Isospin and deformation splittings of the giant dipole resonance for triaxial nuclei

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We study the different mechanisms for the fragmentation of the giant dipole resonance in medium-mass nuclei with triaxial shape, by resorting to group-theoretical techniques. Coupling between low- and high-lying collective modes is considered, together with isospin effects, within the framework of the interacting boson model. An application to titanium isotopes is presented.

It is well known that photon-absorption cross sections for deformed nuclei show, in the energy region of the giant dipole resonance (GDR), a typical fragmentation pattern due to the coupling between low-lying and high-lying collective modes. Moreover, for light- and medium-mass nuclei, where valence neutrons and protons fill the same major shells, a further splitting of the GDR strength originates from isospin effects, owing to the isovector character of the giant dipole excitation.

An algebraic approach,[1-3] as developed in the last few years within the framework of the interacting boson model (IBM),[4] can be useful in elucidating the main mechanisms underlying the GDR fragmentation pattern over a broad nuclear-mass region, where complete shell-model calculations involving 1p-1h excitations across major shell closures are unfeasible. In some cases, when dynamical symmetries arise, group-theoretical techniques allow us to obtain analytic formulas for the GDR splitting, which are particularly suitable to this kind of investigation. Closed-form results have been previously derived for axially symmetric and triaxially deformed nuclei,[1,2] described by SU(3) and SU(3)* IBM dynamical symmetries,[4] respectively. The isospin contribution has been considered for *s-d* shell nuclei in Ref. 5, in the IBM-3 extension[4,6] of the model, which takes explicitly into account neutron and proton degrees of freedom.

In this Brief Report we work out analytic expressions to deal with both deformation and isospin splittings of GDR in triaxial nuclei. This result represents a matter of experimental interest, since it has been recently suggested[7] that nuclei in the *f-p* shell could exhibit a change of symmetry from a deformed axisymmetric to a triaxial shape.

We first consider GDR deformation splitting and recall some results obtained in Ref. 1. In IBM language, the giant dipole excitation is represented by a p boson, with $J^\pi=1^-$, which belongs to the (1,0) irreducible representation (irrep) of group SU(3). It strongly interacts with the usual s and d bosons[4] describing low-lying collective modes, mainly through a quadrupole-quadrupole force.[1-3] The IBM-2 ground-state band of a triaxial nucleus belongs to the $(2N_\nu,2N_\pi)$ irrep of SU(3)*,[4] where N_ν (N_π) is the effective number of neutron (proton) bosons and $N_\nu \geq N_\pi \neq 0$. The coupling between low-lying and GDR states is then given by the following product representation and decomposition:[1]

$$(2N_\nu,2N_\pi)\otimes(1,0)=(2N_\nu+1,2N_\pi)\oplus(2N_\nu-1,2N_\pi+1)\oplus(2N_\nu,2N_\pi-1)\ . \tag{1}$$

Therefore, GDR is split into three components; their eigenvalues can be obtained explicitly[1,2] by exact diagonalization of the following Hamiltonian, considering the associated three-level mixing problem:

$$H=H_{sd}+\epsilon_p\hat{n}_p+b_2(\hat{Q}_\nu^{(2)}+\hat{Q}_\pi^{(2)})\cdot[p^+\times\tilde{p}]^{(2)}\ . \tag{2}$$

Here, H_{sd} is the usual *s-d* boson Hamiltonian[4] in the SU(3)* limit of IBM, the second term in the right-hand side (rhs) is the unperturbed p-boson energy, and the last one is the coupling term given by a quadrupole-quadrupole interaction.[2,3]

