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Reaction cross sections in the four-nucleon system with the multiconfiguration resonating-group method

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(Received 9 April 1990)

The $d+d$ and $p+t$ partial and total reaction cross sections are computed with a multiconfiguration resonating-group method. The model space employed is spanned by $p+t$, $n+h$, $d+d$, and pseudoinelastic configurations involving deuteron pseudoexcited states. The results show that, at energies where the influence of relatively sharp resonance levels is small, the calculated cross sections for the reactions $d(d,p)t$, $d(d,n)h$, and $p(t,n)h$ agree well with experiment. On the other hand, it is also found that, for $d+d$ going into three- and four-particle channels, the calculated reaction cross sections are much too small, suggesting that the inclusion of direct-breakup processes alone through the use of deuteron pseudoinelastic configurations is not sufficient. Because of this, we conclude that the model space employed here is still not extensive enough, especially in the channel spin $S=2$ state, and should be further expanded to include cluster-rearrangement configurations $p+t^*$ and $n+h^*$, where t^* and h^* represent excited states of the three-nucleon systems having $n+d$ and $p+d$ cluster structures with relative orbital angular momentum $l=1$ and spin angular momentum $S=\frac{3}{2}$.

I. INTRODUCTION

Because of computational complexities in multiconfiguration resonating-group studies of nuclear systems,¹ it is essential to have a simple criterion with which one can determine whether or not a sufficient number of cluster configurations has been included in the calculation to yield reliable conclusions. According to results obtained from our recent studies in light nuclei with $A=6, 7$, and 8 , we are of the opinion that such a criterion does exist and is based on the calculated values of the reaction cross sections. That is, if the calculated and measured reaction cross sections agree reasonably with each other, then the chosen model space is extensive enough and the calculated results on binding energies and S -matrix elements can be considered to be sufficiently accurate for practical purposes.

The reaction-cross-section criterion is considered to be useful and practical, because of the following observations.

(i) In a resonating-group study² of ^8Li , performed in a large model space consisting of $n+^7\text{Li}$, $n+^7\text{Li}^*$, $t+^5\text{He}$, and $^4\text{H}+\alpha$ cluster-rearrangement configurations, it was found that the calculated $n+^7\text{Li}$ total reaction cross section at 12.2 MeV has a satisfactory value equal to about 75% of the measured result.

(ii) The ^7Li resonating-group study was previously carried out in a model space spanned by $t+\alpha$, $n+^6\text{Li}$, and $n+^6\text{Li}^*$ cluster-rearrangement configurations.³ However, the calculated $t+\alpha$ total reaction cross section at 15.95 MeV turned out to be rather small, being only around 30% of the measured result. This motivated us to expand the model space by further including the $d+^5\text{He}$ cluster configuration.⁴ The result shows that, as a consequence of this expansion, the calculated $t+\alpha$ total reaction cross section achieves now a satisfactory value equal to about 70% of the experimental value.

(iii) For our six-nucleon investigation,⁵ we have considered $d+\alpha$ and pseudoinelastic configurations consisting of deuteron pseudoexcited states. Although we have taken precaution by including a large number of pseudoinelastic configurations, the calculated $d+\alpha$ total reaction cross section turned out nevertheless to be rather small and is equal to only about 20% of the empirically determined value. This suggests clearly that the model space has to be substantially enlarged. In this particular case, our opinion is that the $p+^5\text{He}$ and $n+^5\text{Li}$ cluster-rearrangement configurations must at least be further included.⁶

In a recent study of the four-nucleon system⁷ (hereafter referred to as KKT), we have adopted a model space that consists of both cluster-rearrangement configurations

(CRC) $p+t$, $n+h$, and $d+d$, and pseudoinelastic configurations (PIC) involving deuteron pseudoexcited states. A discussion of the formulation of this problem is given in KKT, together with selected results on S -matrix elements, and scattering and reaction angular distributions. Here we extend the calculation by computing $d+d$ and $p+t$ partial and total reaction cross sections. The purpose is, of course, to see how well the experimental reaction cross-section data can be explained when both types of cluster configurations (CRC and PIC) are included in the formulation.

The purpose of including PIC in the formulation is to take approximate account of deuteron direct-breakup effects by discretizing the continuum states of the $n+p$ system.⁸⁻¹⁰ In KKT, we adopted, for simplicity in computation, two deuteron pseudoexcited states with five PIC. Although we felt that this number of PIC is adequate for reliable results, there is the question concerning the convergence properties of the calculated quantities. To put this question to rest, we extend the calculation of KKT by using five deuteron pseudoexcited states, resulting in a large number of PIC equal to 20. The results will be discussed in the following sections. Not too surprisingly, we find in fact that they are quite similar to those reported in KKT, thus confirming our belief that, for an adequate description of phenomena in the incident channel, the number of PIC required can be kept reasonably small.

