Quark-gluon-plasma signatures?

Berndt Müller
Department of Physics, Duke University, Durham, North Carolina 27706
and Institute for Nuclear Theory, Seattle, Washington 98195

Stefan Schramm
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125
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We consider the question what constitutes a good signature for the occurrence of an intermediate phase transition in a many-body system. In the context of a simple model for a chiral phase transition we show that, in principle, no rigorous signal for a temporary phase change exists in a finite system. We discuss the relevance of our result to the question of what constitutes an unambiguous signature for the formation of a quark-gluon plasma in relativistic nuclear collisions.

I. INTRODUCTION

Considerable efforts are presently devoted to the study of relativistic nuclear collisions, experimentally as well as theoretically. An important goal of these studies is to find evidence for the formation of a quark-gluon plasma, the hypothetical new state of dense or highly excited hadronic matter that differs from the normal state by the absence of either chiral-symmetry breaking or color confinement, or both.[1–3] The existence of such a phase transition in hadronic matter was conjectured long ago,[4] and a considerable amount of theoretical evidence for it has been obtained through numerical simulations of quantum chromodynamics (QCD) on a lattice.[5]

Although the existence of a high-temperature phase of QCD with the above-mentioned properties is hardly controversial, its observability in nuclear collision events is a matter of intense debate. The problem is that the new phase is not supposed to survive after the collision and thus cannot be detected directly. One therefore has to rely on indirect evidence; i.e., one looks for a "smoking gun" revealing the temporary existence of a quark-gluon plasma. A considerable number of such signatures has been proposed (see Ref. 6 and 7), but most of them appear to be ambiguous.

Here we raise the question whether an unequivocal signal for quark-gluon-plasma formation can exist, in principle, if the criterion is taken as a significant difference in predictions of calculations assuming the presence of a phase change and those that do not. We construct a simple model for fermionic matter exhibiting a chiral phase transition and study what happens if the transition occurs only for a finite time over a finite region of space. We shall find that an unambiguous signature for the temporary phase change does not exist, because the evolution of the system can be described without making explicit reference to it. However, the descriptions with or without reference to the change of phase may differ considerably in their complexity, and it may be more *convenient* to describe the system as developing a phase transition for an intermediate period of time.

II. EQUIVALENCE OF FOCK SPACES

The standard strategy for assessing the validity of a proposed signal for quark-gluon-plasma formation has been to compare the predictions of nuclear collision models with the assumption of the temporary presence of a quark-gluon-plasma phase with the predictions of models based entirely on the dynamics of color-singlet hadrons. We now argue that both types of models must necessarily yield identical results when pushed to their limits. The basic idea underlying our argument is that they are, in principle, two equivalent representations of the same dynamics based on QCD. Here we assume that color confinement is exactly valid; i.e., all asymptotic scattering states of finite energy are composed of separate clusters of color-singlet states, which are called *hadrons*. These hadron states $\{h_\nu\}$, where ν counts all hadronic quantum numbers, such as momentum, spin, parity, isospin, etc., form a complete basis of the Fock space of strong interactions.

Another complete basis of this Fock space is given by the states of noninteracting quarks and gluons coupled to color singlets, $\{(q,g)_\nu;(C=0)_{\rm as}\}$, where the symbol $(C=0)_{\rm as}$ indicates the condition of asymptotic cluster decomposition into color singlets. (We are here not concerned with mathematical subtleties associated with the transition between the Schrödinger and the interaction picture in an interacting relativistic quantum field theory. In the absence of asymptotically charged states, these are a consequence of the ultraviolet divergences of field theory, whereas we are here interested in properties associated with finite particle momenta. In order to simplify notation, we will not continue to denote the condition of asymptotic color neutrality explicitly; however, it is understood to hold at all times.) An immediate consequence of this observation is that every accessible state can be expanded in either basis set:

$$|\Psi\rangle=\sum_\nu A_n|h_\nu\rangle=\sum_\nu a_\nu|(q,g)_\nu\rangle\ . \tag{1}$$

The basis $\{h_\nu\}$ is clearly more suitable for describing the asymptotic in and out states for a given scattering problem, because they correspond to asymptotic eigenstates of the Hamiltonian of strong interactions (the QCD Hamiltonian) by definition. (Our discussion proceeds in the framework of time-dependent scattering theory.) However, at any intermediate time we are free to make a transition between the two bases, since they are connected by the relations

$$A_\nu = \sum_\mu a_\mu \langle h_\nu | (q,g)_\mu \rangle \; , \qquad a_\nu = \sum_\mu A_\mu \langle (q,g)_\nu | h_\mu \rangle \; . \tag{2}$$

One may wonder whether the supposed equivalence of the two Fock spaces holds rigorously. After all, we are confronting a situation where a structural phase transition is predicted to occur, so that nonanalytic behavior of the ground-state properties must be expected. We will discuss this aspect in the subsequent section in the framework of a simple model. There we will show that the Fock bases are equivalent if two conditions are met: (1) The structural change must occur only over a *finite volume,* and (2) the symmetry breaking must be *soft,* in the sense that modifications in the correlation functions are sufficiently damped at high momenta. E.g., when particles acquire a different dynamical mass because of interactions with the medium in which they propagate, this change must vanish at very high momentum. In our conclusion we will argue that this condition is, indeed, satisfied for the chiral-symmetry breaking occurring in QCD. The first of the two requirements is, of course, always fulfilled in the context of nuclear collisions; it is not valid in the context of the very early Universe.