The dipole transition operator $\hat{D}^{(1)}$ can be factorized in a boson $\hat{D}_B^{(1)}$ and an isospin part $\hat{D}_T^{(1)}$, which act on boson and isospin spaces, respectively. It is thus possible to evaluate dipole transition strengths between low-lying and GDR states. Boson matrix elements are proportional to the reduced Wigner coefficients[8] of the $SU_B(3)\supset SO_B(3)$ decomposition and are given by

$$\begin{aligned}
M_1&=\langle L_i^+\|\hat{D}_B^{(1)}\|1_{1B}^-\rangle=D'\langle(2N_\nu,2N_\pi),L_i;(1,0),1\|(2N_\nu+1,2N_\pi),1\rangle\ ,\\
M_2&=\langle L_i^+\|\hat{D}_B^{(1)}\|1_{2B}^-\rangle=D'\langle(2N_\nu,2N_\pi),L_i;(1,0),1\|(2N_\nu-1,2N_\pi+1),1\rangle\ ,\\
M_3&=\langle L_i^+\|\hat{D}_B^{(1)}\|1_{3B}^-\rangle=D'\langle(2N_\nu,2N_\pi),L_i;(1,0),1\|(2N_\nu,2N_\pi-1),1\rangle\ ,
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
D'&=\langle(2N_\nu+1,2N_\pi)\|\hat{D}_B^{(1)}\|(2N_\nu,2N_\pi)\rangle\\
&=\langle(2N_\nu-1,2N_\pi+1)\|\hat{D}_B^{(1)}\|(2N_\nu,2N_\pi)\rangle\\
&=\langle(2N_\nu,2N_\pi-1)\|\hat{D}_B^{(1)}\|(2N_\nu,2N_\pi)\rangle\ .
\end{aligned} \tag{4}$$

L_i is the total spin of low-lying states, and the states $|1_{1B}^-\rangle$, $|1_{2B}^-\rangle$, $|1_{3B}^-\rangle$ belong to the three irreps in the rhs of formula (1).

We now introduce isospin degree of freedom by resorting to the third version of the model (IBM-3).[6] In IBM-3, wave functions of low-lying states are defined by the

product of a boson and an isospin part. Therefore, the usual s and d bosons carry one isospin unit with the third component equal to $+1$ (neutron-neutron pair), 0 (neutron-proton pair), and -1 (proton-proton pair). Thus in isospin space the ground-state band belongs to the (N_ν,N_π)irrep of $SU_T(3)$, since the corresponding boson irrep is $(2N_\nu,2N_\pi)$ and two irreps must have the same symmetry properties, while p boson still belongs to the (1,0) irrep, since the relevant electric dipole excitation has, obviously, isovector character and third component equal to zero. These choices for isospin representations are imposed by the requirement that the nuclear wave function is totally symmetric in the Fock space, due to its bosonic character. The isospin states are then defined in terms of the following decomposition:

$$(N_\nu,N_\pi)\otimes(1,0)=(N_\nu+1,N_\pi)\oplus(N_\nu-1,N_\pi+1)\oplus(N_\nu,N_\pi-1)\ . \tag{5}$$

Each unitary irrep contains two possible components with isospin $T=T_0$ and $T=T_0+1$, respectively, where T_0 is the isospin value of the nuclear ground state. Therefore, the three states arising from deformation splitting are further fragmented into three components; on the whole, nine GDR states result.

It is possible to obtain analytic expressions for the energy splitting of the three states given by formula (5), likewise to the procedure adopted for the boson components. Expressing the total Hamiltonian in terms of quadratic Casimir operators of SU(3), the eigenvalue problem can be easily solved in a closed form. The energy splittings between the first and second state and the first and third one are found to be, respectively,

$$\begin{aligned}\Delta E_{12}&=a\Delta C_2^{12}[\mathrm{SU}(3)]=2a(N_\nu+1)\ ,\\ \Delta E_{13}&=a\Delta C_2^{13}[\mathrm{SU}(3)]=2a(N_\nu+N_\pi+2)\ .\end{aligned} \tag{6}$$

Here, $C_2[\mathrm{SU}(3)]$ are the eigenvalues of quadratic Casimir operators of SU(3):

$$C_2[\mathrm{SU}(3)]=\tfrac{2}{3}[\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)]\ , \tag{7}$$

where (λ,μ) labels SU(3) irreps in Elliott's notation, adopted in this paper, and a in Eq. (6) is an adjustable parameter.

