PHYSICAL REVIEW C VOLUME 43, NUMBER 6 JUNE 1991

Structure of highly deformed ^{102}Zr populated in decay of low- and high-spin isomers of ^{102}Y

John C. Hill, D. D. Schwellenbach, and F. K. Wohn
Ames Laboratory and Department of Physics, Iowa State University, Ames, Iowa 50011

J. A. Winger*
Ames Laboratory and Department of Physics, Iowa State University, Ames, Iowa 50011 and Brookhaven National Laboratory, Upton, New York 11973

R. L. Gill
Brookhaven National Laboratory, Upton, New York 11973

H. Ohm and K. Sistemich
Institut für Kernphysik, Forschungszentrum Jülich, W-5170 Jülich, Germany
(Received 26 December 1990)

The decay of low- and high-spin isomers of ^{102}Y to levels in highly deformed ^{102}Zr was studied from mass-separated activity produced by thermal neutron fission of ^{235}U. Based on γ singles and coincidence measurements, decay schemes for both isomers are deduced. Of special interest are states observed at 894 and 1211 keV that are postulated to be 0_2^+ and 2_2^+, respectively. Possible interpretations of these newly observed levels in terms of β, γ, or spherical bands are given.

I. INTRODUCTION

The evolution of the nuclear shape in the neutron-rich region near ^{100}Zr is characterized by some unusual features,[1] such as an exceptionally abrupt transition from spherical to highly deformed shapes at $N \simeq 60$. The abruptness of the shape transition is especially large for Sr and Zr. This has been attributed to subshell effects (for $Z=38,40$ and $N=56,58$) and to residual proton-neutron interactions among the valence nucleons.[2,3] A second unusual feature of this region is very low-lying excited 0^+ states, which occur at much lower energies in ^{98}Sr and ^{100}Zr than in any other shape transition region. The 0_2^+ states in these two $N=60$ isotones give evidence for shape coexistence and mixing of 0^+ states with very different deformations.[4–7]

Recent studies[7–11] of level lifetimes of Sr, Y, and Zr nuclei in the $A \simeq 100$ region provide additional insights into the development of collectivity in this region. $B(E2)$ and $\rho(E0)$ values in ^{98}Sr and ^{100}Zr led to the conclusion[7] that ^{98}Sr and ^{100}Zr have coexisting bands that are weakly admixed. The mixing analysis[7] showed that the 0^+ ground states are $\sim 90\%$ D band and $\sim 10\%$ S band. The S bands have moderate (~ 0.2) values of $\langle \beta_2^2 \rangle^{1/2}$, but there is insufficient information to completely characterize the S bands. The highly deformed ($\beta \sim 0.4$) prolate D bands are very similar to the yrast bands in the $N=62$ isotones ^{100}Sr and ^{102}Zr, indicating that the deformation "saturates" at the $N=60$ onset of deformation.[7] The striking similarity in the deformed bands of these two $N=60$ isotones extends also to $B(E2)$ rates from their 4_1^+ levels.[8]

An analysis[10] of $E2$ and $M1$ properties of deformed bands in ^{99}Y, ^{99}Sr, and ^{100}Y indicates, within the presently available precision of the data, that the deformation of the deformed even-even core (^{98}Sr) is unaffected by the presence of a valence proton, a valence neutron, or one of each. Odd-even effects, however, show up in the less strongly deformed Mo isotopes. The saturation of deformation (deduced for even-even $A \simeq 100$ Sr nuclei in Ref. 7) was thus shown[10] to be valid also for these three odd-A and odd-odd nuclei. This conclusion about saturation of deformation for Sr nuclei was also reached in Ref. 11, in which a new $B(E2)$ strength was reported for ^{100}Sr. The "immediate" saturation of deformation in the $A \simeq 100$ region can be explained by simple arguments based on the slope of valence Nilsson orbitals at the onset of deformation.[10]

The structure of the $N=62$ isotones of Sr and Zr are of interest in determining whether the two coexisting bands observed in the $N=60$ isotones still persist as a neutron pair is added. Of special interest is the location of the 0_2^+ and 2_2^+ states and their decay properties. The deformed potential-energy minimum, which is only 0.2–0.3 MeV below the more spherical (or slightly oblate) minimum in the $N=60$ isotones of Sr and Zr, is expected to lie significantly lower for the $N=62$ isotones.[3,12,13] Thus the 0_2^+ state in ^{102}Zr or ^{100}Sr would be expected to be substantially higher in energy, perhaps as high as, or higher than, β- or γ-vibrational excitations of the deformed yrast band. No candidates for 0_2^+ or 2_2^+ states have been located in ^{100}Sr. A primary goal of the present study is to locate these states in ^{102}Zr and compare them to corresponding states of the $N=60$ isotones.

