Meson exchange current effects on magnetic dipole moments of *p*-shell nuclei

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It is shown that addition of a two-body magnetic dipole operator arising from the exchange of the isovector pion and rho meson to the well-known one-body operator can give important corrections to the magnetic dipole moments of the $A=4-16$ nuclei. We performed shell-model calculations in complete $0\hbar\omega$ and $(0+2)\hbar\omega$ model spaces, thus investigating simultaneously the effects of extension of the model space and meson exchange currents on the magnetic moments. In the enlarged model space a significant improvement on the description of the magnetic moments is obtained by including exchange currents.

I. INTRODUCTION

Over the last decades it has become increasingly interesting to incorporate non-nucleonic degrees of freedom in the description of nuclei. Several electromagnetic (and weak) properties of nuclei cannot be explained satisfactorily in the impulse approximation (IA), where the assumption is made that the photons (and leptons) interact with the individual nucleons inside a nucleus one at a time. However, the interaction between the nucleons, through the exchange of mesons, has not been accounted for.

Historically, soon after Yukawa postulated the idea that the nucleons within a nucleus are surrounded by a cloud of virtual mesons,[1] it was recognized that these additional degrees of freedom, further on referred to as meson exchange currents (MEC), might give important two-body corrections to the one-body magnetic moment operator used in the impulse approximation. The first attempts to explain the discrepancies between observed magnetic moments and the one-body predictions in terms of exchange current effects were made by Villars[2] and Miyazawa.[3]

One of the almost classical examples to prove the presence of meson exchange currents is provided by the isovector magnetic moment of the trinucleon system. Several authors, using different approaches, were able to explain the discrepancy between the experimental value and the theoretical value obtained in the impulse approximation, by including exchange currents.[4–7] Most convincing is the recent work of Schiavilla, Pandharipande, and Riska,[7] who constructed the exchange current operators directly from the nucleon-nucleon interaction model used to determine the wave functions, thus treating wave functions and exchange current operators in a consistent gauge-invariant way, which is required by the equation of continuity.

Shell-model calculations of corrections to magnetic moments of nuclei were mostly restricted to those situations in which there is one valence nucleon (or hole) outside a closed-shell core.[8,9] For those simple cases very detailed calculations can be performed, examining simultaneously the effects of nucleonic excitations via second-order core polarization and meson effects. Corrections to the single-particle Schmidt values are usually expressed in terms of effective one-body g factors. Dubach, Koch, and Donnelly[10] studied the effects of pion exchange currents in electron scattering from several p-shell nuclei using the wave functions obtained from the work of Cohen and Kurath.[11] The latter authors performed the first systematic shell-model calculations on p-shell nuclei in the smallest possible model space, i.e., the $0\hbar\omega$ model space. However, Towner and Khanna,[12] in their study on magnetic moments of nuclei with an LS-closed core plus or minus one nucleon, showed that effects of core polarization can be quite large and in most cases are of opposite sign compared to exchange current effects. Therefore, it is most desirable, when studying exchange current effects in a shell-model description of p-shell nuclei, to allow admixtures of at least the $2\hbar\omega$ configurations into the wave functions. With present day computer facilities, shell-model calculations on p-shell nuclei in a complete $(0+2)\hbar\omega$ model space, which require the construction and diagonalization of large matrices (dimensions up to 1897), have become feasible. The first systematic shell-model study on p-shell nuclei in this expanded model space has been published recently by Wolters, Van Hees, and Glaudemans.[13] They observed a significant improvement on the description of electric quadrupole moments as a result of the extension of the model space, i.e., the existing discrepancies between experimental and theoretical quadrupole moments in the small $0\hbar\omega$ model space can, to a certain amount, be explained by taking into account nucleonic excitations. However, this does not hold for magnetic dipole moments. Quite large effective isovector g factors had to be introduced in the $(0+2)\hbar\omega$ model-space calculation to obtain a reasonable description of the magnetic dipole moments, whereas the effective isoscalar g factors remained quite close to the bare-nucleon values. This may indicate that one has to include exchange current effects, which are predominantly of isovector nature. Therefore, we investigate exchange current effects on the magnetic moments of p-shell nuclei described in a complete $(0+2)\hbar\omega$ harmonic-oscillator shell-model space, using the interaction of Ref. 13.

Our model for the exchange current operators is based on simple one-pion and one-rho-meson exchange processes. Thus consistency between the nucleon-nucleon interaction used to construct the nuclear wave functions and the exchange current operators is achieved only in an approximate way. However, though a shell-model description of nuclei may be quite crude with regard to the model used for the exchange currents, we believe that the same qualitative features of exchange currents should be observed as obtained in more elaborate exact calculations. This assumption is justified by a simple shell-model calculation of exchange current effects on the trinucleon magnetic moments as will be discussed in Sec. IV.

In Sec. II we will shortly recall the effective nucleon-nucleon interaction model used in Ref. 13 for the calculations in the $0\hbar\omega$ and $(0+2)\hbar\omega$ shell-model spaces. In Sec. III we briefly describe the magnetic moment operator. In Sec. IV the results will be presented and finally in Sec. V a short summary will be given and conclusions will be drawn.

