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Effect of pairing on breathing mode and nuclear matter compressibility

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The systematics of the giant monopole resonance in even-mass Sn isotopes have been studied in the framework of the quasiparticle random-phase approximation. The effect of the inclusion of pairing correlations on the properties of the giant monopole and giant quadrupole resonances in superfluid Sn nuclei has been evaluated. A shift in the monopole energies of these nuclei by about 100–150 keV has been observed. It is shown that the shift in the energy of the breathing mode of finite superfluid nuclei influences the extraction of the nuclear matter compressibility from empirical data only minimally.

I. INTRODUCTION

The study of the breathing-mode giant monopole resonance (GMR) is important due to its connection to the compression properties of nuclei and of nuclear matter.[1–4] The fundamental physical quantity like nuclear matter compressibility has a bearing on the properties of astrophysical objects such as neutron stars[5] and supernovae.[6] Attempts have been made to obtain information on the nuclear matter compressibility using different approaches (see, for a review, Refs. 7 and 8). The GMR has, however, been one of the most reliable means of deducing this quantity in the laboratory. Data on the GMR have been known for years[9,10] but only very recently have precise energy systematics been obtained for spherical isotopic chains of Sn and Sm nuclei.[11] Employing these data in a semiphenomenological approach, the nuclear matter incompressibility (K_∞) has been obtained[4,11] to be 300±25 MeV. This is in contrast with the value[1] of 210±30 MeV deduced from self-consistent microscopic Hartree-Fock calculations based on phenomenological density-dependent effective interactions. The problems associated with the previous conclusions have been discussed in Ref. 4. It has been shown that, in the semiphenomenological approach based upon the liquid-drop-model type of expansion for the finite-nucleus incompressibility K_A, it is possible to disentangle K_A into a volume and a surface part using these precision data. This led to a large surface incompressibility which cannot be accommodated in any of the presently available Skyrme type of interactions.

The nuclei investigated experimentally in Ref. 11 are open-shell nuclei which exhibit a superfluid behavior. This implies that the effect of pairing in the ground state as well as in the spectroscopic properties of these nuclei needs to be taken into account. While the pairing effect in these nuclei manifests itself in many of their properties, it is, however, not known to what extent the GMR energies, and therefore the nuclear matter compressibility, would be influenced by this. In the case of the giant dipole resonance (GDR) it has been shown in Ref. 12 that pairing correlations can produce considerable shifts in the peak energies, nearly all of the theoretical analysis of the GMR has, however, been done without including the pairing correlations. The GMR centroid-energy systematics itself do not provide any indications of the effect of pairing; this is also not known *a priori*. In this work we have, therefore, aimed to ascertain the magnitude of the effect of the pairing correlations upon the observed GMR properties and, consequently, its influence on the nuclear matter incompressibility. We have taken the experimental data obtained by Sharma *et al.*[11] as our main reference. The question which we address here is on the persistence of the pairing effects in the microscopic structure of the GMR. This is particularly relevant for the GMR in even-mass Sn isotopes, where neutron-pairing effects are the dominant degrees of freedom which should be included for the low-energy data.[13]

In Sec. II we show the results of the quasiparticle random-phase approximation (QRPA) calculations of the GMR and the giant quadrupole resonance (GQR) in Sn nuclei and in Sec. III we discuss fits to the experimental data including the shifts caused by the pairing correlations. In Sec. IV we summarize our results.

II. MICROSCOPIC CALCULATION OF PAIRING EFFECTS

The quasiparticle random-phase-approximation equations (for details, see Ref. 14) were solved in a single-particle basis of harmonic-oscillator wave functions. We have included shells up to the major oscillator number $N=7$, both for protons and for neutrons. The single-particle energies were chosen according to Nilsson's formula[15] with parameters $\chi_N=0.0675$, $\chi_Z=0.0671$, $\chi_N\mu_N=0.0278$, and $\chi_Z\mu_Z=0.0363$, respectively. In order to reproduce observed single-particle energy spacings in the neighborhood of the $N=Z=50$ shell closures, only a few single-particle energies near the Fermi level have been adjusted. This has been done by compressing the energy gap in the $N=Z=50$ closure by 0.5 MeV. As residual interaction we have used an isospin-independent δ force with the parameters V_{0N} in the pairing channel for the neutrons and V_{J^π} in the particle-hole channels.[14]

First, we have solved state-dependent BCS equations[16] for neutrons in the neutron number domain $62\le N\le 74$, which corresponds to the even-mass isotopes of Sn nuclei. The coupling constant V_{0N} has been fixed at the value $V_{0N}=110$ MeV fm^4. We have obtained average neutron-gap parameters of the order of 1.16 (^{112}Sn), 1.10 (^{114}Sn), 1.16 (^{116}Sn), 1.13 (^{118}Sn), 1.20 (^{120}Sn), 1.11 (^{122}Sn), and 1.15 MeV (^{124}Sn). The mass dependence of the quasiparticle spectrum is shown in Fig. 1. These values do, in fact, agree nicely with the data for low-lying quasiparticle states in odd-mass Sn isotopes.[13] Experimental energy differences $\delta\varepsilon_{qp}=\varepsilon_{qp}-\varepsilon_{qp}(\text{g.s.})$ between $j^\pi=\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, $\frac{7}{2}^+$ and $\frac{11}{2}^-$ single-quasiparticle states are reproduced by our calculations within a 200-keV limit of acceptance, as shown in Table I.