Levels in ^{102}Zr were first studied by Cheifetz *et al.*[14] by observation of prompt γ rays from fission fragments. Transitions from yrast-band states were observed up to the 8_1^+ level and a large deformation was deduced. These results were confirmed in a similar experiment by Khan, Hofmann, and Hirsch.[15] Delayed-neutron emission from ^{102}Y was observed by Reeder *et al.*[16] The ^{102}Y decay was studied by Shizuma *et al.*,[17] using the JOSEF separator under conditions in which ^{102}Y fission fragments were directly separated; thus, the observed ^{102}Y activity was primarily that of a "high-spin" isomer of ^{102}Y. This study has been repeated under improved experimental conditions. In contrast, in the present experiment at TRISTAN, the ^{102}Y came from the β decay of ^{102}Sr; thus, the decay of a "low-spin" isomer of ^{102}Y is observed. Preliminary results from the present experiment have been presented earlier.[18]

II. EXPERIMENTAL METHODS AND RESULTS

A. Source preparation

At TRISTAN, sources of ^{102}Y were obtained from the $A=102$ mass-separated beam produced by the TRISTAN facility operating on-line to the High-Flux Beam Reactor at Brookhaven National Laboratory. The target consisted of 5 g of enriched ^{235}U in a high-temperature thermal ion source.[19] The neutron flux at the target was 3×10^{10} neutrons/cm^2 s. We assume that the $A=102$ beam consisted primarily of ^{102}Sr ions, with only a minor amount of ^{102}Y ions in the beam, due to the characteristics of the thermal ion source, although the fission yield[20] of ^{102}Sr is much smaller than the yield of ^{102}Y. The ^{102}Y activity studied in this work is therefore assumed to originate primarily from the decay of ^{102}Sr. There is no evidence for cross contamination from adjacent masses.

JOSEF is a recoil separator which has been operated at the Research Reactor DIDO of the Kernforschungsanlage (KFA) Jülich. JOSEF needs no ion source. The number of fission fragments available for spectroscopic studies is thus directly proportional to the fission yield for that fragment. Since the independent fission yields of ^{102}Sr and ^{102}Y in the thermal fission of ^{235}U are estimated[20] to be about 5×10^{-5}% and 1.3×10^{-1}%/fission, respectively, the overwhelming contribution to the $A=102$ beam at JOSEF is due to the latter. Therefore, if a high-spin isomer in ^{102}Y exists, then it may well be populated in the fission process and its decay studied at JOSEF. A previous study[17] at JOSEF of the ^{102}Y decay had indicated the existence of a high-spin isomer. The present JOSEF study was obtained under improved experimental conditions.

B. Measurements

At TRISTAN, the γ-ray measurements were made using two HPGe detectors in 180° geometry. The beam was collected for 1 s and then moved to an intermediate position for a 1-s count. This was necessary to suppress the short-lived ($T_{1/2}=68$ ms) ^{102}Sr parent activity.[21] The tape move time was measured to be about 0.36 s. A thin plastic scintillator provided a coincidence β gate which allowed for background suppression. Due to the low level of activity, only the strong 151-keV γ ray from ^{102}Y could be observed without the β gate. The only background observed with β gating was from Pb x rays and γ rays from capture of neutrons in the Ge detectors. After establishing the presence of ^{102}Y, the main data-taking run began in which the sample decay was followed for 1 s. The 1-s counting period was divided into 20 sequential time bins, each 0.05 s long. After each counting period the tape was moved and the cycle repeated. This time sequence allowed us to suppress short-lived ^{102}Sr and to separate ^{102}Y γ rays from those originating from the decay of longer-lived nuclides (^{102}Zr, ^{102}Nb, etc.) further down the decay chain, thus enabling us to measure the ^{102}Y half-life and obtain γ-γ coincidences with ^{102}Y decay enhanced. Data taking continued for about one week.