When the condition of equivalence of the different Fock spaces is satisfied, the time-evolution operators defined in both spaces are related by a unitary transformation. Even if we insist on using the hadronic basis for the asymptotic in and out states, we may switch from the *hadronic picture* to the *quark-gluon picture* at some intermediate time, say, $t=0$, and then go back to the original picture at a later time $t=T$. This transition will be computationally convenient if at $t=0$ the total system is in such a state that, locally, conditions for strong screening of color interactions are satisfied, i.e., when there is a high density of colored constituents. We may then say that a *quark-gluon-plasma phase has been formed.* But this is a matter of convenience and language; there is no fundamental necessity for switching between the two sets of basis states. We could equally well describe the whole scattering process in the hadronic picture, although it might be more cumbersome.

III. TOY MODEL

Since we cannot solve the realistic case, i.e., QCD dynamics in a relativistic heavy-ion collision, exactly, we proceed to study the practical side of our argument in the context of a simple toy model that captures some essential features of QCD. The problem of color confinement being too difficult and not sufficiently well understood, we concentrate here on the breaking and restoration of chiral symmetry. Let us consider a system of (color-singlet) fermions (henceforth called *current quarks)* with a small bare mass m_0, interacting via a strong static two-body pair potential $V(\mathbf{x})$. The Hamiltonian of the many-body system is given by

$$H = \int d^3x \, \psi^\dagger(\mathbf{x})(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m_0)\psi(\mathbf{x}) + \frac{g_0^2}{2}\int d^3x \, d^3x' {:}\bar\psi(\mathbf{x})\gamma^\mu\psi(\mathbf{x})V(\mathbf{x}-\mathbf{x}')\bar\psi(x')\gamma_\mu\psi(\mathbf{x}'){:} \; , \tag{3}$$

where the colons indicate normal ordering with respect to the current quark vacuum $|0\rangle$. The quark operator $\psi(\mathbf{x})$ is expanded in terms of momentum and helicity eigenstates:

$$\psi(\mathbf{x}) = V^{-1/2}\sum_{\mathbf{k},s}(b_{\mathbf{k}s}u_{\mathbf{k}s}e^{i\mathbf{k}\cdot\mathbf{x}} + d^\dagger_{\mathbf{k}s}v_{\mathbf{k}s}e^{-i\mathbf{k}\cdot\mathbf{x}}) \; , \tag{4}$$

where $b_{\mathbf{k}s}$ and $d_{\mathbf{k}s}$ are annihilation operators for current quarks and antiquarks of momentum $\mathbf{k}$ and helicity s, respectively, and $u_{\mathbf{k}s}$ and $v_{\mathbf{k}s}$ are the familiar unit spinors. We have also introduced a coupling constant g_0, which will be used to tune the strength of the pair interaction. In order to avoid ultraviolet problems, we assume that the interaction falls off sufficiently fast in momentum space; for practical calculations we use the form

$$V(\mathbf{q}) = [\mathbf{q}^2(1+\mathbf{q}^2/\Lambda^2)]^{-1} \; . \tag{5}$$

For $g_0 \geq 1$ and $\mathbf{q}^2 < \Lambda^2$, the potential (5) acts as a strong four-fermion interaction, while the interaction becomes weak for $\mathbf{q}^2 \gg \Lambda^2$. Thus our interaction models the effect of asymptotic freedom present in QCD and avoids some pathologies of the original Nambu–Jona-Lasinio (NJL) model.[8]

When $g_0^2/4\pi$ is of order one, the fermion vacuum spontaneously breaks chiral symmetry[9] and the quark spectrum develops a mass gap M of order Λ. It is then convenient to introduce new quasiparticle operators $B_{\mathbf{k}s}, D_{\mathbf{k}s}$ by a Bogoliubov transformation:

$$B_{\mathbf{k}s} = \cos\theta_k b_{\mathbf{k}s} - s\sin\theta_k d^\dagger_{-\mathbf{k}s} \; , \qquad D_{\mathbf{k}s} = \cos\theta_k d_{\mathbf{k}s} + s\sin\theta_k b^\dagger_{-\mathbf{k}s} \; . \tag{6}$$