The outline of this paper is as follows. In Sec. II we present a brief description of the formulation of the four-nucleon problem. Results for $d+d$ and $p+t$ partial and total reaction cross sections are given in Sec. III and Sec. IV, respectively. Finally, in Sec. V we summarize the important findings of this investigation and make some concluding remarks.

II. FORMULATION OF THE FOUR-NUCLEON PROBLEM

The formulation of the four-nucleon problem is given in KKT; hence only a brief description will be given here. The nucleon-nucleon potential used is the Minnesota or MN potential given by Eqs. (12)–(17) of Ref. 11, with the exchange-mixture parameter $u=1$. The spin-orbit part of this potential is omitted to save computational time, since test calculations indicate that the general features of calculated reaction cross sections are not essentially affected by the omission of the spin-orbit potential.

The depth parameter V_{0s} of the singlet part in the MN potential [see Eq. (15) of Ref. 11] is adjusted from 91.85 to 100.2 MeV, in order that the calculated threshold-energy difference Δ of the $p+t$ and $d+d$ channels be-

comes close to the experimental value. This is an important adjustment, since the strength of coupling between these two channels in the low-energy region is sensitively dependent upon this difference.

The normalized three-nucleon spatial wave function Φ_3 is taken to be a sum of two Gaussian functions, i.e.,

$$\Phi_3 = c_1 \phi_1 + c_2 \phi_2, \quad (1)$$

where

$$\phi_i = \exp \left[-\frac{1}{2} \lambda_i \sum_{j=1}^3 (\mathbf{r}_j - \mathbf{R}_3)^2 \right] \quad (i=1,2), \quad (2)$$

with \mathbf{R}_3 denoting the c.m. coordinate of the three-nucleon cluster. The variational parameters are determined by minimizing the expectation value \tilde{E}_3 of the three-nucleon Hamiltonian. The results obtained with $\hbar^2/2M_n = 20.759$ MeV fm² (M_n is the nucleon mass), together with the calculated values of \tilde{E}_3 and the rms matter radius \tilde{R}_3 , are shown in Table I for both t and h . Because of the necessity of adjusting V_{0s} as mentioned above, the values of \tilde{R}_3 turn out to be about 0.06 fm smaller than the empirically determined values,¹² but are still satisfactory for our purposes.

The spatial wave functions of the deuteron in its ground and pseudoexcited states are obtained by diagonalizing the deuteron Hamiltonian in a space spanned by N_d Gaussian basis functions of the form

$$\chi_i = \exp \left[-\frac{1}{2} \alpha_i \sum_{j=1}^2 (\mathbf{r}_j - \mathbf{R}_d)^2 \right] \quad (i=1-N_d), \quad (3)$$

where \mathbf{R}_d denotes the c.m. coordinate of the deuteron cluster. The nonlinear parameters α_i are determined by minimizing the ground-state energy eigenvalue in a chosen model space. In KKT the value of N_d was chosen as 3 (to be referred to as the $3G_d$ model). Here, for the sake of ascertaining that enough PIC have been included to yield reliable conclusions, we have additionally computed with $N_d=6$ (i.e., the $6G_d$ model), which results in a lengthy calculation involving 20 PIC. In Table II we list the α_i values, and the corresponding energy eigenvalues \tilde{E}_i^d and rms matter radii \tilde{R}_i^d of the deuteron ground and pseudoexcited states. Using this table, we find that the calculated values of the $p+t$ and $d+d$ threshold-energy difference Δ are equal to 4.03 and 3.82 MeV in the $3G_d$ and $6G_d$ models, respectively, which agree quite well with the value experimentally determined.

The cluster configurations included are $p+t$, $n+h$, $d+d$, and PIC; these will be referred to as cluster configurations a , b , c , and d , respectively. In this investigation we compute the partial and total reaction cross

TABLE I. Results for variational parameters, \tilde{E}_3 , and \tilde{R}_3 .

Nucleus	c_1	λ_1 (fm ⁻²)	c_2	λ_2 (fm ⁻²)	\tilde{E}_3 (MeV)	\tilde{R}_3 (fm)
³ H or t	0.003 62	0.213	0.0284	0.681	-8.20	1.568
³ He or h	0.003 73	0.213	0.0278	0.681	-7.41	1.583

TABLE II. Results of variational calculation of the deuteron cluster.