At JOSEF, γ-ray singles spectra were measured with a planar Ge detector of 19 cm^2 area and 1.9 cm depth. These spectra were obtained at several values of the magnetic rigidity of JOSEF in order to identify[22] the γ rays from the decay of ^{102}Y. γ-γ coincidences were studied with the planar Ge detector and a coaxial Ge detector of 130 cm^3 volume. The γ-ray singles and γ-γ coincidence measurements were both made in-beam[22] with irradiation and measurement periods of 4 s. After each period the activity was removed with a transport tape within 0.4 s, and the 4.4-s cycle repeated.

C. Half-life

The half-life of the low-spin isomer of ^{102}Y was determined at TRISTAN by following the decay of the 151-keV γ ray, which was the only ^{102}Y γ ray intense enough to provide sufficient statistics. The resulting decay curve is shown in Fig. 1. A weighted least-squares fit to the above curve yielded a half-life for ^{102}Y of 0.30(1) s, which

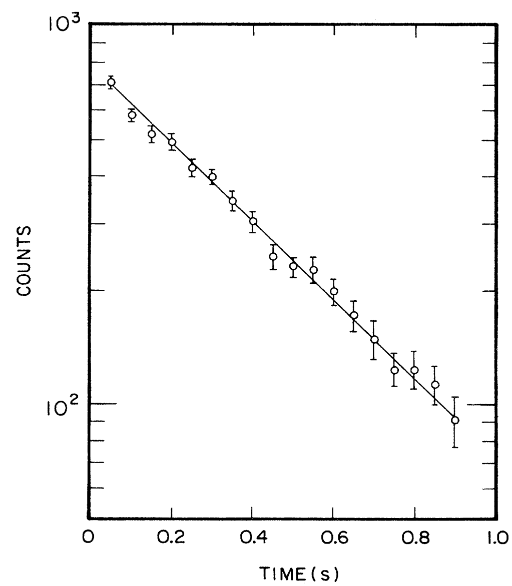

FIG. 1. Decay curve for the strong 151-keV γ ray following the decay of ^{102}Y. The line represents a half-life of 0.30 s.

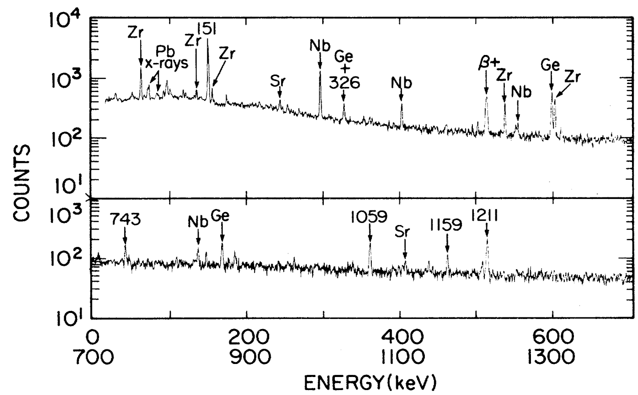

FIG. 2. γ-ray singles spectrum from the $A=102$ source. The counting period for each sample was 1 s. The γ rays from ^{102}Y are indicated by their energies in keV. Strong known γ rays from ^{102}Sr, ^{102}Zr, and ^{102}Nb decay are indicated by Sr, Zr, and Nb, respectively. Ge indicates Ge neutron-capture γ rays.

