Coulomb-dominated low-energy deuteron stripping

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Analysis of a three-body model shows that Coulomb polarization of the deuteron has very little influence on the branching ratio $A(d,p)/A(d,n)$ for transfer reactions on target nucleus A at very low deuteron energies (the Oppenheimer-Phillips effect). We see that polarization effects in transfer reactions are not related to the long range of the Coulomb field, but are caused by the more intense fields near the target nucleus. However, even in that region the induced dipole moment is limited by the deuteron binding, and it is small for low Z targets. We see in addition that the transfer amplitudes tend to be *insensitive* to any polarization admixtures in the entrance channel. On the other hand, the branching ratio can be affected by the Coulomb barrier for the bound final-state wave function of the proton, especially for very weakly bound final states. Brief remarks about the relation of stripping theory to special properties of the $d+d$ system are included.

I. INTRODUCTION

Most papers about Coulomb effects in low-energy (d,p) and (d,n) direct reactions emphasize[1] kinematic conditions that allow fairly large cross sections, despite the Coulomb repulsion. Such conditions occur for (d,p) reactions with low Q, namely, for the case $E_d \approx E_p$, where E_d and E_p are the asymptotic kinetic energies of the deuteron and proton, respectively. However, recent claims[2] about "cold fusion" have revived interest in improved evaluations of the very small cross sections obtained if $E_d \to 0$, originally discussed by Oppenheimer and Phillips (OP).[3] Such energies generally give $E_p \gg E_d$. The present paper shows that the OP picture of low-energy deuteron stripping by light nuclei is misleading.

Similar conclusions are reached by Koonin and Mukerjee (KM),[4] based on a second-order distorted-wave Born approximation (DWBA) analysis of a three-body model of the reaction. KM point out defects in a recent discussion by Cecil, Peterson, and Kunz[5] that supports the traditional OP picture. The present paper has a less perturbative approach than KM, and it has more discussion of the bound states of the product nuclei.

In the OP picture of a Coulomb-dominated collision, polarization of the incident deuteron by the nuclear electric field supposedly enhances (d,p) reactions relative to (d,n) reactions. However, the deuteron ground state has a definite parity (positive); hence it has no permanent electric dipole moment, and therefore polarization depends on an admixture of opposite- (negative-) parity excited states. A first reason for the weakness of the OP effect is that this admixture is inhibited by the weakness of the Coulomb field, relative to the deuteron binding energy, and it is very small. Coulomb effects are more significant at somewhat higher energies,[6,7] where breakup can also occur.[6]

The large values of the classical radius $R_c = Ze^2/E_d$ at which a deuteron with kinetic energy E_d is turned back by Coulomb repulsion can be misleading. It can seem plausible that all important Coulomb effects should take place near this "radius of closest approach." Since the electric field Ze^2/R_c^2 is very small at the radius R_c, it can seem trivial that deuteron polarization and the OP effect should vanish in the limit $E_d \to 0$.

On the other hand, the large R_c only means that deuterons must penetrate a broad potential barrier to reach the target nucleus, which means the reaction rate is small. But our concern about the OP effect is for the rare deuteron that does penetrate the barrier and reach the nucleus, rather than for the average deuteron that turns back at R_c. The Coulomb potential has larger derivatives near the nucleus than at R_c, and these can produce significant polarizations of deuterons that reach this region. Satisfactory understanding requires accurate analysis of Coulomb effects quite near the nucleus, where the nuclear forces also need to be taken into account, where the bound-state wave functions vary more rapidly than the Coulomb wave function, and where the bombarding energy is irrelevant.

In Sec. II a three-body model for the deuteron-nucleus system is discussed. Accurate coupled equations for the $l=0$ and 1 p-n states are obtained, and an approximate solution that takes advantage of the low bombarding energy to produce a localized Green's function is derived. Implications for the stripping cross sections are discussed in Sec. III. It is particularly noted that the weak $l=1$ admixture in the entrance channel must couple to a weak $l=1$ component of relative motion in the exit channel, reducing the cross section. The contribution from polarization is further reduced because the two-nucleon force is weak in triplet odd states. This prediction of small polarization effects resembles an old analysis of the deuteron-bismuth system by Gibson and Kerman;[8] however, that work did not use an appropriate weak triplet-odd two-nucleon force. More recent authors[4] also seem to have treated V_{pn} incorrectly.