Within the above-described single-particle basis, we have diagonalized the QRPA equations for monopole and quadrupole isoscalar modes and calculated $J^\pi=0^+$ and 2^+ energy spectra and strength distributions for monopole and quadrupole transitions, in the isotope chain $112\le A\le 124$ of even-mass Sn nuclei. The coupling constants for neutron (proton) two-quasiparticle (particle-hole) configurations have been fixed at the value

$$V_J(J^\pi=0^+,2^+)=70 \text{ MeV fm}^4$$

for all the isotopes which we have included in our calculations. A cutoff energy, $E_{max}=25$ MeV, was used for all quasiparticle (particle-hole) monopole and quadrupole unperturbed pair configurations. The stability of the numerical results against changes in the cutoff value has been tested and we have found that a very small fraction of the unperturbed energy-weighted sum rules lies outside the energy region limited by the condition $E_{2qp}\le E_{max}$.

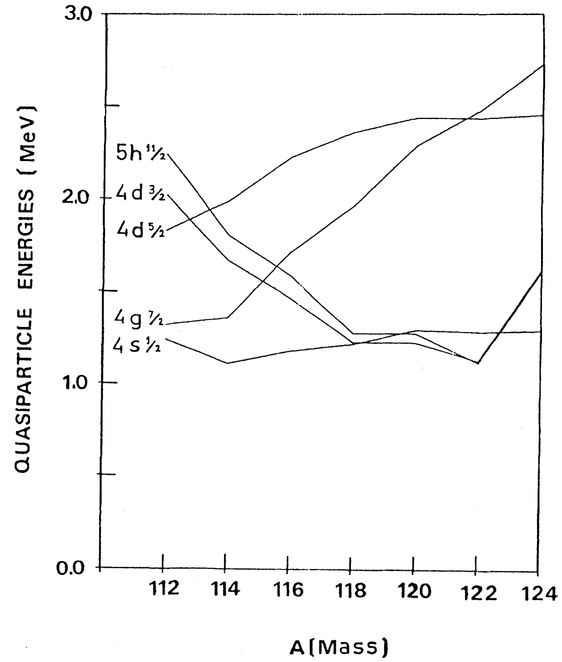

FIG. 1. Mass variation of neutron quasiparticle energies for the mass region $112\le A\le 124$.

In comparing our model-space results for the unperturbed energy-weighted sum rules with the results given by the harmonic-oscillator model, $S(\lambda^\pi=0^+)$ and $S(\lambda^\pi=2^+)$, we have adopted the following values:[1,2]

$$S(\lambda^\pi=0^+)\approx 71.49A^{5/3} \text{ MeV fm}^4$$

and

$$S(\lambda^\pi=2^+)\approx 71.12A^{5/3} \text{ MeV fm}^4 .$$

In order to obtain the centroids and average spreading widths of the resonances, we have accumulated fractions of energy-weighted sum rules, carried by individual RPA roots, in the full energy range divided in bins of 0.5 MeV. The corresponding histograms have been used to evaluate the centroids. Furthermore, we have tested the numerical accuracy of this procedure by calculating the distribution function

TABLE I. (a) theoretical and (b) experimental values for low-lying one-quasiparticle states in odd-mass Sn isotopes. Data are taken from Ref. 13. The values are given in units of MeV.

	113		115		117		119		121		123		125	
J^π	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
$\frac{1}{2}^+$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.06	0.17	0.15	0.14	0.21
$\frac{7}{2}^+$	0.08	0.08	0.25	0.61	0.54	0.71	0.74	0.78	1.07		1.37		1.58	
$\frac{5}{2}^+$	0.59	0.40	0.88	0.98	1.04	1.02	1.14	0.92	1.22		1.33		1.31	
$\frac{3}{2}^+$	0.78	0.50	0.56	0.50	0.28	0.15	0.01	0.02	0.00	0.00	0.01	0.02	0.00(3)	0.03
$\frac{11}{2}^-$	1.00	0.74	0.70	0.71	0.40	0.31	0.06	0.09	0.05	0.01	0.00	0.00	0.00	0.00

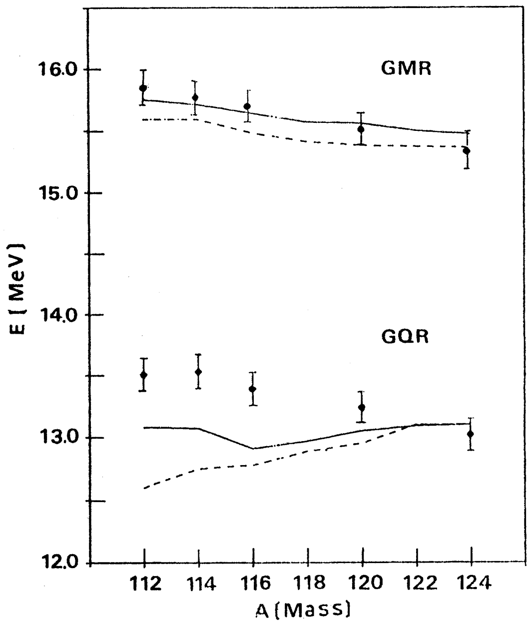

FIG. 2. Comparison between theoretical and experimental values for the energy of giant monopole resonance (GMR) and giant quadrupole resonance (GQR) in even-mass Sn isotopes. Data are taken from Ref. 11. Theoretical results with and without the inclusion of pairing correlations are indicated by solid and dashed lines, respectively.

