factor $A^{1/3}$. The parameters are given in Table I.

Using Eqs. (31a)–(31e), and Eqs. (A8a)–(A8e) and (A17a)–(A17e) in the Appendix, we calculated the central, spin-orbit, Darwin, U_{RR}, and U_{RP} tensor potentials, for the KDP, Proca, and Weinberg equations, respectively. The results are displayed in Figs. 1–5 for $d+{}^{58}$Ni and Figs. 6–10 for $d+{}^{40}$Ca, respectively. We find that the central potentials are almost the same. The Proca and Weinberg spin-orbit potentials are identical under the approximations used. Further, $U_S^K \approx U_S^{W,P}$ as $(1-\alpha_K)\approx 0$ and $g^2 \ll \omega^2$ in Eq. (31b). The U_{RR} potentials for the three equations are very different. The real parts of U_{RR} for the KDP and Weinberg equations first decrease and then increase in magnitude; however, the real part of U_{RR} calculated from the Proca equation is opposite in sign from that of the KDP and Weinberg. Similarly, the imaginary part for the Proca equation first decreases and then increases, which is again opposite from the behavior resulting from the other two equations. The reason for this behavior lies in the absence of some terms in the Proca U_{RR} potential [Eq. (A8d)] which depend on the scalar potential S.

In the present approximation the Darwin potential

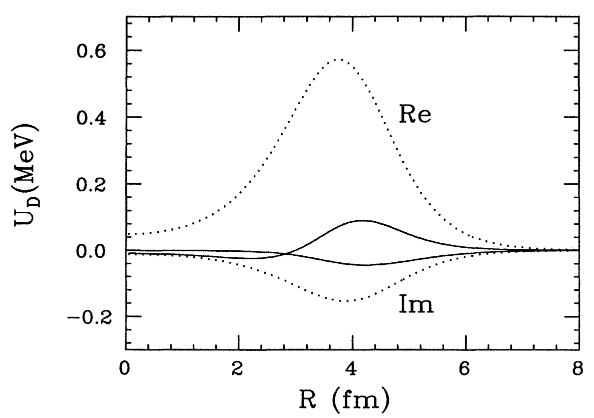

FIG. 4. The effective Darwin potentials for $d+{}^{58}$Ni elastic scattering at a laboratory kinetic energy of 400 MeV. The solid and dotted lines present the KDP, Eq. (31c), and Proca, Eq. (A8c), results, respectively. The Darwin potential from the Weinberg equation vanishes.

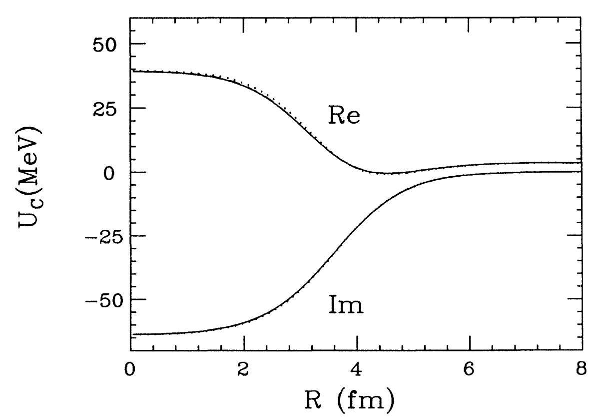

FIG. 6. The effective central potentials for $d+{}^{40}$Ca elastic scattering at a laboratory kinetic energy of 700 MeV. The solid, dotted, and dashed lines present the KDP, Eq. (31a), Proca, Eq. (A8a), and Weinberg, Eq. (A17a), results, respectively.

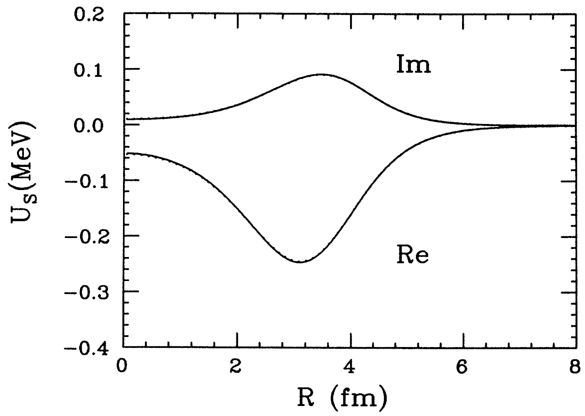

FIG. 7. The effective spin-orbit potentials for $d+^{40}$Ca elastic scattering at a laboratory kinetic energy of 700 MeV. The solid lines present the KDP, Eq. (31b), results and the dotted lines present the identical Proca, Eq. (A8b), and Weinberg, Eq. (A17b), results, respectively.