We will call the quasiparticle states *constituent quarks.* The mixing angle θ_k, which depends only on $k=|\mathbf{k}|$, is determined by the condition that it minimizes the energy of the constituent quark vacuum state $|\Phi_0\rangle$, which is a condensate of spin-singlet current quark-antiquark pairs:

$$|\Phi_0\rangle=\prod_{\mathbf{k},s}(\cos\theta_k+s\sin\theta_k b^\dagger_{\mathbf{k}s}d^\dagger_{-\mathbf{k}s})|0\rangle\equiv U|0\rangle\ , \tag{7}$$

where

$$U=\exp\left[\sum_{\mathbf{k},s}s\theta_k(b^\dagger_{\mathbf{k}s}d^\dagger_{-ks}+b_{\mathbf{k}s}d_{-\mathbf{k}s})\right]\ . \tag{8}$$

The annihilation operators for current and constituent quarks satisfy the conditions

$$b_k|0\rangle=d_k|0\rangle=0,\ \ B_k|\Phi_0\rangle=D_k|\Phi_0\rangle=0\ , \tag{9}$$

and it is easy to show[10] that they are related by the unitary transformation U defined in Eq. (8):

$$B_{\mathbf{k}s}=Ub_{\mathbf{k}s}U^{-1},\ \ D_{\mathbf{k}s}=Ud_{\mathbf{k}s}U^{-1}\ . \tag{10}$$

Minimizing the expectation value $\langle\Phi_0|H|\Phi_0\rangle$ of the Hamiltonian (3) with respect to the coefficients of the Bogoliubov transformation yields a set of integro-differential equations for θ_k and for the single quasiparticle energy $E(k)$, which are given in the Appendix. The equation for $E(k)$ is known as the mass-gap equation, since $M\equiv E(k=0)$ plays the role of the constituent quark mass.[9]

Although the current quark vacuum $|0\rangle$ and the constituent quark vacuum $|\Phi_0\rangle$, as well as the associated creation and annihilation operators, are related by the unitary transformation (8), the two Fock spaces built on $|0\rangle$ and $|\Phi_0\rangle$ may not be equivalent. This is a characteristic property of quantum systems with infinitely many degrees of freedom and is a consequence of the fact that any state with finite particle number in one Fock space may contain infinitely many particles in terms of the other Fock space. In our context the question of equivalence can be reduced to the question whether the two vacua have a nonvanishing overlap.[11] Otherwise, the two Fock spaces are nonequivalent and describe different physical worlds. One easily finds that the overlap is given by

$$\langle\Phi_0|0\rangle=\prod_{\mathbf{k},s}\cos\theta_k\ , \tag{11}$$

Introducing the usual phase-space volume element, the logarithm of the vacuum overlap becomes

$$\ln|\langle\Phi_0|0\rangle|^2=\frac{N_fV}{\pi^2}\int_0^\infty k^2dk\,\ln(\cos^2\theta_k)\ , \tag{12}$$

where V is the volume of the system and N_f is the number of internal quantum numbers of the quarks. Not unexpectedly, the vacuum overlap vanishes for an infinite system, $V\to\infty$. The infinite-volume limit, often called the thermodynamic limit, is well known to be essential for the existence of genuine phase transitions in nonrelativistic many-body systems.

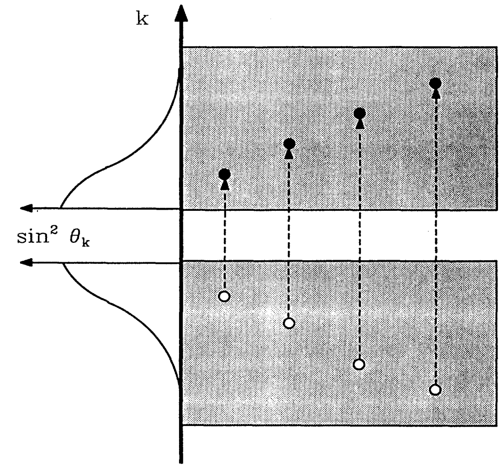

FIG. 1. Vacuum state $|\Phi_0\rangle$ corresponds to a many particle-hole configuration built on the "current quark" vacuum $|0\rangle$. The pair density $\sin^2\theta_k$ is plotted on the left. When it falls sufficiently fast with k, so that its integral over k is finite, the two Fock spaces are equivalent in a finite spatial volume.

Relativistic systems, however, potentially contain an infinite number of particles even in a finite volume. This is most easily understood in terms of the Dirac sea picture (see Fig. 1), where a vacuum rearrangement of the type described by a Bogoliubov transformation affects infinitely many "sea" particles. As a result, the vacuum overlap (12) vanishes also in a finite volume when θ_k falls off more slowly than k^{-2} for large k. This is the case in the original NJL model, where $\theta_k\approx k^{-1}$. When this occurs there is no communication possible between the two Fock spaces built upon the vacua $|0\rangle$ and $|\Phi_0\rangle$, and the system can be described *either* in terms of current quark states built upon $|0\rangle$ *or* in terms of constituent quark states constructed from $|\Phi_0\rangle$, but not in terms of both.