$$S(E)=\frac{1}{2\pi[S_{\mathrm{RPA}}(\lambda^\pi=0^+)+S_{\mathrm{RPA}}(\lambda^\pi=2^+)]}\times\left[\frac{S_{\mathrm{RPA}}(\lambda^\pi=0^+)\Gamma_{\mathrm{GMR}}}{(E-E_{\mathrm{GMR}})^2+\Gamma^2_{\mathrm{GMR}}}+\frac{S_{\mathrm{RPA}}(\lambda^\pi=2^+)\Gamma_{\mathrm{GQR}}}{(E-E_{\mathrm{GQR}})^2+\Gamma^2_{\mathrm{GQR}}}\right], \tag{1}$$

where E_{GMR} and E_{GQR} are the calculated energies for the monopole and the quadrupole giant resonances, Γ_{GMR} and Γ_{GQR} are the corresponding widths, and $S_{\mathrm{RPA}}(\lambda^\pi=0^+)$ and $S_{\mathrm{RPA}}(\lambda^\pi=2^+)$ are the obtained RPA values for the energy-weighted sum rules. We have followed this procedure for each of the isotopes without changing the coupling constants, both with and without the inclusion of neutron-pairing correlations. The results are shown in Fig. 2 where the energetics for GMR and GQR are displayed as a function of the mass number. Agreement with the data are obtained within 250 keV for the lighter masses. From these results it becomes evident that pairing effects are important in the calculation of GMR and GQR energetics. It should be noted that the suppression of pairing produces, as expected, a downward shift of the energies, which is more pronounced for the lighter Sn isotopes with the mass numbers $112 \leq A \leq 116$. An example of the effects associated to pairing correlation is shown in Fig. 3, where the total strength distribution and monopole and quadrupole strength distributions, with and without pairing, are shown for the case of ^{112}Sn. The dependence of total strength distributions, $S(E)$, upon pairing correlations is shown in Fig. 4 where the above-mentioned shift in the energetics of GMR and GQR is clearly visualized.

We can also extract some useful information concerning the role which is played by pairing correlations, in building up the microscopic structure of the giant resonances, from the analysis of the RPA wave functions. For the GMR it is revealed that pairing correlations influence more the structure of the wave functions for light Sn isotopes, namely, for $^{112-116}$Sn, than they do in heavy masses. The explanation of this effect is obvious since the blocking of pairing correlations, for few out-shell particles, produces a drastic reduction in the number of allowed pair configurations. It means that the bulk of the wave functions, for GMR in light Sn isotopes, is composed of one-quasiparticle–one-particle neutron-pair

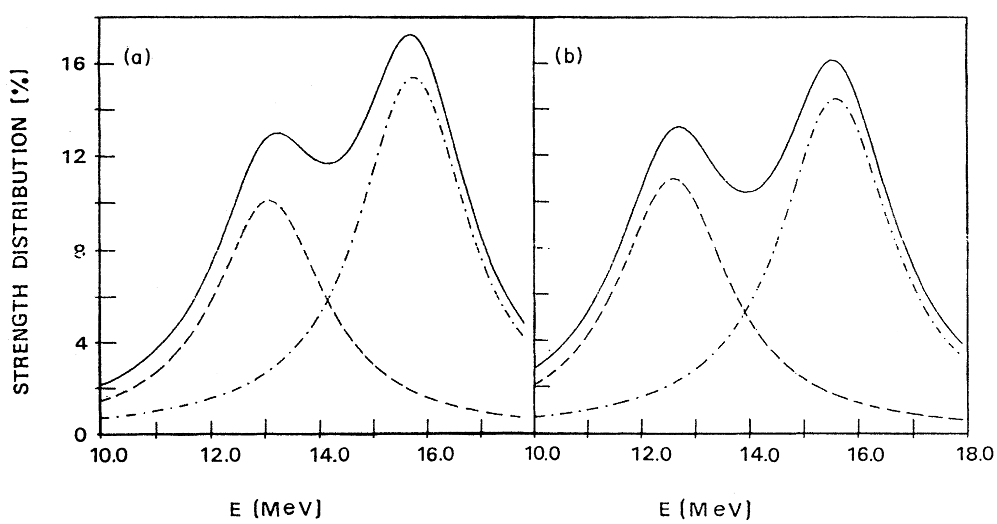

FIG. 3. Strength distributions corresponding to GMR and GQR in ^{112}Sn. Solid lines represent the accumulated strength; dashed-dotted and dashed lines represent GMR and GQR strengths, respectively. (a) and (b) correspond to theoretical results which have been obtained with and without the inclusion of pairing correlations, respectively.