Section IV is both a summary and a sketch of special

properties of transfer reactions in $d+d$ collisions: It is argued that $d+d$ can be treated as a three-body system if the internal motion of one of the deuterons is ignored. However, proper symmetry must be restored.

II. THREE-BODY ANALYSIS

The three-body model of the deuteron-nucleus system is written in standard notation as

$$H=K+V(r)+U_n(r_n)+U_p(r_p)+U_c(r_p)\ , \tag{1}$$

where target nucleus recoil is neglected, and it is assumed that U_p includes all short-range modifications of the Coulomb interaction; so $U_c(r_p)\equiv Ze^2/r_p$.

The exact stripping matrix elements are written in the "post"-representation as

$$T(d,n)=\langle \chi_n^{(-)}(\mathbf{k}_n,\mathbf{r}_n)\psi_p(\mathbf{r}_p)V(r)|\psi\rangle\ , \tag{2}$$

$$T(d,p)=\langle \chi_p^{(-)}(\mathbf{k}_p,\mathbf{r}_p)\psi_n(\mathbf{r}_n)|V(r)|\psi\rangle\ , \tag{3}$$

where ψ is the full solution of the dynamical system governed by H of (1). The other wave functions satisfy

$$\begin{aligned}&(E_n-K_n-U_n)\psi_n=0\ ,\\ &(E_p-K_p-U_p-U_c)\psi_p=0\ ,\end{aligned} \tag{4}$$

$$(E-E_n-K_p-U_p^*-U_c)\chi_p^{(-)}(\mathbf{k}_p,\mathbf{r}_p)=0\ , \tag{5}$$

$$(E-E_p-K_n-U_n^*)\chi_n^{(-)}(\mathbf{k}_n,\mathbf{r}_n)=0\ . \tag{6}$$

DWBA approximations replace ψ on the right-hand side of the T-matrix elements by more calculable expressions that are of sufficient accuracy in the regions $V\psi_n\neq 0$, $V\psi_p\neq 0$.

The principal problem of the three-body model is that the entrance channel is described with the Jacobi variables $\mathbf{R},\mathbf{r}$ and not with the variable $\mathbf{r}_p$ of the Coulomb interaction. We handle this by expanding $U_c(\mathbf{r}_p)$ in multipoles of $\mathbf{R},\mathbf{r}$, an expansion that has good convergence in the present case, since with very low bombarding energy the total energy is negative and there is no three-body breakup. We take the monopole and dipole terms,

$$U_c(\mathbf{r}_p)=U_c(\mathbf{R})+\mu W\ ,$$

with

$$\mu\equiv(\hat{\mathbf{r}}\cdot\hat{\mathbf{R}}),\ W\equiv -Ze^2r_<\,/r_>^2\ , \tag{7}$$

in which $r_<,r_>$ are the lesser and greater of $r/2$ and R. In the absence of breakup, r is comparable to the deuteron radius, and it is sufficient to use $W\approx -Ze^2r/2R^2$.

The dipole interaction adds a small $l=1$ admixture to the $l=0$ internal state of the deuteron. This p wave is induced by the dipole potential; the incident P wave is negligible at our energies. The three-body wave function becomes $\psi\approx\psi_0(r,\mathbf{R})+\mu\psi_1(r,\mathbf{R})$, and the Schrödinger equation becomes

$$\begin{aligned}&[E-K-V(r)-U_n(r_n)-U_p(r_p)-U_c(R)-\mu W]\\ &\qquad\times[\psi_0(r,\mathbf{R})+\mu\psi_1(r,\mathbf{R})]\approx 0\ .\end{aligned} \tag{8}$$

Separating the $l=0$, 1 parts of (8), we obtain coupled equations

$$\begin{aligned}&[E-K-V(r)-U_n(r_n)-U_p(r_p)-U_c(R)]\psi_0(r,\mathbf{R})\\ &\qquad=(W/3)\psi_1(r,\mathbf{R})\ ,\end{aligned} \tag{9}$$

$$\begin{aligned}&[E-K-U_n(r_n)-U_p(r_p)-U_c(R)]\mu\psi_1(r,\mathbf{R})\\ &\qquad=\mu W\psi_0(r,\mathbf{R})\ .\end{aligned} \tag{10}$$

From these equations we determine the magnitude of the $l=1$ admixture ψ_1 and its reaction on the $l=0$ wave function ψ_0.