We are here interested in the description of a system of finite spatial extent, viz., the volume of dense hadronic matter formed in a high-energy nuclear collision. For the interaction (5) the integral in Eq. (12) is finite because of presence of the ultraviolet cutoff Λ. In fact, an analytic investigation of the equations given in the Appendix reveals that asymptotically $\theta_k\propto k^{-5}$, rendering the integral in (12) finite. We have verified this property numerically by solving the equation for θ_k with the interaction (5) for the QCD-motivated choice of parameters $g_0^2/4\pi=\frac{4}{3}$, $\Lambda=207$ MeV, and $m_0=0$. (The value of m_0 is irrelevant in this context, as long as $m_0\ll\Lambda$.) The result for $\sin^2\theta_k$ is shown in Fig. 2; it is seen to fall rapidly for $k>350$ MeV. This shows that the two Fock spaces built on the vacua $|0\rangle$ and $|\Phi_0\rangle$ are equivalent and that the evolution of an initial state in either one makes perfect sense. Changing from the representation in terms of constituent quarks to one in terms of current quarks merely corresponds to a basis change in Fock space.

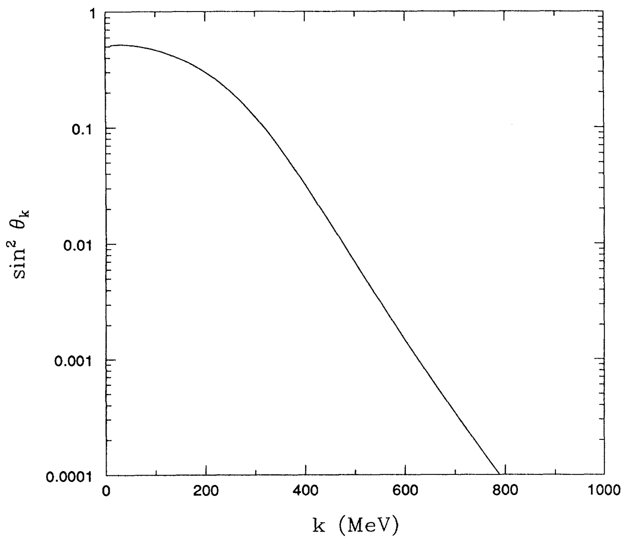

FIG. 2. Coefficient $\sin^2\theta_k$ describing the strength of the fermion pair condensate in the ground state. The calculation was performed for the parameters $\Lambda=207$ MeV, $g_0^2/4\pi=\frac{4}{3}$, and $m_0=0$.

IV. TOY MODEL DYNAMICS

It is illustrative to study the time evolution of our toy system in terms of the two Fock-state bases in order to see explicitly how the same physics is described in the two different representations. Let us consider the following model dynamics: We assume that the Hamiltonian is initially given by Eq. (3) and that the system starts out in the vacuum state $|\Phi_0\rangle$. At time $t=0$, the interaction is switched off, and the system propagates according to the free current quark Hamiltonian H_0. At a later time $t=T$, the interaction is switched on again. Our system then obeys the time-dependent Schrödinger equation

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle=[H_0+g(t)^2H_{\rm int}[|\Psi(t)\rangle \ , \tag{13}$$

$$g(t)=g_0[\theta(-t)+\theta(t-T)] \ , \tag{14}$$

where $\theta(x)$ is the unit step function. The intermediate absence of an interaction among current quarks is supposed to model the screening of color forces at very high density during the nuclear collision. This effect has been imposed by hand, because by starting out in the vacuum state of the interacting Hamiltonian we have neglected the presence of nuclear valence quarks. While our model may appear quite artificial in this respect, it captures the spirit of what one would naively call "a transition to the phase with restored chiral symmetry." During the interval $0<t<T$, the vacuum state of our toy system is indeed given by $|0\rangle$ and not by $|\Phi_0\rangle$.

Now let us ask whether it is possible to deduce from observables at $t\rightarrow\infty$ that the system spent some time in the chirally restored phase or, more precisely, that its Hamiltonian had the chirally unbroken phase as ground state between $t=0$ and T. Unfortunately, this question is still too hard to answer, since after $t=T$ the system will contain a great number of constituent quarks that interact strongly through the residual interaction derived from $H_{\rm int}$. We therefore take recourse to the weaker, but much simpler, question, whether we can detect the presence of the phase change at time $t=T$, i.e., immediately after the Hamiltonian has regained its original form. Obviously, if we cannot detect the transition, then it will hardly be possible at a later time, when the residual final-state interactions have had time to dilute a possible signal.