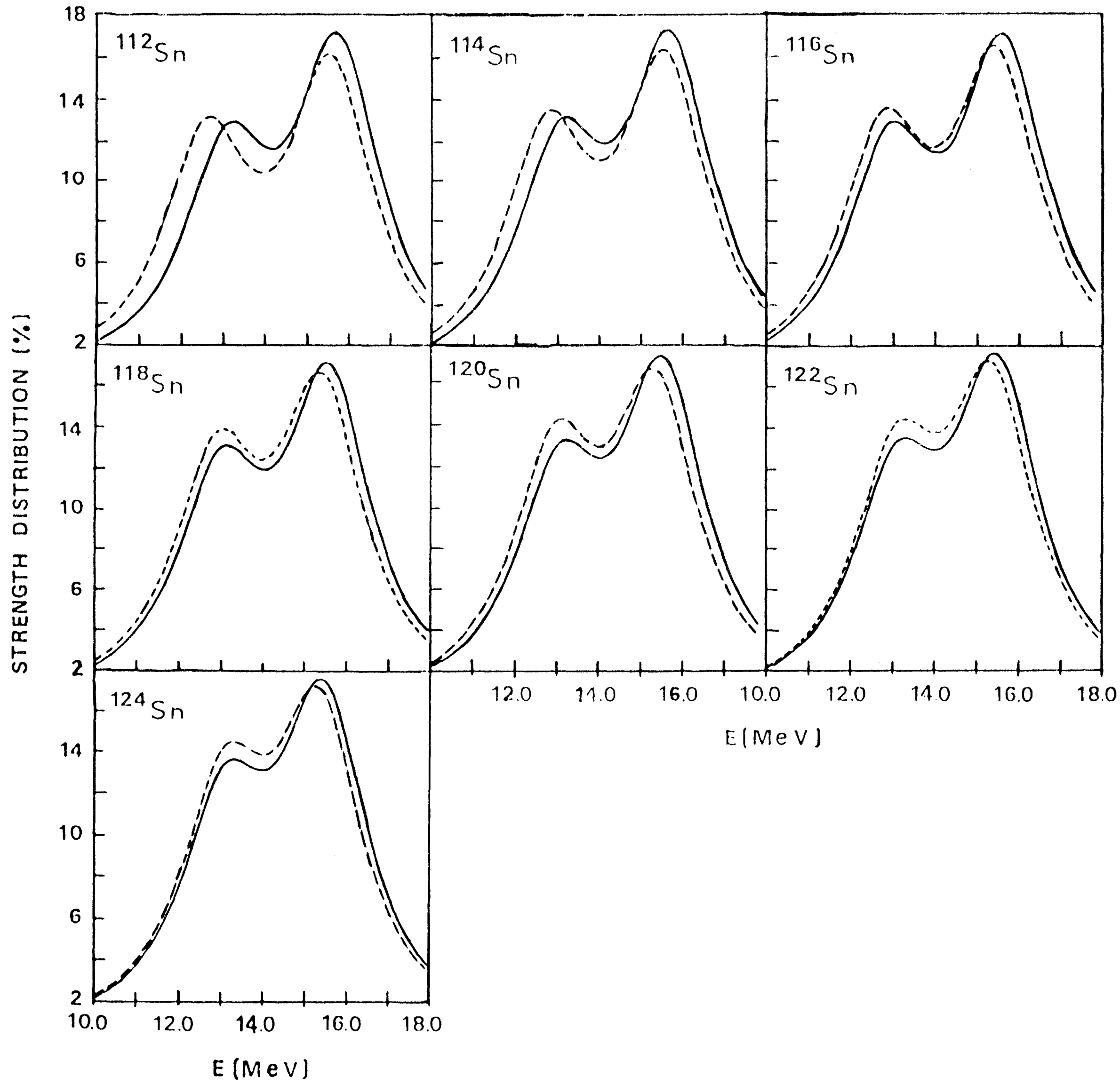

FIG. 4. Accumulated strength distributions for GMR and GQR in even-mass Sn isotopes. Theoretical values, given in percentages for the total GMR + GQR intensity, correspond to RPA calculations performed with (solid lines) and without (dashed lines) the inclusion of pairing correlations.

configurations in addition to the one-particle–one-hole proton-pair configurations. When the pairing correlations are suppressed, dominant amplitudes of the GMR wave function are given by neutron and proton particle-hole pairs nearly in the same amount. This effect is of minor importance for the heavy masses, $^{118-124}$Sn, where the suppression of pairing correlations changes the structure of the wave function less drastically. Dominant contributions to GMR and GQR wave functions for some of the Sn isotopes are shown in Table II.

The results arise due to the interplay of two effects. First, we have, in a superfluid system, enhanced quasiparticle energies. Even without any residual interaction, pairing correlations will therefore increase the energy of the giant resonances as compared with the normal system. Second, pairing correlations produce a broadening of the Fermi surface, thus resembling much the situation of giant resonances on a thermally excited state. In our previous investigations[17,18] on thermal blocking effects, we have found some features on the behavior of giant resonance modes at finite temperature, which are similar to the ones discussed above. From a very crude representation of the GMR and GQR resonances, like single collective modes built on degenerate pair configurations of particle-particle and particle-hole-types, an enhanced collectivity can be expected when the Fermi surface is broadened by thermal excitations. For an attractive force, as for the case of isoscalar excitations, an increase

TABLE II. Wave functions for some dominant states in the GMR energy region. Forward (X) and backward (Y) going RPA amplitudes are shown, for neutron (ν) and proton (π) configurations, for monopole and quadrupole states in ^{112,118,124}Sn. Energies (in MeV) and fractions of the energy-weighted sum rule (EWSR in %) are shown for RPA states calculated (a) with and (b) without the inclusion of pairing correlations. Configurations with $|X| \leq 0.20$ are not included.

