π^- absorption on the diproton in ^{3}He

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Absorption of negative pions on $^1S_0$ proton pairs is studied in a model which treats the two nucleon final-state interactions (including isobar components, which simulate pion p-wave scattering) in an exact way, but leaves the third nucleon an inactive spectator. Pion s-wave rescattering is included by a phenomenological Hamiltonian. Otherwise the initial-state pions are considered undistorted. The influence of different proton pair wave functions, final-state interactions, and pion-nucleon vertex forms is studied. The experimentally observed asymmetry of the cross section about 90° is obtained only if a Galilean-invariant pion absorption operator is used. The final-state interaction is particularly important in the tensor coupled $^3S_1$-$^3D_1$ states, with the final $^3D_1$ state emerging as the dominant one. The normal polarization of outcoming protons is similar to the analyzing power in the reaction $\vec{p}+n\rightarrow(pp)_{S\text{ wave}}+\pi^-$ and is very insensitive on any details of the dynamical input. Also the polarization correlation coefficients show relatively little dependence on the model being rather constrained by kinematics.

I. INTRODUCTION

Major efforts at meson factories have been carried out to investigate the fundamental inelasticities $NN\leftrightarrow NN\pi$, on one hand, and possibly similar processes in a many-body environment, a nucleus, on the other hand.[1,2] One of the principal hopes was originally to find in these reactions a tool to probe nuclear structure and nucleon correlations with high momentum transfers or at short distances. To a large extent these expectations have not been realized yet, partly because of uncertainties in the reaction mechanism itself and because the complexity of the many-nucleon system with explicit mesonic and isobaric (and possibly quark) degrees of freedom has been a formidable challenge for theoretical interpretation of experiments. On the other hand, studies of these new degrees of freedom within nuclei are themselves of utmost interest and a motivation for meson factories in their own right.

Although "nuclear" physics may not have gained terribly much so far by pion production and absorption, the situation with the two-nucleon system is much cleaner. The explicit pionic inelasticity serves to test the meson-exchange nature of nuclear forces. In particular, the basic reaction $pp\leftrightarrow d\pi^+$ and its isobaric analogs can be quite reasonably understood[3-7] in terms of $\pi+\rho$ exchange with an excited isobar intermediate state. However, this study is severely constrained to a single bound state, the deuteron. The deuteron is a much less dense system than nuclear matter and does not necessarily give a correct picture of correlations in nuclei. Also the quantum numbers of this pair have just one value $^3S_1$-$^3D_1$, and one cannot obviously transfer the information from this system to others like absorption on a $^1S_0$ diproton. Therefore, there is considerable interest in quasi-two-body reactions (π,NN) on nuclei with a perspective to widen the variety of available quantum numbers of the "bound" state and its correlation function, and also to see what information can be obtained of nuclear structure in this way. This may be closer to the basic two-body reaction than the more frequently studied single-pion production or absorption in $A+1(\pi,N)A$. Note that even in these reactions the dominant mechanism is expected to be absorption on two nucleons, one of which remains in the residual nucleus. From a theorist's point of view, probably the lightest nuclei ^{3}He and ^{4}He would be the most tractable starting point with minimal initial-state pion interaction complicating the process and because the initial nuclear wave function is, in principle, better known than for heavier systems. Of course, the problem of initial-state interactions is nearly totally removed in electromagnetic reactions, but the reaction mechanism is not necessarily the same as for hadronic probes. In particular, the role of pion-nucleon resonances is more explicit in pionic reactions. Nevertheless, the strong-interaction results can be compared and complemented with the breakup by electrons[8] or muons.

Because absorption on a single nucleon is strongly suppressed, the above quasifree two-nucleon processes are of primary interest as the major contribution to absorption. A considerable amount of experimental[9-11] and theoretical[12-16] work has already been devoted to the cross sections of pion absorption, especially in comparisons of the reactions ^{3}He$(\pi^+,pp)p$ and ^{3}He$(\pi^-,pn)n$, but also in heavier nuclei, looking for explicit signatures of the two-body mechanism, considering the third particle as an inactive spectator. Sometimes the emphasis has been specifically on the total isospin ratio of the two reactions[17,18] or on extracting the contributions of absorption on three or more nucleons in addition to two.[19]

Due to the identity of the final-state particles the former reaction ^{3}He$(\pi^+,pp)p$ cannot mix even- and odd-state nucleon pairs for a given spin. Consequently, the differential cross section must be symmetric about 90° in

the c.m. system of the outcoming two nucleons. The same holds for the pure isospin reaction like $np \leftrightarrow d\pi^0$. In contrast, in negative-pion absorption on a diproton there are two possible isospin components with different parity and spin assignments. In this case, then, an asymmetric cross section results. Existing π^- absorption data at low energies have, indeed, the cross section higher for back angles than forward. The π^+ cross section appears very similar to the free two-body reaction with just a scaling up by a factor of about 1.5, whereas π^- absorption is over an order of magnitude smaller suggesting a different mechanism. Generally theories overestimate absorption cross sections by a factor of 2–5, indicating inadequacy in the treatment (often neglect) of initial- and final-state interactions. Also, the success in predicting the angular dependence has been limited. Experimentally the proton cross section is weighted in the backward direction, whereas models tend to yield the asymmetry the other way. At the present time no theoretical predictions nor experimental data are available for polarization observables in π^- absorption. However, it is interesting to note that the phenomenological analysis of Ref. 20 obtains five qualitatively different sets of amplitudes by fitting the *pn* angular distribution at $T_\pi = 62.5$ MeV with the data of Ref. 9, resulting in widely varying predictions for proton polarization. Therefore, this observable will be necessary in any serious attempt for an unambiguous amplitude analysis.

Some new interest on quasifree two-body absorption has arisen from the prediction[21] that the polarization of the protons in $^3\mathrm{He}(\pi^+,\vec{pp})\vec{p}$ is sensitive on the correlation function of the *np* pair, quasideuteron, on which the pion is absorbed. Polarization data in this reaction will soon be available from the TRIUMF experiment E455.[22] Also similar sensitivity has been predicted for the analyzing power in $^3\vec{\mathrm{He}}(\pi^+,pp)p$.[23] Therefore, it seems that finally the quest as old as meson factories may find pions useful as a probe of nuclear structure and correlations. It is then of interest to have a new look also at the absorption of π^- mesons on $^1S_0$ proton pairs, which has a different isospin structure from the better known $pp \leftrightarrow d\pi^+$, and make a systematic study of the dependence on the pair wave function in the initial and final states as well as on the absorption mechanism itself. Also in this case only cross-section data exist. However, an interesting comparison is possible with the recent analyzing power data[24] for the reaction $\vec{pn} \rightarrow pp\pi^-$ with the final-state protons in a relative $^1S_0$ state.

In pion absorption on an isospin *zero* pair ΔN intermediate states in the $^5S_2$ [coupled to the $^1D_2(NN)$ final state] and $^5P_3$ [coupled to $^3F_3(NN)$] play the dominant role above, say, $T_\pi = 50$ MeV. Also *s*-wave rescattering is important, especially near threshold for the cross section, at all energies for the polarization. The reason for the special importance of the above ΔN configurations is that the two baryon system can, by a tensor transition potential, get from a high *NN* orbital angular momentum two $\hbar$ units lower, thereby saving in the centrifugal barrier energy at short distances $r \leq 1.5$ fm what has been lost in the mass difference. This cannot happen as optimally in absorption on a $^1S_0$ pair. In that case parity conservation and rotational symmetry require $l_\pi = J = L \pm 1$. Therefore, only tensor-coupled two-nucleon final partial waves can participate in the process. Table I shows the lowest partial waves where the transition is possible and also the most important *P*-wave ΔN states. The isospin one states are all odd and so the ΔN components can never be in the most favorable *S* state. Although the $^3F_2$ final wave can, in principle, get to *P*-wave ΔN states, the *P*-wave *NN* channel competes and the ΔN component never gets as important as in $^3F_3 \rightarrow {}^5P_3(\Delta N) \rightarrow d\pi^+$. Furthermore, the transition potential is much weaker than, e.g., in $^3F_3 \rightarrow {}^5P_3(\Delta N)$. Consequently, the ΔN states cannot contribute very much in pion absorption on a $^1S_0$ pair. In fact, it turns out that the dominant mechanism is through the chain $\pi(^1S_0) \rightarrow {}^3S_1 \rightarrow {}^3D_1$, i.e., to a *final D state*, but through an intermediate *S* state. Still, due to the inherent importance of the Δ in the pion-nucleon system and in pionic inelasticities in general, the role of isobars should be checked with as complete a calculation as possible.