The evolution of the state of the system in the interval $0<t<T$ is most easily solved in the current quark basis, where there is no interaction. Beginning with the vacuum state $|\Psi(t)\rangle=|\Phi_0\rangle$ at $t=0$, the state of the system at time $t=T$ is easily expressed in the form

$$|\Psi(t)\rangle=\prod_{\mathbf{k},s}(\cos\theta_k+s\sin\theta_k e^{-2i\epsilon_k t}b^\dagger_{\mathbf{k}s}d^\dagger_{-\mathbf{k}s})|0\rangle \ , \tag{15}$$

since every single-particle operator $b^\dagger_{\mathbf{k}s},d^\dagger_{\mathbf{k}s}$ produces a state of energy $\epsilon_k=(\mathbf{k}^2+m_0^2)^{1/2}$, which propagates freely from $t=0$ to T. It is now simple to calculate the overlap of the state of the system with the constituent quark vacuum state at $t=T$:

$$\langle\Phi_0|\Psi(T)\rangle=\prod_{\mathbf{k},s}(\cos^2\theta_k+\sin^2\theta_k e^{-2i\epsilon_k T}) \ . \tag{16}$$

Introducing the same phase-space volume as in Eq. (12), the logarithm of the (constituent quark) vacuum persistence amplitude becomes

$$\begin{aligned}\ln W_0(T)&=\ln|\langle\Phi_0|\Psi(T)\rangle|^2\\&=\frac{N_f V}{\pi^2}\int_0^\infty k^2dk\,\ln(1-\sin^2 2\theta_k\sin^2\epsilon_k T) \ .\end{aligned} \tag{17}$$

Again, we find the same conditions as for the overlap between the two vacua: The vacuum persistence probability is finite if θ_k falls off more rapidly than $|\mathbf{k}|^{-2}$ and the spatial volume V is finite. Physically, $W_0(T)$ describes the probability that no particle has been produced by the temporary change in the interaction. $W_0(T)$ is shown as a solid line in Fig. 3 for the parameters $V=25\Lambda^{-3}$ and $N_f=6$ (three colors and two flavors). The rapid falloff with T indicates copious production of constituent quarks. For comparison, we have also shown the vacuum overlap $|\langle\Phi_0|0\rangle|^2$ defined in Eq. (12) as a horizontal dotted line in Fig. 3.

The decay of the original vacuum state corresponds to the production of quark-antiquark pairs. The mechanism for this multiparticle production can be expressed in two ways. In the intermediate current quark picture, what happens is that the "hadronic" vacuum state $|\Phi_0\rangle$ corresponds to a complex coherent superposition of current quark states, which rapidly get out of phase. A current quark "plasma" state develops on a time scale Λ^{-1}, governed by the range of energies ϵ_k for which θ_k is significant. Because of the absence of a residual interaction, no approach to thermal equilibrium occurs, but this is not relevant in our context. The projection at time T into the constituent quark Fock space then gives the final particle yield.

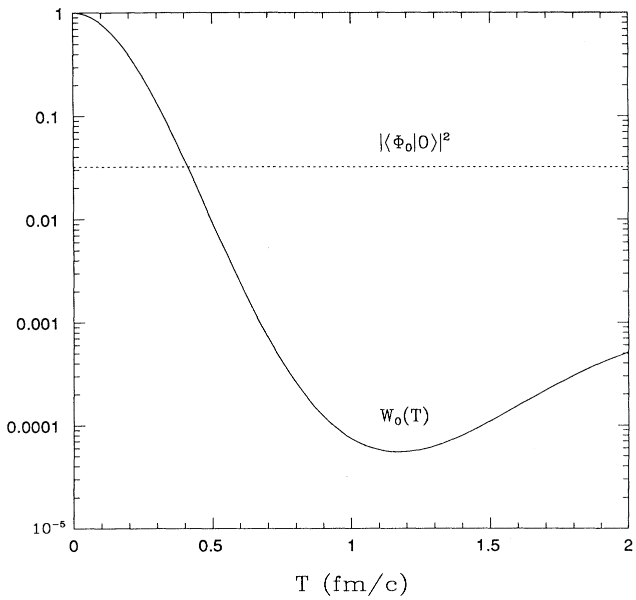

FIG. 3. Solid line shows the probability for survival of the chirally broken vacuum state $|\Phi_0\rangle$ if the interaction responsible for chiral-symmetry breaking is switched off for a time T. The dotted line represents the overlap $|\langle\Phi_0|0\rangle|^2$ between the vacua with and without chiral-symmetry breaking. The interaction parameters are the same as in Fig. 2, and we chose $V=25\Lambda^{-3}=21.7\ \mathrm{fm}^3$, $N_f=6$.

On the other hand, we can describe the evolution of the system entirely in terms of the "hadronic" picture represented by the Fock space of constituent quarks. The particle production is then caused by the presence of a strong interaction $(H_0-H)=-g_0^2H_{\mathrm{int}}$ among the constituent quark states. This interaction is not normal ordered in the quark operators $B_{\mathbf{k}s},D_{\mathbf{k}s}$ and therefore leads to rapid pair production of constituent quarks. The expression $P(T)=1-W_0(T)$ describes exactly this effect.