A		E (MeV)	EWSR (%)	Configuration	Amplitudes X	Amplitudes Y
			Monopole states			
112	(a)	15.88	26.57	$\nu\ (3f_{5/2}\ 5f_{5/2})$	0.749	−0.002
				$\pi\ (2d_{3/2}\ 4d_{3/2})$	0.201	−0.001
		16.44	22.26	$\nu\ (4g_{9/2}\ 6g_{9/2})$	−0.421	0.000(5)
				$\pi\ (3f_{7/2}\ 5f_{7/2})$	−0.656	0.011
	(b)	15.26	14.36	$\nu\ (2s_{1/2}\ 4s_{1/2})$	0.393	0.037
				$\nu\ (3p_{3/2}\ 5p_{3/2})$	0.304	−0.017
				$\pi\ (2s_{1/2}\ 4s_{1/2})$	0.761	0.018
		15.35	22.08	$\nu\ (3f_{5/2}\ 5f_{5/2})$	−0.739	0.004
				$\pi\ (3p_{3/2}\ 5p_{3/2})$	−0.328	0.015
		16.28	27.22	$\nu\ (3f_{7/2}\ 5f_{7/2})$	0.414	0.012
				$\nu\ (3f_{5/2}\ 5f_{5/2})$	−0.335	0.010
				$\nu\ (4g_{9/2}\ 6g_{9/2})$	0.318	−0.005
				$\pi\ (3f_{7/2}\ 5f_{7/2})$	0.510	−0.006
118	(a)	15.88	26.57	$\nu\ (3f_{5/2}\ 5f_{5/2})$	0.749	−0.004
				$\pi\ (3p_{3/2}\ 5p_{3/2})$	0.255	−0.013
		16.37	26.08	$\nu\ (4g_{7/2}\ 6g_{7/2})$	−0.282	0.003
				$\nu\ (4s_{1/2}\ 6s_{1/2})$	0.243	−0.005
				$\nu\ (4g_{9/2}\ 6g_{9/2})$	−0.226	0.003
				$\pi\ (3f_{7/2}\ 5f_{7/2})$	−0.672	0.009
	(b)	14.14	21.48	$\nu\ (4d_{3/2}\ 6d_{3/2})$	0.596	0.007
				$\nu\ (3p_{1/2}\ 5p_{1/2})$	0.544	−0.003
				$\pi\ (3f_{5/2}\ 5f_{5/2})$	0.320	−0.002
		15.35	25.06	$\nu\ (3f_{5/2}\ 5f_{5/2})$	0.759	−0.004
				$\pi\ (3p_{3/2}\ 5p_{3/2})$	0.312	−0.014
		16.38	20.01	$\nu\ (4s_{1/2}\ 6s_{1/2})$	−0.330	0.018
				$\nu\ (3f_{7/2}\ 5f_{7/2})$	0.327	0.008
				$\nu\ (4g_{9/2}\ 6g_{9/2})$	0.305	−0.005
				$\pi\ (2s_{1/2}\ 4s_{1/2})$	0.596	−0.006
				$\pi\ (3f_{7/2}\ 5f_{7/2})$	0.430	−0.006
124	(a)	15.26	17.66	$\nu\ (3f_{5/2}\ 5f_{5/2})$	0.645	−0.006
				$\pi\ (3p_{3/2}\ 5p_{3/2})$	0.401	−0.017
		16.43	16.85	$\nu\ (4g_{7/2}\ 6g_{7/2})$	0.538	0.001
				$\pi\ (3i_{7/2}\ 5f_{7/2})$	0.528	−0.009
	(b)	14.14	19.17	$\nu\ (4d_{3/2}\ 6d_{3/2})$	0.604	0.007
				$\pi\ (3f_{5/2}\ 5f_{5/2})$	0.299	−0.001
		15.33	23.14	$\nu\ (3f_{5/2}\ 5f_{5/2})$	0.739	−0.004
				$\pi\ (3p_{3/2}\ 5p_{3/2})$	0.336	−0.015
		16.30	26.72	$\nu\ (3f_{7/2}\ 5f_{7/2})$	0.288	0.008
				$\nu\ (4g_{9/2}\ 6g_{9/2})$	0.279	−0.004
				$\nu\ (4s_{1/2}\ 6s_{1/2})$	−0.231	0.012
				$\pi\ (3f_{7/2}\ 5f_{7/2})$	0.528	−0.009

TABLE II. (*Continued*).

A		E (MeV)	EWSR (%)	Configuration	Amplitudes X	Y
			Quadrupole states			
112	(a)	12.13	14.63	ν $(4g_{7/2}\ 6i_{11/2})$	−0.811	0.003
				π $(3f_{7/2}\ 5h_{11/2})$	−0.189	−0.002
		12.77	16.22	ν $(5h_{11/2}\ 7j_{15/2})$	0.570	0.002
				ν $(4g_{9/2}\ 6i_{13/2})$	−0.358	−0.001
				ν $(3f_{7/2}\ 5h_{11/2})$	−0.294	−0.009
				π $(3f_{7/2}\ 5h_{11/2})$	−0.443	0.004
	(b)	12.48	8.40	ν $(4p_{3/2}\ 5f_{7/2})$	0.728	0.004
				π $(3p_{3/2}\ 5f_{7/2})$	0.236	0.007
				π $(3f_{7/2}\ 5h_{11/2})$	−0.150	0.000(1)
		12.74	12.64	ν $(3f_{7/2}\ 5h_{11/2})$	−0.264	0.000
				ν $(4g_{9/2}\ 6i_{13/2})$	0.242	0.001
				ν $(3p_{3/2}\ 5f_{7/2})$	0.215	−0.001
				π $(3f_{7/2}\ 5h_{11/2})$	0.804	−0.006
		13.68	11.93	ν $(3f_{5/2}\ 5f_{7/2})$	−0.237	−0.002
				ν $(3f_{7/2}\ 5h_{11/2})$	−0.225	0.007
				π $(3f_{5/2}\ 5h_{9/2})$	0.673	0.003
				π $(2s_{1/2}\ 4d_{5/2})$	0.411	0.004
118	(a)	12.77	24.78	ν $(4g_{9/2}\ 6i_{13/2})$	0.652	0.002
				π $(3f_{7/2}\ 5h_{11/2})$	0.562	−0.005
		12.88	10.09	ν $(3p_{3/2}\ 5f_{7/2})$	0.478	−0.001
				ν $(4d_{5/2}\ 6g_{9/2})$	0.635	0.001
				ν $(3p_{7/2}\ 5h_{11/2})$	0.435	0.002
				π $(3f_{7/2}\ 5h_{11/2})$	−0.232	0.003
	(b)	12.47	21.89	ν $(4g_{9/2}\ 6i_{13/2})$	0.538	0.001
				ν $(3p_{3/2}\ 5f_{7/2})$	0.551	0.004
				ν $(3f_{7/2}\ 5h_{11/2})$	0.489	−0.001
				π $(3p_{3/2}\ 5f_{7/2})$	0.178	0.005
		12.75	12.03	ν $(4g_{9/2}\ 6i_{13/2})$	0.216	0.001
				ν $(3p_{3/2}\ 5f_{7/2})$	0.216	−0.001
				ν $(3f_{7/2}\ 5h_{11/2})$	−0.261	−0.000(1)
				π $(3f_{7/2}\ 5h_{11/2})$	0.803	−0.006
124	(a)	12.69	13.03	ν $(4d_{5/2}\ 6g_{9/2})$	0.653	−0.002
				ν $(3p_{3/2}\ 5f_{7/2})$	0.561	0.005
				π $(3p_{1/2}\ 5f_{5/2})$	0.272	0.003
				π $(3p_{3/2}\ 5f_{7/2})$	0.199	0.004
				π $(2s_{1/2}\ 4d_{5/2})$	−0.123	−0.006
		13.53	10.05	ν $(3p_{1/2}\ 5f_{5/2})$	−0.499	0.001
				π $(2d_{3/2}\ 4g_{7/2})$	0.514	0.000(5)
				π $(3f_{5/2}\ 5h_{9/2})$	0.494	−0.000(1)
	(b)	12.71	22.04	ν $(4g_{9/2}\ 6i_{13/2})$	0.465	−0.000(1)
				π $(3f_{7/2}\ 5h_{11/2})$	0.754	−0.005
		13.58	12.92	ν $(4g_{9/2}\ 6i_{13/2})$	0.293	−0.008
				π $(3f_{5/2}\ 5h_{9/2})$	0.788	0.002