II. MODEL SPACES AND EFFECTIVE INTERACTIONS

Shell-model calculations for the $A=4$–16 nuclei are performed in complete $0\hbar\omega$ and $(0+2)\hbar\omega$ model spaces. For the $0\hbar\omega$ model space only $(0s)^4(0p)^{A-4}$ configurations are allowed. In the enlarged model space all admixtures of $2\hbar\omega$ components are taken into account. For these configurations we can distinguish two types: the 1p1h excitations,

$$(0s)^3(0p)^{A-4}(1s0d)^1 ,$$

$$(0s)^4(0p)^{A-5}(0f1p)^1 \quad (A \geq 5) ,$$

and the 2p2h excitations,

$$(0s)^2(0p)^{A-2} \quad (A \leq 14) ,$$

$$(0s)^4(0p)^{A-6}(1s0d)^2 \quad (A \geq 6) .$$

In both model spaces all nucleons are active, i.e., there is no inert core, although the nucleons may of course form a closed shell. It means that exchange effects between all nucleon pairs can be taken into account. This is unlike the calculations of meson exchange current effects on magnetic moments of closed-shell plus- (or minus-) one nuclei, where only exchange effects between the valence nucleon (hole) and a core nucleon are considered. Note that for the $0\hbar\omega$ calculations on the $A=4$–16 nuclei, where the four $(0s)$ nucleons form a closed shell, the exchange effects among these four nucleons average out to zero.

The effective two-body interaction used is translationally invariant and isospin conserving. It is separated into central spin-singlet, central spin-triplet, spin-orbit, and tensor components. The radial dependencies of these components are not specified and all two-body matrix elements, which enter the calculations when constructing the Hamiltonian matrix, can be written as a linear combination of talmi integrals,[14] which are in fact weighted integrals over the radial parts of the interaction. In the $0\hbar\omega$ model space only 13 talmi integrals can give nonvanishing contributions to the two-body matrix elements. Together with the harmonic-oscillator energy spacing $\hbar\omega$ these talmi integrals are considered as free parameters. This procedure of parametrizing the effective Hamiltonian is outlined in some detail in Ref. 15. All 14 parameters have been optimized simultaneously by means of a least-squares fitting routine using a data set containing 76 experimental binding energies of ground states and excited states of p-shell nuclei, from which the Coulomb energy is removed. Only normal-parity levels, i.e., $\pi=(-1)^A$, can be treated in this model space.

In the $(0+2)\hbar\omega$ model space the same 14 parameters are adjusted to the same data set. Parameter values can be found in Ref. 13. The improvement gained by expanding the model space is reflected in the rms deviations between calculated energy levels and experimental values. In the small $0\hbar\omega$ model space a rms deviation of 0.80 MeV was found, whereas in the $(0+2)\hbar\omega$ model space the smaller value of 0.71 MeV was obtained.[13] A much more significant improvement was achieved for the electric quadrupole moments. In the small model space the average absolute deviation between theoretical and all experimentally known quadrupole moments of p-shell nuclei was found to be 1.45 $e\,\mathrm{fm}^2$, whereas in the expanded model space this value drops to 0.85 $e\,\mathrm{fm}^2$ (assuming no effective charges).[13] Note that in the limit of static nucleons there is no exchange correction to the electric quadrupole moments due to Siegert's theorem.[16]

III. DEFINITION OF THE MAGNETIC MOMENT OPERATOR

The magnetic moment operator for a nucleus consisting of A nucleons may be written as the sum of a one-body and a two-body part,

$$\boldsymbol{\mu}=\sum_{k=1}^{A}\boldsymbol{\mu}_k+\sum_{k<l}^{A}\boldsymbol{\mu}_{kl} . \tag{1}$$

The one-body operator $\boldsymbol{\mu}_k$ is the usual single-particle magnetic moment operator for nucleon k given by (in units of nuclear magnetons)

$$\boldsymbol{\mu}_k=g_k^l\boldsymbol{l}_k+g_k^s\mathbf{s}_k , \tag{2}$$

where g_k^l and g_k^s are the orbital and spin g factors of nucleon k. It is convenient to split this operator into an isoscalar and isovector part using the isospin convention $\tau_z=-1$ for a proton and $\tau_z=+1$ for a neutron,

$$\begin{aligned}\boldsymbol{\mu}_k&=(g_0^l\boldsymbol{l}_k+g_0^s\mathbf{s}_k)-\tau_{kz}(g_1^l\boldsymbol{l}_k+g_1^s\mathbf{s}_k)\\&\equiv\boldsymbol{\mu}_k^S-\boldsymbol{\mu}_k^V .\end{aligned} \tag{3}$$

Here we introduced isoscalar and isovector g factors

$$g_0=\tfrac{1}{2}(g_p+g_n)\ ,$$
$$g_1=\tfrac{1}{2}(g_p-g_n)\ . \tag{4}$$

The free-nucleon values are $g_0^l=g_1^l=0.5$, $g_0^s=0.880$, and $g_1^s=4.706$.