N_d	α_i (fm $^{-2}$)	\bar{E}_i^d (MeV)	\bar{R}_i^d (fm)
3	0.049, 0.224, 1.004	-2.084, 3.158, 27.79	1.987, 4.40, 2.29
6	0.042, 0.154, 0.508 1.460, 3.516, 4.876	-2.190, 2.270, 14.59 60.79, 186.1, 483.7	1.949, 5.08, 3.26 1.97, 1.24, 0.80

sections from configurations a and c to the remaining three configurations. The main purpose is to determine the relative importance of the characteristically different CRC and PIC and, thereby, obtain some important information concerning reaction mechanisms in light systems.

The resonating-group coupled integro-differential equations are solved by adopting a multichannel R -matrix technique.¹³ The resulting intercluster relative-motion functions are used to calculate the S -matrix element S_{if}^i , which describes the coupling between an initial configuration i and a final configuration f (i and f can be configuration a , b , c , or one of the PIC) in a state characterized by the orbital angular momentum l and the spin angular momentum S . Using these matrix elements, we can then compute the various partial reaction cross sections. For instance, in the $d+d$ channel (configuration c), the reaction cross section $\bar{\sigma}_S^{ac}$ for going into the $p+t$ channel (configuration a) in a spin state S is given by

$$\bar{\sigma}_S^{ac} = \frac{2\pi}{k_c^2} \sum_l (2l+1) |S_{lS}^{ac}|^2, \quad (4)$$

where k_c is the wave number in the $d+d$ channel, and the summation is over even- l values for even S and over odd- l values for odd S . Also, a proper account has been made for the fact that identical particles are involved in the incident channel.¹⁴ Similarly, one can easily obtain

$\bar{\sigma}_S^{bc}$ and $\bar{\sigma}_S^{dc}$, and the cross sections $\bar{\sigma}_S^{ba}$, $\bar{\sigma}_S^{ca}$, and $\bar{\sigma}_S^{da}$ in the case where $p+t$ is the incident channel.

III. RESULTS

FOR $d+d$ REACTION CROSS SECTIONS

For the various $d+d$ reaction cross sections, we consider the $S=0$ and 1 contributions separately from the $S=2$ contribution. The reason for making this separate consideration is that in the $S=0$ and 1 states, both CRC (i.e., $p+t$ and $n+h$) and PIC (i.e., $d+d^*$, d^*+d^* , and so on) contribute, while in the $S=2$ states, only PIC make nonzero contributions.

Calculated results for the combined $S=0$ and 1 contributions are shown as a function of E_c , the relative energy of the two deuteron clusters in the c.m. system, in Fig. 1. In this figure the partial reaction cross section $\sigma_{01}(c \rightarrow a)$ is defined as

$$\sigma_{01}(c \rightarrow a) = \frac{1}{4} (\bar{\sigma}_0^{ac} + 3\bar{\sigma}_1^{ac}), \quad (5)$$

and similarly for $\sigma_{01}(c \rightarrow b)$ and $\sigma_{01}(c \rightarrow d)$. Both the $3G_d$ and the $6G_d$ results are plotted; they are given in the left and right sides of the figure, respectively. Also, for clarity in presentation, we have represented these cross sections by either solid circles or dashed curves. In addition,

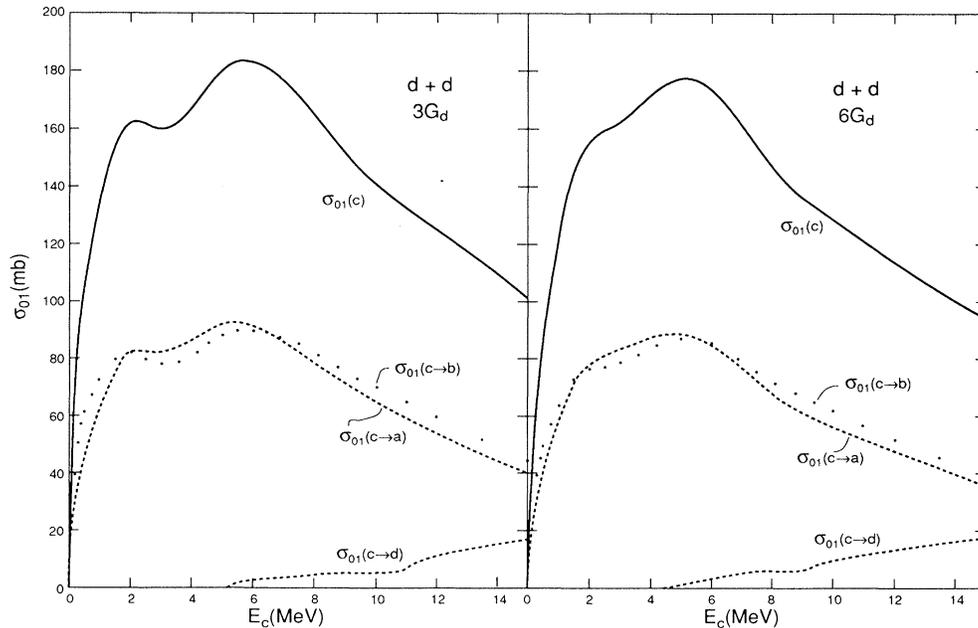


FIG. 1. Combined $S=0$ and 1 contributions to the $d+d$ partial and total reaction cross sections in the $3G_d$ and $6G_d$ models.