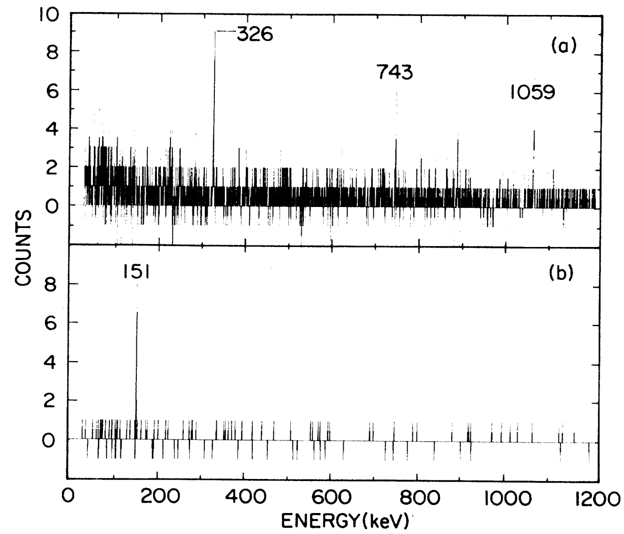

FIG. 3. γ-ray spectra in coincidence with the (a) 151-keV transition and (b) 743-keV transition. The Compton background has been subtracted.

is not in agreement with the value of 0.44(6) s measured from delayed-neutron decay by Reeder *et al.*[16] Our measured value is slightly different from the value of 0.36(4) s for ^{102}Y decay measured[17] at the JOSEF separator where mainly the decay of the high-spin isomer of ^{102}Y is observed, as discussed below. (The γ-ray spectrum multiscaling measurement at JOSEF [for which a half-life of 0.36(4) s for ^{102}Y was reported in Ref. 17] was not repeated in the present JOSEF measurements.)

D. γ-ray energies and intensities

A γ-ray spectrum, obtained by shifting and summing β-gated singles spectra from the two detectors which viewed the source at TRISTAN, is shown in Fig. 2. The γ energies, relative intensities, placements, and coincidence relationships are summarized in Table I. Uncertainties in the energies are due to statistical factors in determining peak centroids and system nonlinearities. The intensity uncertainties are due to statistical factors in determining peak areas and detector efficiencies. γ-ray spectra obtained at TRISTAN in coincidence with the 151- and 743-keV γ rays are shown in Fig. 3. The Compton background has been subtracted.

Inspection of the TRISTAN γ-ray spectrum in Fig. 2 reveals γ rays from ^{102}Sr, ^{102}Zr, and ^{102}Nb decay as well as from ^{102}Y decay. Due to the short counting period no γ rays were observed from longer-lived ^{102}Mo and ^{102}Tc decays. A number of weak γ rays were observed with half-lives longer than that of ^{102}Y. They were distinguished from ^{102}Y decay by comparison of sums of appropriate sequential γ multiscaling spectra. These unidentified γ rays are probably from the decay of ^{102}Zr and ^{102}Nb.

TABLE I. γ transitions observed in ^{102}Y decay.

	Relative γ-ray intensity			Coincident γ rays (keV)	
E_γ (keV)[a]	TRISTAN[b]	JOSEF[c]	High spin[d]	TRISTAN	JOSEF
151.73(14)	100(4)	100(3)	79(10)	326,743,1059	159,326,486,579 743,1059,1091
159.8	<1.1	8.0(8)	8.0(8)		151,579
326.64(15)	8.6(9)	44(3)	42(3)	151	151,486
486.8	<1.9	6.7(11)	6.7(11)		151,326
579.4	<1.1	28(3)	28(3)		151,159
743.01(18)	17(4)	3.5(14)		151	
1059.21(18)	29(3)	14.1(17)	8(3)	151	151
1091.3	<1.3	33(3)	33(3)		151,159,579
1159.49(22)	16.0(19)				
1211.08(16)	40(4)		11(4)		

[a]Energies with uncertainties are from TRISTAN γ-ray singles studies; those without uncertainties are from JOSEF (see Ref. 17).
[b]TRISTAN intensities are from low-spin isomer of ^{102}Y.
[c]JOSEF intensities are from a mix of low-spin and high-spin ^{102}Y isomers.
[d]Intensities of high-spin isomer are deduced by combining TRISTAN and JOSEF studies as explained in Sec. III C.

The γ-ray energy range covered in the TRISTAN experiment was from 18 keV to 3.6 MeV, but no γ rays were observed from ^{102}Y decay with energies greater than 1211 keV. The energy nonlinearities and γ-ray efficiencies of the detector system were determined using standard γ-ray sources. Gamma rays of 159, 486, 579, and 1091 keV were observed at JOSEF (see below) but not at TRISTAN. Upper limits on their intensities from the TRISTAN measurements are given in Table I.