Dipole transition strengths for excitation of GDR states are found by evaluating $\hat{D}^{(1)}_T$ matrix elements, in addition to the boson strengths of Eq. (3):

$$\begin{aligned}N_1&=\langle L_i^+\|\hat{D}^{(1)}_T\|1^-_{1T}\rangle\\ &=D''\langle(N_\nu,N_\pi),T_0;(1,0),1\|(N_\nu+1,N_\pi),T\rangle\langle T_0,T_0;1,0|T,T_0\rangle\ ,\\ N_2&=\langle L_i^+\|\hat{D}^{(1)}_T\|1^-_{2T}\rangle\\ &=D''\langle(N_\nu,N_\pi),T_0;(1,0),1\|(N_\nu-1,N_\pi+1),T\rangle\langle T_0,T_0;1,0|T,T_0\rangle\ ,\\ N_3&=\langle L_i^+\|\hat{D}^{(1)}_T\|1^-_{3T}\rangle\\ &=D''\langle(N_\nu,N_\pi),T_0;(1,0),1\|(N_\nu,N_\pi-1),T\rangle\langle T_0,T_0;1,0|T,T_0\rangle\ ,\end{aligned} \tag{8}$$

where

$$\begin{aligned}D''&=\langle(N_\nu+1,N_\pi)\|\hat{D}^{(1)}_T\|(N_\nu,N_\pi)\rangle\\ &=\langle(N_\nu-1,N_\pi+1)\|\hat{D}^{(1)}_T\|(N_\nu,N_\pi)\rangle\\ &=\langle(N_\nu,N_\pi-1)\|\hat{D}^{(1)}_T\|(N_\nu,N_\pi)\rangle\ .\end{aligned} \tag{9}$$

T_0 and T are, respectively, isospin values of ground state and GDR components ($T=T_0$ or T_0+1) and the $|1^-_{1T}\rangle$, $|1^-_{2T}\rangle$, $|1^-_{3T}\rangle$ states are given by decomposition (5).

The total transition strength for each possible GDR state can then be obtained taking the product of all $\hat{D}^{(1)}_B$ matrix elements (M_1,M_2,M_3) with each $\hat{D}^{(1)}_T$ matrix element (N_1,N_2,N_3). In this way, deformation and isospin splittings of GDR are both taken into account.

Introducing the notation

$$|(\lambda_B,\mu_B);(\lambda_T,\mu_T),T,M_T\rangle\ , \tag{10}$$

the possible GDR states are defined as follows:

$$\begin{aligned}&|1^-_{11}\rangle=|(2N_\nu+1,2N_\pi);(N_\nu+1,N_\pi),T,T_0\rangle\ ,\quad |1^-_{12}\rangle=|(2N_\nu+1,2N_\pi);(N_\nu-1,N_\pi+1),T,T_0\rangle\ ,\\ &|1^-_{13}\rangle=|(2N_\nu+1,2N_\pi);(N_\nu,N_\pi-1),T,T_0\rangle\ ,\quad |1^-_{21}\rangle=|(2N_\nu-1,2N_\pi+1);(N_\nu+1,N_\pi),T,T_0\rangle\ ,\\ &|1^-_{22}\rangle=|(2N_\nu-1,2N_\pi+1);(N_\nu-1,N_\pi+1),T,T_0\rangle\ ,\quad |1^-_{23}\rangle=|(2N_\nu-1,2N_\pi+1);(N_\nu,N_\pi-1),T,T_0\rangle\ ,\\ &|1^-_{31}\rangle=|(2N_\nu,2N_\pi-1);(N_\nu+1,N_\pi),T,T_0\rangle\ ,\quad |1^-_{32}\rangle=|(2N_\nu,2N_\pi-1);(N_\nu-1,N_\pi+1),T,T_0\rangle\ ,\\ &|1^-_{33}\rangle=|(2N_\nu,2N_\pi-1);(N_\nu,N_\pi-1),T,T_0\rangle\ .\end{aligned} \tag{11}$$