the $d+d$ total reaction cross section in the $S=0$ and 1 states (solid curve), i.e.,

$$\sigma_{01}(c) = \sigma_{01}(c \rightarrow a) + \sigma_{01}(c \rightarrow b) + \sigma_{01}(c \rightarrow d), \quad (6)$$

is also shown.

From Fig. 1 one notes that the $3G_d$ and $6G_d$ results differ only in minor details. In fact, even the partial reaction cross sections $\sigma_{01}(c \rightarrow d)$ obtained with these two models are characteristically similar. There are sudden changes in slope at about 10.5 and 8.9 MeV in the $3G_d$ and $6G_d$ cases, respectively; these changes occur as a consequence of the opening of the d^*+d^* channels (see Table II). Thus we can conclude that, at least in the relatively low-energy region under consideration here, the use of the simpler $3G_d$ model seems sufficient. This is actually not too surprising, since the model space of this investigation is also spanned by the energetically favored $p+t$ and $n+h$ configurations, and hence the effects of PIC are expected to be significantly reduced (see also Ref. 7), and even a rather small number of PIC is enough to adequately represent the influence of deuteron breakup in the low-energy region.

Two important features are also exhibited in Fig. 1: (a) The cross sections $\sigma_{01}(c \rightarrow a)$ and $\sigma_{01}(c \rightarrow b)$, represented by dashed curves and solid circles, respectively, differ in a minor way. Because of the structural similarity of the $p+t$ and $n+h$ systems, this is entirely expected. (b) The partial reaction cross section for $d+d$ going into CRC is much larger than that for $d+d$ going into PIC. Even at 15 MeV, it is seen that $\sigma_{01}(c \rightarrow d)$ is only about 20% of the sum of $\sigma_{01}(c \rightarrow a)$ and $\sigma_{01}(c \rightarrow b)$. This is an interesting result and is consistent with previous findings mentioned in items (i)—(iii) of the Introduction.

The observation (b) above concerning the predominance of the CRC indicates that one-nucleon transfer processes make important contributions. In the present case this is especially true because only a loosely bound deuteron cluster needs to be broken up in these processes.

Partial reaction cross sections $\sigma_2(c \rightarrow d)$ or simply σ_2 in the $3G_d$ and $6G_d$ models, defined as

$$\sigma_2 = \frac{5}{9} \bar{\sigma}_2^{dc}, \quad (7)$$

are shown as a function of E_c in the upper and lower parts of Fig. 2, respectively. Here one notes immediately that, in the $3G_d$ case, there are rapid cross-section variations in the energy region between about 8 and 12 MeV, and similarly in the $6G_d$ case. Based on our previous experience, we know that such variations are associated with the presence of spurious resonances which arise from the use of PIC to represent deuteron breakup effects. This can be seen more clearly from Fig. 3 where we show the $d+d$ phase shift δ and reflection coefficient η in the $(l, S) = (0, 2)$ state for the $3G_d$ model.¹⁵ From this figure it is found that such a resonance does exist in the neighborhood of the d^*+d^* threshold and is responsible for the cross-section behavior mentioned above.

The $\sigma_{01}(c \rightarrow d)$ curves in Fig. 1, on the other hand, have rather smooth behavior. The reason for this is that the $p+t$ and $n+h$ configurations contribute to the $d+d$ reaction cross sections in the $S=0$ and 1 states, but not

in the $S=2$ state. Thus we already have here a strong hint that certain important reaction channels are missing in the $S=2$ formulation and our present $S=2$ consideration is likely deficient. This will be further discussed below.