A term $V\mu\psi_1(r,\mathbf{R})$ is omitted from the left-hand side of (10), because V is small for $l=1$ states of relative motion of two nucleons ("Serber exchange"). The short range of V further reduces its overlap with the relative $l=1$ admixture. [These effects also reduce the contributions from $l=1$ admixtures in ψ to the T-matrix elements (2) and (3), frequently mentioned in the older literature.[1,7]] The factor $\frac{1}{3}$ on the right-hand side of (9) is the angle average of μ^2.

The sum of nuclear interactions $U_n(r_n)+U_p(r_p)$ in (9) and (10) tends to be an even function of $\mathbf{r}$; in three-body discussions of nuclear reactions,[9] it is frequently replaced by its $l=0$ part U_0. We also note that the kinetic-energy operator $K=K_r+K_R$ in (10) operates on the angle-dependent factor μ and produces a centrifugal term

$$C=(2\hbar^2/M)(r^{-2}+R^{-2}/4)\ . \tag{11}$$

Insertion of C allows μ to be removed from (10) as a common factor. With these simplifications, (9) and (10) become

$$[E-K-V(r)-U_0-U_c(R)]\psi_0(r,\mathbf{R})=(W/3)\psi_1(r,\mathbf{R})\ , \tag{12}$$

$$[E-K-C-U_0-U_c(R)]\psi_1(r,\mathbf{R})=W\psi_0(r,\mathbf{R})\ . \tag{13}$$

Inserting the solution of (13) into (12), we obtain

$$[E-K-V(r)-U_0-U_c(R)]\psi_0(r,\mathbf{R})=\bar{U}\psi_0(r,\mathbf{R})\ , \tag{14}$$

a modified equation for ψ_0, in which the dipole term produces

$$\begin{aligned}\bar{U}\psi_0(r,\mathbf{R})\equiv&(W/3)[E-K-C-U_0-U_c(R)]^{-1}\\ &\times W\psi_0(r,\mathbf{R})\ ,\end{aligned} \tag{15}$$

with $\bar{U}$ a "polarization potential."

It would be straightforward to solve the coupled partial differential equations (12) and (13) for each partial wave in the variable $\mathbf{R}$ by integrating outward numerically from $R=0$, using some variant of the continuum-discretized coupled-channels (CDCC) approach.[9] Such calculations would combine the effects of the Coulomb multipoles with those of the nuclear interaction U_0. Of course, at the very low energies emphasized in this paper only the $L=0$ partial wave is needed. Because of the low bombarding energy, the total energy E is negative, $E\approx -B$, with $B=2.2$ MeV the binding energy of the deuteron, and this simplifies the boundary conditions for (12) and (13): The solution of (13) reduces asymptotically to a linear combination of decaying exponentials of r,R;

the solution of (12) reduces to an "incoming" exponential in the entrance channel, plus decaying exponentials. We note that near the target nucleus the system has very little sensitivity to the incident deuteron kinetic energy.

Although a numerical solution of the coupled equations is feasible, a discussion in terms of the polarization potential $\bar{U}$ provides rough qualitative insight about modifications caused by coupling to the dipole term. Moreover, $\bar{U}$ can be introduced in standard approximate calculations (e.g., WKB) otherwise used for Coulomb wave functions or penetrability factors.

A rough solution in the important region near the target nucleus can be developed on the basis that outside the nucleus the free local kinetic energies of the active nucleons are dominated by the negative total energy $-B$ and the repulsion U_c, and they tend to be significantly negative. In this situation the Green's function for (13) and (15) does not propagate, and it tends to be of shorter range than the local variations of the source function. Then the contribution of the operator K in (13), for example, is determined by the (comparatively) smooth source function of ψ_1: The kinetic energy is small, and the equation is dominated by the potential terms. We can omit K from the differential equation and from the corresponding Green's function to get

$$\psi_1(r,R)\approx(Ze^2r/2R^2)[B+C+U_0+U_c(R)]^{-1}\psi_0(r,R)\ , \tag{16}$$

or

$$\psi_1\approx(1+BR/Ze^2)^{-1}\psi_0\ , \tag{17}$$

where we insert a typical value $r/2\approx R$ and we disregard the short-range terms $C+U_0$. This p-wave admixture is appreciable near the target nucleus. The sign of the admixture ψ_1 can be checked by noting that the Schrödinger amplitude $\psi_0+\mu\psi_1$ is increased if $\mu>0$, i.e., if the proton is further out than the neutron, as in the Oppenheimer-Phillips picture.

