Chiral solitons with quarks and composite mesons

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We explore the development of a nontopological soliton model for baryons in which effective meson fields are generated from extended quark-antiquark pairs within a nonlocal model of Nambu–Jona-Lasinio type. Meson kinetic and potential energies arise from the Dirac sea. Only scalar-isoscalar and pseudoscalar-isovector meson fields are retained. Functional methods and bilocal field techniques are employed and we work to lowest order in loops for the retained degrees of freedom. We identify a mean-field approximation from the effective action obtained through a Legendre transformation incorporating a chemical-potential constraint. This fixes the baryon number in terms of valence quark configurations and thereby induces a classical configuration for the meson fields. After a localization of the meson sector of the effective action, the model is of the linear sigma type with a nonlocal quark-meson vertex due to the composite extended nature of the internally generated meson fields. With a self-confining dynamical quark mass, the classical meson fields provide a region where quarks can propagate and have a defined eigenenergy. A simple model illustration of this mechanism is provided.

I. INTRODUCTION

The description of baryons in terms of valence quarks with either bag or nontopological soliton methods continues to play an important role in the search for effective low-energy models of QCD. Quark-antiquark fluctuations in the form of meson fields are expected to be particularly efficient degrees of freedom for implementing nonperturbative aspects. Chiral and cloudy bag models[1] emphasize the important role of the pion field, and hence chiral symmetry, for the large-distance physics of the nucleon. In nontopological solitons of the Friedberg-Lee[2] type, the sharp boundary condition of bag models is replaced by coupling the quarks to a dynamical scalar field. The stabilizing mechanism consists of the classical or mean-field configuration of the scalar field approaching a large constant vacuum value as the valence quark density decreases towards the edge of the soliton. Mean-field models of this type have been extensively studied,[3] as have chiral extensions[4] and the inclusion of vector mesons.[5] Covariant versions of quark-meson solitons have also been developed.[6] It is not obvious that the $\bar{q}q$ substructure of the meson collective modes of QCD is unimportant for modeling the structure of a baryon. Investigations along these lines must deal with a nonlocal field-theory model having extended meson fields and a distributed meson-quark vertex. In this paper we explore aspects of the formulation of a static chiral soliton model for a baryon in which the meson fields are generated from quark fields in the form of $\bar{q}q$ fluctuations.

Consider the one-gluon-exchange coupling of local quark currents shown in Fig. 1(a). A Fierz reordering of the quark fields produces two bilocal "currents" $\bar{q}(x)\Lambda^\theta q(y)$ that carry, in the color-singlet sector of the matrices Λ^θ, the quantum numbers of mesons. This is illustrated in Fig. 1(b) and is the same as the quark-interchange mechanism[7] used for initial consideration of meson coupling to bag-model hadrons. The introduction of dynamical meson fields that have such a $\bar{q}q$ origin is most easily accomplished within the Nambu–Jona-Lasinio (NJL) model.[8] This model has proved to be a useful bridge between aspects of QCD and chiral quark-meson degrees of freedom. In the simplest form, a contact four-fermion interaction is employed and the vertex factors Λ^θ are restricted to scalar-isoscalar and

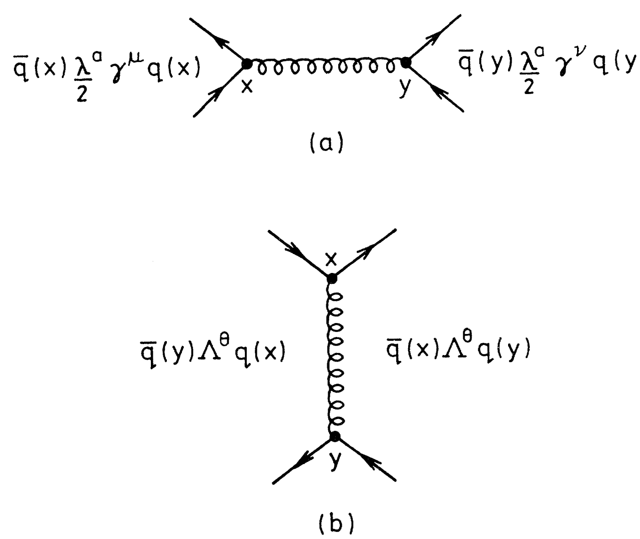

FIG. 1. (a) The one-gluon-exchange interaction between local quark currents. (b) A representation of the same interaction after Fierz reordering in the space of discrete indices. This structure is recast as an integral over bilocal boson fields that have a $\bar{q}q$ content.

pseudoscalar-isovector channels. Auxiliary local meson fields in the sigma and pion channels can be used to recast the Lagrangian so that the basic quark-meson form of the linear sigma model[9] is produced. The meson kinetic and potential terms have their origin in the one fermion loop vacuum polarization integral. Nontopological quark-meson solitons within the NJL framework have been studied recently.[10]

It has previously been pointed out that quark-meson models can be obtained from a nonlocal extension of the NJL framework to allow for an effective gluon propagator of finite range.[11] The required path-integral techniques use bilocal auxiliary fields[12] so that the bilocal current-current interaction is replaced by functional integration over Bose fields $\mathcal{B}^{\theta}(x,y)$ that are Yukawa coupled to quarks. The vacuum configurations $\mathcal{B}^{\theta}(x-y)$ produce a quark dynamical self-energy from gluon dressing.[11] Fluctuations are treated as meson fields, which are bare in the sense of an effective Lagrangian, but have a composite $\bar{q}q$ structure. The associated meson propagators at this level are known[13,14] to contain a meson form factor with a ladder Bethe-Salpeter structure. This form factor corresponds to the quark-meson vertex $\Gamma(k,P)$ illustrated in Fig. 2. The finite extent of the $\bar{q}q$ content provides, through the relative momentum k, a natural regularization for fermion loop integrals.[14]

The prospect of self-regulation of effective nonlocal field models is of interest for nuclear physics. It has recently[15] been argued that the $\bar{q}q$ fluctuation fields for meson collective modes obtained from this type of formalism can be usefully considered as the basis for relativistic nucleon-meson models by inferring meson-nucleon coupling from meson-quark coupling. Meson substructure was ignored in those considerations. If the vertex of Fig. 2 operates for a quark constituent of a nucleon, there is a suppression at large momentum transfer P due to the finite extent of the quark distribution available in the nucleon. At large nucleon momentum, k becomes large and there is a cutoff provided by $\Gamma(k,P)$ due to finite meson extent.

We retain only the scalar and pionic meson modes in the soliton model here. With the $\bar{q}q$ meson degrees of freedom defined, we continue to use functional path-integral methods to formulate the mean-field soliton model. In such an approach, the meson propagators are generated from vacuum loops produced by integrating

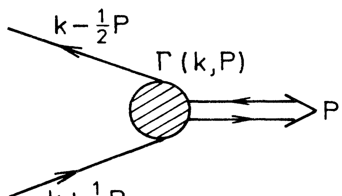

FIG. 2. The quark-meson vertex when the meson is a composite $\bar{q}q$ object with total momentum P and internal relative momentum k.

out the quark fields. The retention of valence quarks to connect such methods with static bag models has been considered before.[11] There the baryon number constraint is imposed through an approximate construction of a three-quark Green's function to identify the minimizing energy of the system through a stationary phase method. We employ here a more direct approach that identifies the effective action $\Gamma[b,n]$ through a functional Legendre transformation of the generator $W[J,\mu]$ of connected Green's functions. Here the quark fields have been integrated out with the constraint of a chemical potential μ to fix the baryon number n, and the fields $b(x)$ for propagation of the fluctuation meson modes have been coupled to an external source $J(x)$ which is eventually set to zero. At the mean-field level the fluctuation meson fields acquire a finite expectation value due to the net baryon number. An advantage of this method is that $\Gamma \propto E$ for a static system, thus identifying the eigenenergy E of the soliton.[16] Also, the loop expansion can be carried to higher order to systematically define radiative corrections in terms of fermion loop effects that have not been absorbed into the meson kinetic and potential terms.

An interesting feature of the type of formalism we are employing is that the dynamical quark self-energy can be absolutely confining if there is sufficient strength in the effective gluon propagator at low momenta.[17] With confinement embodied in the translationally invariant amplitude $B(x-y)$, which is both the scalar piece of the self-energy and the chiral meson-quark vertex, the mechanism for self-confining soliton solutions here can be quite different from that of color dielectric models.[18] No scalar background field is necessary to implement a divergence of the quark self-energy at the edge of the soliton. Rather, the scalar field appearing here is the chiral partner of the $\bar{q}q$ Goldstone pion. With a confining quark self-energy that is translationally invariant, it is the fluctuation meson fields that are necessary to provide a region where quarks can propagate and have a defined energy eigenvalue. A simple illustration of this mechanism is provided.

In Sec. II A the existing bilocal field methods we employ here are summarized in the context of the global color symmetry model (GCM).[19] This model of QCD is of the nonlocal NJL type and is parametrized by an effective two-point gluon propagator having a running mass with logarithmic falloff characteristic of asymptotic freedom. The transformation of quark fields to incorporate the chemical-potential constraint is made in Sec. II B and the coupling to σ and π meson fluctuations in chiral format is also discussed. The energy functional for the static soliton solution is obtained from the Legendre transformation in Sec. III at the lowest level of the loop expansion. The equations of motion for both the meson and quark sectors are also obtained. Particular attention is paid to the self-consistency in the valence quark eigenenergies that arises from the dynamical content of the self-energy, and the corresponding wave-function renormalization factor is incorporated. In Sec. IV the soliton formation mechanism in the presence of a self-confining quark self-energy is illustrated schematically. Summary and discussion follow in Sec. V.