Next, we compare our calculated $6G_d$ results with experiment. In the $d+d$ case the most complete set of reaction cross-section data was obtained by Hegland and Brown¹⁶ at $E_c = 8.75$ MeV and is given as follows:

$$\begin{aligned} \sigma_t &= 62.6 \pm 0.5 \text{ mb}, \\ \sigma_h &= 65.1 \pm 2.0 \text{ mb}, \\ \sigma_3 &= 236.7 \pm 10.7 \text{ mb}, \\ \sigma_4 &= 14.8 \pm 7.4 \text{ mb}, \end{aligned} \quad (8)$$

where σ_t , σ_h , σ_3 , and σ_4 represent reaction cross sections for $d+d$ going into $p+t$, $n+h$, $d+p+n$, and $p+p+n+n$, respectively. The experimental values of σ_t and σ_h can be compared with our calculated values of $\sigma_{01}(c \rightarrow a) = 63.6$ mb and $\sigma_{01}(c \rightarrow b) = 68.8$ mb. Quite obviously, there is a good agreement. As for σ_3 and σ_4 , their sum is to be compared with the sum $\sigma(c \rightarrow d)$ of $\sigma_{01}(c \rightarrow d)$ and $\sigma_2(c \rightarrow d)$. This comparison is, however,

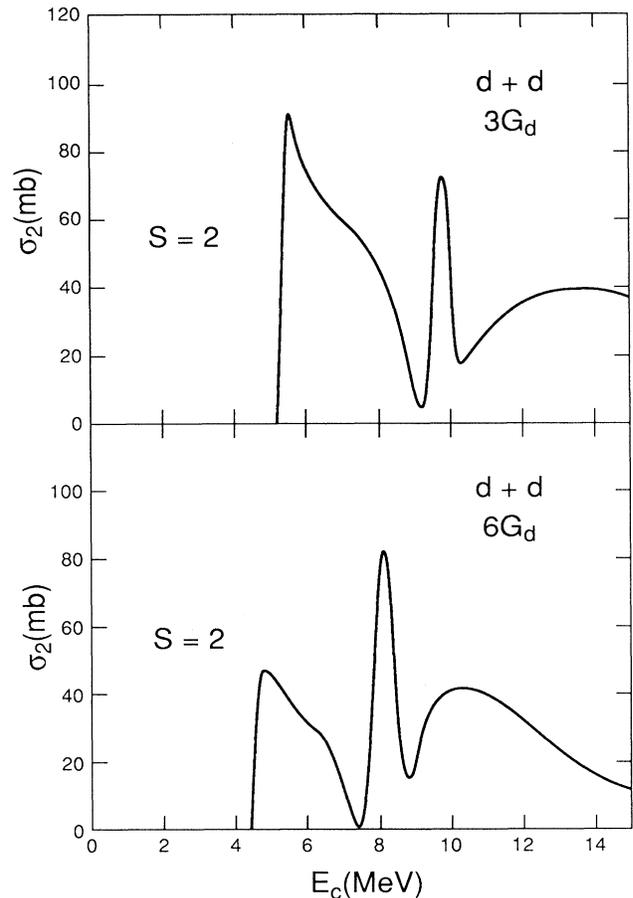


FIG. 2. $S=2$ contributions to the $d+d$ total reaction cross sections in the $3G_d$ and $6G_d$ models.

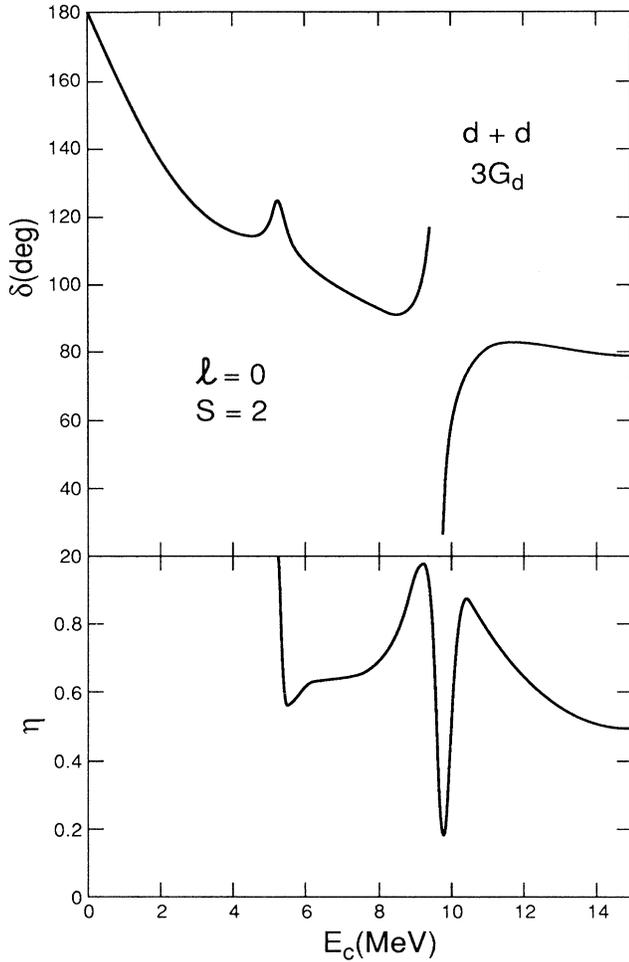


FIG. 3. Phase shift δ and reflection coefficient η in the $(l,S)=(0,2)$ state of the $d+d$ system in the $3G_d$ model.