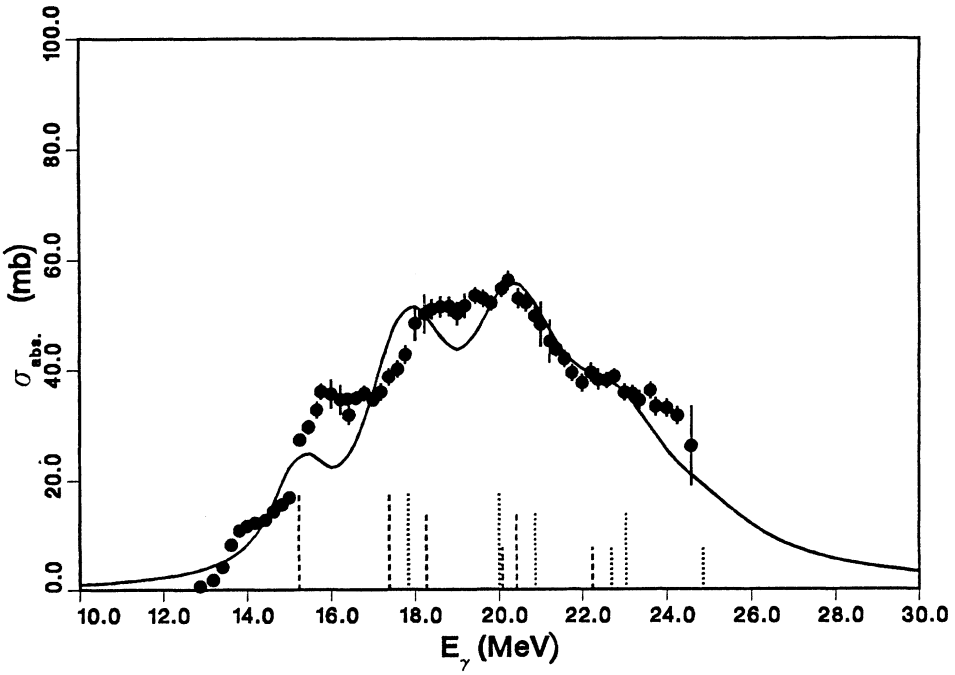

FIG. 1. Experimental (Ref. 12) and calculated $[SU_B(3)^* \otimes SU_T(2)]$ photoabsorption cross section of ^{46}Ti. Bars at the bottom represent calculated dipole strengths (in arbitrary units); dashed line, $T=1$ components; dotted line, $T=2$ components.

Moreover, the dipole transition strengths between these states and the low-energy ones are

$$S_{ijk} = |\langle L_i^+ \| \hat{D}^{(1)} \| 1_{jk}^- \rangle|^2 = |\langle L_i^+ \| \hat{D}_B^{(1)} \| 1_{jB}^- \rangle \langle L_i^+ \| \hat{D}_T^{(1)} \| 1_{kT}^- \rangle|^2 , \quad (12)$$

with $j,k=1$, 2, and 3. The two constants $\mathbf{D}'$ and $\mathbf{D}''$ appearing in the $\hat{D}_B^{(1)}$ and $\hat{D}_T^{(1)}$ matrix elements can be merged into only one constant D_0 in the definition of S_{ijk}.

The Hamiltonian can be diagonalized in the SU(3)* basis (5) only if adiabatic approximation holds, that is if the energy spacing of states with different T values belonging to the same SU(3)* irrep is small with respect to the energy splitting between GDR states with $T=T_0$ and $T=T_0+1$. The first quantity is connected with the symmetry term in the Weiszacker mass formula:[9]

$$\Delta E(T,T_0) = \left[\frac{134-238A^{-1/3}}{A} \right] [T(T+1)-T_0(T_0+1)] \text{ MeV} , \quad (13)$$

while the second one is given by[10]

$$\Delta E = \frac{60(T_0+1)}{A} . \quad (14)$$

If adiabatic approximation does not hold, the isospin coupling has to be introduced at the $SU_T(2)$ level and, therefore, the isospin Hamiltonian is expressed in terms of quadratic Casimir operators of SU(2) rather than SU(3), whose eigenvalues are

$$C_2[SU(2)] = J(J+1) , \quad (15)$$

with J label of SU(2) irreps.

In this situation, there are only two GDR states arising from isospin coupling for given isospin value, whose energy spacing is given by

$$\Delta E_{23} = b\Delta C_2^{23}[SU(2)] = 2b(T_0+1) . \quad (16)$$

Therefore, only six GDR states persist altogether for each isospin component (respectively, T_0 and T_0+1). Dipole transition strengths are still given by Eq. (12), but now with $k=2,3$ for each j value and only Clebsch-Gordan coefficients make different matrix elements (N_1,N_2,N_3). It is worth noticing that the analytic ex-

TABLE I. Interacting boson model parameters.