II. FORMALISM

A. Global color symmetry model and bilocal fields

The GCM is defined in Euclidean metric by the action[19]

$$S[\bar{q}q]=-\int d^4x\, d^4y \left[\bar{q}(x)(\gamma\cdot\partial+m)\delta(x-y)q(y) + \frac{g^2}{2}j^a_\mu(x)D_{\mu\nu}(x-y)j^a_\nu(y)\right], \tag{2.1}$$

where the quark current is $j^a_\mu(x)=\bar{q}(x)(\lambda^a/2)\gamma_\mu q(x)$. In the limit as the small current quark mass $m\to 0$, the GCM has, for the case of two quark flavors employed here, $SU(2)_L\otimes SU(2)_R$ chiral symmetry. Also, as the name indicates, the GCM has global color symmetry. The GCM can be looked upon as a nonlocal version of the NJL model. The color algebra, chiral symmetry, and the association of the function $D_{\mu\nu}$ appearing in (2.1) with an effective two-point gluon function make this model capable of describing some aspects of QCD.[11,13] For convenience we take the effective two-point gluon propagator to be diagonal in Lorentz indices $D_{\mu\nu}(x-y)=\delta_{\mu\nu}D(x-y)$. The point of view of the GCM is that $D(x-y)$ is a parameter function for the model which is to contain at least a running coupling constant $\alpha_s(q^2)$ to incorporate the asymptotic freedom of QCD at large Euclidean momenta. A variety of forms have been employed for numerical work within the GCM (Refs. 11 and 13) and in other studies.[17] For present purposes the typical form

$$D(q)=\int d^4x\, e^{-iq\cdot x}D(x)=\frac{4\pi}{q^2}\frac{\alpha_s(q^2)}{g^2} \tag{2.2}$$

shall suffice with $\alpha_s(q^2)=12\pi/33\ln(1+\tau+q^2/\Lambda^2)$ where $\Lambda\sim 200$ MeV is the QCD scale parameter and τ parametrizes the infrared strength. The GCM has proved successful in the description of the low-mass meson spectrum and dynamics.[13]

The Euclidean space generating functional for fermion Green's functions is given by

$$Z[\bar{\eta},\eta]=N\int D\bar{q}\, Dq \exp\left\{S[\bar{q}q]+\int d^4x(\bar{\eta}q+\bar{q}\eta)\right\}. \tag{2.3}$$

In the limit of vanishing quark sources $\bar{\eta}$ and η, this is equivalent to the canonical partition function of statistical mechanics. The implicit couplings of quarks to composite $q\bar{q}$ fields carrying meson quantum numbers could be revealed by perturbative construction of the quark propagator through functional differentiation. However, the auxiliary bilocal field method provides an efficient and nonperturbative way to expose such degrees of freedom.[19] Fierz reordering applied to the quark Grassmann fields transforms the current-current term so that

$$\frac{1}{2}\int d^4x\, d^4y\, j^a_\mu(x)D(x-y)j^a_\mu(y) = -\frac{1}{2}\int d^4x\, d^4y\, \mathcal{J}^\theta(x,y)D(x-y)\mathcal{J}^\theta(y,x), \tag{2.4}$$

where $\mathcal{J}^\theta(x,y)=\bar{q}(x)\Lambda^\theta q(y)$ acts as a bilocal source. These two forms are illustrated in Figs. 1(a) and (b). The discrete index θ ranges over the terms of distinct transformation character in Lorentz, flavor, and color space produced by the Fierz reordering relation

$$\left[\frac{\lambda^a}{2}\gamma_\mu\right]_{ij}\left[\frac{\lambda^a}{2}\gamma_\mu\right]_{lm}=(\Lambda^\theta)_{im}(\Lambda^\theta)_{lj}. \tag{2.5}$$

The Λ are direct products of Lorentz, flavor, and color matrices. In the present case wherein $D_{\mu\nu}$ is taken to be diagonal, terms that Lorentz-transform as scalar, vector, pseudoscalar, and axial vector are produced. With two flavors of quarks each Λ is either isoscalar or isovector. The Fierz reordering of the color λ matrices yields color-singlet and color-octet terms. We follow the bosonization procedure[11,12] in which the bilocal quark current structure of Eq. (2.4) is translated into a functional integration over auxiliary Bose fields carrying the quantum number sets θ. Fluctuations in these fields will be interpreted as effective meson fields. For the fluctuations we will ignore the color-octet sector and deal only with color-singlet effective meson fields. This is not completely satisfactory, since part of the color structure of the model action is thereby discarded. However, it has been shown[14] that the complete color structure may be kept by the use of a further Fierz reordering so that bilocal combinations of quark fields that are not color singlets appear only in the form of diquark fields $q(x)q(y)$ and $\bar{q}(x)\bar{q}(y)$. In this work we seek a static mean-field quark-meson model, and the retention of just color-singlet composite fields can be viewed as ignoring correlations that are expressible as diquark degrees of freedom.

With the bilocal form (2.4) for the quartic dependence on quark fields, the generating functional is

$$Z[\bar{\eta},\eta]=N\int D\bar{q}\, Dq \exp\left[-(\bar{q}G_0^{-1}q) + \frac{g^2}{2}(\mathcal{J}^\theta D\tilde{\mathcal{J}}^\theta)+(\bar{\eta}q+\bar{q}\eta)\right], \tag{2.6}$$

where the bracket notation is meant to signify the usual space-time integrations, $\tilde{\mathcal{J}}(x,y)=\mathcal{J}(y,x)$, and $G_0^{-1}(x,y)=(\gamma\cdot\partial+m)\delta(x-y)$. The quartic current-current term can be translated into a functional integration over auxiliary Bose fields by first inserting into (2.6) the constant factor

$$N'=[\det(1/g^2D)]^{-1/2} = \int D\mathcal{B}\exp\left[-\int d^4x\, d^4y\frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(y,x)}{2g^2D(x-y)}\right] \tag{2.7}$$

where the notation $\int D\mathcal{B}$ stands for $\int D\mathcal{B}^{\theta_1}D\mathcal{B}^{\theta_2}\ldots$, and θ is summed in the integrand. With a shift $\mathcal{B}^\theta(x,y)\to\mathcal{B}^\theta(x,y)+g^2D(x-y)\mathcal{J}^\theta(y,x)$, the current-

current term is eliminated in favor of a term quadratic in $\mathcal{B}$ and two terms bilinear in $\mathcal{J}$ and $\mathcal{B}$. The two bilinear terms are equal due to symmetric integration, and, after adjustment of the normalization constant, the generating functional is reexpressed in the equivalent form

$$Z[\bar{\eta},\eta]=N\int D\bar{q}\,Dq\,D\mathcal{B}\exp\left[-(\bar{q}G^{-1}q)-\frac{1}{2}\left[\mathcal{B}^{\theta}\frac{1}{g^2D}\tilde{\mathcal{B}}^{\theta}\right]+(\bar{\eta}q+\bar{q}\eta)\right], \tag{2.8}$$

where

$$G^{-1}(x,y)=(\gamma\cdot\partial+m)\delta(x-y)+\Lambda^{\theta}\mathcal{B}^{\theta}(x,y)\ . \tag{2.9}$$

The quarks are Yukawa coupled to the auxiliary Bose field variables of integration with bare vertices Λ^{θ}. The action in (2.8) is now bilinear in the quark fields allowing the Grassmann integration to be performed in the standard way. The result is a generating functional which is written completely in terms of the bilocal Bose fields

$$Z[\bar{\eta},\eta]=N\int D\mathcal{B}\exp\{S[\mathcal{B}]+(\bar{\eta}G\eta)\}\ , \tag{2.10}$$

where the bosonized action is

$$S[\mathcal{B}]=\mathrm{Tr\,Ln}G^{-1}[\mathcal{B}]-\int d^4x\,d^4y\frac{\mathcal{B}^{\theta}(x,y)\mathcal{B}^{\theta}(y,x)}{2g^2D(x-y)}\ . \tag{2.11}$$

In the limit of a local NJL model, $D(x-y)\rightarrow\delta(x-y)$ in (2.4), and the corresponding limit of (2.11) becomes $[D(x-y)]^{-1}\rightarrow\delta(x-y)$. At this point we take $\bar{\eta}=\eta=0$ and consider the vacuum-to-vacuum amplitude, since we are not prepared to investigate the quark propagator until the level of approximation for the Bose fields $\mathcal{B}$ has been defined.