For the two-body operator $\boldsymbol{\mu}_{kl}$ we consider two types of exchange currents: (i) those associated with the static components of the isospin-dependent single-pion and single-rho-meson exchange potentials and (ii) exchange currents due to the processes where one of the nucleons is excited to the intermediate Δ_{33} resonance and a pion or rho meson is exchanged (isobaric currents). The former type of exchange currents are called model independent, since they are directly related, through the continuity equation, to the nucleon-nucleon interaction, whereas the latter type of exchange currents are called model dependent, since they are purely transverse and thus are not constrained by the nucleon-nucleon potential. All exchange mechanisms mentioned are of isovector nature in the static-nucleon limit. They are depicted in Fig. 1. The exchange current due to the pion (rho-meson) exchange interaction is a sum of the "seagull" current [diagram (a)] and the "mesonic" current [diagram (b)].

The isovector two-body magnetic moment operator $\boldsymbol{\mu}_{12}^V$, derived from the exchange currents considered, is separated into a term depending only on the relative coordinate $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ of the two particles, and a term depending on the center-of-mass coordinate $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$ as well. The former part, which we denote by $\boldsymbol{\mu}_{12}^V(\text{rel})$, can be written in the following form in the coordinate space, using the general classification of Chemtob and Rho:[4]

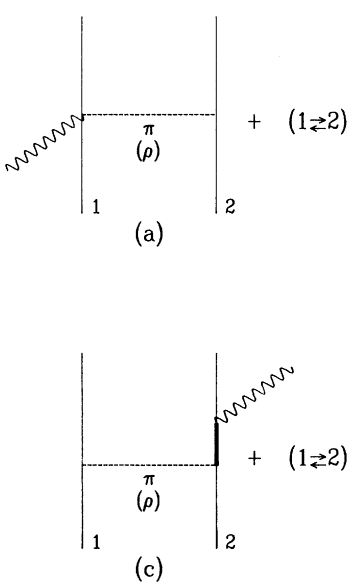

FIG. 1. Feynman diagrams representing the exchange currents included in our calculations: (a) the seagull, (b) the mesonic, and (c) the isobaric currents.

$$\begin{aligned}\boldsymbol{\mu}_{12}^V(\text{rel})=\tfrac{1}{2}G\{&(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z[g_I(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)+g_{\rm II}T_{12}^{\times}]\\ &+(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)_z[h_I(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)+h_{\rm II}T_{12}^{-}]\\ &+(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)_z[j_I(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)+j_{\rm II}T_{12}^{+}]\}\ ,\end{aligned} \tag{5}$$

where $G=e\hbar/2Mc=\mu_N$ is the nuclear magneton, M being the nucleon mass. The operators $T_{12}^{\odot}$ are given by

$$\begin{aligned}T_{12}^{\odot}&=(\boldsymbol{\sigma}_1\odot\boldsymbol{\sigma}_2)\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}-\tfrac{1}{3}(\boldsymbol{\sigma}_1\odot\boldsymbol{\sigma}_2)\\ &=-\tfrac{1}{3}\sqrt{8\pi}[Y_2(\hat{\mathbf{r}})\times(\boldsymbol{\sigma}_1\odot\boldsymbol{\sigma}_2)]^{(1)},\quad \odot=\times,\pm\ .\end{aligned} \tag{6}$$

For the center-of-mass dependent part of the exchange magnetic moment operator, mostly referred to as the Sachs magnetic moment operator, we write, using the notation of Hyuga, Arima, and Shimizu,[17]

$$\begin{aligned}\boldsymbol{\mu}_{12}^V(\text{Sachs})=G(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z(\hat{\mathbf{r}}\times\hat{\mathbf{R}})[&F_{\rm I}+F_{\rm II}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\\ &+F_{\rm III}S_{12}]\ ,\end{aligned} \tag{7}$$

where $S_{12}=3(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ is the tensor operator. The real scalar functions $g_{\rm I},\ldots,j_{\rm II}$ in Eq. (5) depend only on the relative coordinate $r=|\mathbf{r}|$, whereas the functions $F_{\rm I},\ldots,F_{\rm III}$ in Eq. (7) depend also on the center-of-mass coordinate $R=|\mathbf{R}|$. The explicit expressions for the radial functions $g_{\rm I},\ldots,F_{\rm III}$ for the meson exchange processes mentioned above, we take the following from the work of Riska,[18] assuming no form factors at the meson-nucleon vertices.

(a) One-pion exchange

$$\begin{aligned}g_{\rm I}^{\pi}(r)&=\frac{2}{3}\frac{f_\pi^2}{4\pi}\frac{M}{m_\pi}(2x_\pi-1)Y_0(x_\pi)\ ,\\ g_{\rm II}^{\pi}(r)&=-2\frac{f_\pi^2}{4\pi}\frac{M}{m_\pi}(x_\pi+1)Y_0(x_\pi)\ ,\\ F_{\rm II}^{\pi}(r,R)&=\frac{1}{3}\frac{f_\pi^2}{4\pi}\frac{M}{m_\pi}x_\pi y_\pi Y_0(x_\pi)\ ,\\ F_{\rm III}^{\pi}(r,R)&=\frac{1}{3}\frac{f_\pi^2}{4\pi}\frac{M}{m_\pi}x_\pi y_\pi Y_2(x_\pi)\ .\end{aligned} \tag{8}$$