TABLE I. Quantum numbers of the states participating in $\pi(^1S_0) \rightarrow NN$.

$l_\pi = J$	Final state	Intermediate ΔN
0	$^3P_0$, $T=1$	$^3P_0$
1	$^3S_1$-$^3D_1$, $T=0$	
	$^3D_1$-$^3S_1$, $T=0$	
2	$^3P_2$-$^3F_2$, $T=1$	$^3P_2$,$^5P_2$
	$^3F_2$-$^3P_2$, $T=1$	$^3P_2$,$^5P_2$

The expected relative unimportance of ΔN intermediate states has raised further interest in this reaction as a possibility of searching for six quark cluster effects in nuclear interactions.[15] In $pp \leftrightarrow d\pi^+$ these may be masked under the ΔN contribution, whose threshold effects were at some time interpreted as arising from exotic six quark states, so called "dibaryons." Furthermore, in the denser nuclear systems this kind of short-range mechanism could be enhanced compared to the deuteron. In the reaction $pp \leftrightarrow d\pi^+$ the deuteron, as a loosely bound system, is so extended that compared with it, nucleons can be considered as point particles. The nucleon form factor can be introduced as a factor slightly modifying the momentum dependence at the highest energies. This is not necessarily so for ^{3}He or especially ^{4}He. There the extension of the nucleon internal structure approaches in order of magnitude that of the nucleus itself, and could have more profound effects. Also with this point of view in mind, calculations with the conventional hadron level dynamics should be extended as far as possible.

In Sec. II the model of the present calculation is briefly presented and in Sec. III the results are laid out for the cross section as well as for the final-state polarization with a discussion of their meaning as compared with experimental results and expectations.

II. MODEL

The theory used in this work for pion absorption on a nucleon pair has been presented in some detail in Ref. 3

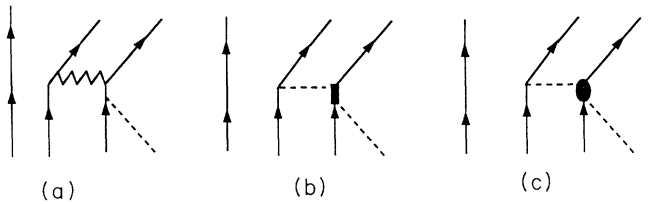

FIG. 1. The mechanisms included in pion absorption on the nucleon pair in ^{3}He: (a) absorption on a nucleon pair with a final-state correlation generated by a potential; (b) absorption with an intermediate Δ; (c) absorption involving an s-wave pion-nucleon rescattering.

for the deuteron case and will not be repeated here except for a few essentials. Figure 1 shows the basic absorption mechanisms considered in this work. The first one is absorption on a nucleon with a final-state correlation or interaction [Fig. 1(a)]. The pion-nucleon coupling is taken to be of the nonrelativistic Galilean-invariant form

$$H_{\pi NN}=\sum_{i=1,2}\frac{f}{\mu}\boldsymbol{\sigma}_i\cdot\left[\mathbf{q}\boldsymbol{\tau}_i\cdot\boldsymbol{\phi}(\mathbf{x}_i)-\frac{\omega_q}{2M}[\mathbf{p}_i\boldsymbol{\tau}_i\cdot\boldsymbol{\phi}(\mathbf{x}_i)+\boldsymbol{\tau}_i\cdot\boldsymbol{\phi}(\mathbf{x}_i)\mathbf{p}_i]\right], \tag{1}$$

with $f^2/4\pi=0.081$ and μ and M the pion and nucleon masses. In the case of absorption with an intermediate Δ formation [Fig. 1(b)], for the $\pi N\Delta$ vertex the Pauli matrices $\boldsymbol{\sigma}_i$ and $\boldsymbol{\tau}_i$ are replaced by the transition spin and isospin operators $\mathbf{S}_i$ and $\mathbf{T}_i$. As the coupling constant I use $f^{*2}/4\pi=0.35$, which can be obtained from the free $\Delta(1232)$ decay width.[25] Equation (1) follows directly from the relativistic pseudovector coupling. However, the latter term is not completely unambiguous in the Foldy-Wouthuysen transformation.[26]

The final-state interactions involving nucleons and Δ's are treated in an exact way (in the two-body sense) by solving the coupled-channels two-nucleon scattering equation involving also intermediate ΔN states. This is an improvement over, e.g., Ref. 14, where the final-state interaction was treated perturbatively and the nucleon correlations were neglected. That work established the crucial importance of the tensor effect of one-pion exchange (OPE) in the final state by the dominance of the D state. This suggests that a careful evaluation of the final-state correlation would be, indeed, warranted. Numerically, some of the overestimation of the cross section by a factor of 3–5 could be removed by the more exact treatment. The feedback effect of the ΔN channels on the NN wave function, renormalizing it at short distances, decreases the contribution from both. Similarly, the short-range correlations in the NN wave functions and the tensor force are undoubtedly important.

However, the poorly (if at all) known initial-state pion-nuclear distortions would complicate very much the partial wave expansion of the operator (1) and they will be neglected at present. If the distortion factor is similar in different pion partial waves, then the effect of this approximation would have a negligible effect on relative quantities. Hopefully this approximation, presumably accounting for most of the still remaining excess of theory versus experiment, can be improved in further studies of the reactions. However, an exact treatment of the NN and ΔN dynamics is *a priori* more essential, since it is clearly very state dependent, in particular isospin dependent. Only after a reliable treatment of this interaction can one confidently test approximations in final states. Both OPE and ρ exchange are used in the ΔN-NN transition potential. The $\rho\Delta N$ coupling constant can be related by the quark model to the ρNN and $\pi\Delta N$ couplings $f_{\rho\Delta N}=f_{\rho NN}f^*/f$. The ρNN coupling is given in Ref. 27 as $f_{\rho NN}=g_{\rho NN}(1+K_{\rho NN})$, with $g^2_{\rho NN}/4\pi=0.55$ and $K_{\rho NN}=6$. The diagonal NN interaction is the Reid soft-core potential[28] modified to avoid doubly counting the ΔN box contribution in $T=1$ states when necessary.

The last mechanism considered is pion-nucleon scattering in a relative s wave followed by absorption on a second nucleon [Fig. 1(c)]. This is incorporated by a phenomenological Hamiltonian

$$H_s=4\pi\frac{\lambda_1}{\mu}\boldsymbol{\phi}\cdot\boldsymbol{\phi}+4\pi\frac{\lambda_2}{\mu^2}\boldsymbol{\tau}\cdot\boldsymbol{\phi}\times\boldsymbol{\pi}\ . \tag{2}$$

The strength parameters are allowed a simple energy dependence fitted to give approximately πN s-wave scattering amplitudes. Just at pion threshold the scattering lengths would give $\lambda_1=0.0054$ and $\lambda_2=0.0445\mu/\omega_q$. The dominant second term of Eq. (2) leads to a two-nucleon term proportional to $\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2$, which necessarily changes the isospin.[3] Therefore, this term *does not contribute* to the s-wave pion absorption on a diproton (into the final isospin one $^3P_0$ state) as it does in the reaction $pp\leftrightarrow d\pi^+$. Similarly it vanishes for the ΔN intermediate states. Consequently, also the s-wave rescattering effect can be expected to be much less important in absorption on $^1S_0$ pairs than on $^3S_1$ pairs where the λ_2 term survives. This is contrary to some conjectures that s-wave rescattering would be important in the present case, since the Δ is unimportant.

In the operators (1) and (2) the isospin, spin, and spatial variables can be separated to a convenient form of four symmetric terms. Specifically, the first (non-Galilean) term becomes

$$\frac{\mu}{f}H^{\mathrm{nG}}=\tfrac{1}{4}[\mathbf{q}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}+\mathbf{q}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}](e^{i\mathbf{q}\cdot\mathbf{r}/2}+e^{-i\mathbf{q}\cdot\mathbf{r}/2})$$
$$+\tfrac{1}{4}[\mathbf{q}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}+\mathbf{q}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}](e^{i\mathbf{q}\cdot\mathbf{r}/2}-e^{-i\mathbf{q}\cdot\mathbf{r}/2})\ . \tag{3}$$

In this form now the spatial dependence of the pion field is explicitly implied in the plane waves and $\boldsymbol{\phi}$ carries only the

isospin properties of the pion. Clearly only the spin-changing terms proportional to $\sigma_1-\sigma_2$ survive in absorption on a diproton. Also it is easy to see that the matrix elements of the isospin factors $\langle [NN]T|(\tau_1\pm\tau_2)\cdot\phi|pp\rangle$ are $2\sqrt{2}$ for $T=1$ and $-2\sqrt{3}$ for $T=0$. Furthermore, the parity assignments for nucleons in the transition are very transparent. Obviously in pion absorption on an isospin zero pair the terms proportional to $\tau_1-\tau_2$ must be chosen. In the same way the separation can be made for the other terms of the interaction defined by Eqs. (1) and (2).