B. Valence quarks and effective meson fields

To deal with a ground-state configuration with valence quarks, we add a constraint on the baryon number through a chemical potential μ. After the canonical transformation of quark fields

$$q(x)\rightarrow q'(x)=e^{\mu x_4}q(x)\ , \tag{2.12}$$

the ground-state amplitude from (2.8) is reexpressed as

$$Z[\mu]=N\int D\bar{q}'\,Dq'\,D\mathcal{B}\exp\left\{-[\bar{q}'G^{-1}(\mu)q']-\frac{1}{2}\left[\mathcal{B}^{\theta}\frac{1}{g^2D}\tilde{\mathcal{B}}^{\theta}\right]\right\}, \tag{2.13}$$

where the chemical-potential dependence induced for the inverse propagator is

$$\begin{aligned}G^{-1}(\mu;x,y)&=e^{\mu x_4}G^{-1}(x,y)e^{-\mu y_4}\\&=(\gamma\cdot\partial+m-\gamma_4\mu)\delta(x-y)\\&\quad+e^{\mu x_4}\Lambda^{\theta}\mathcal{B}^{\theta}(x,y)e^{-\mu y_4}\ .\end{aligned} \tag{2.14}$$

The constraint for the baryon number n_b is imposed through

$$n\equiv n_b\int dx_4=\frac{\partial}{\partial\mu}\ln Z[\mu]\ . \tag{2.15}$$

In the case of statistical mechanics, where the range of x_4 integration is the inverse temperature β, (2.12) is the fugacity transformation that takes the canonical partition function into the grand canonical partition function with appropriate adjustment of boundary conditions. Besides the familiar shift of the time derivative, the additional μ dependence in (2.14) is due to the nonlocality of the Bose fields. The four-fermion interaction that is recovered upon integration over $\mathcal{B}$ remains independent of μ. In later developments, a propagator G associated with (2.14) will be required. With appropriate boundary conditions, the μ dependence of G will serve to shift the pole structure in the momentum component conjugate to x_4-y_4, so that valence and vacuum configurations are treated together in the usual way. Upon Grassmann integration over the quark fields, the ground-state amplitude (2.13) becomes

$$Z[\mu]=N\int D\mathcal{B}\exp S[\mu,\mathcal{B}]\ , \tag{2.16}$$

where the bosonized action can be written as

$$S[\mu,\mathcal{B}]=\mathrm{Tr}(\mathrm{Ln}G^{-1}[\mu,\mathcal{B}]-\mathrm{Ln}G^{-1}[\mu=0,\mathcal{B}])+S[\mathcal{B}]\ , \tag{2.17}$$

with the vacuum action given by

$$S[\mathcal{B}]=\mathrm{Tr\,Ln}G^{-1}[\mu=0,\mathcal{B}]-\frac{1}{2}\left[\mathcal{B}^{\theta}\frac{1}{g^2D}\tilde{\mathcal{B}}^{\theta}\right]. \tag{2.18}$$

We choose this separation in order to define effective meson fields from an expansion of the vacuum action.

For expansion of $S[\mathcal{B}]$, the vacuum configuration of $\mathcal{B}(x,y)$ should be identified so that propagating boson fluctuations above that may be exposed. We follow the treatment of Cahill and Roberts[11] and expand the action about the saddle point $\mathcal{B}_0(x,y)$, defined by $\delta S/\delta\mathcal{B}_0=0$. We work only to zero order ($\hbar^0$) in $\mathcal{B}$ field loops, and at this level the vacuum expectation value and the saddle-point configuration $\mathcal{B}_0$ coincide. Since from (2.9)

$$\frac{\delta}{\delta\mathcal{B}_0^{\theta}(x,y)}\mathrm{Tr\,Ln}G^{-1}[\mathcal{B}_0]=\mathrm{tr}(\Lambda^{\theta}G[\mathcal{B}_0;y,x])\ , \tag{2.19}$$

where tr denotes the trace on discrete indices, translationally invariant solutions for each $\mathcal{B}_0^{\theta}$ satisfy the self-consistent equation

$$\mathcal{B}_0^{\theta}(x-y)=g^2D(x-y)\mathrm{tr}(\Lambda^{\theta}G[\mathcal{B}_0;x-y])\ . \tag{2.20}$$

The vacuum quark self-energy produced at this level is therefore identified from (2.9) to be $\Sigma(x-y)=\Lambda^{\theta}\mathcal{B}_0^{\theta}(x-y)$, and (2.20) produces the self-consistent equation

$$\Sigma(x-y)=g^2D(x-y)\frac{\lambda^a}{2}\gamma_{\mu}G[\mathcal{B}_0;x-y]\frac{\lambda^a}{2}\gamma_{\mu}\ . \tag{2.21}$$

Here the Fierz reordering has been reversed to emphasize that the structure is a Schwinger-Dyson equation for Σ with bare quark-gluon vertices and a self-consistent quark propagator. The momentum space form of (2.21) is

$$\Sigma(p)=i\not{p}[A(p^2)-1]+B(p^2)$$
$$=g^2\int\frac{d^4q}{(2\pi)^4}D(p-q)\frac{\lambda^a}{2}\gamma_\mu\frac{1}{i\not{q}+m+\Sigma(q)}\frac{\lambda^a}{2}\gamma_\mu\ , \tag{2.22}$$

and the structure is illustrated in Fig. 3. Numerical solutions for the amplitudes A and B have been obtained in recent studies.[13,17] For the remaining formal developments we shall assume that convenient forms for the amplitudes A and B are available. We are particularly interested in retaining the dependence of the final soliton field equations upon these amplitudes for two reasons. First, for suitable infrared strength of the effective gluon propagator D, the resulting quark self-energy can be self-confining.[17] This provides the possibility of a cavity formation without the assistance of the additional scalar field that is often employed for soliton models. Second, the amplitude B is directly related to the distributed vertex for coupling of quarks to the $q\bar{q}$ Goldstone modes. This feature is of interest to us even in the circumstance that the self-energy contribution is approximated by a constant constituent mass.

The nature of the $q\bar{q}$ fluctuations above the $\mathcal{B}_0(x-y)$ configurations can be investigated from the associated propagators. The latter can be constructed to the desired level in the loop expansion through functional expansion of the vacuum action $S[\mathcal{B}]$ in powers of the fluctuations $\hat{\mathcal{B}}$ defined by

$$\hat{\mathcal{B}}^\theta(x,y)=\mathcal{B}^\theta(x,y)-\mathcal{B}_0^\theta(x-y)\ . \tag{2.23}$$

We shall limit our considerations to the composite fluctuation Bose fields that are scalar isoscalar and pseudoscalar isovector in character. The corresponding pair of bare vertices Λ^θ from the Fierz reordering (2.5) are

$$\Lambda^\theta=\frac{\sqrt{2}}{3}\{1,i\gamma_5\tau\}\ , \tag{2.24}$$

and the four associated fields $\hat{\mathcal{B}}^\theta$ may be denoted by

$$\hat{\mathcal{B}}^\theta(x,y)=\hat{\mathcal{B}}^\theta(r;R)=\frac{3}{\sqrt{2}}\{\hat{\sigma}(r;R),\pi(r;R)\}\ . \tag{2.25}$$

It is convenient to use a relative coordinate $r=x-y$ and a center-of-mass coordinate $R=(x+y)/2$ for the bilocal meson fields. We wish to limit the number of degrees of freedom by restricting consideration to the internal meson eigenstates of lowest mass. The dependence of the fields $\hat{\sigma}$ and π upon r will then be described by the associated form factor, and the remaining dependence upon R will be taken up by local dynamical field variables.

A general procedure for this localization is the following.[14] The vacuum action from (2.18), with an explicit lowest-order fluctuation term, can be written

$$S[\mathcal{B}]=S[\mathcal{B}_0]-\tfrac{1}{2}\int\frac{d^4p\,d^4q\,d^4P}{(2\pi)^{12}}\hat{\mathcal{B}}^\theta(p;-P)$$
$$\times\mathcal{D}_\theta^{-1}(p,q;P)\hat{\mathcal{B}}^\theta(q;P)-R[\hat{\mathcal{B}}]\ , \tag{2.26}$$

where $R[\hat{\mathcal{B}}]$ contains the remaining higher-order terms and θ is summed over the σ and π channels. In (2.26) we have used a momentum representation in which p and P are Euclidean momenta conjugate to r and R, respectively. Analysis reveals that $\mathcal{D}_\theta^{-1}$ is Hermitian, with eigenfunctions Γ_n and real eigenvalues Δ_n^{-1} satisfying[14]

$$\int\frac{d^4q}{(2\pi)^4}\mathcal{D}_\theta^{-1}(p,q;P)\Gamma_n^\theta(q;P)=\Delta_{\theta,n}^{-1}(P^2)\Gamma_n^\theta(p;P)\ . \tag{2.27}$$

The total momentum of the propagating meson mode is P while p or q are relative momenta for the $q\bar{q}$ content. The eigenfunctions Γ_n describe the internal structure for the nth mode. The condition for on-mass-shell free meson modes is evidently $\Delta_{\theta,n}^{-1}(-M^2)=0$, where M is the invariant mass. For these physical modes (2.27) becomes a ladder Bethe-Salpeter equation for bound states. This structure is dictated by the inherent ladder content of the self-consistent equation (2.20) for the bilocal field configurations that determine the self-energy Σ.