At JOSEF the γ-ray singles spectra allowed γ rays of 151, 326, and 579 keV to be unambiguously assigned to the decay of ^{102}Y using the isotope identification procedure[22] used at JOSEF. (Other γ rays were assigned to the ^{102}Y decay based on γ-γ coincidences.) The relative γ-ray intensities which result from these singles spectra are given in Table I. As discussed below, the γ-ray intensities obtained at JOSEF were combined with those obtained at TRISTAN to deduce the separate decays of the two isomers of ^{102}Y.

E. γ-γ coincidence relationships

At TRISTAN, γ-γ coincidence measurements were carried out with the front face of the HPGe detectors 1.5 cm from the $A=102$ source. Because of the low level of the $A=102$ activity, all γ-γ events were β gated. The coincidence events were recorded as address triplets representing the γ-ray energies and time separation. The timing signal was derived from a standard fast coincidence system with a time resolution full width at half maximum (FWHM) of about 20 ns. Table I gives the deduced coincidence relationships.

At JOSEF, the coincidence pattern that was obtained is given in Table I. The present study resulted in improved coincidence statistics compared to the previous JOSEF study of Ref. 17. It confirms most of the relationships given in Ref. 17, except for the possible coincidence[17] between the 326- and 579-keV transitions. The present JOSEF study shows that these two γ rays are not in coincidence. An additional, although weak, coincidence between the 159- and 1091-keV transitions is deduced from the present results.

The relative intensities of γ rays observed at JOSEF in coincidence with the 151-keV γ ray were deduced from the areas of the peaks in the spectrum gated by the 151-keV γ ray. They have been normalized to the 326-keV γ ray, whose intensity was observed in the singles spectrum to be 44(3) relative to an intensity of 100 for the 151-keV γ ray. We point out that the relative intensities are more precise than those given in Ref. 17. However, a deviation beyond the quoted uncertainties occurs only for the 579-keV γ ray, which must be attributed to a contaminant in the previous study of Ref. 17.

III. LEVEL SCHEME FOR ^{102}Zr

A. Evidence for two β^- decaying isomers in ^{102}Y

There is little doubt that two β^- decaying ^{102}Y isomers exist. The ratio of γ-ray intensities $I(2^+\rightarrow0^+)/I(4^+\rightarrow2^+)$ observed at JOSEF is 2.3(2) compared to 11.6(17) at TRISTAN. This ratio can be understood if both a low-spin and a high-spin isomer exist for ^{102}Y. The amount of ^{102}Y in the $A=102$ beam at TRISTAN is very small due to the refractory nature of Y compared to Sr, but this is not a factor for the JOSEF fragment analyzer. Thus the ^{102}Sr ($J^\pi=0^+$) activity[21] in the TRISTAN ion beam β^- decays only to the low-spin isomer of ^{102}Y, whereas both isomrs of ^{102}Y were observed at JOSEF. As additional support for this argument, the 965-keV 6^+ state in ^{102}Zr was not populated at TRISTAN but was observed at JOSEF. Another factor adding strength to the above argument that a low-spin decay mode is seen at TRISTAN is the fact that the "γ strength" observed at TRISTAN for the decay of ^{102}Sr is much greater than for the decay of ^{102}Y, suggesting β^- decay directly to the ^{102}Y ground state. It is interesting to note that the half-lives of the two β^- decaying states are very similar. Furthermore, the energy difference between the two isomers is unknown and there is no information about which isomer is the ^{102}Y ground state. It is possible that the higher-lying isomer may decay only by β^- decay.

B. ^{102}Zr from β^- decay of low-spin ^{102}Y

The level scheme for ^{102}Zr populated in ^{102}Y decay (low-spin isomer) is shown in Fig. 4(a). The scheme is based on the TRISTAN γ-ray singles and γ-γ coincidence measurements.

The ground-state band. The first two excited yrast states are the 2_1^+ state at 151 keV and the 4_1^+ state at 478 keV. Their assignments are well established from previous studies of ^{102}Zr decay[17] and prompt decay of fission fragments.[14,15] The ratio $E(4_1^+)/E(2_1^+)$ for the energies of the first two yrast states is 3.15, which is close to the limit of 3.33 for a rigid rotor.