After some algebra, the expansion of Eqs. (1) and (2) into partial waves leads finally to the transition matrix elements for pion absorption evaluated in the two nucleon subsystem

$$\left\langle \phi^{\mathrm{SM}} \left| H_{\pi NN}+H_{\pi NN}\frac{1}{E-H_0}H_s \right| {}^1S_0,\pi \right\rangle$$

$$=\frac{2\sqrt{8\pi}}{\sqrt{2\omega_q}}\frac{f}{\mu}\frac{q}{k}\sum_{L_fJ}(-1)^{L_f-S-M}\sqrt{(2J+1)(2L_f+1)}\begin{bmatrix} L_f & S & J \\ 0 & M & -M \end{bmatrix}\begin{bmatrix} J & J & 0 \\ -M & M & 0 \end{bmatrix}C_{JM}(\hat{q})$$

$$\times\sum_L(-1)^{(L_f-J+1)/2}[\tfrac{1}{2}(2J+1)(J+L+1)(3-T)]^{1/2}$$

$$\times\left\{\left[1-\frac{\omega_q}{2M}\right]\int_0^\infty u_{JL}^*(r)j_L\left[\frac{qr}{2}\right]v(r)dr\right.$$

$$+(-1)^{(J-L+1)/2}\frac{\omega_q}{Mq}\int_0^\infty u_{JL}^*(r)j_J\left[\frac{qr}{2}\right]v'(r)dr$$

$$+(-1)^{(J+L-1)/2}\left[\frac{\lambda_1}{\mu}+\frac{3}{2}\frac{\lambda_2}{\mu}\frac{\omega_q}{\mu}\right]\frac{1}{q}$$

$$\times\left[\left[2-\frac{\omega_q}{2M}\right]\int_0^\infty u_{JL}^*(r)f'(r)j_J\left[\frac{qr}{2}\right]v(r)dr\right.$$

$$\left.\left.+(-1)^{(J+L+1)/2}\frac{\omega_q}{M}\int_0^\infty u_{JL}^*(r)f(r)j_J\left[\frac{qr}{2}\right]v'(r)dr\right]\right\}. \tag{4}$$

Here the quantum numbers are obvious: S the final spin (one), M its z component, T the isospin; J is the final total angular momentum ($=l_\pi$), and L_f the orbital one. The (possibly tensor-coupled) intermediate angular momentum is L. Initial and final momenta are q and k, respectively, in the πNN center-of-mass system. The $^1S_0$ pair wave function is $v(r)$, whereas the final (or intermediate) wave function is $u_{JL}(r)$ and j_L is the spherical Bessel function. The derivative of the wave function is defined as $v'(r)=r\,d/dr[v(r)/r]$. For the $^1S_0$ pair wave function I use first the square root of the $T=0$ pair-correlation function from Ref. 29 and study the dependence on this function later. In this respect the present work is similar to Refs. 21 and 23, which studied polarization observables in positive-pion absorption on a quasideuteron in ^{3}He. The free-nucleon wave functions $u_{JL}(r)$ are normalized so that asymptotically they approach spherical Riccati-Bessel functions.

The s-wave rescattering propagator function is[3]

$$f(r)=\frac{\exp(-\mu' r)}{r}\quad\text{and}\quad f'(r)=\frac{d}{dr}f(r)\ , \tag{5}$$

with $\mu'^2=\frac{1}{4}(3\mu^2-q^2)$. For $q>\sqrt{3}\mu$ this becomes complex with the incoming pion boundary condition. In fact, a dipole form factor with $\Lambda=700$ MeV is also included in $f'(r)$. The model so specified describes the genuine two-body reaction $pp\leftrightarrow d\pi^+$ quite satisfactorily. Furthermore, it was already noted above that for $T=1$ final states the λ_2 term vanishes.

Equation (4) is directly generalizable to include also ΔN channels in the L sum. Due to the spin operator $\mathbf{S}_i$ only triplet ΔN channels can directly contribute to transitions from a singlet state. From the difference between the isospin and spin $\frac{3}{2}$ particles versus nucleons and the coupling constants, these channels then need an additional factor $-(3\sqrt{2})^{-1}f^*/f$. Also in this case the λ_2 term must be omitted in s-wave rescattering.

Now the nucleon pair must be combined with the spectator nucleon. The spin-isospin structure of the ^{3}He nucleus is taken to be of the antisymmetric S-wave form

$$\Psi_{ts}=\frac{1}{\sqrt{2}}([[[\tfrac{1}{2}\times\tfrac{1}{2}]^{01}\times\tfrac{1}{2}]^{1/2\,1/2,\,1/2\,1/2}$$

$$-[[\tfrac{1}{2}\times\tfrac{1}{2}]^{10}\times\tfrac{1}{2}]^{1/2\,1/2,\,1/2\,1/2})v(r)w(\rho)\ , \tag{6}$$

where the square brackets denote the standard Clebsch-Gordan coupling to a good isospin and spin state, e.g.,

$$[\tfrac{1}{2}\times\tfrac{1}{2}]^{TT_z,SS_z}=\sum_{\sigma_1\sigma_2\tau_1\tau_2}\langle\tfrac{1}{2}\sigma_1\tfrac{1}{2}\sigma_2|SS_z\rangle\langle\tfrac{1}{2}\tau_1\tfrac{1}{2}\tau_2|TT_z\rangle$$

$$\times|\tfrac{1}{2}\sigma_1;\tfrac{1}{2}\tau_1\rangle|\tfrac{1}{2}\sigma_2;\tfrac{1}{2}\tau_2\rangle\ . \tag{7}$$

After this step also a tensor-coupled D-state component can be allowed as a part of the correlations of a $TS=01$

pair. Obviously Eq. (6) is a drastic simplification (made also in previous works on this topic) from the true multiterm Faddeev wave functions. Since the relative motion $v(r_{12})$ is based on the two-nucleon correlation function, however, this should be a reasonable starting point.[30] The spectator degrees of freedom are all integrated over, so the main effect of its wave function $w(\rho_3)$ is to change the kinematics by absorbing on average some 5 MeV of energy due to the spectator Fermi motion, the value at which its Fourier transform approximately peaks. So effectively, then, the relevant ^{3}He "binding" energy is about 10 MeV more than that of the deuteron in $\pi d \to NN$ (one 5 MeV arises from the normal binding energy difference).

Using the above wave function in the matrix element calculation and allowing then for three different possible pairings, one gets the weight factors 1.5 for absorption cross section on a quasideuteron in ^{3}He (i.e., on average there are 1.5 isospin 0 pairs in a ^{3}He nucleus), 1 for absorption on a diproton and 0.5 for an isospin 1 neutron-proton pair. In principle, a positive pion can also be absorbed on the last isospin one pair in Eq. (6), mixing with the quasideuteron absorption. However, in this case the final state (two protons) would be purely an isospin one and this small symmetric contribution would be indistinguishable in the much larger quasideuteron cross section. This component could possibly contribute to polarization. In the present reaction the final state in absorptions on a neutron-proton pair is different and irrelevant here.

In addition, also the final three-nucleon state should be properly symmetrized, which gives another statistical factor of $\frac{1}{3}$ for the cross section in the present case where a final proton-neutron pair is observed. If wanted, this coefficient can be factorized into two parts. First, there is a $\frac{2}{3}$ probability for finding a good isospin 0 or 1 pair in addition to the spectator neutron. Another $\frac{1}{2}$ comes then from the two-nucleon isospin wave functions.