It is instructive to calculate the number of particles at a given time t in both pictures. In the "plasma" picture all particles are produced instantaneously during the transition between the two pictures; because of the neglect of residual interactions, no production occurs during the high-density phase. The number of current quarks at time t is given by the constant expression

$$\begin{aligned} N_q &= \sum_{\mathbf{k},s} \langle \Psi(t)|b^\dagger_{\mathbf{k}s}b_{\mathbf{k}s}|\Psi(t)\rangle \\ &= \frac{N_f V}{\pi^2}\int_0^\infty k^2dk\,\sin^2\theta_k\ . \end{aligned} \tag{18}$$

On the other hand, in the constituent quark picture, particle production starts gradually after the change in the Hamiltonian, but then continues until an asymptotic value is reached which is higher than the constant value (18). The part of the Hamiltonian responsible for pair production of constituent quarks is

$$H_{\mathrm{pair}}=\sum_{\mathbf{k},s}\epsilon_k\sin2\theta_k\,B^\dagger_{\mathbf{k}s}D^\dagger_{-\mathbf{k}s}\ ; \tag{19}$$

the number of quarks created by it can be calculated exactly with the help of Eqs. (6) and (15):

$$\begin{aligned} N_Q(t) &= \sum_{\mathbf{k},s}\langle\Psi(t)|B^\dagger_{\mathbf{k}s}B_{\mathbf{k}s}|\Psi(t)\rangle \\ &= \frac{N_f V}{\pi^2}\int_0^\infty k^2dk\,\sin^2 2\theta_k\sin^2\epsilon_k t\ . \end{aligned} \tag{20}$$

This agrees with the result derived in perturbation theory from the interaction (19) for small $t=T$.

In Fig. 4 we show the evolution of the particle numbers N_q and N_Q for the parameters mentioned above. As we argued before, the number of "constituent quarks" begins to rise steadily as the interaction responsible for chiral-symmetry breaking is switched off, exceeding the number of "current quarks" only after a certain time $t\approx(3\Lambda)^{-1}$. We note that in our model the constituent quarks are strongly interacting during the interval $0<t<T$ and that our calculation for $N_Q(t)$ takes this interaction fully into account. In the current quark picture, a major fraction of the final particle number is produced at the moment of restoration of chiral-symmetry breaking, $t=T$. This effect is reminiscent of the mechanism of particle production through fragmentation, which has been found to be an essential aspect of models for the rehadronization of a quark-gluon plasma.

Having established that the evolution of the system can, in principle, be described in both Fock bases, we now turn to the discussion of a typical "signature" of the

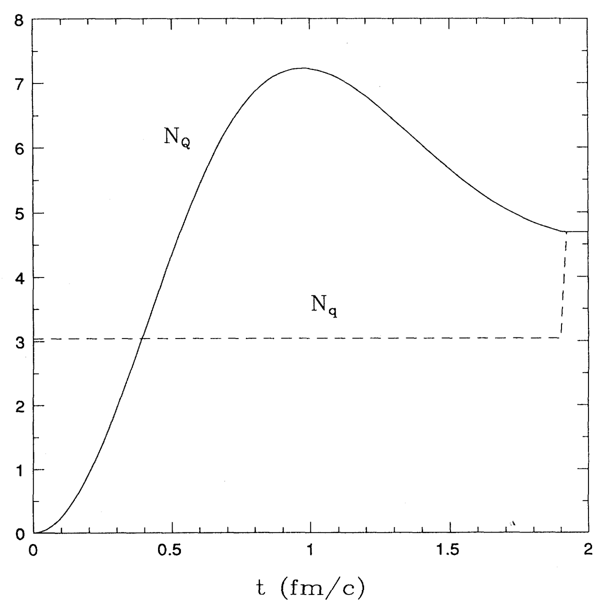

FIG. 4. Number of constituent quarks N_Q (solid line) and current quarks N_q (dashed line) as function of time. The interaction was switched off between $t=0$ and $T=1.19$ fm/c. Other parameters are the same as in Fig. 2.

plasma state. In nuclear collisions observables based on electromagnetic interactions have been extensively studied, such as hard photon emission or lepton pair creation. These observables are sensitive to the number of charged-particle pairs present in the system at early times which can interact electromagnetically. Similarly, the proposed signal of J/ψ depletion depends critically on the density of scatterers in the system immediately after the collision. Indeed, when we calculate the number of current quarks (N_q) and constituent quarks (N_Q), respectively, we obtain the very different results shown in Fig. 4.

