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Charge-exchange effects in elastic scattering with radioactive beams

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The elastic scattering of nuclei belonging to an isobaric multiplet is discussed. These reactions involve at least one unstable nucleus. Such experiments are becoming feasible with the advent of radioactive beams. It is suggested that strong resonances can be found in the scattering of (isobaric) mirror nuclei at energies close to the Coulomb barrier. A simple model for the collision of mirror nuclei composed of identical, inert cores and valence nucleons is considered and the coupled equations, whose solutions yield the direct and charge-exchange amplitudes, are derived.

I. INTRODUCTION

The effect of charge exchange is known to be very important in nucleon-nucleus scattering.[1] This has led to the identification of isobaric analog states in various light and heavy nuclei.[2] The importance of charge exchange in reactions of the type (^{3}He,t) has been studied by Robson and Contanch[3] who concluded that the effect is very small. It has been argued[4] that charge-exchange effects in heavy-ion collisions should become much smaller and can be ignored in the lowest approximation. This argument is based on the assumption that charge exchange is a volume effect and as such its strength will depend inversely on the mass of the colliding nuclei.

During recent years, the elastic exchange process in the collision of nuclei differing by a single nucleon or a cluster has been observed in reactions involving stable nuclei.[5] These involve reactions of type (^{12}C, ^{13}C), (^{12}C, ^{16}C), etc., and significant progress has been made in the critical interpretation of elastic scattering,[6] in particular by Imanishi and Von Oertzen[7] who utilize a dynamical molecular orbital approach. The fusion of such systems has also been studied and the interesting feature of fusion oscillations at low energies[8] has been interpreted in terms of a parity dependence of the optical potential.[9] Imanishi and Von Oertzen[10] have also suggested that a strong spin dependence observed in the optical potentials at energies near the Coulomb barrier arises from the exchange of the valence nucleon between the two identical cores.

We had suggested some time ago[11] that one may be able to study charge dependence in nucleus-nucleus potentials by the analyses of the elastic scattering of isobaric mirror nuclei such as (^{13}C, ^{13}N) or (^{17}O, ^{17}F) at energies close to the Coulomb barrier. Once again, one has two nuclei with identical ("inert") cores but now there are two valence nucleons which differ in the Z component of the isospin. There would be strong coupling of the two nuclei due to different possible exchanges of the valence nucleons (as well as the cores) and, at energies close to the Coulomb barrier, the explicit charge exchange part of the neutron-proton interaction will have an important effect. This expectation is based on the assumption that at these energies the absorption effects will be small and unlikely to swamp the effect we wish to observe. In addition, at energies close to the Coulomb barrier, recoil effects are small and reaction times are large so that it is possible to study the importance of multiple charge exchange between the colliding nuclei, which may exhibit itself as a resonance. Evidence of strong coupling in near barrier energy collisions has been observed in a number of cases[12] and the important role of the valence nucleon in the collision of ^{17}O by ^{208}Pb has recently been pointed out.[13] However, to our knowledge multiple charge-exchange effects have not been observed in elastic scattering of strongly interacting systems. The detailed dependence of these expected large cross sections and the possible resonances will provide a sensitive tool to explore the density distributions of valence nucleons and charge-exchange mechanisms. Resonant charge exchange in the scattering of atomic systems has been known for many years[14] and serves as a provocative analog.

In this paper we develop the formalism for the charge-exchange reaction referred to in Ref. 11 in detail. We consider a model comprised of two identical (inert) cores and two valence nucleons. With the aid of simplifying assumptions, we obtain a set of coupled equations whose solutions yield the direct and charge-exchange amplitudes. With a further, no-recoil, approximation, we show how those coupled equations can be decoupled into eigenchannels. The expressions for the direct and exchange integrals as well as the nonorthogonality overlap are gathered together in the Appendix.