(b) One-rho exchange

$$\begin{aligned}g_{\rm I}^{\rho}(r)&=2\frac{f_\rho^2}{4\pi}(1+K_V)^2\frac{M}{m_\rho}(x_\rho-1)Y_0(x_\rho)\ ,\\ F_{\rm I}^{\rho}(r,R)&=4\frac{f_\rho^2}{4\pi}\left[\frac{M}{m_\rho}\right]^3x_\rho y_\rho Y_0(x_\rho)\ ,\\ F_{\rm II}^{\rho}(r,R)&=\frac{2}{3}\frac{f_\rho^2}{4\pi}(1+K_V)^2\frac{M}{m_\rho}x_\rho y_\rho Y_0(x_\rho)\ ,\\ F_{\rm III}^{\rho}(r,R)&=-\frac{1}{3}\frac{f_\rho^2}{4\pi}(1+K_V)^2\frac{M}{m_\rho}x_\rho y_\rho Y_2(x_\rho)\ .\end{aligned} \tag{9}$$

(c) One-pion exchange with an intermediate Δ_{33}

$$\begin{aligned} g_{\mathrm{I}}^{\pi\Delta}(r) &= \tfrac{2}{3}\xi_{\Delta}^{\pi}Y_0(x_\pi)\ , \\ g_{\mathrm{II}}^{\pi\Delta}(r) &= -\xi_{\Delta}^{\pi}Y_2(x_\pi)\ , \\ h_{\mathrm{I}}^{\pi\Delta}(r) &= j_{\mathrm{I}}^{\pi\Delta}(r) = -g_{\mathrm{I}}^{\pi\Delta}(r)\ , \\ h_{\mathrm{II}}^{\pi\Delta}(r) &= j_{\mathrm{II}}^{\pi\Delta}(r) = 2g_{\mathrm{II}}^{\pi\Delta}(r) \end{aligned} \tag{10}$$

(d) One-rho exchange with an intermediate Δ_{33}

$$\begin{aligned} g_{\mathrm{I}}^{\rho\Delta}(r) &= \tfrac{4}{3}\xi_{\Delta}^{\rho}Y_0(x_\rho)\ , \\ g_{\mathrm{II}}^{\rho\Delta}(r) &= \xi_{\Delta}^{\rho}Y_2(x_\rho)\ , \\ h_{\mathrm{I}}^{\rho\Delta}(r) &= j_{\mathrm{I}}^{\rho\Delta}(r) = -\tfrac{1}{2}g_{\mathrm{I}}^{\rho\Delta}(r)\ , \\ h_{\mathrm{II}}^{\rho\Delta}(r) &= j_{\mathrm{II}}^{\rho\Delta}(r) = 2g_{\mathrm{II}}^{\rho\Delta}(r)\ . \end{aligned} \tag{11}$$

In these expressions m_π and m_ρ represent the pion mass and rho-meson mass, respectively, $x_\alpha = m_\alpha r$, $y_\alpha = m_\alpha R$ ($\alpha=\pi,\rho$), and f_π, f_ρ, K_V are the meson-nucleon coupling constants. In our calculations we assume the following values:[8] $f_\pi^2/4\pi = 0.079$, $f_\rho^2/4\pi = 0.092$, and $K_V = 6.6$ ($f_\alpha/m_\alpha = g_\alpha/2M$). The Yukawa functions $Y_0(x)$ and $Y_2(x)$ are given by $Y_0(x) = e^{-x}/x$ and $Y_2(x) = (1+3/x+3/x^2)Y_0(x)$. Finally, the factors ξ_Δ^α are given by

$$\begin{aligned} \xi_\Delta^\pi &= \frac{16}{25}\frac{f_\pi^2}{4\pi}(\mu_p-\mu_n)\frac{m_\pi}{M_\Delta - M}\ , \\ \xi_\Delta^\rho &= \frac{16}{25}\frac{f_\rho^2}{4\pi}(1+K_V)^2(\mu_p-\mu_n)\frac{m_\rho}{M_\Delta - M}\ , \end{aligned} \tag{12}$$

where $\mu_p - \mu_n = 4.706$ is the isovector nucleon magnetic moment and M_Δ is the Δ_{33} mass. In these expressions the $\alpha N\Delta$ and $\gamma N\Delta$ coupling constants have been related to the corresponding αNN and γNN coupling constants by using the static quark model.[19]

Many other exchange currents like, for instance, the frequently used model-dependent $\rho\pi\gamma$ (isoscalar) and $\omega\pi\gamma$ (isovector) exchange currents, could be included in the calculations, but since these currents have often proven to be of little importance[8] and because of the uncertainty that has to be attributed to our many-body harmonic-oscillator wave functions, we decided to neglect these processes.