From (2.26) and (2.27), only the projections of $\hat{\mathcal{B}}$ onto the eigenstates Γ enter the quadratic term of (2.26). With a truncation to the internal states of lowest invariant mass, the resulting projected field variables shall be denoted by $\hat{\sigma}(P)$ and $\pi(P)$. In the pion channel, for example, the eigenfunction expansion is truncated to the single term

$$\pi(q;P)=\Gamma_\pi(q;P)\pi(P)\ , \tag{2.28}$$

or, in coordinate space,

$$\pi(r;R)=\int d^4R'\,\Gamma_\pi(r;R-R')\pi(R')\ . \tag{2.29}$$

A corresponding form is used for the $\hat{\sigma}$ field. For zero-current quark mass, $\Delta_\pi^{-1}(P^2=0)=0=m_\pi^2$, and Eq. (2.27) for the form factor reduces to the Schwinger-Dyson equation (2.22) for the scalar component B of the self-energy, that is, $\Gamma_\pi(q;P^2=0)=B(q^2)$. This $q\bar{q}$ fluctuation is thus the Goldstone mode associated with the breaking of chiral symmetry through the generation of dynamical quark mass. The $\hat{\sigma}$ and π components mix freely under a chiral transformation, and only a single form factor $\Gamma_\pi(q)=\Gamma_\sigma(q)=B(q^2)$ is required.[20]

In this limit, fluctuation fields with the form

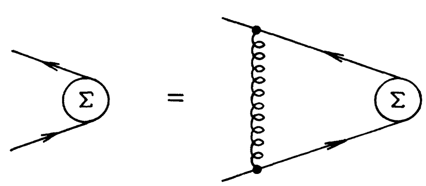

FIG. 3. An illustration of the self-consistent equation of the Schwinger-Dyson type that arises for the quark self-energy when the bilocal boson fields are evaluated at the saddle-point level. See Eqs. (2.21) and (2.22).

$$\Lambda^\theta \hat{B}^\theta(x,y) = B(x-y)\left[\hat{\sigma}\left[\frac{x+y}{2}\right] + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}\left[\frac{x+y}{2}\right]\right], \tag{2.30}$$

used in an earlier consideration[11] of soliton models of the present type, are seen to be appropriate for the zero-momentum Goldstone modes. The localized meson fields in (2.30) are dimensionless. The factorized form in $x-y$ and $(x+y)/2$ is an approximation due to use of the $P^2=0$ vertex for all P. The expansion (2.26) for the vacuum action becomes

$$S[\mathcal{B}] = S[\mathcal{B}_0] - \tfrac{1}{2}\int \frac{d^4P}{(2\pi)^4}[\hat{\sigma}(-P)\Delta_\sigma^{-1}(P^2)\hat{\sigma}(P) + \boldsymbol{\pi}(-P)\Delta_\pi^{-1}(P^2)\boldsymbol{\pi}(P)] - R[\hat{\sigma},\boldsymbol{\pi}], \tag{2.31}$$

where now

$$\Delta_\theta^{-1}(P^2) = \int \frac{d^4p\, d^4q}{(2\pi)^8} B(p^2)\mathcal{D}_\theta^{-1}(p,q;P)B(q^2). \tag{2.32}$$

With retention of the first two terms in the momentum expansion about $P^2=0$, we have $\Delta_\theta^{-1}(P^2) \simeq f_\theta^2(P^2+m_\theta^2)$ in terms of decay constants f_θ and masses m_θ. The remainder $R[\hat{\sigma},\boldsymbol{\pi}]$ collects terms beyond second order in the fields and contains derivative as well as local terms. Contact with local models of the linear sigma[9] type may be made after neglect of the derivative terms of R. In this case, and with a zero-current quark mass, (2.31) reduces to the local form[11]

$$S[\mathcal{B}] = S[\mathcal{B}_0] + \int d^4R\,[\tfrac{1}{2}f_\pi^2 \hat{\sigma}(R)\Box_R \hat{\sigma}(R) + \tfrac{1}{2}f_\pi^2 \boldsymbol{\pi}(R)\Box_R \boldsymbol{\pi}(R) - V(\chi^2(R))], \tag{2.33}$$

where $\sigma(R) = 1+\hat{\sigma}(R)$ and $\chi^2(R) = \sigma^2(R)+\boldsymbol{\pi}^2(R)$. Here $m_\pi = 0$, and a closed-form expression for the self-interaction $V(\chi^2(R))$, which contains the σ mass term, is given in Ref. 11. It has turning points at $\chi^2=0$ and at the vacuum configuration $\chi^2=1$ corresponding to a local maximum and an absolute minimum, respectively. A simplified form that respects these properties is $V(\chi^2) = c(\chi^2-1)^2$, where $c = f_\pi^2 m_\sigma^2/8$. With a small current mass m included in the quark propagators that make up V, a pion mass term with $m_\pi \propto m$ is generated. The quark loop integrals that give the meson parameters f_π, m_π, m_σ and the interaction $V(\chi^2)$ from expansion of the vacuum action are automatically regulated by the meson form factors. Explicit expressions can be found in Refs. 11 and 19. Typical values that can be obtained this way[11] are $f_\pi = 72$ and $m_\sigma = 940$ MeV, while the experimental pion mass is fitted through use of a current quark mass of about 10 MeV.

The vacuum portion of the action that we shall take for a soliton model is therefore

$$\begin{aligned} S[\hat{\sigma},\boldsymbol{\pi}] &= S[\mathcal{B}] - S[\mathcal{B}_0] \\ &= -\int d^4R\,\{\tfrac{1}{2}f_\pi^2 \hat{\sigma}(R)[-\Box_R + m_\sigma^2]\hat{\sigma}(R) + \tfrac{1}{2}f_\pi^2 \boldsymbol{\pi}(R)[-\Box_R + m_\pi^2]\boldsymbol{\pi}(R) + U(\hat{\sigma},\boldsymbol{\pi})\}, \end{aligned} \tag{2.34}$$

where

$$U(\hat{\sigma}(R),\boldsymbol{\pi}(R)) = V(\chi^2(R)) - \tfrac{1}{2}f_\pi^2 m_\sigma^2 \hat{\sigma}^2. \tag{2.35}$$

When the fields are rescaled to absorb the decay constant f_π, the complete action from (2.17), including the valence quark component, is

$$S[\mu,\hat{\sigma},\boldsymbol{\pi}] = \mathrm{Tr}(\mathrm{Ln}G^{-1}[\mu,\hat{\sigma},\boldsymbol{\pi}] - \mathrm{Ln}G^{-1}[0,\hat{\sigma},\boldsymbol{\pi}]) - \tfrac{1}{2}(\hat{\sigma},\Delta_\sigma^{-1}\hat{\sigma}) - \tfrac{1}{2}(\boldsymbol{\pi},\Delta_\pi^{-1}\boldsymbol{\pi}) - U[\hat{\sigma},\boldsymbol{\pi}], \tag{2.36}$$

where the constant $S[\mathcal{B}_0]$ has been discarded, and the notation

$$(\hat{\sigma},\Delta_\sigma^{-1}\hat{\sigma}) = \int d^4R\, \hat{\sigma}(R)(-\Box + m_\sigma^2)\hat{\sigma}(R) \tag{2.37}$$

has been used. The chemical potential appears only in the fermion Tr Ln term of the action in (2.36) and this term will generate a meson source from valence quarks. The inverse quark propagator occurring in (2.36) is, for $\mu=0$,

$$G^{-1}(x,y) = G_0^{-1}(x,y) + B(x-y)f_\pi^{-1} \times \left[\hat{\sigma}\left[\frac{x+y}{2}\right] + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\left[\frac{x+y}{2}\right]\right], \tag{2.38}$$

where

$$G_0^{-1}(x,y) = \gamma\cdot\partial_x A(x-y) + m\delta(x-y) + B(x-y) \tag{2.39}$$

$$= (\gamma\cdot\partial_x + m)\delta(x-y) + \Sigma(x-y). \tag{2.40}$$

The scalar term $B(x-y)$ in G_0^{-1} serves as the vacuum value of the total bilocal scalar field. For $\mu\neq 0$, the inverse propagator is obtained from the above via

$$G^{-1}(\mu;x,y) = e^{\mu x_4}G^{-1}(x,y)e^{-\mu y_4}, \tag{2.41}$$

and one may identify the valence quark self-energy in the absence of meson fields as

$$\Sigma(\mu,x-y) = e^{\mu x_4}\Sigma(x-y)e^{-\mu y_4} = e^{\mu x_4}\Lambda^\theta \mathcal{B}_0^\theta(x-y)e^{-\mu y_4}. \tag{2.42}$$

In a representation in terms of the momentum component or frequency conjugate to x_4-y_4, there will be a μ-dependent shift of the frequency for the self-energy function. This induces a self-consistency for the frequency location for poles of the associated propagator that will be retained in the subsequent analysis.