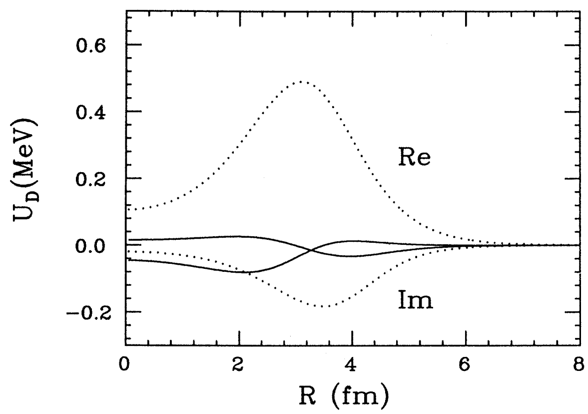

FIG. 9. The effective Darwin potentials for $d+^{40}$Ca elastic scattering at a laboratory kinetic energy of 700 MeV. The solid and dotted lines present the KDP, Eq. (31c), and Proca, Eq. (A8c), results, respectively. The Darwin potential from the Weinberg equation vanishes.

vanishes for the Weinberg equation. For the KDP and Proca, their shapes are similar but magnitudes are different. The Darwin term has not been included in the calculation of observables but, just as in the Dirac equation, it can be transformed away resulting in a comparatively small contribution to the central potential. This should not affect the observables significantly. The U_{RP} tensor potentials vanish for the Proca and Weinberg equations. For the KDP equation, it is also of the order of 10^{-3} MeV and so can be justifiably neglected. The U_{RR} potentials are included in all calculations.

Next we present calculations of the differential cross sections and the vector and tensor analyzing powers, A_y and A_{yy} in Figs. 11(a)–(c) and 12(a)–(c) for $d+^{58}$Ni and $d+^{40}$Ca, respectively, together with the experimental data.[41] The computer code SNOOPY8Q was used in the calculations.[42] For σ and A_y, all three equations give similar results. The curves for A_{yy} show some differences between the Proca and the other two predictions. Qualitative agreement with the differential cross section and A_y data is obtained; however, the A_{yy} predictions, while having roughly the correct shape, are too small in overall magnitude. The differences between the A_{yy} predictions from the Proca equation and the other two equations is due to the U_{RR} terms.

The major source of the difference between the KDP results presented here and the cases given in Refs. 11–13 is due to the various approximations made. For example, in Ref. 12 we fold over the deuteron wave function, where, in this work as well as in Refs. 11 and 13, no folding is done. This difference is most noticeable in the cross-section calculations. The spin observables are, not

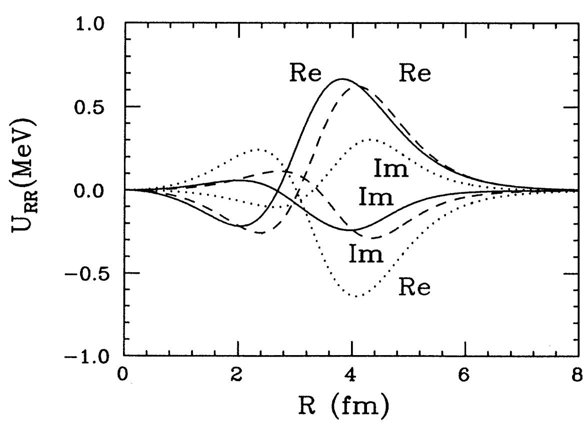

FIG. 8. The effective RR tensor potentials for $d+^{40}$Ca elastic scattering at a laboratory kinetic energy of 700 MeV. The solid, dotted, and dashed lines present the KDP, Eq. (31d), Proca, Eq. (A8d), and Weinberg, Eq. (A17d) results, respectively.