The simple "two-body" kinematics described above can be extended to better account for the motion of the spectator (at rest in the laboratory) in the c.m. system of the active participants. First one needs to relate the initial pion laboratory momentum to its momentum relative to the pair, i.e., in the system moving with the center of mass of the final-state pair as used in Eq. (4) and in the genuine two-nucleon reaction.[3] The square of the total center-of-mass energy is the invariant

$$s = 2M_3 T_\pi + (M_3+\mu)^2 , \tag{8}$$

where M_3 is the ^{3}He mass and T_π the pion kinetic energy in the laboratory with $\omega_L = \mu + T_\pi$. In the final state this can also be expressed as

$$s = M_s^2 + s_2 + 2M_s(s_2+q_L^2)^{1/2} , \tag{9}$$

where M_s is the spectator mass and q_L the pion laboratory momentum transferred to the pair. The "invariant" s_2 is the square of the total internal energy of the pair subsystem, i.e., the energy of the final-state pair in its own center-of-mass system

$$s_2 = s + M_s^2 - 2M_s(s+q_L^2)^{1/2} = (M_3+\omega_L-M_s)^2 - q_L^2 . \tag{10}$$

With s_2 so calculated, obviously in the laboratory the center of mass of the pair gains the velocity

$$v_{2L} = \frac{q_L}{(s_2+q_L^2)^{1/2}} = \frac{q_L}{M_3+\omega_L-M_s} . \tag{11}$$

Now the velocity of the pion in the system boosted by v_{2L} is

$$v_2 = \frac{v_L - v_{2L}}{1 - v_L v_{2L}} = q_L \frac{(s_2+q_L^2)^{1/2}-\omega_L}{\omega_L(s_2+q_L^2)^{1/2}-q_L^2} \tag{12}$$

and its momentum

$$q_2 = q_L \frac{(s_2+q_L^2)^{1/2}-\omega_L}{\sqrt{s_2}} = q_L \frac{M_3-M_s}{\sqrt{s_2}} . \tag{13}$$

This momentum is to be used as q in the above equation for the transition matrix element. In practice, the momentum q_2 obtained by the two-body argument is indistinguishable from the more exact result (13). Of course, in calculating the cross section the relevant incident velocity is the pion velocity in the laboratory $v = q_L/\omega_L$.

III. RESULTS

From the form of the integrals of Eq. (4) it can be expected that the dominant contribution would have the final orbital angular momentum equal to zero (i.e., absorption of p-wave pions). Then the dominant first term of direct production (non-Galilean) has $j_0(qr/2)$ in the integral vs $j_1(qr/2)$ in s-wave absorption. This is also the case for the reaction $pp \leftrightarrow d\pi^+$, except in the neighborhood of the pion threshold. However, for $L_f = 0$ the isospin zero NN wave function $u_{10}(r)$ has a node situated so close to the origin that the oscillatory integrand nearly cancels off. There has to be an "extra" node because of the existence of a bound state in this wave. The cancellation is much less complete for the *intermediate S-wave* component ($L=0$) emerging from $L_f = 2$. This term then turns out to be the dominant one.

To give an idea of the relative importance of different partial waves Table II shows at 62.5 MeV the reduced matrix elements of the pion-production operator similar to those defined in Ref. 3 [the part of Eq. (4) after $C^*_{JM}(\hat{q})$]. The $^3D_1$ final state is distinctly dominant and even more conspicuously so in the total cross section as seen in Table III. Except for the $^3P_2$-wave amplitudes the present results are not unlike solution 1 of Ref. 20. In that particular solution other odd waves but $^3P_0$ were omitted, so slight differences are not surprising. Table II clearly shows that the widely used restriction to only s- and p-wave absorption is not necessarily valid even at this low energy. The reason is that the pion wave function appearing in the normally dominant first term of the matrix elements (4) is $j_L(qr/2)$, *the same for both the* P_0 *and* $^3P_2$ *states*. The angular momentum $L=1$ can be

TABLE II. Partial-wave amplitudes at 62 MeV for some interactions used in this work (fm$^{1/2}$). These are the reduced results obtained from the innermost sum of Eq. (4).

	$s\to{}^3P_0$	$p\to{}^3S_1$	$p\to{}^3D_1$	$d\to{}^3P_2$	$d\to{}^3F_2$	$f\to{}^3D_3$	$f\to{}^3G_3$
Reid, full: Re	0.078	−0.144	0.707	0.109	−0.023	0.058	0.035
Im	−0.043	−0.111	−0.313	0.028	0.004	0.003	−0.011
Reid, non-Galilean	−0.050	−0.055	0.600	0.130	−0.062	0.065	0.029
	−0.001	−0.103	−0.275	0.032	0.004	0.005	−0.012
Reid, no s	0.048	−0.162	0.767	0.107	−0.025	0.058	0.036
	−0.017	−0.118	−0.347	0.028	0.004	0.003	−0.012
Modified Reid	0.078	−0.216	0.645	0.109	−0.023	0.058	0.035
	−0.043	−0.065	−0.287	0.028	0.004	0.003	−0.011
Reid+$N\Delta$	0.091	−0.144	0.707	0.112	−0.003	0.058	0.035
	−0.065	−0.111	−0.313	0.031	0.005	0.003	−0.011

coupled with the pion momentum in the operator $\mathbf{q}\cdot\sigma$ to form a total $l_\pi = J = 0$ or 2. Therefore, it is in fact inconsistent to include in this term only the former $l_\pi = 0$ but not the latter, in spite of the formally different pion angular momenta. Table II also confirms the expectation of Sec. II that the contribution to the s-wave absorption via s-wave rescattering is much less important in the present case than in $\pi d \to pp$. There the s-wave absorption amplitude was qualitatively changed by this rescattering. In the analysis of Ref. 20 the three solutions, where *all five* amplitudes are included, are dominated by P waves and therefore are qualitatively different from the present result and from the other two of the same analysis when only three amplitudes were included. Of course, the three coefficients of Legrendre polynomials fitted from the pn angular distribution[9] and used in Ref. 20, should not be enough constraint for a unique determination of five amplitudes even qualitatively. Anyway, the wide variation of the fitted amplitudes suggests that theoretical estimates are needed also for d-wave absorption.

Another uncertainty in interpreting the data lies in the early truncation of the Legendre series in fitting the cross section and in the determination of the Legendre coefficients a_n in the expansion

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi} \sum_{n=0}^{\infty} a_n P_n(\cos\theta) \ . \tag{14}$$

This particular normalization is chosen here in accordance with Ref. 31 to have a_0 directly the total cross section. The factor 4π is not used in the analysis of Refs. 9 and 11. Only the first three terms $n = 0$, 1, and 2 are used there. My calculation described above also gives a sizable a_3, which in an experimental fit is embedded in a_1. Note that, due to the properties of the Legendre polynomials, near 90° the a_3 term contributes similarly to the a_1 term but with the sign reversed. The two can be distinguished only over a wider range than presently available. In the more intuitive old parametrization of the reaction $\pi d \to pp$

$$\frac{d\sigma}{d\Omega} = \frac{1}{32\pi} \sum_{0}^{\infty} \gamma_n \cos^n\theta \ , \tag{15}$$

the model predicts that γ_3 can be very nearly the same as γ_1, although around 90° it is effectively suppressed in comparison with γ_1 by a factor $\cos^2\theta$. These parameters are given in Table IV. Clearly an early truncation of the Legendre series may not be physically justified or meaningful, at all, except as a compact way to present the data. The use of the experimental a_n coefficients to "exactly" extract amplitudes can then be misleading. Of course, it is true that at present the data do not allow a determination of more coefficients. In Table IV one should also note that the total cross section $\sigma_{abs} = a_0$ is still a gross overestimate as compared with the experimental result.

Because of the uncertainties and mutual inconsistencies in a comparison with a_n's it is best to compare the theory directly with data. This is done in Figs. 2–6 for the pion laboratory energy 62.5 MeV. In the two-nucleon system this corresponds to the relative energy of 180 MeV or 390 MeV in the system in which one of the nucleons is at rest ("two-nucleon laboratory system"). The solid curve is the full calculated result with purely nucleonic wave

TABLE III. The total absorption cross sections at 62 MeV for some bound pair wave functions discussed in the text (mb).

Wave function	σ_{abs}	Final-state separation $^3P_0$	$^3S_1$	$^3D_1$
Basic correlation	2.96	0.11	0.15	2.66
Modified deuteron	2.00			
Hajduk	2.02			
Deuteron	0.86			

TABLE IV. The coefficients a_n of the Legendre polynomial expansion of $d\sigma(pn)/d\Omega$ (mb/sr) for some final-state interactions discussed in the text. Note that the experimental results are given in the original papers with a different normalization as $A_n = a_n/4\pi$. Also the parametrization in terms of γ_n's is given.

	a_0	a_1	a_2	a_3	γ_0	γ_1	γ_2	γ_3
Reid	2.96	−1.19	3.53	0.99	8.9	−21.0	48.8	18.0
Reid+$N\Delta$	2.99	−1.39	3.57	0.83	8.9	−20.5	50.1	14.1
Modified $J=1$	2.60	−1.13	3.71	0.93	5.5	−19.8	49.0	16.8
Experiment	0.69	0.57	1.12					

functions calculated using the Reid soft-core potential.[28] The theoretical results have been scaled to produce the experimental total absorption cross section 690±40 μb of Ref. 9. The angular dependence prediction is quite good. It is interesting to study the origin of this angular distribution.