However, we must bear in mind that the particle number is not directly observable at some intermediate time t; only matrix elements of physical operators entering into the Hamiltonian are. For instance, the electromagnetic interaction is determined by matrix elements of the current operator; utilizing Eq. (4), we have

$$j^{\mu}_{fi}(\mathbf{x},t)=\langle\Psi_f(t)|\bar{\psi}(\mathbf{x})\gamma^{\mu}\psi(\mathbf{x})|\Psi_i(t)\rangle$$
$$=\sum_{\mathbf{k},s}\sum_{\mathbf{k}',s'}e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{x}}[\bar{u}_{\mathbf{k}'s'}\gamma^{\mu}u_{\mathbf{k}s}\langle\Psi_f(t)|b^{\dagger}_{\mathbf{k}'s'}b_{\mathbf{k}s}|\Psi_i(t)\rangle+\cdots], \tag{21}$$

where the dots indicate similar terms involving other combinations of the current quark operators $b_{\mathbf{k}s},d_{\mathbf{k}s}$. This expression can be written in terms of the constituent quark operators $B_{\mathbf{k}s},D_{\mathbf{k}s}$ with the help of our unitary transformation (8):

$$j^{\mu}_{fi}(\mathbf{x},t)=\sum_{\mathbf{k},s}\sum_{\mathbf{k}',s'}e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{x}}[\bar{u}_{\mathbf{k}'s'}\gamma^{\mu}u_{\mathbf{k}s}\langle\Psi'_f(t)|B^{\dagger}_{\mathbf{k}'s'}B_{\mathbf{k}s}|\Psi'_1(t)\rangle+\cdots], \tag{22}$$

where $|\Psi'_{i/f}\rangle=U|\Psi_{i/f}\rangle$. The existence of the unitary transformation and the equivalence of the two Fock-space descriptions ensures that the result for the transition matrix element of the current operator will be the same whether Eq. (21) or (22) is used to evaluate it. The electromagnetic interactions of "current" and "constituent" quarks differ significantly, resulting in the same current density, even though the expectation value of the number of particles is not the same in the two descriptions. We conclude that it would be erroneous to assume that an observable that appears to be sensitive to the number of particle pairs present in the system can distinguish between the two pictures, even though this may be contrary to our naive expectation.

V. CONCLUSIONS

Do the arguments advanced in the context of our toy model for chiral-symmetry breaking apply to QCD? We first note that a similar suppression of chiral-symmetry breaking at large momentum transfer is found in more realistic approximations to QCD.[12] However, the decrease of the QCD interaction in the ultraviolet is only logarithmic and not strong enough to render the integral (12) finite. But note that this is not the issue of interest here: We do not claim that it would have no unequivocal observable consequences if the QCD interaction were completely switched off. Instead, we claim that the *screening* of the QCD interaction in dense hadronic matter cannot be observed unambiguously in a finite-size system. Hence we have to compare the short-range behavior of the QCD coupling constants with and without medium effects. Only considering the influence of temperature, we find in the static limit for the pure SU(3) gauge theory in Coulomb gauge:[1]

$$\alpha(\mathbf{q},T)\xrightarrow{|\mathbf{q}|\to\infty}\alpha(\mathbf{q},0)\left[1+\alpha(\mathbf{q},0)\frac{2\pi T^2}{3|\mathbf{q}|^2}\right]; \tag{23}$$

i.e., the medium-dependent part of the interaction has the same high-momentum behavior $|q|^{-2}$ as our model interaction (5). It is therefore reasonable to expect that the mixing angle θ_k describing the modification of the single-quark states in the presence of the medium falls off sufficiently fast with k.

While the equivalence of the hadronic and quark-gluon pictures is valid as a matter of principle, if our argument is also correct in full QCD, there are several difficulties that presently prevent its complete realization in practical calculations. First, the detailed structure of hadrons in terms of quarks and gluons is still not well understood. Second, we do not yet know sufficiently well how to derive effective interactions between hadrons from the underlying QCD dynamics. Finally, little is known about the hadronization of a dense system of quarks and gluons. Models for one picture or the other are usually based on some approximation to the exact QCD dynamics, e.g., perturbation theory for the quark-gluon basis and chiral meson exchange models for the hadronic basis. Therefore, it is presently impossible to show the exact equivalence of complete calculations carried out in the two pictures.

Nonetheless, it cannot be surprising if the best available models for the dynamics of nuclear reactions in the context of both pictures yield almost identical results for a given observable. Indeed, the virtual agreement found in several carefully studied cases indicates that the models correctly embody the dominant aspects of QCD dynamics relevant for the description of the observable in question. If both descriptions exhibit the presence of, e.g., a very high energy density at an intermediate time, this is a strong indication that an exact solution of QCD dynamics, if it were possible, would also yield a transient phase of high-energy density. However, one cannot deduce that the observable proves the existence of a quark-gluon-plasma phase. In particular, the usual strategy of calculating the same observable in either framework and then asking whether the results differ significantly must necessarily lead to disappointment. When the calculations are complete and exact, the results *must* agree. The best one can do is to determine in which picture the calculation is *simpler*.