III. ENERGY FUNCTIONAL FOR A STATIC SOLITON

We shall use $b^\alpha(x) = [\hat{\sigma}(x),\boldsymbol{\pi}(x)]$ to denote the four mesonic fields so that the action (2.36) can be written in the compact form

$$S[\mu,b] = \mathrm{Tr}(\mathrm{Ln}G^{-1}[\mu,b] - \mathrm{Ln}G^{-1}[0,b]) - \tfrac{1}{2}(b^\alpha,\Delta_\alpha^{-1}b^\alpha) - U[b], \tag{3.1}$$

where $\Delta_\alpha^{-1}(x,y)=(-\Box_x+m_\alpha^2)\delta(x-y)$ with a common mass for the three π components. To initiate a treatment via a loop expansion, we introduce external sources $J^\alpha(x)$ for the meson fields and identify the standard generating functionals Z and W through[21]

$$Z[J,\mu]=N\int Db\,\exp S[\mu,b,J]=e^{W[J,\mu]}\ , \tag{3.2}$$

where

$$S[\mu,b,J]=S[\mu,b]+(J^\alpha,b^\alpha)\ . \tag{3.3}$$

The loop expansion for $W[J,\mu]$ is developed from the expansion of $S[\mu,b,J]$ about the saddle point $b_0^\alpha(x)$ defined by

$$\frac{\delta}{\delta b_0^\alpha(x)}S[\mu,b_0,J]=0\ , \tag{3.4}$$

in the form

$$S[\mu,b,J]=S[\mu,b_0,J]+\hat{S}[\mu,b_0,\hat{b}]\ , \tag{3.5}$$

where the shifted action is

$$\hat{S}[\mu,b_0,\hat{b}]=-\tfrac{1}{2}(\hat{b}^{\,\alpha},\hat{\Delta}_\alpha^{-1}\hat{b}^{\,\alpha})-\hat{U}[b_0,\hat{b}]\ . \tag{3.6}$$

Here $\hat{b}^{\,\alpha}(x)=b^\alpha(x)-b_0^\alpha(x)$ are the shifted fields, and $\hat{U}$ collects all terms of second and higher order in $\hat{b}$ except the explicit second-order term that is diagonal in α. The saddle-point fields b_0 are functionals of μ and J which, by definition, vanish in the vacuum limit. However, for finite μ, the fields b_0 retain condensed values due to the net baryon number. The inverse propagator $\hat{\Delta}_\alpha^{-1}$ is a second-order functional derivative of S and includes the bare inverse propagator Δ_α^{-1}, contributions from the self-interaction term $U[b]$ and a $q\bar{q}$ loop from the fermion Tr Ln terms. With (3.5), the generating functional $Z[J,\mu]$ from (3.2) becomes

$$Z[J,\mu]=e^{S[\mu,b_0,J]}N\int D\hat{b}\,e^{\hat{S}[\mu,b_0,\hat{b}]} \tag{3.7}$$

$$=e^{S[\mu,b_0,J]}Ne^{W_1[\mu,b_0]}\ . \tag{3.8}$$

The functional W_1 defined above is at least of order $\hbar$ in meson loops, and of order $\hbar^2$ in fermion loops. The saddle-point action is of order $\hbar^0$ in meson contributions and of order $\hbar$ in fermion contributions (from the Tr Ln terms). The lowest term in the loop expansion is obtained by discarding W_1 for the result

$$W[J,\mu]\simeq\ln N+S[\mu,b_0,J]\ . \tag{3.9}$$

A. Legendre transformation

The variables from which an energy functional for a mean-field soliton is to be constructed are the baryon number

$$n\equiv n_b\int dx_4=\frac{\delta W}{\delta\mu} \tag{3.10}$$

and the field expectation values

$$\tilde{b}_c^{\,\alpha}(x)=\frac{\delta W}{\delta J^\alpha(x)}\equiv\langle 0^+|\Phi^\alpha(x)|0^-\rangle_{J\mu}/\langle 0^+|0^-\rangle_{J\mu}\ , \tag{3.11}$$

where $\Phi^\alpha(x)$ is the quantum field operator corresponding to the c-number field $b^\alpha(x)$ occurring in the functional path integral of (3.2). Both n and $\tilde{b}_c^{\,\alpha}$ are functionals of J and μ. Inversion of these relations allows n and $\tilde{b}_c^{\,\alpha}$ to become the independent variables of the effective action $\Gamma[\tilde{b}_c,n]$ introduced through the Legendre transformation

$$\Gamma[\tilde{b}_c,n]=W[J,\mu]-(J^\alpha,\tilde{b}_c^{\,\alpha})-\mu n\ . \tag{3.12}$$

Since the source that produces a given classical field $\tilde{b}_c^{\,\alpha}(x)$ is determined by

$$\frac{\delta\Gamma[\tilde{b}_c,n]}{\delta\tilde{b}_c^{\,\alpha}(x)}=-J^\alpha(x)\ , \tag{3.13}$$

the physical limit $J\to 0$ identifies the required physical ground-state expectation value $b_c^\alpha(x)$ as a saddle point of the effective action Γ. The equation of motion for $b_c^\alpha(x)$ in the physical ground state constrained by n is, therefore,

$$\frac{\delta\Gamma[b_c,n]}{\delta b_c^\alpha(x)}=0\ . \tag{3.14}$$

To impose time-translation invariance (ensuring a defined total energy E) the fields $b_c^\alpha(x)$ are taken to be static: $b_c^\alpha(x)=b_c^\alpha(\mathbf{x})$, and Γ is directly proportional to the energy functional,[16] viz.,

$$\Gamma[b_c,n]=-E[b_c,n]\int dx_4\ . \tag{3.15}$$

The explicit form for Γ to lowest order in the loop expansion is now required. Combination of (3.9) and (3.12) yields

$$\Gamma[\tilde{b}_c,n]=\ln N+S[\mu,b_0]+(J^\alpha,b_0^\alpha-\tilde{b}_c^{\,\alpha})-\mu n\ , \tag{3.16}$$

where the source-dependent terms have been collected together. The saddle-point fields $b_0^\alpha(x)$ and the classical fields $\tilde{b}_c^\alpha(x)$ are defined differently and do not cancel in general. However, the difference $b_0^\alpha-\tilde{b}_c^{\,\alpha}$, which originates from the functional W_1 defined in (3.7) and (3.8), is of higher order in $\hbar$ than the level at which we are presently working. In particular, the result $\tilde{b}_c^\alpha=b_0^\alpha(x)$ follows directly from the source derivative of the lowest-order $W[J,\mu]$ given by (3.9) after employing the saddle-point condition (3.4) that defines $b_0^\alpha[J,\mu]$. The effective action at the (Hartree) level of zero meson loops and one fermion loop is therefore

$$\Gamma[b_0,n]=S[\mu,b_0]-\mu n \tag{3.17}$$

$$=\mathrm{Tr}(\mathrm{Ln}G^{-1}[\mu,b_0]-\mathrm{Ln}G^{-1}[0,b_0])-\mu n$$
$$-\tfrac{1}{2}(b_0^\alpha,\Delta_\alpha^{-1}b_0^\alpha)-U[b_0]\ . \tag{3.18}$$

Here we have chosen the normalization constant so that the vacuum value of Γ is zero. The functional $\mu[b_0,n]$ is to be treated as the inversion of the baryon number expression (3.10), which at the present level is

$$n=\frac{\delta}{\delta\mu}S[\mu,b_0,J]\equiv\frac{\partial}{\partial\mu}\mathrm{Tr}\,\mathrm{Ln}G^{-1}[\mu,b_0]\ . \tag{3.19}$$

Although b_0 is a functional of μ, the saddle-point condition (3.4) has been used to obtain the second equality with a partial derivative. From (3.15) and (3.18), the energy functional can be separated into a quark component E_q and a meson component E_m. That is,

$$E[b_0,n]=E_q[b_0,n]+E_m[b_0,n] , \quad (3.20)$$

where

$$E_q[b_0,n]\left[-\int dx_4\right] =\mathrm{Tr}(\mathrm{Ln}G^{-1}[\mu,b_0]-\mathrm{Ln}G^{-1}[0,b_0])-\mu n , \quad (3.21)$$

and

$$E_m[b_0,n] =\int d^3x\{\tfrac{1}{2}b_0^\alpha(\mathbf{x})[-\Box_x+m_\alpha^2]b_0^\alpha(\mathbf{x})+U(b_0(\mathbf{x}))\} . \quad (3.22)$$

B. Quark sector

The valence quark energy contribution can be expressed in terms of eigenenergies through spectral decompositions of the fermion propagators required by (3.21).[22] With static meson fields, $G^{-1}(x,y)$ depends on time only through the variable $\tau=x_4-y_4$, and it is convenient to use the Fourier representation

$$G^{-1}(\omega;\mathbf{x},\mathbf{y})=\int d\tau\, e^{-i\omega\tau}G^{-1}(\tau;\mathbf{x},\mathbf{y}) . \quad (3.23)$$

The explicit form can be written as

$$\begin{aligned}G^{-1}(\omega;\mathbf{x},\mathbf{y})&=(\gamma_4 i\omega+\boldsymbol{\gamma}\cdot\boldsymbol{\partial}_x+m)\delta(\mathbf{x}-\mathbf{y})\\&\quad+\Sigma(\omega;\mathbf{x}-\mathbf{y})+V(\omega^2;\mathbf{x},\mathbf{y})\\&=\gamma_4 i\omega\delta(\mathbf{x}-\mathbf{y})+\gamma_4 H(\omega;\mathbf{x},\mathbf{y}) , \quad (3.24)\end{aligned}$$

where a Hamiltonian has been defined. The term describing interaction with the meson fields is

$$V(\omega^2;\mathbf{x},\mathbf{y})=B(\omega^2;\mathbf{x}-\mathbf{y})f_\pi^{-1} \times\left[\hat\sigma\left[\frac{\mathbf{x}+\mathbf{y}}{2}\right]+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\left[\frac{\mathbf{x}+\mathbf{y}}{2}\right]\right] , \quad (3.25)$$

and the self-energy is

$$\Sigma(\omega;\mathbf{x}-\mathbf{y})=B(\omega^2;\mathbf{x}-\mathbf{y})+(\gamma_4 i\omega+\boldsymbol{\gamma}\cdot\boldsymbol{\partial}_x) \times[A(\omega^2;\mathbf{x}-\mathbf{y})-\delta(\mathbf{x}-\mathbf{y})] . \quad (3.26)$$

The time-translation invariance of $G^{-1}(x,y)$ allows stationary eigenstates and associated eigenvalues $\hat\lambda_j(\omega)$ which satisfy

$$\int d^4y\, G^{-1}(x_4-y_4;\mathbf{x},\mathbf{y})\psi_{j,\omega}(y)=\hat\lambda_j(\omega)\psi_{j,\omega}(x) , \quad (3.27)$$

where

$$\psi_{j,\omega}(x)=(2\pi)^{-1/2}u_j(\mathbf{x})e^{i\omega x_4} . \quad (3.28)$$

With (3.23), the states u_j satisfy

$$\int d^3y\, G^{-1}(\omega;\mathbf{x},\mathbf{y})u_j(\mathbf{y})=\hat\lambda_j(\omega)u_j(\mathbf{x}) . \quad (3.29)$$

If we set $\hat\lambda_j(\omega)=i\gamma_4\lambda_j(\omega)$, then the relation to energy eigenvalues is

$$\lambda_j(\omega)=\omega-i\varepsilon_j(\omega) , \quad (3.30)$$

where (3.29) is equivalent to

$$\int d^3y\, H(\omega;\mathbf{x},\mathbf{y})u_j(\mathbf{y})=\varepsilon_j(\omega)u_j(\mathbf{x}) . \quad (3.31)$$

The index j labels the set of distinct states of the spectrum for a given value of ω. The physical values of the frequency variable ω will subsequently be identified from the pole structure of the contour-integral form of the quark Tr Ln terms in (3.21). Equation (3.31) will need to be solved explicitly only for the few ω values necessary to provide the valence states to reproduce the baryon number and flavor of the ground state.