The polarization potential of (15) is made explicit by choosing the familiar factored ansatz $\psi_0=\psi_d(\mathbf{r})F_0(\mathbf{R})$ for the $l=0$ channel. We get

$$\begin{aligned}\bar{U}&\approx-(Z^2e^4/12R^4)\int d^3r\,r^2[B+U_c(R)]^{-1}|\psi_d(\mathbf{r})|^2\ ,\\ &\approx-(1/48\gamma^2R^2)U_c(R)(1+BR/Ze^2)^{-1}\ ,\end{aligned} \tag{18}$$

where a zero-range deuteron wave function $\phi_d\approx(\gamma/2\pi)^{1/2}r^{-1}e^{-\gamma r}$ is used (effective range normalization[1] would improve ϕ_d), and again the short-range terms $C+U_0$ are disregarded. Evidently, near the nucleus polarization slightly reduces the strength of the Coulomb repulsion, a reasonable second-order OP effect.

Of course, the approximation used above for the Green's function is only valid just outside the target nucleus, where the local kinetic energy is large and negative. For greater accuracy, or to continue into the nuclear interior, the coupled equations (12) and (13) must be used.

III. STRIPPING MATRIX ELEMENTS

The matrix elements (2) and (3) may be separated into two parts:

$$\begin{aligned}T(d,p)&=T^{(0)}(d,p)+T^{(1)}(d,p)\ ,\\ T(d,n)&=T^{(0)}(d,n)+T^{(1)}(d,n)\ .\end{aligned} \tag{19}$$

In each case these are, respectively, the contributions from the $l=0,1$ parts of ψ. The $l=1$ contributions for p and n are of opposite sign. We first consider the $l=0$ matrix elements:

A. $l=0$

In terms of notation previously used, the $T^{(0)}(d,p)$ matrix element, e.g., can be written

$$T^{(0)}(d,p)=\langle\chi_p^{(-)}(\mathbf{k}_p,\mathbf{r}_p)\psi_n(\mathbf{r}_n)|V(r)|\psi_d(\mathbf{r})F_0(\mathbf{R})\rangle\ , \tag{20}$$

and in zero-range approximation this becomes

$$T^{(0)}(d,p)=D_0\int d^3R\,\chi_p^{(-)*}(\mathbf{k}_p,\mathbf{R})\psi_n^*(\mathbf{R})F_0(K,R)\ , \tag{21}$$

Elementary WKB expressions for entrance channel radial partial waves in the region $R<R_c$ are[10]

$$F_L(K,R)\approx q_L^{-1/2}\exp\left[-\int_R^{R_c}dR'q_L(R')\right]\ , \tag{22}$$

$$\begin{aligned}q_L(R)\equiv(4M/\hbar^2)^{1/2}[&U_c+U_0+\bar{U}\\ &+\hbar^2(L+\tfrac{1}{2})^2/4MR^2]^{1/2}\ .\end{aligned} \tag{23}$$

The bombarding energy is omitted, since it is negligible in the region of application of (23). Only the location of the turning point R_c and the overall normalization depend on that energy. Since the centrifugal term in (23) is negligible at large R, compared to the Coulomb term, the turning point R_c is nearly independent of L. Moreover, because $q_L(R)$at small R increases rapidly with L, we only need the $L=0$ partial wave.

The integrand of (21) has a maximum, determined by the overlap of the wave functions $\psi_n^*(\mathbf{R})$ and $F_0(K,R)$. The outgoing proton wave function $\chi_p^{(-)*}(\mathbf{k}_p,\mathbf{R})$ does not have much effect on the location of this maximum, because for typical exothermic reactions it oscillates with nearly constant amplitude in the region of interest. The radius of maximum contribution to the integral tends to be determined by the final-state bound functions, essentially exponentials in the region of overlap

$$\psi_p\propto\psi_n\propto e^{-\kappa R}\ . \tag{24}$$

The radius of the maximum is seen to be of the order $R_{\max}\approx\kappa^{-1}$. Although the Coulomb wave function $F_0(K,R)$ suffers a tremendous reduction of magnitude as it penetrates to the nucleus, this reduction takes place over a considerable distance, and in the region of overlap $F_0(K,R)$ varies more slowly than the bound-state wave functions.