II. FORMALISM

We consider a reaction of the type

$$\underbrace{(C_1+n)}_{A}+\underbrace{(C_2+p)}_{B}\rightarrow\underbrace{(C_1+n)}_{A}+\underbrace{(C_2+p)}_{B}$$
$$\rightarrow\underbrace{(C_1+p)}_{B'}+\underbrace{(C_2+n)}_{A'},$$

where C_1 and C_2 are two identical cores and the colliding system is left in the elastic channel. We shall refer to the first reaction as a direct process and the second as a charge-exchange process. The two processes are experimentally indistinguishable and the physical elastic amplitude will be a superposition of the two. The nuclei A and B are mirror nuclei.

The coordinate system is shown in Fig. 1. We shall make the approximation of ignoring the shift in the centers of mass of nuclei A and B from those of the cores C_1 and C_2 (the no-recoil approximation[15]). We thus approximate the separations of the centers of mass A and B by $\mathbf{R}$. The general formalism remains the same even without invoking the no-recoil approximation. It only results in a coupled integro-differential equation to be solved where the rearrangement kernel becomes nonlocal.

The total Hamiltonian of the system can be written as

$$H=(H_{C_1}+K_{C_{1n}}+V_{C_{1n}})+(H_{C_2}+K_{C_{2p}}+V_{C_{2p}})+K_{AB}+(V_{C_{1p}}+V_{C_{2n}}+V_{C_1C_2}+V_{np}) \quad (1)$$

$$=(H_{C_1}+K_{C_{1p}}+V_{C_{1p}})+(H_{C_2}+K_{C_{2n}}+V_{C_{2n}})+K_{B'A'}+(V_{C_{1n}}+V_{C_{2p}}+V_{C_1C_2}+V_{np}) , \quad (2)$$

where H_{C_1} and H_{C_2} are the internal Hamiltonians of the cores C_1 and C_2, respectively. $K_{\alpha\beta}$ is the kinetic-energy operator for the relative motion of the fragments α and β and $V_{\alpha\beta}$ is their interaction potential. Since we treat the cores as inert we shall omit their internal Hamiltonians in the discussions below.

For the exact scattering wave function of the system we shall make a two-state approximation:

$$\Psi^{(+)}=[\Phi_A C_1\otimes n)\Phi_B(C_2\otimes p)]F_1(\mathbf{R}_i)+[\Phi_{B'}(C_1\otimes p)\Phi_{A'}(C_2\otimes n)]F_2(\mathbf{R}_f) , \quad (3)$$

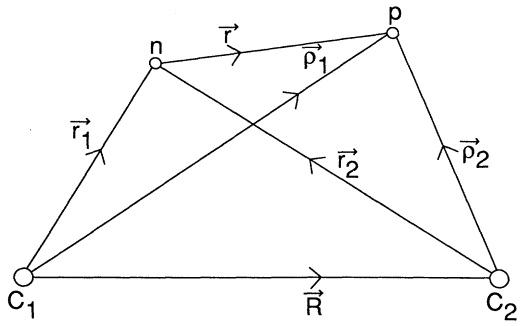

FIG. 1. Coordinates used in the description of the charge-exchange reaction. C_1 and C_2 represent the cores of the two nuclei while n and p represent the valence neutron and the valence proton or the valence neutron hole and the valence proton hole as appropriate to the system.

where $\mathbf{R}_i$ and $\mathbf{R}_f$ are the channel vectors for the initial (direct) and final (charge-exchange) channels. In the no-recoil approximation we have

$$\mathbf{R}_i\simeq\mathbf{R} , \quad \mathbf{R}_f\simeq-\mathbf{R} . \quad (4)$$