We choose the normalization condition

$$\int d^3x\, u_{j'}^\dagger(\mathbf{x})u_j(\mathbf{x})=\delta_{j'j} . \quad (3.32)$$

The spectral expansions of the propagator and its inverse are

$$G(x,y)=-i\sum_j\int d\omega\,\psi_{j,\omega}(x)\frac{1}{\lambda_j(\omega)}\bar\psi_{j,\omega}(y) \quad (3.33)$$

and

$$\gamma_4 G^{-1}(x,y)=i\sum_j\int d\omega\,\psi_{j,\omega}(x)\lambda_j(\omega)\psi_{j,\omega}^\dagger(y) . \quad (3.34)$$

The standard Feynman boundary conditions are incorporated in the Euclidean form of $G(x,y)$ with the integration contour C taken along the real ω axis with closure in the lower half plane, as shown in Fig. 4. In the presence of a chemical potential μ, the appropriate boundary conditions for $G(\mu;x,y)$ are implemented through use of the same contour with the replacement $\lambda_j(\omega)\rightarrow\lambda_j(\omega+i\mu)$. The Tr Ln term of the valence quark energy functional becomes

$$\mathrm{Tr}[\mathrm{Ln}G^{-1}(\mu)-\mathrm{Ln}G^{-1}(0)] =\sum_j\int\frac{d\omega}{2\pi}e^{-i\omega\eta}\ln\left[\frac{\lambda_j(\omega')}{\lambda_j(\omega)}\right]\int dx_4 , \quad (3.35)$$

where $\omega'=\omega+i\mu$, and the limit $\eta\rightarrow0^+$ is implied. Integration by parts yields

$$\mathrm{Tr}[\mathrm{Ln}G^{-1}(\mu)-\mathrm{Ln}G^{-1}(0)] =\sum_j\int\frac{d\omega}{2\pi}e^{-i\omega\eta}\left[\frac{\omega\lambda_j'(\omega')}{\lambda_j(\omega')}-\frac{\omega\lambda_j'(\omega)}{\lambda_j(\omega)}\right]\left(-\int dx_4\right) \quad (3.36)$$

where $\lambda_j'(\omega)=d\lambda_j(\omega)/d\omega$.

The term μn, which must be subtracted from (3.36) to obtain the valence quark energies, can be cast in a similar form. From (3.19) and (3.35) we have

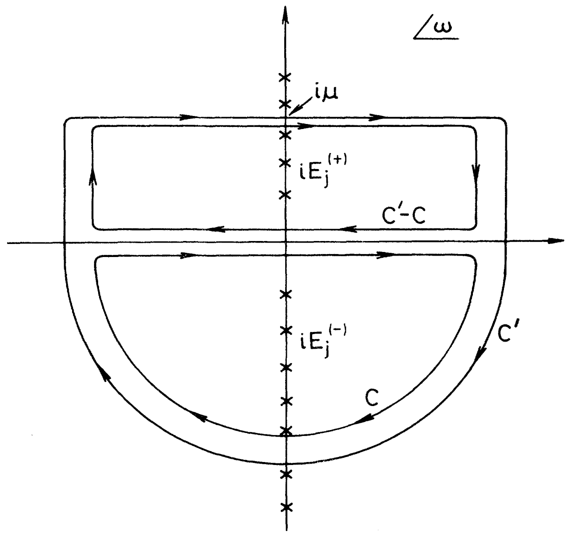

FIG. 4. Integration contours in the Euclidean frequency (or fourth component of momentum) plane for evaluation of the quark contribution to the soliton energy.

$$n \equiv n_b \int dx_4 = \frac{\delta}{\delta\mu} \sum_j \int \frac{d\omega}{2\pi} e^{-i\omega\eta} \ln \lambda_j(\omega') \int dx_4 \ ,$$

$$= i \sum_j \int \frac{d\omega}{2\pi} e^{-i\omega\eta} \frac{\lambda_j'(\omega')}{\lambda_j(\omega')} \int dx_4 \ . \quad (3.37)$$

Then from (3.21), the valence quark energy functional is

$$E_q[b_0,n] = \sum_j \int_c \frac{d\omega}{2\pi} e^{-i\omega\eta} \left[\frac{\omega'\lambda_j'(\omega')}{\lambda_j(\omega')} - \frac{\omega\lambda_j'(\omega)}{\lambda_j(\omega)} \right] , \quad (3.38)$$

where the first term of the integrand is related to the second by the frequency shift $\omega'=\omega+i\mu$. The integration contour C is the real ω axis with closure in the lower half plane. A shift of integration variable to ω' for the first term of (3.38) produces a shifted contour C' with the result

$$E_q[b_0,n] = \sum_j \int_{c'-c} \frac{d\omega}{2\pi} e^{-i\omega\eta} \frac{\omega\lambda_j'(\omega)}{\lambda_j(\omega)} \ . \quad (3.39)$$

The contours C, C' and $C'-C$ are shown in Fig. 4 along with the positions of poles given by $\lambda_j(\omega_p)=0$, which is equivalent to

$$\omega_p = i\varepsilon_j(\omega_p) \ . \quad (3.40)$$

This condition identifies the physical eigenvalues $E_j=-i\omega_p$ to be obtained from the self-consistent Dirac equation, which from (3.31) is

$$\int d^3y \, H(iE_j;\mathbf{x},\mathbf{y}) u_j(\mathbf{y}) = E_j u_j(\mathbf{x}) \ . \quad (3.41)$$

Only the positive energies in the interval $[0,\mu]$ are required. This is the equation of motion for the valence quark states.

To evaluate the residues of the spectral integral in (3.39), one may expand $\lambda_j(\omega)$ about ω_p given by (3.40) to obtain

$$\lambda_j(\omega) = \omega - i\varepsilon_j(\omega) = (\omega-\omega_p) Z_j(\omega,\omega_p) \ , \quad (3.42)$$

where

$$Z_j(\omega,\omega_p) = \lambda_j'(\omega_p) + \tfrac{1}{2}(\omega-\omega_p)\lambda_j''(\omega_p) + \cdots \ . \quad (3.43)$$

Thus $\lambda_j'(\omega)/\lambda_j(\omega)$ has unit residue at the poles, and (3.39) yields

$$E_q[b_0,n] = \sum_j E_j^{(+)} \theta(\mu - E_j^{(+)}) \ . \quad (3.44)$$

The required chemical potential μ is identified as the highest occupied single-particle level consistent with the baryon number, spin, and isospin of the system. The Dirac-sea contributions present in the individual terms of the formal expression (3.21) have canceled out to produce the finite result in (3.44). That is, both $W[J,\mu]$ and μn can be divergent due to contributions from the Dirac sea, but the effective action Γ is produced from their difference via (3.12) and is finite. The final result is equivalent to formulating the Legendre transformation with the finite subtracted quantities $\tilde{W}[J,\mu] = W[J,\mu] - \mu n[\mu=0]$ and $\tilde{n} = n[\mu] - n[\mu=0]$. The set of constants

$$Z_j = Z_j(\omega_p,\omega_p) = \lambda_j'(\omega_p) = 1 - i\varepsilon_j'(\omega_p) \quad (3.45)$$

produce wave-function renormalization so that the residue of the propagator $G(x,y)$ from (3.33) involves states $Z_j^{-1/2}\psi_{j,\omega}(x)$. If the self-energy functions $A(x-y)$ and $B(x-y)$ as well as the interaction $V(x,y)$ with mesons are taken to be static, the familiar result $Z_j \to u_j^\dagger A u_j$ is obtained.