Because of the strong repulsion at short internucleon distances, nuclear wave functions should vanish at the origin. However, harmonic-oscillator wave functions (in particular, *s*-wave orbits) do not have the required behavior at the origin. Therefore, to simulate the repulsive core, we introduce a short-range correlation function $f(r)$, which suppresses the wave functions in the relative coordinate. This implies, that when two-body matrix elements of the exchange magnetic moment operator are evaluated, one has to include the correlation function in the radial integral in the relative coordinate, but not in the center-of-mass coordinate. In order to perform this, one should first make the usual transformation from single-particle coordinates to relative and center-of-mass coordinates. We use a short-range correlation function of the following type:

$$f(r) = 1 - j_0(q_c r),\quad q_c = 3.93\ \mathrm{fm}^{-1}\ . \tag{13}$$

This function was used by Brown *et al.*[20] The parameter q_c was adjusted to reproduce the short-range structure of the Reid soft-core potential in an acceptable way. Many other choices for the function $f(r)$ are possible and we realize that our treatment of the short-range part of the exchange current operator is rather arbitrary. Therefore, we investigate the sensitivity of our results to the parameter q_c as will be discussed in the next section. For the calculation of two-body matrix elements of the exchange magnetic moment operator we use harmonic-oscillator wave functions with the size parameter b_{HO} set at 1.8 fm. The oscillator parameter b_{HO}, which we assume to be mass (and state) independent, was obtained from a least-squares fit of calculated charge radii, in the $(0+2)\hbar\omega$ model space, to the experimental values, taking into account the effect of the finite nucleon size.[13]

The magnetic dipole moment of a nucleus in a state with total spin J and isospin T is given by (in units of nuclear magnetons)

$$\begin{aligned} \mu &= \langle JM=J, TT_z | \mu_z | JM=J, TT_z \rangle \\ &= \langle JM=J, TT_z | \sum_{k=1}^{A} (\mu_k^S - \mu_k^V)_z \\ &\quad - \sum_{k<l}^{A} (\mu_{kl}^V)_z | JM=J, TT_z \rangle\ . \end{aligned} \tag{14}$$

The minus sign in front of the isovector terms is a consequence of the isospin convention mentioned before.

IV. RESULTS AND DISCUSSION

A comparison between calculated magnetic dipole moments of *p*-shell nuclei and experimental values is given in Table I. For the small model space, denoted as $0\hbar\omega$, only calculated values in the impulse approximation (IA) are shown, assuming free-nucleon *g* factors. In the larger model space, indicated as $(0+2)\hbar\omega$, we also present the effects of including exchange currents by successively adding the one-pion exchange currents, the one-rho-meson exchange currents, and the isobaric exchange currents, again assuming free-nucleon *g* factors for the one-body part. In the last two columns we treat the magnetic moment operator as an effective operator, first consisting of only a one-body part (eff 1b), then adding a two-body part (eff 1+2b). For the one-body effective magnetic moment operator the four nucleon *g* factors are considered as free parameters and they are determined from a least-squares fitting procedure to the 18 experimentally known magnetic moments of normal-parity states in *p*-shell nuclei. For the effective one- plus two-body magnetic moment operator, not only the four *g* factors are treated as free parameters, but also an overall strength factor λ of the exchange magnetic moment operator (all exchange processes included, i.e., π, ρ, and Δ).

For all cases considered in Table I an rms deviation

TABLE I. Magnetic moments (in μ_N) of p-shell nuclei calculated with and without (IA) meson exchange current corrections in complete $0\hbar\omega$ and $(0+2)\hbar\omega$ model spaces. Only in the enlarged model space MEC are included as indicated. The rms deviation of the calculated moments from their experimental values is denoted by $\Delta\mu_{\rm rms}$.

			$0\hbar\omega$	$(0+2)\hbar\omega$					
Nucleus	J^π	Expt.[a]	IA	IA	$+\pi$	$+\rho$	$+\Delta$	eff 1b	eff 1+2b
^{6}Li	1^+	0.82	0.87	0.83	0.83	0.83	0.83	0.82	0.81
^{7}Li	$\frac{3}{2}^-$	3.26	3.21	3.05	3.23	3.30	3.38	3.29	3.34
^{8}Li	2^+	1.65	1.36	1.42	1.61	1.67	1.76	1.36	1.59
^{8}B	2^+	1.04	1.32	1.24	1.05	0.99	0.90	1.32	1.03
^{9}Be	$\frac{3}{2}^-$	−1.18	−1.16	−1.04	−1.06	−1.10	−1.10	−1.34	−1.21
^{9}Li	$\frac{3}{2}^-$	3.44	3.20	3.01	3.23	3.39	3.48	3.36	3.44
^{10}B	1^+	0.63(12)	0.84	0.76	0.76	0.76	0.76	0.76	0.74
^{10}B	3^+	1.80	1.82	1.83	1.83	1.83	1.83	1.85	1.80
^{11}B	$\frac{3}{2}^-$	2.69	2.31	2.26	2.33	2.45	2.48	2.55	2.51
^{11}C	$\frac{3}{2}^-$	−0.96	−0.61	−0.55	−0.62	−0.74	−0.77	−0.81	−0.82
^{12}B	1^+	1.00	0.62	0.56	0.64	0.79	0.81	0.73	0.80
^{12}N	1^+	0.46	0.76	0.82	0.73	0.58	0.56	0.65	0.55
^{13}B	$\frac{3}{2}^-$	3.18	3.02	2.94	3.12	3.34	3.39	3.35	3.39
^{13}C	$\frac{1}{2}^-$	0.70	0.86	0.55	0.49	0.44	0.51	0.54	0.51
^{13}N	$\frac{1}{2}^-$	−0.32	−0.48	−0.19	−0.13	−0.08	−0.16	−0.16	−0.15
^{14}N	1^+	0.40	0.38	0.35	0.35	0.35	0.35	0.37	0.35
^{15}N	$\frac{1}{2}^-$	−0.28	−0.26	−0.27	−0.18	−0.09	−0.17	−0.22	−0.18
^{15}O	$\frac{1}{2}^-$	0.72	0.64	0.67	0.58	0.49	0.57	0.64	0.57
$\Delta\mu_{\rm rms}$			0.22	0.26	0.19	0.16	0.14	0.16	0.12