not quite straightforward, because $E_c=8.75$ MeV happens to be in the energy region where $\sigma_2(c \rightarrow d)$ has a rapid, but unrealistic, cross-section variation. Thus what we need to do is to perform a reasonable averaging of the calculated values for $E_c=6-10$ MeV. In this way we obtain a crude estimate for $\sigma_2(c \rightarrow d)=32$ mb. Adding this to $\sigma_{01}(c \rightarrow d)=7$ mb, we find that $\sigma(c \rightarrow d)=39$ mb, which is equal to only 16% of the sum of σ_3 and σ_4 .

Another way of comparison may yield more interesting information. For this comparison we make the assumption that the division of the experimental cross section into contributions from channel-spin states is purely according to $(2S+1)$ statistical weights. Thus, in the $S=0$ and 1 states, the experimental $d+d$ total reaction cross section $\sigma_{01}(c)$ is taken to be $\frac{4}{9}$ of the sum of σ_t , σ_h , σ_3 , and σ_4 ; that is, $\sigma_{01}(c)$ has an experimental value of 168 mb. This compares reasonably well with our calculated value of $\sigma_{01}(c)=139$ mb. On the other hand, the experimental value of the $d+d$ total reaction cross section in the $S=2$ state is $\sigma_2(c)=211$ mb, which is much larger than the calculated value of 32 mb mentioned in the preceding paragraph.

The situation in the $S=2$ state is quite similar to that in the $d+\alpha$ system. As was mentioned in item (iii) of the Introduction, a resonating-group calculation using the $d+\alpha$ configuration and PIC yielded only about 20% of the empirically determined $d+\alpha$ total reaction cross section. However, by enlarging the model space to include also the cluster-rearrangement configurations $p+{}^5\text{He}$ and $n+{}^5\text{Li}$, a much improved value equal to about 85% can be obtained.⁶ Thus, in the present case regarding especially the $S=2$ state, we are certain that what is necessary is to further incorporate the $p+t^*$ and $n+h^*$ cluster-rearrangement configurations into the formulation (see also the discussion in Ref. 17), with $t^*(h^*)$ being the excited state of $t(h)$ having an $n+d$ ($p+d$) cluster structure with relative orbital angular momentum $l=1$ and spin angular momentum $S=\frac{3}{2}$. Empirically, there is indeed evidence for the presence of such three-nucleon excited states. The phase-shift analysis of experimental data on $p+d$ scattering and reaction cross sections by Arvieux¹⁸ shows convincingly that a broad ${}^4\text{P}$ resonance does exist in the ${}^3\text{He}$ system.

The above suggestion of adding the $p+t^*$ and $n+h^*$ configurations into the formulation is consistent with our finding concerning the importance of one-nucleon transfer processes. These particular configurations can be reached from the $d+d$ configuration through such processes, again with only the breakup of a loosely bound deuteron cluster.

IV. RESULTS FOR $p+t$ REACTION CROSS SECTIONS

In this section we discuss the case where $p+t$ is the incident channel. The results, obtained in both the $3G_d$ and $6G_d$ models, are shown in Fig. 4 as a function of E_a , the relative energy of the proton and the t cluster in the c.m. system. In this figure the partial reaction cross section $\sigma_{01}(a \rightarrow b)$ is defined as

$$\sigma_{01}(a \rightarrow b) = \frac{1}{4}(\bar{\sigma}_0^{ba} + 3\bar{\sigma}_1^{ba}), \quad (9)$$

and similarly for $\sigma_{01}(a \rightarrow c)$ and $\sigma_{01}(a \rightarrow d)$. The $p+t$ total reaction cross section, denoted as $\sigma_{01}(a)$, is given by

$$\sigma_{01}(a) = \sigma_{01}(a \rightarrow b) + \sigma_{01}(a \rightarrow c) + \sigma_{01}(a \rightarrow d). \quad (10)$$

Also, it should be noted that, in this case, only $S=0$ and 1 states contribute.

From Fig. 4 one notes the following features.

(a) The $3G_d$ and $6G_d$ results are similar in their essential characteristics, with only relatively minor differences in the magnitudes of the various cross sections. This indicates that, for a general understanding of the $p+t$ scattering and reaction properties, the procedure of using a rather small number of deuteron pseudoexcited states can be adopted.