B. $l=1$

A corresponding WKB approximation for the p-wave admixture in ψ is obtained by multiplying F_0 of (22) by $\mu(1+BR/Ze^2)^{-1}$, as in (17). The p-wave contribution is not large; for $Z=1$ and $R=2.8\times10^{-13}$ cm, the

coefficient in the correction factor has the value 0.2.

It was mentioned previously that the two-nucleon force tends to be weak in triplet-odd states. It is also very spin dependent.[11] A full calculation of $T^{(1)}(d,x)$ requires low-energy matrix elements of $V(r)$ that take these effects into account. However, an important qualitative property of $V(r)$ is that it conserves parity; it does not couple $l=0,1$ states of relative motion. In the end a nonvanishing value for the matrix element $T^{(1)}(d,p)$ depends on the $l=1$ part[4] of the product

$$\chi_p^{(-)}(\mathbf{k}_p,\mathbf{r}_p)\psi_n(\mathbf{r}_n) ,$$

on the left-hand side of (20).

Thus the polarization effect in the stripping cross section is weak for three reasons: (a) polarization of the entrance channel wave function is weak; (b) the two-body interaction is weak in odd states; (c) a weak relative odd wave in the entrance channel must couple to a weak relative odd wave in the exit channel.

IV. SUMMARY: THE $d+d$ REACTION

We have seen that deuteron polarization causes very little modification of the branching ratio for the reactions $A(d,p)/A(d,n)$ at very low deuteron energies (the OP effect), because for low Z nuclei polarization of the entrance channel wave function is small, and because the operator that links the exit channel to the polarized part of the entrance channel is weak.

A more significant Coulomb modification of the branching ratio can enter through the bound final-state wave function (the stripping form factor) of the proton, $\psi_p(\mathbf{r}_p)$; this wave function experiences a Coulomb barrier, which causes it to differ from the bound wave function of the neutron $\psi_n(\mathbf{r}_n)$. This barrier effect is more important for weakly bound states. It could matter for $(d,x\gamma)$reactions through excited states of heavier target nuclei.

Although the reaction rate at low energy is drastically reduced by the Coulomb barrier for the deuteron at large distances, calculations of the rate must use overlaps near the target nucleus and, therefore, must have wave functions that are accurate in this region. These can be obtained from the coupled equations in Sec. II, with suitable normalization to account for the long-range repulsion.

Although a straightforward application of the above three-body stripping theory to the $d+d$ system is very unsymmetrical; with one deuteron treated as inert, the theory can be employed if it is symmetrized. We then wonder whether the reactions might be dominated by important selection rules. For example, it is known[12] that at low energy the reaction $d+d\rightarrow\alpha+\gamma$ encounters a selection rule against $E1$ emission; $M1$ is also greatly reduced. Therefore, the process is dominated by $E2$ emission, to the extent that it can measure the D state admixtures in the d and α wave functions. However, there does not seem to be a selection rule of similar importance for the main transitions in deuteron rearrangement reactions.

Selection rules seem possible in $d+d$ rearrangement because we limit the collision of two deuterons at low energy to orbital angular momentum zero, and we typically assume the nuclei involved have relative $l=0$ internal wave functions. Then all the angular momentum in the system comes from the spins. Consider an exit channel $^3\mathrm{H}+p$: The two neutrons in $^3\mathrm{H}$ are coupled in a singlet spin state. Since the two protons in the system are assumed to have zero relative orbital angular momentum, antisymmetrization also requires a spin singlet for them, and the overall exit channel wave function has a very restricted form. However, this restricted exit channel wave function still has good overlap with the direct product of triplet spin functions in the entrance channel and it does not limit the cross section.

Any three-body model of a deuteron-nucleus reaction treats the target nucleus as structureless, and thereby it omits a subtle exchange contribution to the normalization of the cross section. Correction for this effect can increase cross sections by roughly 25%.[13]

Finally, recoil is important in the structure of the Hamiltonian of the $d+d$ system. For example, a correct reduced mass must be used to calculate the relative wave function $F_0(K,R)$.

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