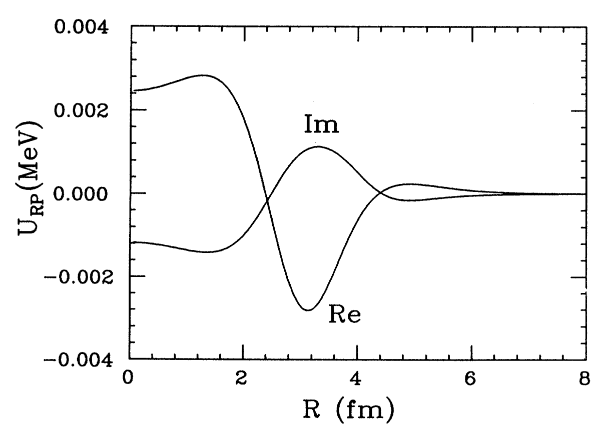

FIG. 10. The effective RP tensor potentials for $d+^{40}$Ca elastic scattering at a laboratory kinetic energy of 700 MeV. The solid line gives the KDP results from Eq. (31e). The Proca and Weinberg effective RP tensor terms vanish.

surprisingly, most affected by the treatment of the spin-orbit term. As described in Ref. 13, the various ways of choosing to write the mixed tensor term constitute different approximations. The observables obtained from the KDP formalism used here are almost identical to those given by the dashed line in Fig. 1 of Ref. 13. The major source of difference between the results here and those given by the various cases discussed in Refs. 11–13 lies in the approximations made in obtaining an effective second-order equation for the wave function. In this work we formulated the problem so that similar approximations could be made for all three equations. In the

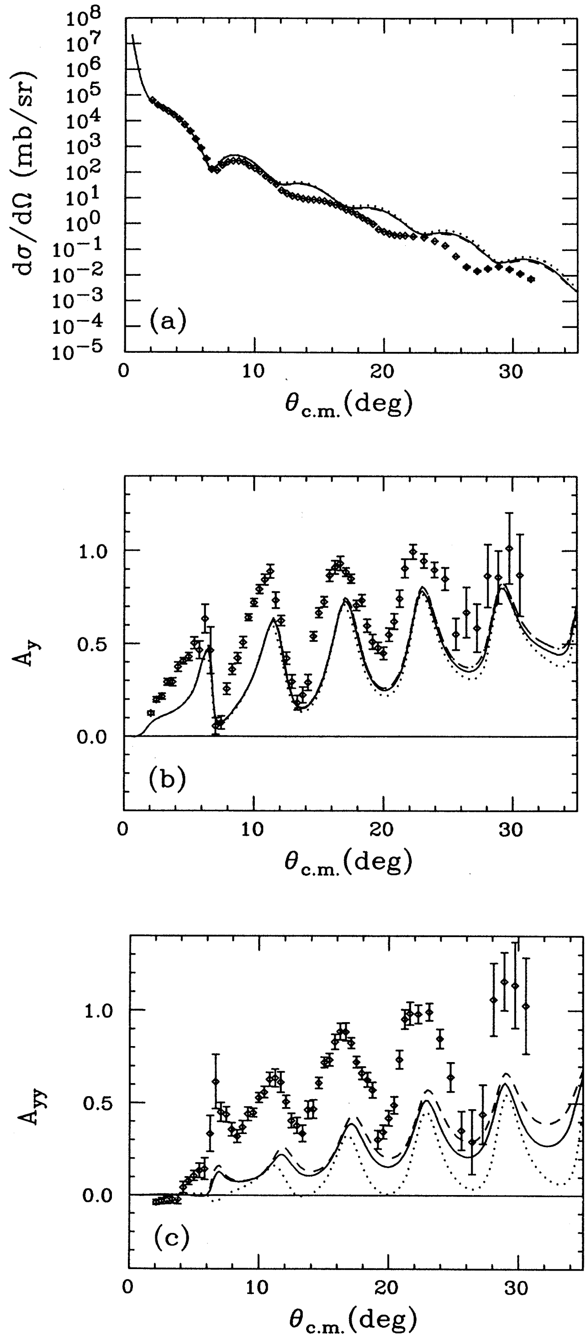

FIG. 11. The differential cross sections (a), vector analyzing powers (b), and tensor analyzing powers (c) for $d+{}^{58}Ni$ elastic scattering at a laboratory kinetic energy of 400 MeV. The experimental data are from Ref. 41 and solid, dotted, and dashed curves are predictions from the KDP, Proca, and Weinberg equations, respectively.