C. Meson sector

According to (3.14), the equation of motion for the meson fields $b_0^\alpha(\mathbf{x})$ is

$$\frac{\delta\Gamma[b_0,n]}{\delta b_0^\alpha(\mathbf{z})} \equiv \frac{\delta}{\delta b_0^\alpha(\mathbf{z})} (E_m[b_0,n] + E_q[b_0,n]) = 0 \ . \quad (3.46)$$

With (3.22) for E_m and (3.44) for E_q, we obtain

$$0 = [-\Box_z + m_\alpha^2] b_0^\alpha(\mathbf{z}) + U_\alpha'(b_0(\mathbf{z})) + \frac{\delta}{\delta b_0^\alpha(\mathbf{z})} \sum_j \theta(\mu - E_j^{(+)}) E_j^{(+)} \ , \quad (3.47)$$

where the last term provides the valence quark source for the meson fields. Both the chemical potential μ and the energy eigenvalues $E_j^{(+)}$ have a functional dependence upon the meson fields. The dependence of $E_j^{(+)}$ upon the meson fields can be obtained from the interaction term of the Dirac equation (3.41), but due to the self-consistency evident in the energy dependence of the Hamiltonian appearing there, this can be ambiguous. The physical E_j is the special value of the function $\varepsilon_j[\omega,b_0]$ given by $E_j = \varepsilon_j[\omega = iE_j, b_0]$, where there is both explicit and implicit dependence upon b_0. A reliable procedure is to return to the contour-integral form for the valence quark energies, such as (3.39). There one deals with the quantity $\lambda_j[\omega,b_0]=\omega-i\varepsilon_j[\omega,b_0]$, where ω is an independent variable and the b_0 dependence is simpler. We show in the Appendix that this procedure yields

$$\frac{\delta}{\delta b_0^\alpha(\mathbf{z})}E_q[b_0,n]=\sum_j \theta(\mu-E_j^{(+)})\frac{1}{Z_j}\frac{\delta\varepsilon_j(\omega)}{\delta b_0^\alpha(\mathbf{z})}\Bigg|_{\omega=\omega_p}, \tag{3.48}$$

where ω_p is the location of the pole whose residue gives the physical $E_j^{(+)}$, and Z_j is the wave-function renormalization constant identified earlier in (3.45). Thus the above-mentioned ambiguity is eliminated by leaving ω as a free variable until after differentiation with respect to $b_0^\alpha(\mathbf{z})$. From the Dirac equation (3.31), together with the normalization condition (3.32), we obtain

$$\frac{\delta\varepsilon_j(\omega)}{\delta b_0^\alpha(\mathbf{z})}=\frac{\delta}{\delta b_0^\alpha(\mathbf{x})}\langle u_j|H(\omega,b_0^\alpha)|u_j\rangle \tag{3.49}$$

$$=\langle u_j|\frac{\delta}{\delta b_0^\alpha(\mathbf{z})}H(\omega,b_0^\alpha)|u_j\rangle\ . \tag{3.50}$$

In (3.49) we have employed a braket notation for three-space matrix elements, and in (3.50) the normalization condition (3.32) and the Hermitian nature of H have been used to eliminate the terms involving derivatives of the states. The only term of H which depends on b_0^α is the linear interaction term $\gamma_4 V$. We thus have, for the σ and π equations of motion, the explicit forms

$$0=[-\Box_z+m_\sigma^2]\sigma(\mathbf{z})+U'_\sigma(\sigma(\mathbf{z}),\pi(\mathbf{z}))+Q_\sigma(\mathbf{z}) \tag{3.51}$$

and

$$0=[-\Box_z+m_\pi^2]\pi(\mathbf{z})+\mathbf{U}'_\pi(\sigma(\mathbf{z}),\pi(\mathbf{z}))+\mathbf{Q}_\pi(\mathbf{z})\ , \tag{3.52}$$

where the meson sources provided by valence quarks are

$$Q_\sigma(\mathbf{z})=\sum_j\frac{1}{f_\pi Z_j}\int d^3x\, d^3y\, \bar{u}_j(x)B(-E_j^2;\mathbf{x}-\mathbf{y})\times\delta\left[\frac{\mathbf{x}+\mathbf{y}}{2}-\mathbf{z}\right]u_j(\mathbf{y}) \tag{3.53}$$

and

$$\mathbf{Q}_\pi(\mathbf{z})=\sum_j\frac{1}{f_\pi Z_j}\int d^3x\, d^3y\, \bar{u}_j(\mathbf{x})B(-E_j^2;\mathbf{x}-\mathbf{y})i\gamma_5\tau\times\delta\left[\frac{\mathbf{x}+\mathbf{y}}{2}-\mathbf{z}\right]u_j(\mathbf{y})\ . \tag{3.54}$$

In the limit of point coupling where the above function B is replaced by $\bar{B}\delta(\mathbf{x}-\mathbf{y})$, the sources reduce to the usual local form.

IV. SOLITON FORMATION WITH SELF-CONFINING DYNAMICAL QUARK MASS

The present soliton model departs from standard models in two main ways. The meson field equations (3.51) and (3.52) contain nonlocal coupling to valence quark sources, and the quark equation of motion (3.41) contains a translationally invariant dynamical self-energy. The scalar self-energy component $B(x-y)$ also plays the role of a distributed vertex for meson-quark coupling. The mechanism for soliton formation reduces to the familiar one in the local limit $B(x-y)\to\bar{B}\delta(x-y)$. To investigate the soliton mechanism in the presence of distributed coupling and a dynamical self-energy, we consider here the valence quark Dirac equation in momentum space with zero current mass and only a scalar meson field. This is

$$[i\not{p}A(p^2)+B(p^2)]u(\mathbf{p})+f_\pi^{-1}\int\frac{d^3p'}{(2\pi)^3}B\left[\frac{p+p'}{2}\right]\hat{\sigma}(\mathbf{p}-\mathbf{p}')u(\mathbf{p}')=0\ , \tag{4.1}$$

where $p_4=p'_4=iE$ and E is the energy eigenvalue. The self-energy amplitudes $A(p^2)$ and $B(p^2)$ depend upon the phenomenological gluon propagator employed. An interesting situation is created when the employed gluon propagator has sufficient strength in the low-momentum region to produce an absolutely confining dynamical mass.[17] In that case, there is no solution (discrete or continuum) to $i\not{p}A(p^2)+B(p^2)=0$ for timelike $p^2<0$. Hence $p^2+M^2(p^2)\neq 0$, where $M(p^2)=B(p^2)/A(p^2)$ is the dynamical mass which prevents quark propagation in the vacuum. A condensed meson field can modify the mass and so provide a region where quarks can propagate so that an energy eigenvalue is possible. Consider the simplified case where the meson field is spatially constant, so that in momentum space $\hat{\sigma}(\mathbf{p}-\mathbf{p}')\to(2\pi)^3 f_\pi\bar{\sigma}\delta(\mathbf{p}-\mathbf{p}')$, where $\bar{\sigma}$ is a dimensionless constant to characterize the strength. Then the eigenvalue equation (4.1) becomes simply

$$p^2+M^2(p^2)(1+\bar{\sigma})^2=0\ . \tag{4.2}$$

A physical eigenvalue E is obtained if there is a solution to (4.2) for timelike $p^2\equiv -E^2+\mathbf{p}^2<0$. This requires knowledge of $M(p^2)$ in the non-Euclidean region. For the sake of discussion, consider the behavior $M^2(p^2)=C^2-p^2$ shown in Fig. 5 as the upper solid line. This absolutely confining form obtains in the extreme limit of an effective gluon propagator that has only a zero momentum mode,[11,23] viz., $g^2D(q)\approx(2\pi)^4 3C^2/4\delta^4(q)$. The strength parameter C can be thought of as a characterization of the gluon condensate.[24] We use $C=0.5$ GeV corresponding to the strength of the dynamical mass obtained from typical numerical solutions[13,17] of the Schwinger-Dyson equation (2.22).

In this schematic model, then, the effective dynamical mass is $\hat{M}(p^2)=M(p^2)(1+\bar{\sigma})$ and a scalar meson field with strength below the vacuum value, i.e., $\bar{\sigma}<0$, leads to a reduced slope for $\hat{M}^2(p^2)$. This is illustrated in Fig. 5 by the lower solid line with $\bar{\sigma}=-\frac{1}{2}$. For $p^2>0$, a more realistic behavior is indicated by the long dashed lines in Fig. 5 to represent typical results from recent numerical solutions of the Schwinger-Dyson equation. We are interested only in the timelike region for the present discussion and we take the behavior shown in Fig. 5 to be typi-

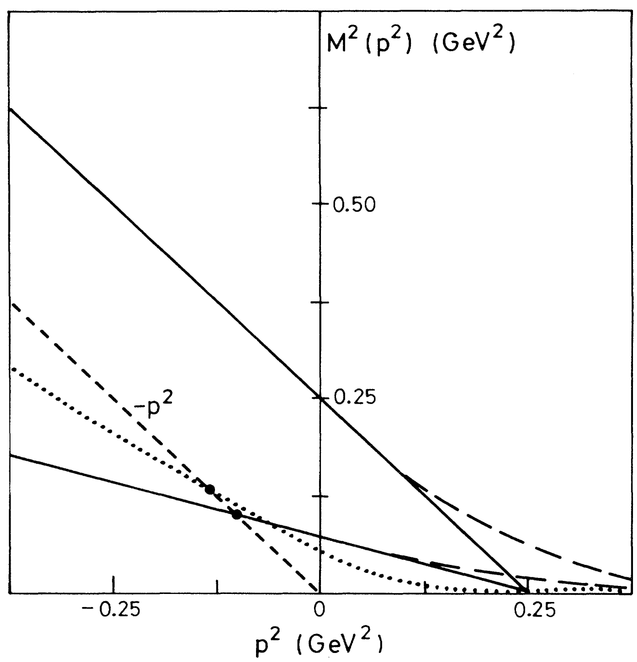

FIG. 5. The square of the dynamical quark mass is plotted versus Euclidean p^2. The upper solid line corresponds to no dynamical meson field and employs a frequently used schematical model for the self-energy which is absolutely confining and described in the text. Distributed coupling to a spatially constant and negative scalar meson field value produces the lower solid line for the effective mass squared with an intercept indicating physical propagation and a defined energy. The dotted line obtains in the limit of point coupling as described in the text. The long dashed lines indicate a behavior more realistic than that of the schematic model.

cal. Physical solutions to (4.2) occur for $-2<\bar{\sigma}<0$ at $p^2=-M_c^2$, where

$$M_c^2=C^2\frac{(1+\bar{\sigma})^2}{1-(1+\bar{\sigma})^2}\ , \tag{4.3}$$

and M_c may be considered a constituent mass. The quark energies in the medium characterized by $\bar{\sigma}$ are given by $E^2(\mathbf{p})=\mathbf{p}^2+M_c^2$. With a meson field of finite extent a number of discrete states of finite extent should be possible for the valence quarks in a self-consistent way. The confining dynamical mass should induce a large distance decay for the quark wave function that is faster than the characteristic exponential form of constant mass solutions and a baglike behavior might be expected.