Figure 2 shows contributions from individual terms of Eq. (4). The dash-dotted curve gives the result with the *non-Galilean operator alone* [the first term in Eqs. (1) and (4)]. This has a wrong asymmetry about 90° in the center-of-mass system of the outgoing fast nucleons, as well as does the non-Galilean result with *s*-wave pion rescattering included (dotted curve). By varying interactions and wave functions it was not possible to produce the correct asymmetry without introducing the Galilean-*invariance term* dependent on the nucleon momentum [the second term in Eqs. (1) and (4)]. It may be noted that in the reaction $pp \leftrightarrow d\pi^+$ this controversial term[26] did not give significant effects in *relative observables*, merely scaling the total cross section by some 20%.[3] This finding about the importance of Galilean invariance in π^- absorption is in line with Ref. 15, where it was also imposed in a quark bag model. Once the Galilean term is included, the correct asymmetry follows easily. A quick study of the partial-wave amplitudes in Table II reveals

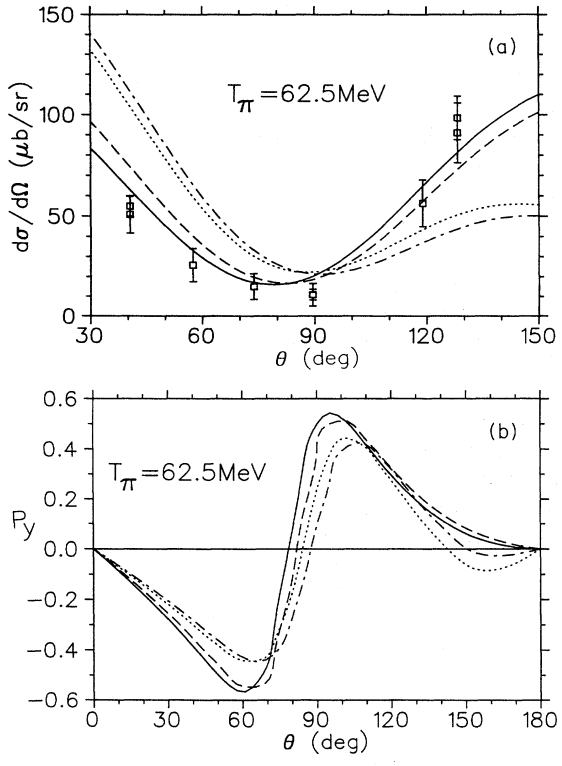

FIG. 2. Effect of different terms in Eq. (4) on the angular distribution (a) and proton polarization (b). Solid curve: full calculation including all the terms. Dashed curve: the Galilean-invariant result without *s*-wave rescattering. Dotted curve: the non-Galilean result. Dash-dotted curve: the non-Galilean result without *s*-wave rescattering. Here as in the following figures the differential cross sections are scaled to produce the experimental total cross section.

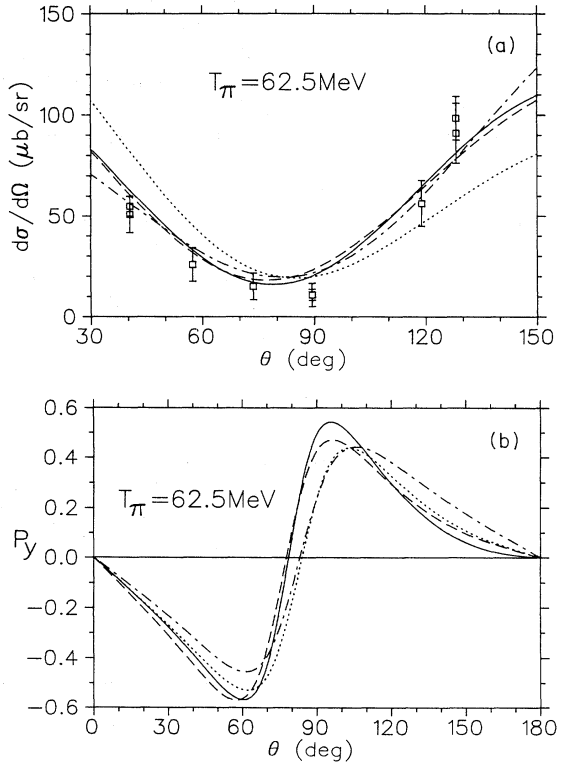

FIG. 3. Effect of various partial waves on the angular distribution (a) and proton polarization (b). Solid curve: All waves up to $J=3$ (same as Fig. 2). Dashed curve: $J=3$ absorption omitted. Dash-dotted curve: only $J=0,1$ absorption included. Dotted curve: only $J=1,2$ included.

that its effect is large enough in the s-wave absorption (with a $^3P_0$ final state) to change the sign of that amplitude. Finally, in the dashed curve the s-wave rescattering is neglected in an otherwise full calculation. This term is in the right direction but is not enough to reverse the asymmetry by itself. It is much less important at higher energies as could be expected from the factor q^{-1} in Eq. (4) and the experience with $pp \leftrightarrow d\pi^+$. The full calculation including all the terms in Eq. (4) agrees best with experiment.

Figure 3 shows explicitly the importance of various partial waves. Since omitting either all even or odd waves would result in a trivially symmetric cross section, it makes sense only to keep at least the dominant even wave and some odd waves. The solid curve is the same as in Fig. 2 and includes all amplitudes up to $J = l_\pi = 3$. In the dashed curve the highest f partial wave is omitted resulting in a negligible change. Similarly omitting d-wave absorption ($^3P_2$-$^3F_2$ final state; dash-dotted) makes little difference, thus justifying in some sense the phenomenological neglect of this wave. In view of the sizable d-wave absorption amplitudes shown in Table II this result is not obvious. Of course, literally this only shows the relative insensitivity of the observables on variations of this particular amplitude. However, if instead the s-wave absorption amplitude ($^3P_0$) is left out (but keeping the d wave) the correct asymmetry is lost, so this partial wave is most essential (the dotted curve).

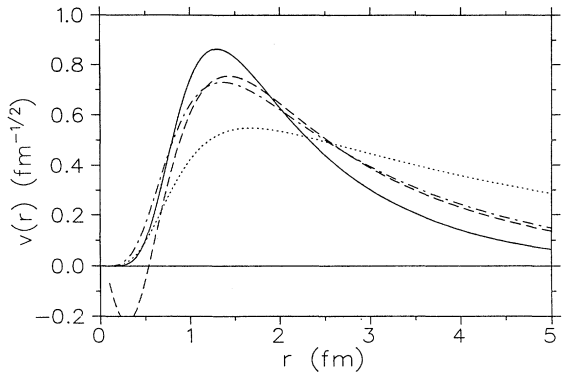

FIG. 5. Different pair wave functions used in studying the dependence on initial state. The notation is as in Fig. 4.

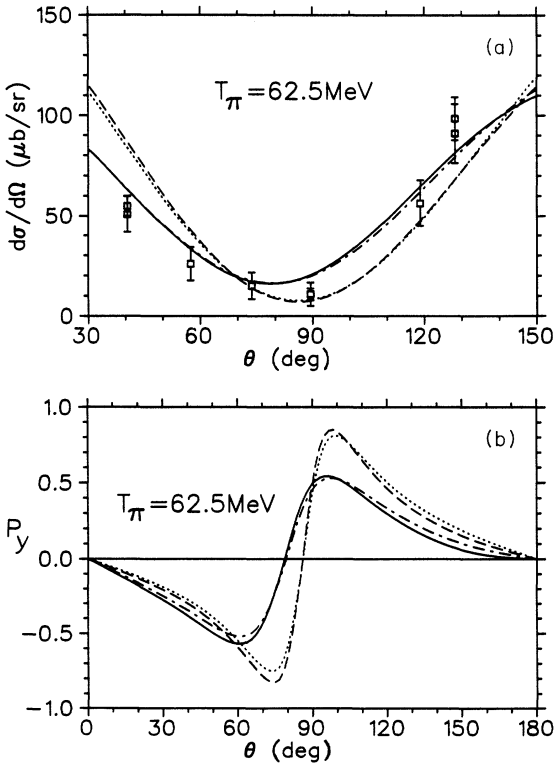

FIG. 4. Dependence of the angular distribution (a) and proton polarization (b) on the bound pair wave function. Solid curve: based on the correlation function of Ref. 29 as in previous figures. Dashed curve: pair wave function of Ref. 32 obtained by factorization of the relative coordinates from the Faddeev wave function. Dash-dotted curve: the Reid deuteron wave function modified for range as explained in the text. Dotted curve: the Reid deuteron wave function.