$\Psi^{(+)}$ is assumed to be a solution of the equation

$$(H-E)\Psi^{(+)}=0 . \quad (5)$$

The equations for F_1 and F_2 are obtained by multiplying Eq. (5) by $[\Phi_A(C_1\otimes n)\Phi_B(C_2\otimes p)]$ and by $[\Phi_{B'}(C_1\otimes p)\Phi_{A'}(C_1\otimes n)]$ and integrating over all the internal variables. This results in coupled equations for the functions $F_1(\mathbf{R})$ and $F_2(-\mathbf{R})$ given by

$$[K_R+V_{C_1C_2}(R)+\Delta V^D(\mathbf{R})-\epsilon]F_1(\mathbf{R})=-[K_R+V_{C_1C_2}(R)-\epsilon]S(\mathbf{R})F_2(-\mathbf{R})-\Delta V^E(\mathbf{R})F_2(-\mathbf{R}) , \quad (6)$$

$$[K_R+V_{C_1C_2}(R)+\Delta V^D(\mathbf{R})-\epsilon]F_2(-\mathbf{R})=-[K_R+V_{C_1C_2}(R)-\epsilon]S(\mathbf{R})F_1(\mathbf{R})-\Delta V^E(\mathbf{R})F_1(\mathbf{R}) , \quad (7)$$

where ϵ is the channel energy. $\Delta V^D(\mathbf{R})$ and $\Delta V^E(\mathbf{R})$ are the direct and exchange potentials defined by

$$\Delta V^D(\mathbf{R})=\langle[\Phi_A(C_1\otimes n)\Phi_B(C_2\otimes p)]|V_{C_{1p}}+V_{C_{2n}}+V_{np}|\Phi_A(C_1\otimes n)\Phi_B(C_2\otimes p)\rangle=\langle[\Phi_{B'}(C_1\otimes p)\Phi_{A'}(C_2\otimes n)]|V_{C_{1n}}+V_{C_{2p}}+V_{np}|[\Phi_{B'}(C_1\otimes p)\Phi_{A'}(C_2\otimes n)]\rangle , \quad (8)$$

$$\Delta V^E(\mathbf{R})=\langle[\Phi_{B'}(C_1\otimes p)\Phi_{A'}(C_2\otimes n)]|V_{C_{1p}}+V_{C_{2p}}+V_{np}|[\Phi_A(C_1\otimes n)\Phi_B(C_2\otimes p)]\rangle=\langle[\Phi_A(C_1\otimes n)\Phi_B(C_2\otimes p)]|V_{C_{1n}}+V_{C_{2p}}+V_{np}|[\Phi_{B'}(C_1\otimes p)\Phi_{A'}(C_2\otimes n)]\rangle , \quad (9)$$

and $S(\mathbf{R})$ is the nonorthogonality overlap defined by

$$S(\mathbf{R})=\langle[\Phi_{B'}(C_1\otimes p)\Phi_{A'}(C_2\otimes n)]|[\Phi_A(C_1\otimes n)\Phi_B(C_2\otimes p)]\rangle . \quad (10)$$

In Eqs. (8)–(10), the integration is only over the internal variables of the nuclei A and B. In general, the direct and exchange potentials as well as the nonorthogonality overlap will depend upon the vector $\mathbf{R}$. In the case where the spin of the mirror nuclei is $\frac{1}{2}$, they are functions only of the magnitude of the vector.

The coupled equations (6) and (7) can be decoupled by the introduction of new functions

$$F_{\pm}(\mathbf{R})=[1\pm S(\mathbf{R})][F_1(\mathbf{R})\pm F_2(-\mathbf{R})] . \tag{11}$$

$F_{\pm}(\mathbf{R})$ are solutions of the eigenchannel equations

$$\left[K_R+V_{C_1C_2}(R)+\frac{\Delta V^D(\mathbf{R})\pm\Delta V^E(\mathbf{R})}{1\pm S(\mathbf{R})}-\epsilon\right]F_{\pm}(\mathbf{R})=0 \tag{12}$$

and satisfy the boundary conditions

$$F_{\pm}(\mathbf{R})\underset{R\to\infty}{\to}\exp(i\mathbf{K}\cdot\mathbf{R}) +[f_d(\theta,\phi)\pm f_{ce}(\pi-\theta,\pi+\phi)]\frac{e^{ikR}}{R} . \tag{13}$$