of the resonance energy is expected.[17]

Figure 5 shows, for the case of the GMR in ^{112}Sn, the temperature dependence of the strength distributions. We have obtained these results from the temperature-dependent RPA treatment of the same interaction.[18] Clearly the effects, spreading of the intensities, and shift of the energies, are more pronounced for GQR than the GMR.

III. PAIRING EFFECTS ON NUCLEAR COMPRESSIBILITY

The semiphenomenological procedure employed to extract K_∞ from breathing-mode GMR energies consists of using scaling assumption (for details see Ref. 4). Under this assumption, the finite-nucleus incompressibility K_A can be obtained from the breathing-mode energy E by

$$E=\left[\frac{\hbar^2 K_A}{m\langle r^2\rangle}\right]^{1/2}, \tag{2}$$

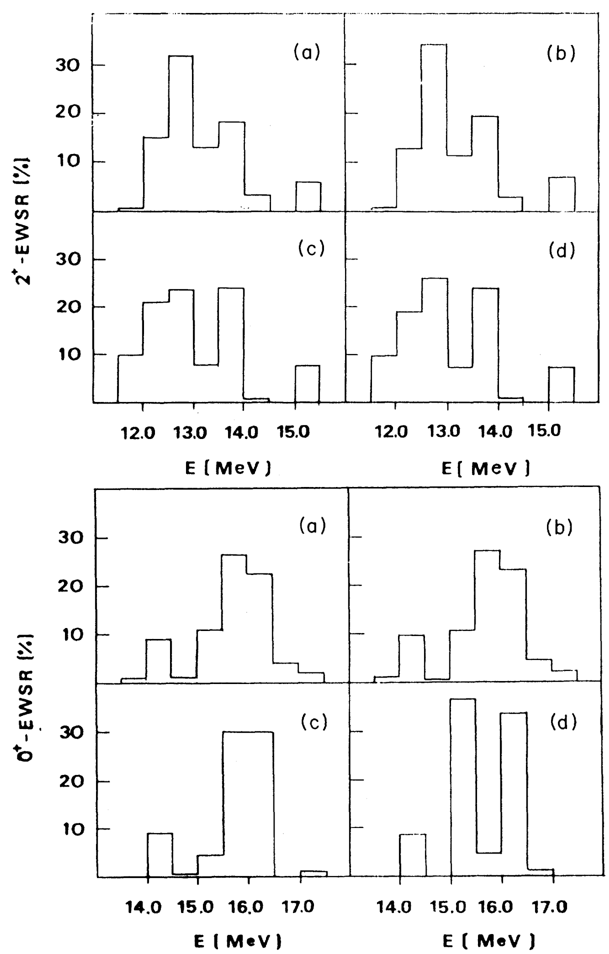

FIG. 5. Histograms corresponding to percentages of quadrupole ($\lambda^\pi=2^+$) and monopole ($\lambda^\pi=0^+$) energy-weighted sum rules in ^{112}Sn for different values of the nuclear temperature. (a)–(d) have been obtained with temperatures $T=0$, 0.4, 0.8, and 1.2 MeV, respectively.

where $\langle r^2\rangle$ is the mean-square radius. The extraction of K_∞ from the calculated values of K_A goes about using a liquid-drop-model type of expansion[1] for K_A:

$$K_A=K_\infty+K_S A^{-1/3}+K_\Sigma\left[\frac{N-Z}{A}\right]^2 +K_{\mathrm{Coul}}Z^2A^{-4/3}+K_{\mathrm{curv}}A^{-2/3}\,. \tag{3}$$