Nucleus	^{46}Ti	^{48}Ti
N_ν	2	3
N_π	1	1
ϵ_p $(T=T_0)$ (MeV)	17.25	17.00
b_2 (keV)	0.24	0.24
b (keV)	0.27	0.38
D_0 (fm)	3.6	4.3
Γ_0 (MeV$^{-1/2}$)	0.026	0.015

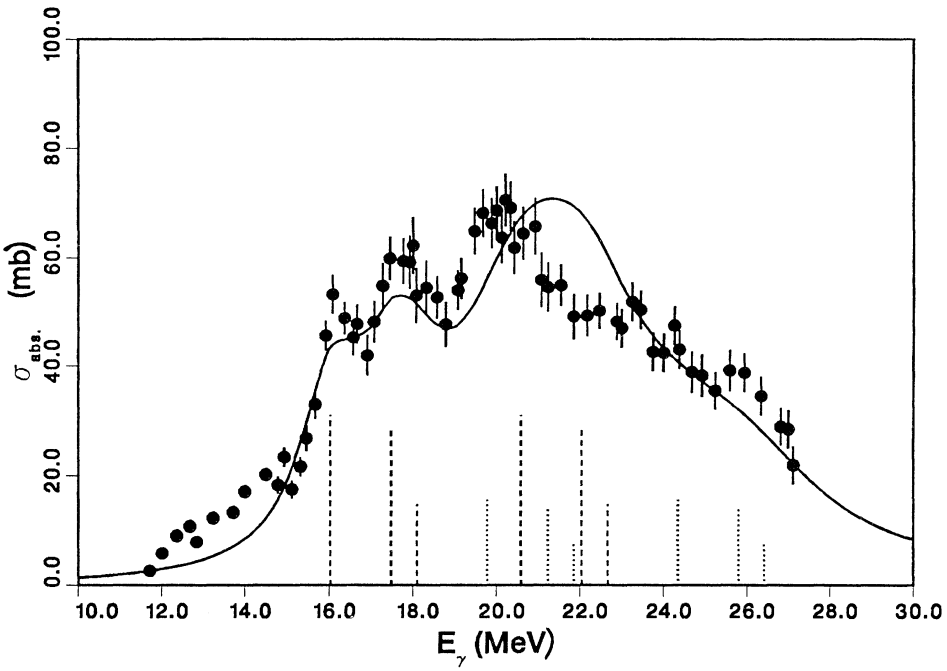

FIG. 2. Experimental (Ref. 13) and calculated $[SU_B(3)^* \otimes SU_T(2)]$ photoabsorption cross section of ^{48}Ti. Bars at the bottom represent calculated dipole strengths (in arbitrary units); dashed line, $T=2$ components; dotted line, $T=3$ components.

pressions derived above are totally independent of any microscopic interpretation of the model and are thus quite general.

They can be applied, in principle, to any (nonmagic) even-even nucleus with triaxial shape through the periodic table. As an example of possible applications of the above formalism, in addition to the intrinsic interest, we present the results obtained for photoabsorption cross sections of ^{46}Ti and ^{48}Ti (Figs. 1 and 2), postponing a deeper discussion of the adopted Hamiltonian and the coupling between low- and high-energy degrees of freedom to a forthcoming paper, actually in preparation.

For these nuclei, adiabatic approximation does not hold and, therefore, the GDR isospin coupling has to be introduced at the $SU_T(2)$ level. Photoabsorption cross sections have been calculated by means of standard formulas,[11] associating to each dipole strength of the six GDR states an intrinsic width given by a phenomenological power law,[11] $\Gamma(E)=\Gamma_0 E^{3/2}$ MeV. The parameters adopted in the present calculations are listed in Table I.

The results are in quite good agreement with experimental data and they do not seem to be incompatible with the triaxiality hypothesis, proposed for heavier nuclei in this mass region.[7] Further calculations for titanium isotopes, as regards elastic and inelastic photon-scattering cross sections, together with a detailed discussion of the fit to low-energy spectra will be presented in a forthcoming publication.

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