In writing down Eq. (13) we have utilized the property that $S(\mathbf{R})$ goes to zero asymptotically. f_d and f_{ce} are the direct and charge exchange amplitudes, respectively. If the nonorthogonal term $S(\mathbf{R})$ is negligible (compared to unity) and if ΔV^E is a small perturbation, then Eq. (7) can be solved in DWBA; i.e., the charge exchange is evaluated from the purely outgoing solution of the equation

$$[K_R+V_{C_1C_2}(R)+\Delta V^D(\mathbf{R})-\epsilon]F_{20}(-\mathbf{R}) =-\Delta V^E(\mathbf{R})F_{10}(\mathbf{R}) , \tag{14}$$

where $F_{10}(\mathbf{R})$ is the (zero-order) elastic-scattering solution of

$$[K_R+V_{C_1C_2}(R)+\Delta V^D(\mathbf{R})-\epsilon]F_{10}(-\mathbf{R})=0 . \tag{15}$$

By a comparison of the scattering amplitudes obtained from Eqs. (14) and (15) with those from Eq. (12), one can assess the importance of multiple charge exchange.

Finally, a remark about the two functions $F_{\pm}(\mathbf{R})$. In the actual collision, there is only one physical elastic-scattering amplitude. Its separation into a "direct" and "charge-exchange" part is artificial. Only one combination (f_d+f_{ce}) or (f_d-f_{ce}) will represent the physical amplitude. If the cores C_1 and C_2 are bosons, it is (f_d+f_{ce}) and one needs to determine only the function $F_+(\mathbf{R})$.

Expressions for the direct and exchange potentials $\Delta V^D(\mathbf{R})$ and $\Delta V^E(\mathbf{R})$ as well as the nonorthogonality overlap $S(\mathbf{R})$ are given in the Appendix. It is easiest to evaluate them by the use of the Fourier transforms of the potentials, densities, and density matrices.

In the collision of mirror nuclei such as $^{13}C+^{13}N$, the valence nucleons are very weakly bound. At energies close to the Coulomb barrier, one would expect the dominant effect to be due to the neutron-proton interaction because the densities of ^{13}C and ^{13}N at large distances will be primarily those of the valence nucleons. It would thus be possible to estimate the charge exchange part of the n-p interaction.

III. SUMMARY AND CONCLUSION

The scattering of isobaric mirror nuclei was considered as a four-body system comprised of two inert cores and two valence nucleons. It was shown that in the no-recoil approximation, the coupled equations could be exactly decoupled. The method utilized is identical to that used in the study of nucleon elastic exchange collisions or charge exchange in atomic collisions. The two-state approximation made in Sec. II may, in general, be unrealistic. It may be important to include the excited states of the nuclei. This was observed to be important in the scattering of ^{17}O by ^{208}Pb where the nucleon transfer and inelastic channels were strongly coupled.[13] The same is true in the elastic exchange reactions discussed by Imanishi and Von Oertzen[7] where the coupling of inelastic and transfer channels built by the molecular orbitals was found to be significant.

In the particular case of the scattering of isobaric mirror nuclei, the inclusion of excited states poses no additional problem. Instead of the new functions $F_{\pm}(\mathbf{R})$ being solutions of "optical potentials," they will be solutions of coupled equations.

With the aid of sophisticated and comprehensive programs such as FRESCO,[16] the solutions to these systems of equations can be attempted. Work on some model calculations is in progress.

We eagerly await experimental data on reactions with radioactive beams to test the speculations we have presented here.

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APPENDIX: Expressions for $S(\mathbf{R})$, $\Delta V^D(\mathbf{R})$, and $\Delta V^E(\mathbf{R})$

Treating the cores C_1 and C_2 as inert we use the terminology

$$\begin{aligned}&\Phi_A(C_1\otimes n)=\phi_\alpha(\mathbf{r}_1),\quad \Phi_{A'}(C_2\otimes n)=\phi_\alpha(\mathbf{r}_2) ,\\ &\Phi_B(C_2\otimes p)=\phi_\beta(\boldsymbol{\rho}_2),\quad \Phi_{B'}(C_1\otimes p)=\phi_\beta(\boldsymbol{\rho}_1) ,\end{aligned} \tag{A1}$$

where the vectors are depicted in Fig. 1.