(b) Gross features of the $\sigma_{01}(a)$ curve can be easily understood. The large peak near $E_a=2$ MeV is associated with the presence of resonance states in ${}^4\text{He}$, which have cluster structures of one nucleon plus three nucleon with relative orbital angular momentum $l=1$. Changes in slope at about 4.0 and 9.3 MeV in the $3G_d$ case, and at

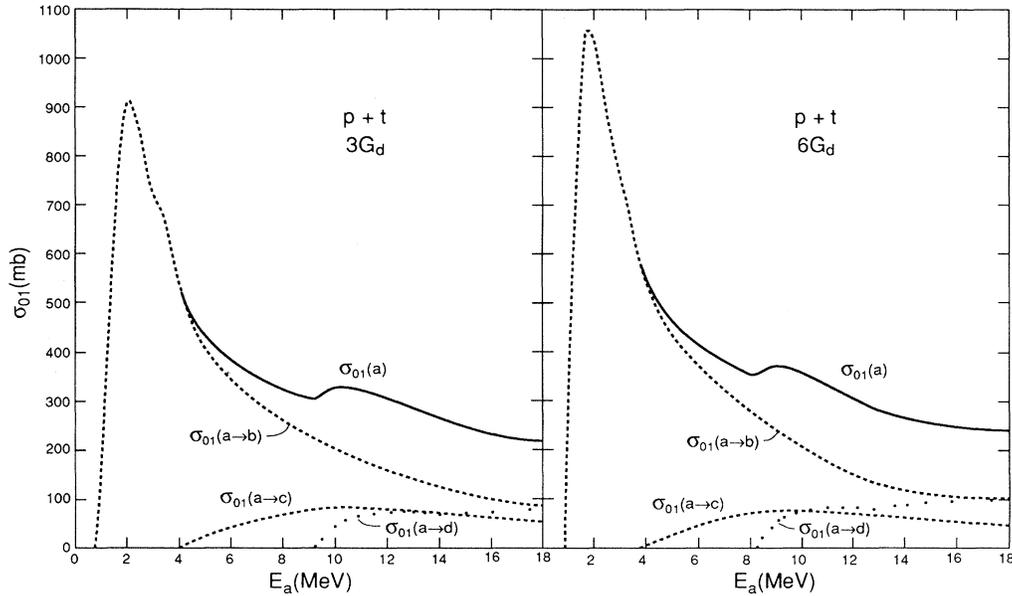


FIG. 4. Combined $S=0$ and 1 contributions to the $p+t$ partial and total reaction cross sections in the $3G_d$ and $6G_d$ models.

about 3.8 and 8.3 MeV in the $6G_d$ case, are related to the opening of $d+d$ and $d+d^*$ channels.

(c) The contribution of PIC is more important in the $p+t$ case than in the $d+d$ case. To see this let us consider the $6G_d$ model. At $E_a=17.3$ MeV, which corresponds to $E_c=13.5$ MeV, $\sigma_{01}(a \rightarrow d)$ is 39% of $\sigma_{01}(a)$, while $\sigma_{01}(c \rightarrow d)$ is only 16% of $\sigma_{01}(c)$. This means that PIC with deuteron pseudoexcited states are coupled more strongly to the $p+t$ configuration than to its parent $d+d$ configuration. The reason for this is not quite clear to us, although we suspect that it has something to do with the large difference in structures between the deuteron ground and pseudoexcited states. In this respect it is interesting to mention that a similar finding has also been noted in the five-nucleon system.¹⁷

The calculated results for the reaction cross section of $p+t$ going into $n+h$ will now be discussed. Since we have, for simplicity, omitted noncentral effects in this calculation and since even a rather extensive multiconfiguration study of the type employed here will not precisely yield the positions and widths of the compound-nucleus resonances, it is prudent that one should appropriately avoid the low-energy region and examine the results only at energies where the reaction cross-section curves have nonoscillatory and rather slow energy variation. Thus, in the following discussion, we shall consider E_a values larger than about 9 MeV and compare the calculated values of $\sigma_{01}(a \rightarrow b)$ with the measured results of McDaniels *et al.*¹⁹

In the $6G_d$ model the calculated values of $\sigma_{01}(a \rightarrow b)$ at 9.75 and 12.0 MeV are equal to 214 and 150 mb, respectively, which are larger than the corresponding experimental values of 155 and 136 mb by about 38% and 10%. The fact that there is a rather large discrepancy at the lower energy is not pleasing; a possible explanation for

this is that, even at 9.75 MeV, there may still be appreciable influence from compound-nucleus resonances. The comparison at 12.0 MeV is, on the other hand, much more favorable, suggesting that our present calculation does contain most of the important ingredients.