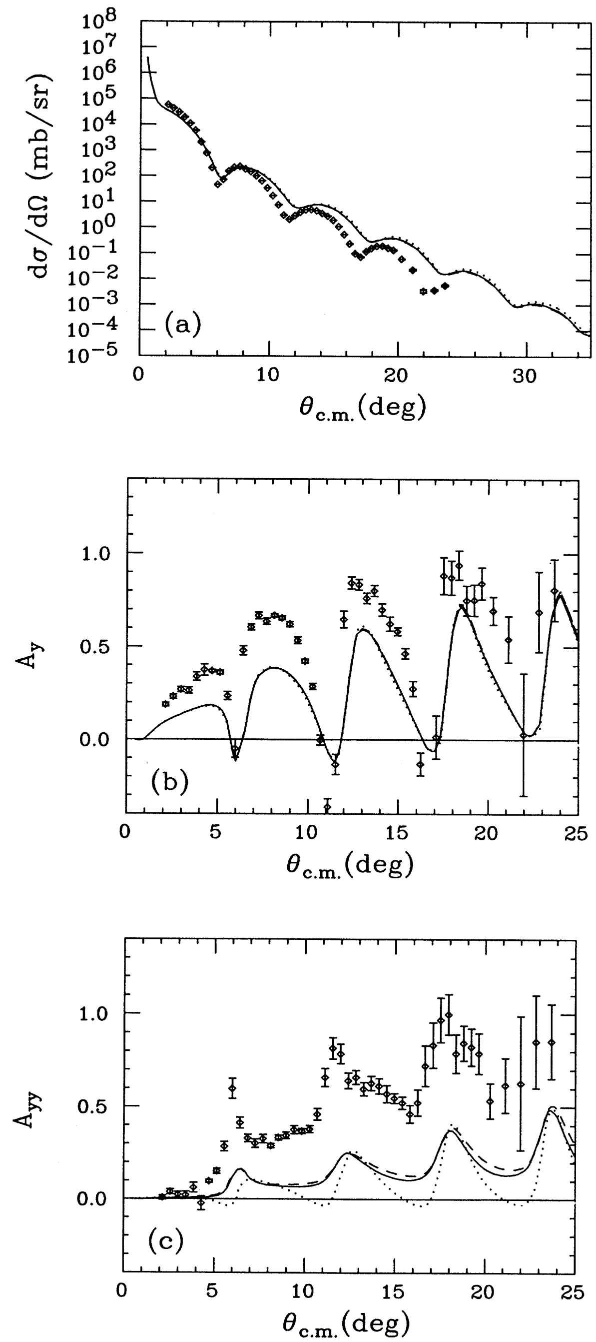

FIG. 12. The differential cross sections (a), vector analyzing powers (b), and tensor analyzing powers (c) for $d+{}^{40}Ca$ elastic scattering at a laboratory kinetic energy of 700 MeV. The experimental data are from Ref. 41 and solid, dotted, and dashed curves are predictions from the KDP, Proca, and Weinberg equations, respectively.

earlier work we dealt only with the KDP equation starting with an exact equation for the wave function given by Eq. (3.6) in Ref. 12. The second-order equation obtained from this equation can produce a somewhat larger spin-orbit term and improved agreement with experiment. Which approximation method gives results closer to the exact solution of the full KDP equation can only be determined by solving that equation. Work on this project is underway.

V. CONCLUSIONS

In this work we have presented an approximate method for solving the KDP, Proca, and Weinberg equations for a massive spin-one particle interacting with scalar and vector potentials. The second-order equations that result from approximating the three RWE's were used to make parameter-free predictions for deuteron-nucleus elastic-scattering observables. A plausible model for obtaining the deuteron-nucleus potentials from proton-nucleus Dirac phenomenology was used which is analogous to the usual Watanabe[37] model used in nonrelativistic analyses. The predictions obtained from each of the three spin-one equations are similar and qualitatively agree with experiment. It is quite likely to be important to incorporate the internal structure of the deuteron in our calculations before attempting quantitative comparisons with data. Folding the proton-nucleus optical potentials with the deuteron wave function and including breakup corrections in the effective potentials is one method for doing this. Another involves the solution of the Bethe-Salpeter equation in external fields. The Bethe-Salpeter equation cannot be solved exactly and it will be necessary to invoke approximations and to reduce it to an effective one-body equation if possible.[10] It will be interesting to see which one of the above equations arises most naturally in such an approach. Work along these lines is in progress.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation Grants PHY-8600702 and PHY-8822550, and the U.S. Department of Energy. We acknowledge useful conversations with F. Santos, P. Schwandt, and E. Stephenson.