[a]Taken from Ref. 21; experimental errors are smaller than the last digit given except where indicated otherwise.

$\Delta\mu_{\rm rms}$ of calculated magnetic moments from their experimental values is given. Large improvements are obtained in the $(0+2)\hbar\omega$ model space by successively including the different exchange processes. The resulting rms deviation decreases from $0.26\mu_N$, in case of the impulse-approximation calculation, to $0.14\mu_N$, when meson exchange current contributions are added. This value appears to be even smaller than the value of $0.16\mu_N$ obtained by introducing an effective one-body magnetic moment operator. If also the two-body part is parametrized, i.e., the strength factor λ and the four g factors are treated as free parameters, only a small improvement is achieved compared to the result obtained using free-nucleon coupling constants, i.e., the rms deviation decreases from $0.14\mu_N$ to $0.12\mu_N$.

If we examine the contributions coming from the different exchange mechanisms to the magnetic moments of p-shell nuclei, as indicated in Table I, we see that the effects of the π and ρ exchange enhance each other. The contributions coming from the isobaric currents lead to further enhancement, except for the $A=13$ and 15 magnetic moments, where these contributions cancel those arising from the π and ρ exchange. Generally speaking, we may say that one-pion exchange currents and one-rho-meson exchange currents roughly contribute the same amount, whereas the isobaric contributions are somewhat smaller. However, the ρ and Δ contributions are much more sensitive to the range of the correlation function given in Eq. (13), as we will show later on. Of course the magnetic moments of $T=0$ nuclei are not affected by the inclusion of exchange currents, since these are purely isovector in our calculations and therefore cannot contribute to magnetic moments of $T=0$ nuclei. Note that in the impulse approximation these moments are computed very close to the experimental values.

The effects on the magnetic moments of the p-shell nuclei coming from the expansion of the model space on one side and inclusion of exchange currents on the other cancel each other in most cases, as follows from Table I. Exceptions are the magnetic moments of the $A=8$ and 13 nuclei. Nevertheless, the statement, made by several authors, that corrections to magnetic moments arising from configuration mixing of $2\hbar\omega$ (and higher-order) components and from meson exchange currents are large and, in most cases, of opposite sign,[8,9] is to some extent confirmed by our calculations.

Within the p shell there are five cases where the experimental values of the magnetic dipole moments are known for analog states in pairs of mirror nuclei. For these nuclei isoscalar (S) and isovector (V) magnetic moments can be calculated, with the following definition:

$$\mu_{S/V}\equiv\tfrac{1}{2}[\mu(T_z=+T)\pm\mu(T_z=-T)]\ , \qquad (15)$$

TABLE II. Isoscalar (μ_S) and isovector (μ_V) magnetic moments (in μ_N) of p-shell nuclei calculated with and without (IA) meson exchange current corrections in complete $0\hbar\omega$ and $(0+2)\hbar\omega$ model spaces. Only in the larger model space MEC are included as indicated.

			$0\hbar\omega$	$(0+2)\hbar\omega$					
	A	Expt.[a]	IA	IA	$+\pi$	$+\rho$	$+\Delta$	eff 1b	eff 1+2b
μ_S	8	1.35	1.34	1.33	1.33	1.33	1.33	1.34	1.31
	11	0.86	0.85	0.86	0.86	0.86	0.86	0.87	0.85
	12	0.73	0.69	0.69	0.69	0.69	0.69	0.69	0.67
	13	0.19	0.19	0.18	0.18	0.18	0.18	0.19	0.18
	15	0.22	0.19	0.20	0.20	0.20	0.20	0.21	0.20
μ_V	8	0.31	0.02	0.09	0.28	0.34	0.43	0.02	0.28
	11	1.83	1.46	1.41	1.48	1.60	1.62	1.68	1.66
	12	0.27	−0.07	−0.13	−0.04	0.11	0.12	0.04	0.13
	13	0.51	0.67	0.37	0.31	0.26	0.34	0.35	0.33
	15	−0.50	−0.45	−0.47	−0.38	−0.29	−0.37	−0.43	−0.38

[a]See footnote of Table I.

where the plus and minus sign denote the isoscalar and isovector parts, respectively.