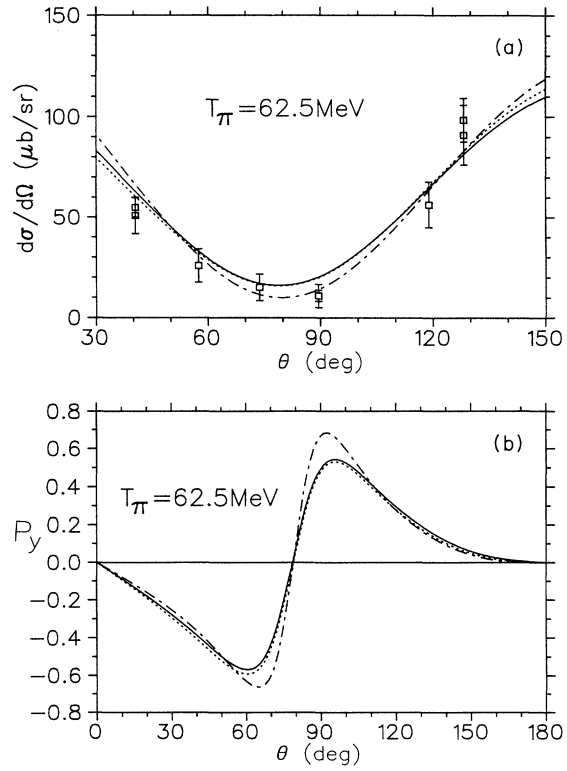

FIG. 6. Dependence of the angular distribution (a) and proton polarization (b) on the final-state interaction. Solid curve: the Reid potential. Dotted curve: the Reid potential supplemented in isospin one states by $N\Delta$ admixture. Dash-dotted curve: the Reid potential modified in the $J=1$ states to better produce the mixing parameter ϵ_1 of Ref. 35 as explained in the text.

Earlier work on π^+ absorption on a quasideuteron indicated strong sensitivity on the bound pair wave function.[21,23] The compression of the quasideuteron in ^{3}He had a significant effect on the polarization of the outcoming protons as well as on the analyzing power. Figure 4 shows the dependence of negative-pion absorption on the *pp* correlation function in ^{3}He. The solid curve is the same as above and is based on the correlation of Ref. 29 as used earlier in Ref. 21 for π^+ absorption. As mentioned before, the relative diproton wave function is taken to be simply the square root of the two-nucleon correlation function. This correlation is, in fact, calculated for $T=0$ pairs but also should be a fair description of the diproton. The dashed-dotted curve has been calculated using a wave function obtained by multiplying the *S*-wave part of the Reid deuteron wave function by $\exp(-0.27r$ fm$^{-1})$. This has similar asymptotic behavior as, e.g., the relative three-body wave-function parametrization of Hajduk, Green, and Sainio,[32] but lacks the node appearing there. It extends to shorter ranges, as well as it is somewhat longer ranged than the one based only on the correlation. By the dotted curve a much longer-ranged and unrealistic wave function is also depicted—the *S*-wave part of the Reid deuteron wave function. The two "realistic" wave functions both give mutually indistinguishable angular distributions, although the total cross section varies. For the deuteron wave function the correct asymmetry is lost.

In calculating the dashed curve a parametrization of the actual Faddeev three-body wave functions of Ref. 32 is used. This parametrization expresses the wave function as the sum of three different pairings, which are each a product of functions of only one relative coordinate as in Eq. (6). The pair wave function in this case has a node generated by the presence of the third particle also obtained in Ref. 33 (see Fig. 5). The final result is similar to the one using the deuteron wave function, with the correct asymmetry lost, and does not agree with the realistic wave functions. The reason is apparently the present use of just one term instead of all three as the wave function. In a truly consistent treatment all three should be considered also in a nondiagonal situation, i.e., the pion absorbing on a "wrong" pair in the coordinate wave function. This would be analogous to calculating the nodeless correlation function such as Ref. 29 from the Faddeev wave functions including nodes. As argued above, in the simplistic two-body approach the use of the correlation wave function may be more realistic. Clearly much more work remains to be done to extend the theory to incorporate the Faddeev wave functions in a consistent way.

Still another pair wave function can be used in an attempt to complete the comparison with the calculation of Maxwell and Cheung.[14] They employed a fit in terms of Gaussians due to Fearing.[34] This function—although decaying fast in the tail region—peaks half a Fermi further out than the others and also produces a wrong asymmetry in the angular distribution. As anticipated, the total cross section is $\sigma_{abs}=1.5$ mb, considerably less than the value 3.2 mb obtained in Ref. 17. This difference is an indication of the importance of the final-state correlations. Although the final-state interaction is different in Ref. 17 (one meson exchange) from the present Reid potential, variations of this interaction do not give such large effects either in Ref. 17 or below.

A problem with the present results is that they grossly overestimate the total cross section by a factor of 2–4. This is somewhat more than the factor of ≈ 2 or less for π^+ absorption.[21] With longer-ranged bound pair wave functions the cross section gets smaller as seen in Table III. However, as was seen above in Fig. 4, then the correct asymmetry may be lost, although the simple range-modified deuteron wave function works well. It is of great interest to check here also the effect of varying the final-state interactions, ΔN components as well as the $^3S_1$-$^3D_1$ mixing. For the standard Reid soft-core potential the mixing parameter at $E_{\rm lab}=390$ MeV is $\epsilon_1=9.2°$, whereas the phase-shift analysis of Arndt *et al.*[35] gives $\epsilon_1=5.7°$. Even this may be too high a value.[36] With the dominance of the $^3D_1$ final state one might expect that correcting this 50% overestimate of the tensor coupling would be crucial in getting a lower total cross section. The mixing parameter can be decreased simply by adjusting the *S*- and *D*-wave central potentials without actually touching the tensor part. The dash-dotted curve in Fig. 6 shows the result with a refitted Reid $^3S_1$-$^3D_1$ potential [adding an extra $-230\exp(-4\mu r)/\mu r$ MeV to V_C and $-200\exp(-4\mu r)/\mu r$ MeV to V_{LS}]. This modification gives $(\delta_S,\delta_D,\epsilon_1)=(-1.7°,-26.2°,5.6°)$, quite a satisfactory fit with the values $(-1.3°,-27.7°,5.7°)$ of Ref. 35 at this energy. The angular dependence does not change much, but disappointingly neither does the total cross section, as seen from Table IV. However, this modified interaction will be mainly used in the following energy-dependence studies. In the dotted curve the ΔN components have been incorporated by coupled channels in the isospin one final states to the "basic" model represented by the solid curves. Also this change has very little effect at this energy for the reasons discussed earlier. Naturally, the total cross section is not much affected by slight changes in the minor $T=1$ odd partial waves caused by isobar admixtures.

The energy dependence of the total cross sections in Fig. 7 shows much overestimation at low energies and a quite rapid decrease with increasing energy. This suggests that the reason for the overestimation may well be the neglect of pion distortions expected to be more important at low energies. Especially for the standard Reid potential as the final-state interaction (solid curve), this behavior is even quantitatively very similar to the $^3D_1$ contribution of Ref. 14. However, in that work higher pion partial waves (very small here) get important and give contributions of even 0.5 mb at high energies keeping the cross section as nearly a constant around 3 mb.