APPENDIX

In this Appendix we derive the second-order approximations to the Proca and Weinberg equations when interactions are included. The Proca equation with interactions is given by

$$D_\mu(D^\mu\psi^\nu - D^\nu\psi^\mu) + \tilde{m}^{\,2}\psi^\nu = 0 , \tag{A1}$$

where the wave function is

$$\psi^\mu = (\phi, \mathbf{A}) , \tag{A2}$$

with ϕ and $\mathbf{A}$ as the scalar and vector components, respectively. For $\nu = 0, i$, the above equation gives

$$(\nabla^2 - \tilde{m}^{\,2})\phi = -i\omega(\Theta\hat{\mathbf{r}}\cdot\mathbf{A} - \nabla\cdot\mathbf{A}) , \tag{A3a}$$

$$(\nabla^2 + \omega^2 - \tilde{m}^{\,2})\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - i\omega\nabla\phi , \tag{A3b}$$

where $\Theta = V'/\omega$ as before. As in the KDP case, we transform the $\mathbf{A}$ field by letting

$$\mathbf{A} = \frac{E}{\omega}\mathbf{A}_1 , \tag{A4}$$

so that the $\hat{\mathbf{r}}\cdot\mathbf{A}$ term is eliminated from the Eq. (A3a). Then we get

$$(\nabla^2 - \tilde{m}^{\,2})\phi = iE\nabla\cdot\mathbf{A}_1 , \tag{A5a}$$

$$\begin{aligned}(\nabla^2 + \omega^2 - \tilde{m}^{\,2})\mathbf{A}_1 = &-\left[\Theta' + \Theta^2 + \frac{2}{r}\Theta\right]\mathbf{A}_1 - 2\Theta(\hat{\mathbf{r}}\cdot\nabla)\mathbf{A}_1 \\ &+ \nabla(\nabla\cdot\mathbf{A}_1) + \frac{\Theta}{r}[\mathbf{r}\nabla\cdot\mathbf{A}_1 + \nabla(\mathbf{r}\cdot\mathbf{A}_1)] \\ &+ \left[\Theta' + \Theta^2 - \frac{\Theta}{r}\right]\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{A}_1) + R_P ,\end{aligned} \tag{A5b}$$

where

$$R_P = -i\frac{\omega^2}{E}\nabla\phi . \tag{A5c}$$

We formally invert Eq. (A5a) to rewrite the above as

$$R_P = -\omega^2\nabla\left[\frac{1}{\omega^2 - \hat{Z}}\nabla\cdot\mathbf{A}_1\right] , \tag{A6}$$

where $\hat{Z}$ is given by Eq. (24b). We expand the operator above and keep the leading term only as before. Then we obtain the following tensor-decomposed expression for the final second-order equation

$$\left[\frac{\mathbf{p}^2}{2E} + U_C^P + U_S^P\mathbf{S}\cdot\mathbf{L} + U_D^P(\mathbf{r}\cdot\nabla) + U_{RR}^P\left[\frac{T_{RR}}{r^2}\right] + iU_{RP}^P T_{RP}\right]\mathbf{A}_1 = \frac{k^2}{2E}\mathbf{A}_1 , \tag{A7}$$

where

$$U_C^P = \left[\frac{1}{2E}\right]\left\{-\omega^2 + \tilde{m}^{\,2} + k^2 - \frac{2}{3}\left[\Theta' + \Theta\left[\Theta + \frac{2}{r}\right]\right]\right\} , \tag{A8a}$$

$$U_S^P = \left[\frac{1}{2E}\right]\frac{\Theta}{r} , \tag{A8b}$$

$$U_D^P = -\left[\frac{1}{2E}\right]\frac{2\Theta}{r} , \tag{A8c}$$

$$U_{RR}^{P}=-\left[\frac{1}{2E}\right]\left[\Theta'+\Theta\left[\Theta-\frac{1}{r}\right]\right], \tag{A8d}$$