If the point coupling limit is applied to the above uniform model, the effective dynamical mass becomes $\hat{M}(p^2)\rightarrow M(p^2)+C\bar{\sigma}$. The behavior of $\hat{M}^2$ is no longer linear, and for $\bar{\sigma}=-\frac{1}{2}$, the result is shown by the dotted line in Fig. 5, where the upper dashed line tail has been used for $M(p^2)$. A physical intercept in the timelike region occurs for $\bar{\sigma}<0$ and is given by

$$M_c^2=C^2\left[\frac{1-\bar{\sigma}^2}{2\bar{\sigma}}\right]^2 . \tag{4.4}$$

For the case considered here, the point coupling limit produces a 30% change in the constituent mass. In a realistic finite-range situation similar effects might be expected for the quark energies. A program of numerical investigations of this soliton mechanism in the finite-range case is under way.

V. SUMMARY AND DISCUSSION

In this work we have studied the development of a chiral quark-meson soliton model in which the meson degrees of freedom are composite objects produced from a previously developed bosonization of the action of the global color symmetry model of QCD. The only dynamical meson fields that we retain are the σ and π, and they include form factors that describe internal structure. We adopt the approximation of Cahill and Roberts wherein the scalar portion of the dynamical quark self-energy provides the quark-meson vertex or form factor for both σ and π in the limit of zero momentum and exact chiral symmetry. For the development of a soliton with valence quark configurations, we depart from previous work through the use of a chemical-potential constraint to define the energy functional from the effective action functional and we retain the nonlocality of the quark-meson vertex. In this way the valence quark contributions can be separated from those quark sea contributions that make up the meson kinetic and potential terms.

Only the lowest (Hartree) level of the loop expansion of the effective action functional is retained. No attention has been paid to the topic of spurious center-of-mass motion which is characteristic of mean-field approximations. The meson field equations of motion for a mean-field static soliton are given in Eqs. (3.51)–(3.54). The source terms due to valence quarks have a nonlocality due to meson substructure and also contain wave-function renormalization factors arising from the dynamically generated quark self-energy. Without these elements, the soliton equations reduce to the form expected for solitons from a linear sigma model.

Due to the bilocal nature of the employed boson fields, the dynamical quark mass is translationally invariant and depends self-consistently upon the quark energy. If the dynamical mass is self-confining, then solitons with absolute confinement are possible without the assistance of an additional scalar background field beyond the chiral partner of the pion. In Sec. IV, we have examined the Dirac equation for valence states with nonlocal meson coupling and a crude model for a self-confining dynamical mass. The role of the meson field in providing a region where quarks can propagate and have a defined energy eigenvalue is illustrated.

In point coupling relativistic field models of nuclear matter such as the Walecka model,[25] in which nucleons interact with scalar σ and vector ω mesons, the high-momentum behavior of quantum baryon loops generates troublesome instabilities and difficulties with convergence of techniques such as the loop expansion.[26] An example is provided by the incorrect singularity structure of the σ propagator from loop momenta at the nucleon mass scale.[27] At such short distances the relevant degrees of freedom lie outside the assumed elementary hadronic fields of the model. An alternative strategy is to consider effective Lagrangians in which the hadronic fields are accountable to some aspects of QCD substructure through derived distributed vertices. The soliton model for baryons in terms of quarks and composite mesons that we consider here may be useful in that regard.

The quantum $\hat{\sigma}$ field that arises from the $q\bar{q}$ correlations in the procedure we have adopted is not likely to correspond closely to the empirical σ field employed in nonchiral nucleon-meson models such as the Walecka $\sigma-\omega$ model. There the σ mass is required to be in the range 500–600 MeV for a successful description of the bulk properties of saturated nuclear matter. As mentioned earlier, the chiral σ mass arising from (2.32) is typically 1 GeV. This is characteristic also of the chiral σ model description of low-energy πN dynamics. The self-interactions imposed by $U[\hat{\sigma},\pi]$ to produce scalar $(\pi\pi)$ correlations appear to provide a net effective scalar field with the lower mass needed for successful phenomenology of the midrange NN attraction.[28] At this stage we are concerned with just the formation of a soliton model for a single nucleon in which the nonlocality $[B(x-y)\neq\bar{B}\delta(x-y)]$ of meson-quark coupling is included. The above considerations will be of concern if σN coupling is to be deduced from such a model.

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APPENDIX

We provide here details of the derivation of (3.48) for the meson source contribution $\delta E_q/\delta b_0(\mathbf{z})$ due to valence quarks. From (3.21), (3.35), and (3.37), the valence quark energy functional can be written as

$$E_q[b_0,n]=-\sum_j\int_C\frac{d\omega}{2\pi}e^{-i\omega\eta}\ln\left[\frac{\lambda_j(\omega')}{\lambda_j(\omega)}\right]+\mu n_b\ . \quad (A1)$$

Here the integration is along the μ-independent contour C shown in Fig. 4, and we recall that $\omega'=\omega+i\mu$ and $\lambda_j(\omega)=\omega-i\varepsilon_j(\omega)$. The dependence upon the $b_0^\alpha(\mathbf{z})$ resides in $\mu[b_0,n]$ (so that n_b is a constant) and in the energy eigenvalues $\varepsilon_j[\omega,b_0]$. With $b(\mathbf{z})$ denoting any one of the meson field components $b_0^\alpha(\mathbf{z})$, the source term is

$$\frac{\delta E_q}{\delta b(\mathbf{z})}=-\sum_j\int_C\frac{d\omega}{2\pi}e^{-i\omega\eta}\left[\lambda_j^{-1}(\omega')\frac{\delta\lambda_j(\omega')}{\delta b(\mathbf{z})}-\lambda_j^{-1}(\omega)\frac{\delta\lambda_j(\omega)}{\delta b(\mathbf{z})}\right]+\frac{\delta\mu}{\delta b(\mathbf{z})}n_b\ . \quad (A2)$$

For the first term, the derivative of $\lambda_j(\omega')$ receives contributions from the explicit $b(\mathbf{z})$ dependence of $\varepsilon_j[\omega',b]$ and also from the implicit $b(\mathbf{z})$ dependence introduced through μ via $\omega'=\omega+i\mu$. Thus

$$\frac{\delta}{\delta b(\mathbf{z})}\lambda_j[\omega',b]=\lambda'_j[\omega',b]i\frac{\delta\mu}{\delta b(\mathbf{z})}-i\frac{\partial}{\partial b(\mathbf{z})}\varepsilon_j[\omega',b]\ , \quad (A3)$$

where $\lambda'_j[\omega,b]=\partial\lambda_j[\omega,b]/\partial\omega$ and the partial derivative in the second term operates only on the second argument of ε_j. When (A3) is substituted into the (A2), the contribution from the first term of (A3) cancels the last term of (A2). This follows from use of the contour-integral expression for n_b from (3.37). We thus obtain

$$\begin{aligned}\frac{\delta E_q}{\delta b(\mathbf{z})}&=i\sum_j\int_C\frac{d\omega}{2\pi}e^{-i\omega\eta}\left\{\lambda_j^{-1}(\omega')\frac{\partial\varepsilon_j(\omega')}{\partial b(\mathbf{z})}-\lambda_j^{-1}(\omega)\frac{\partial\varepsilon_j(\omega)}{\partial b(\mathbf{z})}\right\}\\&=i\sum_j\int_{C'-C}\frac{d\omega}{2\pi}e^{-i\omega\eta}\lambda_j^{-1}(\omega)\frac{\partial\varepsilon_j(\omega)}{\partial b(\mathbf{z})}\ .\end{aligned} \quad (A4)$$

Here $C'-C$ is the μ-dependent contour shown in Fig. 4. Since from (3.40) and (3.42) we have $\lambda_j(\omega)=(\omega-\omega_p)Z_j(\omega,\omega_p)$ where the pole position is at $\omega_p=i\varepsilon_j(\omega_p)$, evaluation of (A4) produces

$$\frac{\delta}{\delta b_0^\alpha(\mathbf{z})}E_q[b_0,n]=\sum_j\theta(\mu-E_j^{(+)})\frac{1}{Z_j}\frac{\delta\varepsilon_j(\omega)}{\delta b_0^\alpha(\mathbf{z})}\bigg|_{\omega=\omega_p}\ , \quad (A5)$$

which is the result given in Eq. (3.48) of the text.

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