Figure 8 shows the full angular distribution with the modified (in $J=1$ states) Reid potential (dash-dotted curves) at four energies as compared with the experimental fits of Ref. 11 (solid curves). Again, at low energies the agreement is good, especially if one considers that all the data points are within the range 40°–140° making the forward and backward tails suspect, since the $P_3(\cos\theta)$

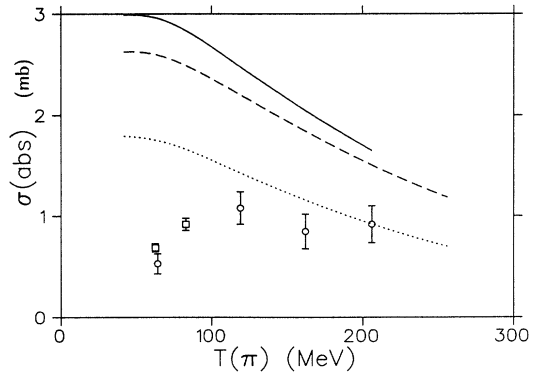

FIG. 7. Energy dependence of the total cross section. Solid curve: the Reid potential. Dashed curve: the Reid potential modified in the $J=1$ states. Dotted curve: as the dashed curve but with the range-modified deuteron wave function. The squares are the data of Ref. 9 and the circles of Ref. 11.

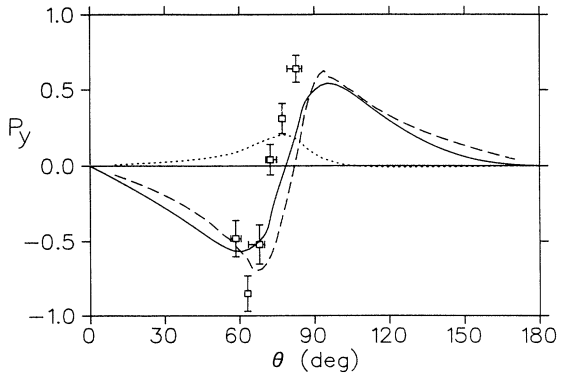

FIG. 9. Comparison of the proton polarization at 62.5 MeV calculated using the Reid potential (solid curve) with the analyzing power data in the reaction $\vec{p}n \to \pi^- pp(^1S_0)$. Also shown are predictions of solution 1 (dashed curve) and solution 2 (dotted curve) by Piasetzky *et al.* from an amplitude fit to the cross section.

term is not included in the fits. Outside this range the theoretical result becomes nearly symmetric due to about equal and opposite contributions from the $P_1(\cos\theta)$ and $P_3(\cos\theta)$ terms. Above about 100 MeV the theoretical results become somewhat forward peaked. Also experimental results get less and less asymmetric but much slower than theory. The theoretical prediction is slightly improved if one uses the modified deuteron wave function (dotted curves). A numerical comparison at various energies is given in Table V by means of ratios a_n/a_0. In comparing with experiment one should get an "effective" a_1 valid near 90° by subtracting the calculated a_3 from the calculated a_1. In that comparison even at 119 MeV the effective asymmetry is of the correct sign but far too small. At still higher energies even this agreement is lost. Even the inclusion of the isobar at high energies does not help and its effect is, in general, small. However, except for 206 MeV the ratio a_2/a_0 is quite well produced indicating that the pure isospin zero final-state contribution is reasonably well described in the model.

Along with the angular distributions of the cross section, Figs. 2–6 show also the polarizations of the proton for the same interactions and wave functions. It can immediately be seen that this observable is remarkably robust against changes in the model. So far there are no data for the polarization in this reaction, although some are soon to appear for positive-pion absorption from the TRIUMF experiment E455.[22] In that case a sensitivity on the pair wave function has been theoretically predicted.[21] However, there are some new data[24] on an analogous inverse two-body reaction $\vec{p}n \to \pi^- pp(^1S_0)$ at the proton laboratory energy 400 MeV. This is close enough to the "equivalent" two-nucleon energy 390 MeV in the quasifree absorption of 62-MeV pions to perform a meaningful comparison. This is done in Fig. 9 using the result of solid curve in Figs. 2–6. The agreement is surprisingly good considering that in the experiment the pair is in the continuum, whereas in the theory it is bound. Of course, this may be a reflection of the insensitivity of P_y on this wave function.

Also shown in Fig. 9 are the polarization predictions obtained by Piasetzky, Ashery, Moinester, Miller, and Gal[20] from fitting the cross section with three amplitudes. The present prediction excludes solution 2 (dotted curve), whereas solution 1 is very close to the model (dashed curve). A direct comparison of the amplitudes in Table II of this paper and Ref. 20 shows a qualitative but not fully quantitative similarity. However, the present result is even more dominated by the $^3D_1$ final state than Ref. 20. A comparison of the amplitudes also excludes all the five amplitude fits of Ref. 20, which are dominated by d-wave pion absorption, although one of them gives the correct P_y. The implications of Fig. 9 are contrary to the suggestion by Vigdor, Jacobs, and Korkmaz[37] that the $A(\vec{p},\pi^{\pm})A+1$ continuum analyzing powers would favor the choice of solution 2.

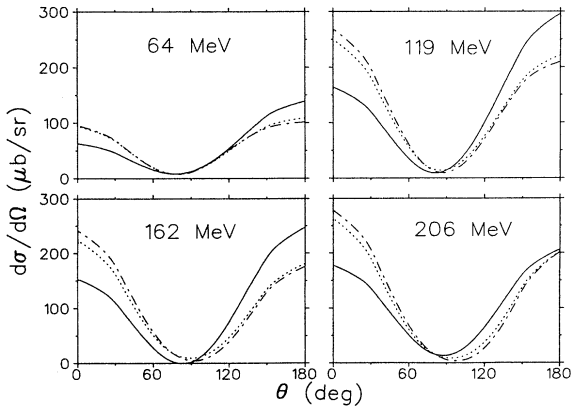

FIG. 8. Full angular distributions at four energies calculated using the modified Reid potential (dash-dotted curves) versus the experimental fits of Ref. 11 (solid curves). The dotted curves use the range-modified Reid deuteron wave function.

TABLE V. Relative angular distributions for some energies. Unless specifically mentioned these are calculated using the Reid potential with the $J=1$ modification as the final-state interaction and the bound pair wave function based on the correlation function as discussed in the text.

Energy/Model	a_1/a_0	a_2/a_0	a_3/a_0	a_4/a_0
42 MeV	−0.72	1.23	0.36	−0.07
Reid	−0.66	1.00	0.33	−0.07
62 MeV	−0.44	1.42	0.36	−0.07
Reid	−0.40	1.19	0.33	−0.08
Modified deuteron	−0.46	1.45	0.28	−0.03
Only $J=1$	0	1.53	0	0
Expt.[a]	−0.81	1.62		
Expt.[b] (64 MeV)	−0.90	1.40		
83 MeV	−0.22	1.58	0.33	−0.06
Expt.[a]	−0.79	1.68		
119 MeV	0.09	1.79	0.24	0.00
Expt.[b]	−0.77	1.68		
162 MeV	0.37	1.97	0.12	0.13
Expt.[b]	−0.69	1.94		
206 MeV	0.58	2.05	−0.00	0.27
Reid	0.55	1.92	−0.02	0.21
with $N\Delta$	0.71	1.93	−0.06	0.32
Modified deuteron	0.38	1.95	+0.06	0.25
Expt.[b]	−0.19	1.14		
256 MeV	0.74	2.01	−0.14	0.43

[a]Reference 9.
[b]Reference 11.

Further, Fig. 10 presents the polarization as a function of energy using the $J=1$ modified Reid potential. There is a slight systematic trend of the angular dependence of the polarization becoming sharper with increasing energy. However, qualitatively P_y remains similar over the range of energies.

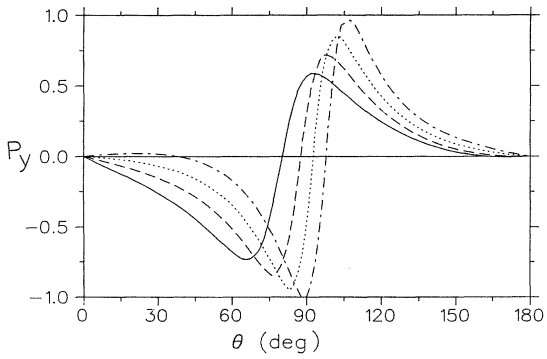

FIG. 10. The proton polarization at four energies using the modified Reid potential. Solid curve: 64 MeV. Dashed curve: 119 MeV. Dotted curve: 162 MeV. Dash-dotted curve: 206 MeV.