$$U_{RP}^{P}=0\ , \tag{A8e}$$

with

$$k^2=E^2-m^2\ . \tag{A9}$$

In a similar manner, the Weinberg equation can be put into the above form. We basically follow Refs. 8 and 9 but express the effective potentials in terms of the previous notation. The starting equation is

$$\left[\left[p_\mu-V\delta_{\mu 0}\right]\left[p_\nu-V\delta_{\nu 0}\right]\gamma^{\mu\nu}-\tilde{m}^{\,2}\right]\Psi_W=0\ , \tag{A10}$$

which can be rewritten using the representations of $\gamma^{\mu\nu}$ as

$$\begin{bmatrix}\omega^2-\tilde{m}^{\,2}-\mathbf{p}^2+2X^2 & -\{\omega,X\}\\ +\{\omega,X\} & -\omega^2-\tilde{m}^{\,2}+\mathbf{p}^2-2X^2\end{bmatrix}\begin{bmatrix}\mathbf{A}\\ \mathbf{B}\end{bmatrix}=0\ , \tag{A11}$$

where

$$X\begin{bmatrix}\mathbf{A}\\ \mathbf{B}\end{bmatrix}=\mathrm{curl}\begin{bmatrix}\mathbf{A}\\ \mathbf{B}\end{bmatrix}, \tag{A12}$$

and the curly brackets stand for anticommutators. Now the lower component can be eliminated from the above equation to yield

$$\left[(\nabla^2+\omega^2-\tilde{m}^{\,2})+2\,\mathrm{curl}\,\mathrm{curl}-\{\omega,\mathrm{curl}\}\left[\frac{1}{2\omega^2-\hat{Z}_W}\right]\{\omega,\mathrm{curl}\}\right]\mathbf{A}=0\ , \tag{A13}$$

where

$$\hat{Z}_W=\nabla^2-2\nabla(\nabla\cdot)+\omega^2-\tilde{m}^{\,2}\ . \tag{A14}$$

Expanding Eq. (A13) in zeroth order in $\hat{Z}_W$, we find

$$(\nabla^2+\omega^2-\tilde{m}^{\,2})\mathbf{A}=\left[\Theta'+\Theta\left[\frac{\Theta}{2}+\frac{1}{r}\right]\right]\mathbf{A}+\frac{\Theta}{r}[\mathbf{r}\times(\nabla\times\mathbf{A})+(\mathbf{r}\cdot\nabla)\mathbf{A}-\mathbf{r}(\nabla\cdot\mathbf{A})]-\left[\Theta'+\Theta\left[\frac{\Theta}{2}-\frac{1}{r}\right]\right]\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{A})\ . \tag{A15}$$

In the present approximation the $\nabla\times(\nabla\times\mathbf{A})$ term vanishes. After some algebra, the tensor-decomposed second-order equation can be written as

$$\left[\frac{\mathbf{p}^2}{2E}+U_C^W+U_S^W\mathbf{S}\cdot\mathbf{L}+U_D^W(\mathbf{r}\cdot\nabla)+U_{RR}^W\left[\frac{T_{RR}}{r^2}\right]+iU_{RP}^W T_{RP}\right]\mathbf{A}=\frac{k^2}{2E}\mathbf{A}\ , \tag{A16}$$

where

$$U_C^W=\left[\frac{1}{2E}\right]\left\{-\omega^2+\tilde{m}^{\,2}+k^2+\frac{1}{3}\left[2\Theta'+\Theta\left[\Theta+\frac{4}{r}\right]\right]\right\}, \tag{A17a}$$

$$U_S^W=\left[\frac{1}{2E}\right]\left[\frac{\Theta}{r}\right], \tag{A17b}$$

$$U_D^W=0\ , \tag{A17c}$$

$$U_{RR}^W=\left[\frac{1}{2E}\right]\left[\Theta'+\Theta\left[\frac{\Theta}{2}-\frac{1}{r}\right]\right], \tag{A17d}$$

and

$$U_{RP}^W=0\ . \tag{A17e}$$

We note that the Proca and Weinberg spin-orbit expressions are identical. Expressions similar to the above have been derived in Ref. 9 assuming a plane-wave approximation for the lower component of the wave function.

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