In this expansion, the contributing terms correspond to bulk, surface, asymmetry, Coulomb, and curvature parts, respectively. Since the equilibrium condition of the nuclear ground state has been taken into account,[1] the coefficients K_S and K_Σ are no more the second derivatives of the respective coefficients in the mass formula[19] and the Coulomb coefficient K_{Coul} is modified to

$$K_{\mathrm{Coul}}=\frac{3e^2}{5r_c}(1-27R)\,, \tag{4}$$

here R is related to the third derivative of the volume energy density with respect to the bulk density (see Ref. 4 for details). The expansion (3) does not depend on any assumption like scaling nor on any other approximation such as a mean-field theory with Skyrme-type interactions. Under the scaling assumption, in particular, the volume term of expansion (3) is synonymous with infinite nuclear matter. The microscopic calculations[1] have indeed revealed the scaling nature of the breathing mode. Using various Skyrme interactions and calculating breathing-mode energies in a density variational framework,[21] it has been shown in Ref. 4 that expansion (3) holds well. This expansion has, therefore, been extensively used[4,8,11] to fit the empirically obtained K_A values for various nuclei to extract various compressibility coefficients. It may be worth mentioning that this fitting procedure gives the correct value of K_∞ for a given interaction even in a compressional mode, where the inertial parameter is different from the scaling case.[4] This is due to the *leptodermous* behavior of the breathing nuclear densities which display a division into a homogeneous bulk part and a surface skin. Thus, expansion (3) serves a useful basis to extract K_∞ from experimental data.

The expansion (3) includes all important terms contributing to K_A and it converges fast in the scaling assumption. The nonanalytic nature of the pairing term does not, however, allow its explicit introduction in (3). The expansion (3) was used[4,8,11] without taking into account the pairing effects. The set of nuclei included in the above work were the isotopes of Sn and Sm and the closed-shell nuclei ^{208}Pb and ^{90}Zr. The even-mass isotopes of Sn being open-shell nuclei display a superfluid behavior with a large value of the pairing gap parameter Δ. In the present work, we have observed that the pairing increases the centroid energies of the GMR in Sn nuclei by about 100–150 keV. This effect is obviously minimal. We are, however, interested to find how much this shift in energy of the GMR due to pairing influences K_∞.

We have fit the experimental data from Refs. 4 and 11 using expansion (3) in its present form. The mean-square radius $\langle r^2\rangle$ used in Eq. (2) has been taken from empirical

TABLE III. Results of fits of experimental breathing-mode data (Refs. 11 and 22–24) on Sn and Sm isotopes, ^{90}Zr, ^{208}Pb, and ^{24}Mg from KVI, Groningen to Eq. (3); (a) without corrections for pairing, (b) with corrections for pairing on Sn and Sm nuclei, (c) with pairing corrections only for Sn nuclei. See text for further details.

Sets	K_∞ (MeV)	K_S (MeV)	K_Σ (MeV)	red. χ^2
(a)	300±23	−750±83	−316±192	0.17
(b)	290±24	−731±89	−301±199	0.20
(c)	294±23	−749±84	−295±197	0.85

data.[20] In order to compensate for the pairing effect, which is not included in (3), we have reduced the GMR energies taken from the experimental data[11] of Sn and Sm nuclei by 150 keV. We have assumed that this effect is of the same magnitude for Sm nuclei too. The energies for ^{208}Pb and ^{90}Zr, the other nuclei also included in the fit, are taken to be the same as the experimental one. These are closed-shell nuclei and do not show any superfluidity.

The results of a three-parameter fit of Eq. (3) including the nuclei ^{24}Mg (Ref. 22), ^{90}Zr (Ref. 23), Sn and Sm isotopes (Refs. 4 and 11), and ^{208}Pb (Ref. 24) are shown in Table III. The curvature term has been fixed at a reasonable value $K_{\text{curv}} \sim 350$ MeV. Part (a) shows the results of a fit on all nuclei without any reference to the pairing as in Refs. 4 and 11. On considering the pairing effect in Sn and Sm nuclei (i.e., by reducing the centroid energies by 150 keV as mentioned above), the results are indicated in part (b). The value of K_∞ is reduced by ~ 10 MeV ($\sim 3\%$) from the value obtained without pairing in part (a). This is well within the experimental error bars of 300±25 MeV. The surface and the asymmetry terms also do not show any significant variation and the change is much less than the error bars. The quality of fit (b) remains as good as that of (a) as is evident from the relative χ^2 values obtained. The spherical samarium nuclei ^{144}Sm and ^{148}Sm taken here also have some amount of superfluidity, but lesser than the tin nuclei. Now, if we neglect the pairing effect in Sm isotopes and consider it only for Sn nuclei, the resulting fit is shown in part (c). The parameters of the fit are not different from those of fit (b). However, the fit quality seems to have degraded following the increase in the χ^2 value. This, in a way, seems to demand that the pairing is important for Sm nuclei too if one wants a good fit as in the case (b), where the fit quality is as good as the best fit of (a).

IV. CONCLUSIONS

The effects due to pairing correlations upon the structure of the GMR and the GQR in even-mass Sn isotopes have been discussed. Theoretical results on the energy systematics, which are based on the RPA treatment of a schematic residual two-body interaction of zero range, are found for the GMR in good agreement with the experimental data. Without altering the strength of the interaction in going through the isotopic chain of even-mass Sn nuclei, the calculations show that pairing correlations enforce a shift in the centroid energy of the GMR. This amounts to an overestimation of its value by $\sim 100-150$ keV as compared with the case without pairing correlations. In the case of the GQR, however, the pairing effects are more strongly manifest.