Results for the isoscalar and isovector magnetic moments are shown in Table II. The isoscalar magnetic moments, which are of course not influenced by exchange currents in our calculations, are predicted very well in the impulse approximation, deviating no more than a few hundredths of a μ_N from experiment. The isoscalar magnetic moments hardly change when the model space is expanded from $0\hbar\omega$ to $(0+2)\hbar\omega$. However, for the isovector magnetic moments the situation is completely different. In most cases very poor agreement between theoretical and experimental values is obtained in the impulse approximation in both model spaces. Deviations from experiment up to about $0.4\mu_N$ occur. Note that the $A=12$ isovector magnetic moment is calculated even with the wrong sign in the impulse approximation. Many other shell-model calculations fail in this respect.[11,22,23] In fact, only reasonable values are obtained for the $A=13$ and 15 isovector magnetic moments in the impulse approximation, although especially the $A=13$ isovector magnetic moment appears to be very sensitive to the wave-function details, as might be concluded from the strong change in value due to the expansion of the model space. Comparing the impulse-approximation predictions in both model spaces for isoscalar as well as for isovector magnetic moments, it can be concluded that isovector magnetic moments are much more affected by configuration mixing of the $2\hbar\omega$ components into the wave functions than isoscalar magnetic moments.

Inclusion of the exchange currents yields a considerable improvement in case of the $A=8$ to 12 isovector magnetic moments, whereas the agreement with experiment for moments at the end of the p shell, i.e., for the $A=13$ and 15 nuclei, slightly deteriorates. Nevertheless, the necessity to include exchange currents in order to get a fairly reasonable description of isovector magnetic moments of ground-state p-shell nuclei is obvious. The same conclusion may be drawn from the behavior of the g factors in case of effective magnetic dipole operators. Values for the isoscalar and isovector g factors as well as for the strength factor λ of the two-body part are presented in Table III. All parameters are obtained in the $(0+2)\hbar\omega$ model space. The effective g factors given in case of a one-body operator only are somewhat different from those given in Ref. 13 because we adjusted them to the experimental magnetic moments of p-shell nuclei, whereas in Ref. 13 they are determined from a fit to the corresponding measured g factors. Very large effective isovector g factors have to be introduced, whereas the isoscalar g factors remain very close to the free-nucleon values. In case of an effective one- plus two-body operator the optimized isovector g factors are smaller, coming much closer to the bare-nucleon values. The two-body strength parameter λ is fitted at 0.67, which indicates that exchange currents are important, although the realistic value of 1.0 is not reached.

As mentioned before, a weak point in this kind of calculations is the rather *ad hoc* approach to the short-range part of the exchange currents. In all results presented so far, the parameter q_c in the short-range correlation function, given in Eq. (13), was kept fixed at the value 3.93 fm^{-1}, as suggested by Brown *et al.*[20] In Table IV we show the results obtained in the larger model space by using other values of q_c, i.e., $q_c=2.0$ and 6.0 fm^{-1}. Again bare-nucleon g factors are used for the one-body part (IA). The contributions from pion exchange currents ap-

TABLE III. Parameters of the effective magnetic dipole operator obtained for the calculations in the $(0+2)\hbar\omega$ model space. The parameter λ denotes the strength factor of the two-body magnetic dipole operator.

	Bare nucleon	eff 1b	eff 1+2b
g_0^l	0.500	0.514	0.497
g_0^s	0.880	0.872	0.857
g_1^l	0.500	0.720	0.601
g_1^s	4.706	5.254	4.894
λ	1.000	0.000	0.673

TABLE IV. Dependence of the magnetic moments on the parameter q_c. Magnetic moments (in μ_N) are given for the calculations in the $(0+2)\hbar\omega$ model space.

			IA+π		IA+π+ρ q_c (fm^{-1})		IA+π+ρ+Δ	
Nucleus	J^π	Expt.[a]	2.0	6.0	2.0	6.0	2.0	6.0
^{7}Li	$\frac{3}{2}^-$	3.26	3.22	3.22	3.26	3.31	3.28	3.43
^{8}Li	2^+	1.65	1.61	1.61	1.63	1.68	1.66	1.81
^{8}B	2^+	1.04	1.06	1.06	1.03	0.99	1.00	0.86
^{9}Be	$\frac{3}{2}^-$	−1.18	−1.06	−1.06	−1.08	−1.11	−1.08	−1.11
^{9}Li	$\frac{3}{2}^-$	3.44	3.22	3.23	3.30	3.41	3.33	3.54
^{11}B	$\frac{3}{2}^-$	2.69	2.33	2.33	2.39	2.46	2.40	2.51
^{11}C	$\frac{3}{2}^-$	−0.96	−0.62	−0.62	−0.68	−0.75	−0.69	−0.80
^{12}B	1^+	1.00	0.64	0.64	0.72	0.81	0.73	0.83
^{12}N	1^+	0.46	0.73	0.73	0.65	0.56	0.65	0.54
^{13}B	$\frac{3}{2}^-$	3.18	3.11	3.11	3.23	3.37	3.24	3.44
^{13}C	$\frac{1}{2}^-$	0.70	0.48	0.49	0.46	0.43	0.54	0.50
^{13}N	$\frac{1}{2}^-$	−0.32	−0.13	−0.13	−0.10	−0.07	−0.18	−0.15
^{15}N	$\frac{1}{2}^-$	−0.28	−0.18	−0.18	−0.13	−0.08	−0.20	−0.17
^{15}O	$\frac{1}{2}^-$	0.72	0.57	0.58	0.53	0.48	0.60	0.56

[a]See footnote of Table I.

pear to be almost insensitive to the range of the correlation function, as is expected, since pion exchange is typically of long-range character. However, contributions from rho-meson exchange currents are much more affected by variations of q_c, since the heavier rho meson generates a much shorter-ranged interaction than the pion. Increasing the range of the correlation function, i.e., taking a smaller value for q_c, leads to quenching of the rho-meson exchange contributions. We may conclude from this that exchange current contributions, other than from pion exchange, are very much dependent from the short-range phenomenology used in the calculations.