Does this imply that the experimental search for the quark-gluon plasma is futile? Not necessarily, but the goal has to be defined differently. Experimental data can reveal information about hadronic matter at high-energy density formed in relativistic nuclear collisions, and they can probe those of its properties which can be expressed in terms of physical observables. Heavy-ion collisions may not be able to detect the quark-gluon plasma unambiguously, but they can be used to study its physical properties. If QCD matter exhibits a phase transition, it may be detected, e.g., through the observation of changes in the kinematic variables of the final state. The experiment, which measures certain combinations of elements of the S matrix, is concerned with the Hamiltonian governing the dynamics of strongly interacting matter and not with its representation in Hilbert space. After all, it is reassuring that the physics is independent of the language used for its description.

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APPENDIX

We here give the integro-differential equation that determines the mixing angle θ_k of the Bogoliubov transformation (6):

$$\tan 2\theta_k \left[k^0 - \frac{g_0^2}{4\pi^3}\int d^3q \frac{V(\mathbf{k}-\mathbf{q})}{k^0 q^0}\sin\theta_q \left[\sin\theta_q (2m_0^2 + \mathbf{k}\cdot\mathbf{q}) + \cos\theta_q \frac{m_0}{q}(\mathbf{k}\cdot\mathbf{q} - 2\mathbf{q}^2)\right]\right]$$
$$= \frac{g_0^2}{4\pi^3}\int d^3q \frac{V(\mathbf{k}-\mathbf{q})}{k^0 q^0}\sin\theta_q \left[\cos\theta_q \left[2kq + \frac{m_0^2}{kq}\mathbf{k}\cdot\mathbf{q}\right] + \sin\theta_q \frac{m_0}{k}(\mathbf{k}\cdot\mathbf{q} - 2\mathbf{k}^2)\right] .$$

Writing the Hamiltonian in normal-ordered form with respect to the chirally broken vacuum $|\Phi_0\rangle$,

$$H = \langle \Phi_0 | H | \Phi_0 \rangle + \sum_{\mathbf{k},s} E(k)(B^\dagger_{\mathbf{k}s}B_{\mathbf{k}s} + D^\dagger_{\mathbf{k}s}D_{\mathbf{k}s}) + \cdots ,$$

the mass-gap equation has the form

$$E(k) = k^0 \cos 2\theta_k - \frac{g_0^2}{4\pi^3}\int d^3q \frac{V(\mathbf{k}-\mathbf{q})}{k^0 q^0}\sin\theta_q \left\{ (k\cos 2\theta_k - m_0 \sin 2\theta_k)\frac{\mathbf{k}\cdot\mathbf{q}}{kq}(q\sin\theta_q + m_0\cos\theta_q) \right.$$
$$\left. + 2(m_0\cos 2\theta_k + k\sin 2\theta_k)(m_0\sin\theta_q - q\cos\theta_q) \right\} .$$

The numerical techniques used for the solution of these integral equations will be discussed in a separate publication.

[1] B. Müller, *The Physics of the Quark Gluon Plasma* (Lecture Notes in Physics, Vol. 225) (Springer, Heidelberg, 1985).

[2] L. McLerran, Rev. Mod. Phys. **58**, 1021 (1986).

[3] Proceedings of the Quark Matter conferences: Nucl. Phys. **A461**, (1987); Z. Phys. C **38**, (1988); Nucl. Phys. **A498**, (1989).

[4] P. Carruthers, Collect. Phenom. **1**, 147 (1973); J. C. Collins and M. J. Perry, Phys. Rev. Lett. **34**, 1336 (1975); E. V. Shuryak, Phys. Rep. **61**, 71 (1980).

[5] For a recent review, see, e.g., J. Engels, in *The Nuclear Equation of State: Part B*, edited by W. Greiner and H. Stöcker (NATO Advanced Study Institute, Series B: Physics, Vol. 216B) (Plenum, New York, 1990), p. 17.

[6] K. Kajantie and L. McLerran, Annu. Rev. Nucl. Part. Sci. **37**, 293 (1987).

[7] For timely discussion of the major proposed signatures, see *Quark Gluon Plasma*, edited by R. C. Hwa (World Scientific, Singapore, 1990).

[8] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[9] M. Finger, D. Horn, and J. E. Mandula, Phys. Rev. D **20**, 3253 (1979); J. R. Finger and J. E. Mandula, Nucl. Phys. **B199**, 168 (1982); A. Amer, A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D **28**, 1530 (1983).

[10] See, e.g., S. Schramm, B. Müller, and W. Greiner, J. Phys. G **13**, L69 (1987); S. Schramm and W. Greiner, Univ. Frankfurt report (1991).

[11] See, e.g., F. Strocchi, *Elements of Quantum Mechanics of Infinite Systems* (World Scientific, Singapore, 1985), Chap. 2.3.

[12] K. Higashijima, Phys. Rev. D **29**, 1228 (1984).

[13] J. Schaffner *et al.*, Univ. Frankfurt report (1991).