Choosing the $\mathbf{r}_1$, $\boldsymbol{\rho}_2$, and $\mathbf{R}$ as the linearly independent vectors, we have the relations

$$\mathbf{r}_2=\mathbf{r}_1-\mathbf{R},\quad \boldsymbol{\rho}_1=\boldsymbol{\rho}_2+\mathbf{R},\quad \mathbf{r}=\boldsymbol{\rho}_2-\mathbf{r}_1+\mathbf{R} . \tag{A2}$$

Thus we obtain

$$S(\mathbf{R})=\left|\int d\mathbf{q}\, e^{i\mathbf{q}\cdot\mathbf{R}}|\phi_\alpha(\mathbf{q})|^2\right|^2 , \tag{A3}$$

and for the direct potential we obtain

$$\Delta V^D(\mathbf{R})=(2\pi)^{3/2}\int d\mathbf{q}\,\rho_\alpha(\mathbf{q})\rho_\beta(-\mathbf{q})V_{np}e^{-i\mathbf{q}\cdot\mathbf{R}} + \int d\mathbf{q}\,\rho_\alpha(\mathbf{q})V(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{R}} + \int d\mathbf{q}\,\rho_\beta(\mathbf{q})V(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{R}}\ , \tag{A4}$$

where

$$\phi_\alpha(\mathbf{q})=(2\pi)^{-3/2}\int d\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}\phi_\alpha(\mathbf{r})\ , \tag{A5}$$

$$\rho_\alpha(\mathbf{q})=(2\pi)^{-3/2}\int d\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}\rho_\alpha(\mathbf{r})\ , \tag{A6}$$

$$V_{np}(q)=(2\pi)^{-3/2}\int d\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}V_{np}(r)\ , \tag{A7}$$

and

$$\rho_\alpha(\mathbf{r})=|\phi_\alpha(\mathbf{r})|^2\ . \tag{A8}$$

The exchange potential is

$$\Delta V^E(\mathbf{R})=\left[\int d\mathbf{q}\,e^{i\mathbf{q}\cdot\mathbf{R}}|\phi_\alpha(\mathbf{q})|^2\right]\int d\mathbf{q}\,e^{-i\mathbf{q}\cdot\mathbf{R}}\left[\epsilon-\frac{\hbar^2q^2}{2\mu}\right]|\phi_\beta(\mathbf{q})|^2 + \left[\int d\mathbf{q}\,e^{-i\mathbf{q}\cdot\mathbf{R}}|\phi_\beta(\mathbf{q})|^2\right]\int d\mathbf{q}\,e^{i\mathbf{q}\cdot\mathbf{R}}\left[\epsilon-\frac{\hbar^2q^2}{2\mu}\right]|\phi_\alpha(\mathbf{q})|^2+(2\pi)^{3/2}\int d\mathbf{q}|F_{\mathbf{q}}(\mathbf{r})|^2V_{np}(\mathbf{q})\ , \tag{A9}$$

where

$$F_{\mathbf{q}}(\mathbf{R})=(2\pi)^{-3/2}\int d\mathbf{r}\,\phi^*(\mathbf{r})\phi(\mathbf{r}-\mathbf{R})e^{i\mathbf{q}\cdot\mathbf{r}}\ . \tag{A10}$$

Equations (2)–(10) constitute the complete expressions for the various terms needed to evaluate the potentials and overlaps. The details of the angular momentum of the single-particle states have not been specified. The label α, for instance, represents the quantum numbers n, l, j, and m where n is the principal quantum number, l and j are the orbital and total angular momentum, and m is the z component of the total angular momentum of the valence nucleon. If $j=\frac{1}{2}$, the overlap $S(R)$ and the direct and exchange potentials $\Delta V^D(R)$ and $\Delta V^E(R)$ are dependent only on the magnitude of R. (They are monopole in character.)

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