The effect of pairing upon the centroid energies of the isoscalar giant resonances studied in this paper is in contrast with the pairing effect on the isovector giant dipole resonance observed in Ref. 12. The isoscalar resonances show a much less pronounced effect than the isovector case. A larger shift in the isovector case can be traced back to the two reasons. First, because of the nonvanishing gap parameter, the relevant two-quasiparticle energies in superfluid nuclei are larger than the corresponding particle-hole energies found without pairing, and second, the particle-particle and hole-hole pairs present in the case with pairing produce an enhanced collectivity as compared to the case without pairing, where only particle-hole excitations are possible. Due to the repulsive character of the residual interaction, this enhanced collectivity is shifted to higher energies, which is of the order of magnitude of the pairing-gap parameter.[12] The situation for the isoscalar resonances is, however, different. Because of the attractive nature of the residual interaction, the enhanced collectivity produces a lowering of the peak energy and thus canceling to a large extent the upward shift in the energy caused by the gap in the quasiparticle energies. The final effect is non-negligible although small.

We have included the effect of pairing on the energy of the GMR in the analysis of the experimental data for extracting nuclear matter compressibility. The semiphenomenological method based upon the liquid-drop-model expansion of the finite-nucleus incompressibility has been employed to determine K_∞. Using the precision data on the GMR of Sn and Sm nuclei[4,11] and several other data from Kernfysisch Versneller Instituut (KVI), Groningen, the effect of pairing on Sn and spherical Sm nuclei has been taken into account in the fit of experimental data. The resulting incompressibility parameters show a very small change in its values. Thus, the pairing seems to show only a marginal effect on the value of the nuclear matter incompressibility and it is further corroborated that K_∞ obtained empirically is about 300 MeV.

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[1]J. P. Blaizot, Phys. Rep. **64**, 171 (1980).
[2]O. Bohigas, A. L. Lane, and J. Martorell, Phys. Rep. **51**, 267 (1979).
[3]J. Treiner, H. Krivine, O. Bohigas, and J. Martorell, Nucl. Phys. **A371**, 253 (1982).
[4]M. M. Sharma, W. Stocker, P. Gleissl, and M. Brack, Nucl. Phys. **A504**, 337 (1989); M. M. Sharma, in *Proceedings of NATO Advanced Study Institute on Nuclear Equation of State, Peñiscola, Spain, 1989,* edited by W. Greiner and H. Stöcker (Plenum, New York, 1990), Vol. 216A, p. 661.
[5]N. K. Glendenning, Phys. Rev. Lett. **57**, 1120 (1986).
[6]E. A. Baron, J. Cooperstein, and S. Kahana, Phys. Rev. Lett. **55**, 126 (1985).
[7]N. K. Glendenning, Phys. Rev. C **37**, 2713 (1988).
[8]M. M. Sharma, in *Proceedings of the Third International Summer School on Nuclear Astrophysics, La Rabida, Spain, 1988,* Nuclear Astrophysics, Research Reports in Physics, edited by M. Lozano *et al.* (Springer-Verlag, Berlin, 1989), p. 306.
[9]D. H. Youngblood, P. Bogucki, J. D. Bronson, U. Garg, Y. W. Lui, and C. M. Rosza, Phys. Rev. C **23**, 1997 (1981).
[10]See, for a review, M. Buenerd, in *Proceedings of the International Symposium on Highly Excited States in Nuclear Structure, Orsay, France, 1983* [J. Phys. (Paris) Colloq. **45**, C4-115 (1984)].
[11]M. M. Sharma, W. T. A. Borghols, S. Brandenburg, S. Crona, A. van der Woude, and M. N. Harakeh, Phys. Rev. C **38**, 2562 (1988); M. M. Sharma, *Proceedings of the XXVI International Winter Meeting on Nuclear Physics, Bormio, Italy, 1988* (University of Milano, Milano, 1988), Vol. 63, p. 510.
[12]P. Ring, L. M. Robledo, J. L. Egido, and M. Faber, Nucl. Phys. **A419**, 261 (1984).
[13]Nucl. Data Sheets **26**, 207 (1979); **26**, 385 (1979); **29**, 453 (1980); **30**, 431 (1980); **32**, 497 (1981); **33**, 1 (1981); **50**, 63 (1987).
[14]P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New York, 1980).
[15]S. G. Nilsson *et al.,* Nucl. Phys. **A131**, 1 (1969).
[16]M. Baranger, Phys. Rev. **120**, 957 (1960).
[17]F. Alasia, O. Civitarese, and M. Reboiro, Phys. Rev. C **39**, 1012 (1989).
[18]O. Civitarese, A. G. Dumrauf, and M. Reboiro, Phys. Rev. C **41**, 1785 (1990).
[19]W. D. Myers and W. J. Swiatecki, Ann. Phys. (N.Y.) **84**, 186 (1974).
[20]A. M. Bernstein, in *Advances in Nuclear Physics,* edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Vol. 3, p. 325.
[21]P. Geissl, M. Brack, J. Meyer, and P. Quentin, Ann. Phys. (N.Y.) **197**, 205 (1990).
[22]H. J. Lu, S. Brandenburg, R. De Leo, M. N. Harakeh, T. D. Poelhekken, and A. van der Woude, Phys. Rev. C **33**, 1116 (1986).
[23]W. T. A. Borghols, Ph.D. thesis, University of Groningen, 1988 (unpublished); W. T. A. Borghols, S. Brandenburg, J. H. Meier, J. M. Schippers, M. M. Sharma, A. van der Woude, M. N. Harakeh, A. Lindholm, L. Nilsson, S. Crona, A. Hakansson, L. P. Ekström, N. Olsson, and R. De Leo, Nucl. Phys. **A504**, 263 (1989).
[24]S. Brandenburg, W. T. A. Borghols, A. G. Drentje, L. P. Ekström, M. N. Harakeh, A van der Woude, A. Hakansson, L. Nilsson, N. Olsson, M. Pignanelli, and R. De Leo, Nucl. Phys. **A446**, 29 (1987).