There is the possibility that adding the $p+t^*$ and $n+h^*$ configurations into the formulation, as suggested in Sec. III, may worsen the comparison between calculated and experimental $p+t$ reaction cross sections. However, we are not overly concerned, because the direct coupling between $p+t(n+h)$ and $p+t^*(n+h^*)$ configurations may be rather weak, owing to the rather different spatial structures of the ground and excited states of the three-nucleon cluster. Of course, we cannot be positive about this point; a confirmation of our belief will certainly require an explicit calculation, which we plan to carry out in the not-too-distant future.

V. CONCLUSION

In this investigation the $d+d$ and $p+t$ partial and total reaction cross sections are computed with a multiconfiguration resonating-group method. The model space employed is spanned by $p+t$, $n+h$, $d+d$, and pseudoinelastic configurations (PIC) involving deuteron pseudoexcited states. To ascertain that the results are reliable, we have made calculations with two models, the $3G_d$ and $6G_d$ models, which differ by the number of pseudoinelastic configurations contained, being equal to 5 and 20, respectively.

The reason for calculating with both the $3G_d$ and $6G_d$ models is to see the effects of better representing the energy continuum of the deuteron cluster. It is found, however, that for all the calculated reaction cross sections, the $3G_d$ and $6G_d$ results turn out to be characteristically

similar. In fact, in the important spin $S=0$ and 1 states, even the differences in absolute magnitudes are rather minor. This suggests that, in a calculation where energetically more-favored cluster-rearrangement configurations are already included, it is a reasonable procedure to adopt only a relatively small number of deuteron pseudoexcited states.²⁰ From a practical viewpoint this is an important finding, since to perform a multiconfiguration resonating-group study generally requires a large amount of computational time.

At energies where the influence of relatively sharp resonance levels is small, it is found that the calculated cross sections for the reactions $d(d,p)t$, $d(d,n)h$, and $t(p,n)h$ agree well with experiment. Considering the fact that there are no adjustable parameters in this calculation, we are of the opinion that this successful finding is a confirmation of the basic soundness of the resonating-group approach in treating light nuclear systems.

The results indicate strongly that one-nucleon transfer processes make important contributions, especially in the present case where only a loosely bound deuteron cluster needs to be broken up. This is gratifying, since in recent investigations of the six- and seven-nucleon systems,^{4,6} similar conclusions have also been reached.

This investigation is, however, not without its deficiency, which occurs mainly in the spin $S=2$ state. For $d+d$ going into three- and four-particle channels, we have found that the calculated reaction cross sections are much too small. This indicates that the inclusion of direct-breakup processes alone through the use of pseudoinelastic configurations is not sufficient. Because of this, we are prompted to suggest that the cluster-rearrangement configurations $p+t^*$ and $n+h^*$, where t^* and h^* represent excited states of the three-nucleon systems having $n+d$ and $p+d$ cluster structures with relative orbital angular momentum $I=1$ and spin angular momentum $S=\frac{3}{2}$, must be further incorporated into the formulation. Quite clearly, such an expansion of the model space will result in a very lengthy project; however, in view of the far-reaching consequences that the results will have, we feel that it is certainly worth the effort to carry out the calculation.

Suppose that the further inclusion of the $p+t^*$ and $n+h^*$ configurations does greatly improve the results, then we can make an important conclusion. Because the decay of $d+d$ into $d+p+n$ through these particular cluster configurations, which involve real excited states of the three-nucleon systems, is sequential in nature [i.e., $d+d \rightarrow p+t^*(n+h^*) \rightarrow p+n+d$], we can conclude that, at low energies in the $d+d$ channel, the process of sequential decay is more important than the process of direct breakup. This is certainly a very significant statement, since it gives a clear indication regarding reaction mechanisms. In the future we plan to further investigate the validity of this statement by examining other $d+nucleus$ systems.

In spite of the above discussion, we must emphasize that deuteron pseudoinelastic configurations are not unimportant. The contribution of these configurations becomes larger as the energy increases. Already at an energy of 17.3 MeV, it was mentioned in Sec. IV that these pseudoinelastic configurations contribute an appreciable amount equal to 39% of the $p+t$ total reaction cross section.

For simplicity in calculation, noncentral forces are not included in the formulation. These forces are certainly important if one wishes to study the features of vector and tensor analyzing powers in $d+d$ and $p+t$ scattering; however, they are not expected to be greatly significant in studying reaction cross sections, which is the main purpose of this investigation. For the nucleon-nucleon spin-orbit interaction, we have in fact explicitly found that its inclusion affects the reaction-cross-section results in a very minor manner.

In conclusion, we wish to mention that this investigation is part of a large project where we utilize the multiconfiguration resonating-group method to examine the properties of light nuclei with $A=4-8$. By itself the present $A=4$ study may have only limited significance. However, together with results obtained in other cases, we are confident that we will eventually achieve a general understanding of the intricate behavior exhibited by light nuclear systems.

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