COMMENTS

Comments are short papers which comment on papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Triton model calculation test of the Bonn W-matrix rank-one approximation"

H. Haberzettl
Department of Physics, The George Washington University, Washington, D.C. 20052

U. Kerwath
Physikalisches Institut, Universität Bonn, Endenicher Allee 11-13, D-5300 Bonn 1, Germany
(Received 4 September 1990)

We report that a recent conclusion by Gibson, Pearce, and Payne indicating a violation of the conjectured variational principle underlying the calculation of three-body binding energies within the W-matrix approach is in error.

In Refs. 1 and 2 it was shown that the one-term separable approximation, which follows from the W-matrix representation of the two-body T matrix introduced in Ref. 3, is fully capable of reproducing the binding energy and scattering results of direct solutions of the three-nucleon equations. The two-body interactions used in this context were the semirealistic local Malfliet-Tjon potentials[4] MT I and MT III.

A particularly interesting, and for practical purposes important, finding[1] was that the triton binding energy $E_3(k)$, as a function of the W-matrix parameter k (see Refs. 1 and 3 for details), has a minimum at some value of k which compares very well with the full solution of the three-nucleon equation. For all other values of k the function $E_3(k)$ is bounded below by this exact result. This behavior is very reminiscent of variational methods; it was therefore conjectured in Ref. 1 that a variational principle may be underlying these findings. It was proposed that, within the W-matrix approach, three-body binding energies for other interactions should be calculated by searching for the minimum of $E_3(k)$; if the variational principle holds true, this would guarantee that it is bounded below by the corresponding exact value and thus ensure the best possible approximate binding energy.

Following this prescription, in a recent article[5] Gibson, Pearce, and Payne compare W-matrix results of triton binding energies for several other potentials with the results from direct solutions with the full interactions. For the MT I-III interactions they confirm the findings of Ref. 1. For several other versions and combinations of the Malfliet-Tjon potentials[4] their results show that the conjectured variational principle is upheld in all cases considered. (In some cases, Gibson *et al.* find a rather large difference, up to some 10%, between the approximate and exact values; we will comment on this below.)

A particularly disappointing result, however, is reported for a modified Reid-soft-core (RSC) potential.[6] Employing the RSC spin-singlet interaction with a modified midrange attraction for a three-boson model, Gibson *et al.* give an exact binding energy of $E_3=-7.1$ MeV and a corresponding minimal W-matrix value of $E_3(k)=-9.1$ MeV. This result is not only much worse than any of the other approximate values, it moreover is in clear violation of the conjectured variational principle.

In view of the fact that this is the only result known to us that contradicts our conjecture, we felt compelled to repeat the calculation ourselves. Using the modified values for the midrange attraction V_2 and the parameter k as quoted explicitly in Ref. 5, $V_2=-1815.66$ MeV and $k=0.85$, respectively, we find a minimal W-matrix result of $E_3(k)=-9.2$ MeV ($\pm 3\%$), which is in agreement with the value given by Gibson *et al.* (The numerical error of our calculation is rather large; we were not interested in great accuracy but only in quick results.) We noticed, however, that the midrange attraction quoted is not—as stated in the text of Ref. 5, p. 2879—a factor 1.08 larger than the original RSC value[6] (-1650.6 MeV), but larger by a factor 1.1. We therefore repeated the calculation with V_2 corresponding to the factor 1.08, i.e., $V_2=-1782.65$ MeV; we find $E_3(k)=-6.8$ MeV ($\pm 3\%$).

Note that this latter number compares quite favorably with the exact value of $E_3=-7.1$ MeV reported in Ref. 5. We suspected, therefore, that there had been an inadvertent mix-up of the respective midrange parameters V_2 corresponding to the factors 1.08 and 1.1, and that the exact binding energy of $E_3=-7.1$ MeV has to be com-

pared with our W-matrix result of $E_3(k)=-6.8$ MeV for the factor 1.08, and not with the value of $E_3(k)=-9.1$ MeV quoted in Ref. 5.

Subsequently Gibson recalculated the exact and the W-matrix prescription binding energies for both midrange factors. His new results[7] confirm our values given above and indeed show that there had been a mix-up of parameters in Ref. 5. In other words, the violation of the conjectured variational principle reported in Ref. 5 is in error.

We conclude, therefore, that all numerical tests known to us uphold the variational principle.

In the meantime, after the investigations reported here were finished, a mathematical proof for the variational principle was provided by one of the present authors (H.H.).[8] As it turns out, the variational principle is valid not just for the W-matrix representation but for all well-defined separable expansions of the two-body T matrix which have a block-diagonal structure and are controlled by a free parameter.

Let us add one final remark concerning the quantitative agreement between exact binding energies and the corresponding W-matrix values. Our own experience is that, in general, we find that the more realistic the interactions the closer the approximate values are to the exact ones. This experience is also borne out by the numbers given in Ref. 5, where rather large differences in binding energies are only found for models with unrealistic (i.e., unrealistic for three-nucleon calculations) purely attractive potentials. Elsewhere we shall report on calculations for the best nucleon-nucleon interactions available that show complete agreement between exact and approximate binding energies.

We thank Dr. B. F. Gibson for his cooperation in clearing up this discrepancy and for providing numerical checks of our results. The work of H.H. was supported in part by the U.S. Department of Energy, Grant No. DE-FG05-86-ER40270.

[1]E. A. Bartnik, H. Haberzettl, Th. Januschke, U. Kerwath, and W. Sandhas, Phys. Rev. C **36**, 1678 (1987).
[2]T. N. Frank, H. Haberzettl, Th. Januschke, U. Kerwath, and W. Sandhas, Phys. Rev. C **38**, 1112 (1988).
[3]E. A. Bartnik, H. Haberzettl, and W. Sandhas, Phys. Rev. C **34**, 1520 (1986).
[4]R. A. Malfliet and J. A. Tjon, Nucl. Phys. **A127**, 161 (1969); Ann. Phys. (N.Y.) **61**, 425 (1970).
[5]B. F. Gibson, G. C. Pearce, and G. L. Payne, Phys. Rev. C **40**, 2877 (1989).
[6]R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).
[7]B. F. Gibson, private communication.
[8]H. Haberzettl, submitted to Phys. Rev. C.