In the same way also the polarization correlations (or analyzing powers in the inverse process) depend only slightly on the model input or energy. The reason will be discussed below. Figure 11 shows the energy dependence of two nonzero observables, if the polarizations of both final-state particles are observed. The relations to other observables are given below. Of course, in an experiment it may be impractical to measure both the proton and neutron polarizations, but one should keep in mind also the possibilities of analogous inverse processes using, e.g., a polarized ^{3}He target as a "polarized neutron target" to obtain quasifree $\vec{p}\,\vec{n}\to\pi^-(pp)$ results.[38]

It is worth noting that the polarization is nearly antisymmetric about 90°. The reason for this is the dominance of one isospin state as for the near symmetry of the cross section. If absorption took place into a single isospin state, the polarization in this reaction would indeed, be antisymmetric and the general kinematic relation would reduce to

$$P_{0y}(\theta)=-P_{y0}(180°-\theta)=P_{y0}(\theta)\ . \tag{16}$$

A trivial example of this kind of reaction would be $pp\to\pi^0(pp)_{S\text{ wave}}$. Similar symmetries are rather closely fulfilled also by other polarization observables, correlations. Adhering to the Madison convention,[39] the polar-

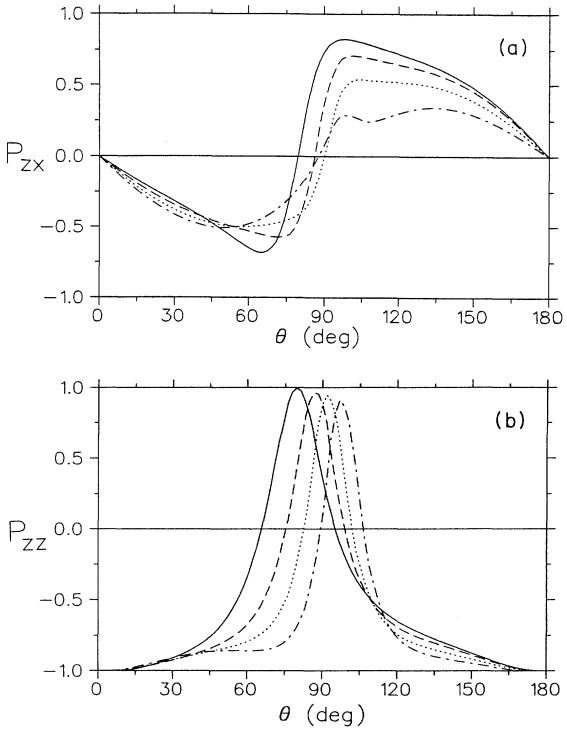

FIG. 11. The final-state polarization correlations P_{zx} (a) and P_{zz} (b) at four energies. The notation is as in Fig. 10.

ization correlations satisfy in negative-pion absorption on a diproton identically the relations

$$P_{yy}=1, \quad P_{zz}=-P_{xx}, \quad P_{zz}(0°)=P_{zz}(180°)=-1 \ . \tag{17}$$

If only one isospin state participated in the final state, P_{zz} would be symmetric and P_{zx} as well as P_y antisymmetric about 90°. Furthermore, for $T=0$ the quantity $P_{zz}(90°)$ would be identically $+1$, whereas for $T=1$ it would be -1. A deviation of $P_{zz}(90°)$ from unity can clearly be used as an indication of mixing of isospin states. Since the $^3S_1$-$^3D_1$ final states dominate, kinematics already sets some quite strong constraints on polarization correlations causing the relative model insensitivity.

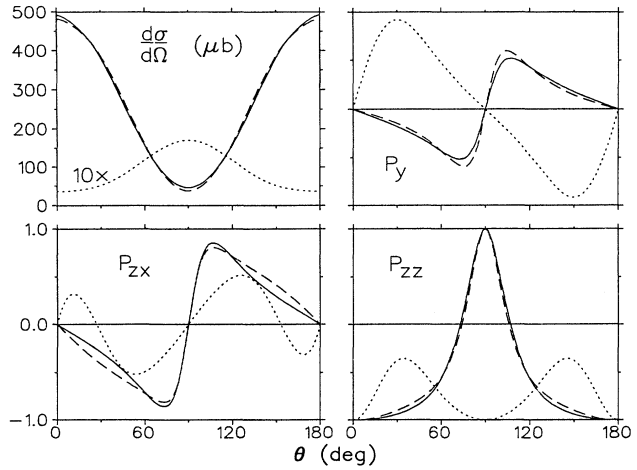

FIG. 12. The cross section and polarization observables at 62 MeV for pure isospin final states. Solid curve: the $^3S_1$-$^3D_2$ states only. Dashed curve: total $T=0$ result. Dotted curve $T=1$ states only (both $J=0$ and 2). The cross section of the $T=0$ states has been multiplied by 10 to make it distinguishable in the figure.

It may be of interest to see how the observables would look if only a single isospin in the final state is taken into account. A comparison of the $T=0$ and 1 states is performed in Fig. 12 at 62 MeV using the modified Reid potential. The dashed curves present the total $T=0$ result including the $J=1$ and 3 final states, whereas in the solid curves only the $J=1$ states are included. There is no significant effect from the addition of the higher partial wave, and it can be seen from Table V that the ratio a_2/a_0 is quite well produced with the $J=1$ contribution alone. In contrast, the dotted curves for only $T=1$ show a qualitative difference. However, in the final physical results their effect is not large because of the small amplitude reflected also in the small cross section.

IV. CONCLUSION

The angular distribution of negative-pion absorption on a diproton in ^{3}He can be theoretically understood at low energies, if a Galilean-invariant pion-nucleon absorption vertex is used. Galilean invariance appears to be the only way to achieve the experimentally observed slightly backward peaking. However, the early truncation of the Legendre expansion in fitting the data prohibits its direct comparison with theory. To check reliably the prediction of the model of a sizable component of form $P_3(\cos\theta)$ would require an extension of the angular range of experiments. The ratio a_2/a_0 was produced quite well over a wide range of energies, even if only the dominant $J=1$ states were included. The present calculation indicates a dominance of the final $^3D_1$ wave, by a factor of 2–5 over the $^3S_1$ state at the amplitude level, even more over the ^3P_J waves. This result could possibly be used to reduce ambiguities in amplitude analyses of the reaction. Furthermore, in the calculations it was established by an exact two-body treatment that the role of the Δ isobar is, indeed, negligible even at energies normally considered to be at the top of the resonance. Because of the smallness of the Δ effect one could speculate about the Roper resonance $N'(1440)$ with the nucleon quantum numbers. However, one would not expect much effect from this either in the odd partial waves causing the angular asymmetry. In the $^3S_1$ states it could be significant, in principle, but in practice changes in this amplitude do not alter the observables very much. Preliminary attempts to incorporate also the $N'(1440)$ did not give effects of any importance. However, one should acknowledge that the transition potential $NN \rightarrow NN'(1440)$ is very poorly known and uncertain at best.

In spite of the success with the angular distribution, the total cross sections are overestimated by a factor of 2–4, depending on the final-state interaction and the bound pair wave function. The lower value can be ob-

tained using a pair wave function obtained by simply changing the range of the deuteron wave function and is about the same as similar overestimation in positive-pion absorption.[21] This wave function also gives a slightly better—though not completely satisfactory—angular distribution at high energies, where the experimental asymmetry of the cross sections is lost. The trend of the angular distribution as a function of energy is, however, similar to experiment. A plausible reason for the overestimation of the cross section is the neglect of the incident pion distortions. Another possibility could be the need for a more exact treatment of the ^{3}He wave function with multiterm Faddeev wave functions.

The polarization observables of the final-state nucleons have also been calculated. This is clearly a necessity if any sensible amplitude analyses are to be made of the reaction. The present result for P_y appears to exclude most of the solutions obtained in Ref. 20 and also agrees well with the analyzing power data in the reaction $\bar{p}n \to \pi^- pp(^1S_0)$ of Ponting *et al.*[24] At present there are no data for P_y in π^- absorption. Although the present results suggest that the polarization is very insensitive to the model input (contrary to the case with π^+ absorption on a quasideuteron), this observable has its interest. If data on this in actual negative-pion absorption on ^{3}He would clearly disagree with theory, it would be a strong indication that the model is basically missing something. Because of the insensitivity there would be no apparent resort to slight changes in interaction parameters and wave functions. Rather a serious discrepancy could mean, e.g., the need for a more exact treatment of the three-body bound state on the basis of the Faddeev formalism and that the present simplified correlation-based approach is inadequate in quasifree absorption—a question already raised above in the context of the overestimation of the total cross section. This need does not arise in the two-nucleon reactions and, therefore, the agreement of the theoretical P_y with the existing data does not *necessarily* imply knowledge about the three-body bound state. If, on the other hand, the present simple quasi-two-body theory is valid for ^{3}He, then it can find applications in absorption on more complex nuclei.

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