Corrections to the isovector magnetic moment of the $A=15$ ground-state nuclei due to exchange currents computed in the $0\hbar\omega$ model space refer to the single-particle Schmidt value. These calculations can be compared with the corresponding results obtained by Towner,[8] who performed extensive shell-model calculations on nuclei with a valence nucleon (or hole) outside a closed-shell core. This is done in Table V. All contributions given are expressed as percentages of the Schmidt values. The correction due to the configuration mixing, denoted as CM, is simply the impulse-approximation prediction in the $(0+2)\hbar\omega$ model space in our calculation, whereas in Ref. 8 this correction is computed via second-order core polarization, using a one-boson-exchange potential. Of course in the latter calculation also excitations higher than $2\hbar\omega$ above the valence orbits are involved. Corrections due to exchange currents are shown for different values of the range parameter q_c of the correlation function. It should be noted that Towner included meson-nucleon vertex form factors in the evaluation of the exchange current contributions. Inclusion of these form factors gives rise to an extra quenching of the contributions from heavier mesons, like the rho meson.[12] This may be an indication why the results computed with the longer-ranged correlation function, i.e., with $q_c=2.0$ fm^{-1}, are very similar to the results obtained by Towner.

Finally, to illustrate the predictive power of a simple shell-model calculation of exchange current effects, we computed the corrections to the $A=3$ magnetic moments in both model spaces, again using the effective interactions given in Ref. 13. Results are presented in Table VI. The $0\hbar\omega$ calculations are, of course, expectation values between $(0s)^3$ configurations. The predicted isoscalar magnetic moment is in very good agreement with the experimental value, whereas the discrepancy between the calculated isovector moment in the impulse approximation and the experimental isovector moment should be attributed to exchange current effects. The same qualitative features of exchange current effects are observed in this simple shell-model calculation as in the more accurate calculations of Schiavilla, Pandharipande, and Riska,[7] who treated the nuclear wave functions on the same footing as the exchange currents, which is required by the equation of continuity.

TABLE V. Corrections to the isovector magnetic moment of the $A=15$ nuclei coming from exchange currents and configuration mixing expressed as a percentage of the single-particle Schmidt value. For b_{HO} the value 1.8 fm was used.

	Present work			
q_c (fm^{-1})	2.0	3.93	6.0	Towner
π	−24.4	−24.1	−24.5	−20.5
ρ	−9.5	−17.3	−19.0	−10.7
Δ	15.0	18.2	20.6	16.9
CM	4.4	4.4	4.4	6.5

TABLE VI. Magnetic moments (in μ_N) of the trinucleon systems calculated with and without (IA) the MEC corrections in both the $0\hbar\omega$ and the $(0+2)\hbar\omega$ model space. For $b_{\rm HO}$ we adopted the value 1.8 fm in both spaces ($q_c=3.93$ fm^{-1}).

		Expt.	IA	$+\pi$	$+\rho$	$+\Delta$
$0\hbar\omega$	^{3}H	2.98	2.79	2.94	2.99	3.04
	^{3}He	−2.13	−1.91	−2.06	−2.11	−2.16
	Isoscalar	0.43	0.44	0.44	0.44	0.44
	Isovector	2.55	2.35	2.50	2.55	2.60
$(0+2)\hbar\omega$	^{3}H	2.98	2.76	2.91	2.98	3.06
	^{3}He	−2.13	−1.89	−2.03	−2.11	−2.19
	Isoscalar	0.43	0.44	0.44	0.44	0.44
	Isovector	2.55	2.33	2.47	2.55	2.63

V. SUMMARY AND CONCLUSION

Meson exchange current effects on magnetic dipole moments of p-shell nuclei have been investigated within a $(0+2)\hbar\omega$ shell-model space. The model applied for the exchange currents is derived from one-pion and one-rho-meson exchange, using the static nucleon limit. We have shown that, in order to get a proper description of the magnetic dipole moments of the $A=4$–16 nuclei, it is necessary to include exchange currents.

The exchange currents involved are purely isovector and their presence can explain, to a certain amount, the large discrepancies between theoretical isovector magnetic moments calculated in the impulse approximation and experimental values. The isoscalar magnetic moments are well described in the impulse approximation. However, our treatment of the exchange currents is model dependent, especially with regard to the short-range behavior. A more consistent approach is required, which means that one and the same interaction should be used to construct the nuclear wave functions and the exchange current operators. Until now such an approach has only been applicable to the very light nuclear systems. Nevertheless, shell-model calculations have proven to be a powerful tool to illustrate qualitatively the effects of exchange currents in heavier nuclei.

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