894-keV level. We postulate a level at 894 keV based on a strong coincidence between the 743- and 151-keV γ rays. A crossover γ ray to the ground state was not observed and the upper limit on its intensity is 0.7 compared to an intensity of 17 for the 743-keV γ ray. Based on the absence of a transition to the ground state, we postulate a tentative assignment of 0_2^+ for the 894-keV level. This is quite different from the situation in ^{100}Zr where the energy of the 0_2^+ state is at 331 keV. The significance of this result is discussed in the next section. The above conclusion is also consistent with our postulate that the decay of only the low-spin isomer of ^{102}Y was observed at TRISTAN and that the 743-keV transitions is only weakly seen at JOSEF.

1159- and 1211-keV levels. The 1159-keV level is based on a relatively intense γ ray at 1159 keV that was not in coincidence with any other γ rays but whose decay characteristics indicated it originated from ^{102}Y decay. No stopover transitions from the 1159-keV level were observed. Since only one depopulating transition was observed, the level is dashed in Fig. 4(a). The level postulated at 1211 keV is well established based on observation of a crossover transition at 1211 keV and a strong coincidence betwen the 151- and 1059-keV γ rays. The 1211-keV level is strongly fed in β^- decay since 44% of the β feeding to excited states populates this level. A ten-

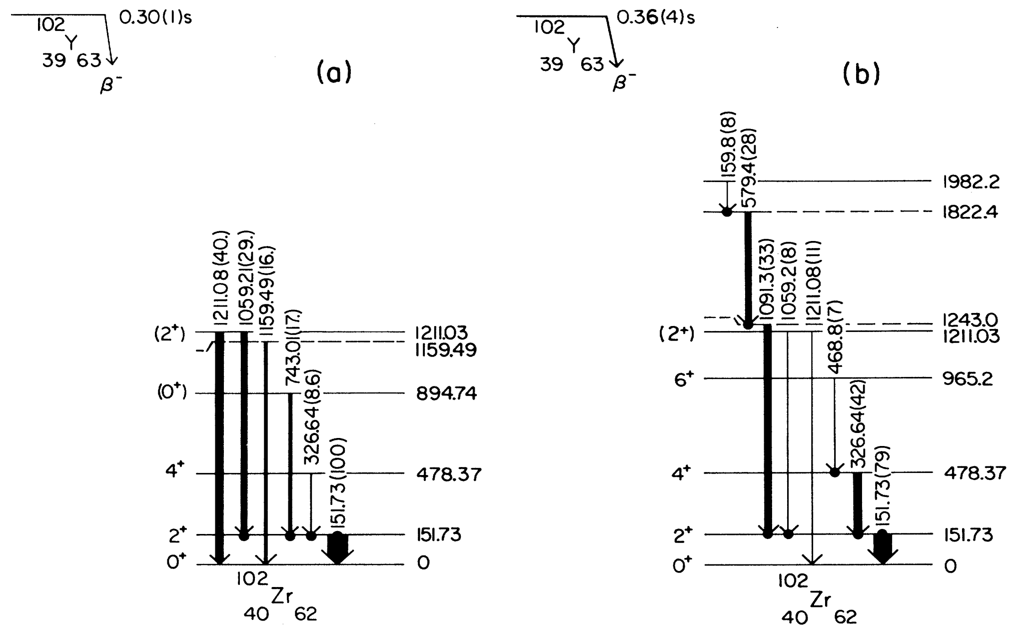

FIG. 4. Decay scheme for (a) low-spin and (b) high-spin isomers of ^{102}Y with energies in keV. The intensities in (a) and (b) are from the TRISTAN and high-spin columns, respectively, in Table I.

tative assignment of 2^+ is given for the 1211-keV level based on observation of transitions to both the ground and 2_1^+ states and the fact that 1^+ states are not expected to lie this low in energy for deformed nuclei.

Neither β^- feeding nor logft values are given in the decay of the low-spin ^{102}Y isomer shown in Fig. 4(a). The β^- feeding to the ground state of ^{102}Zr is not zero. This is supported by a comparison of "γ strengths" in an $A=102$ equilibrium spectrum which indicates that the γ strength for ^{102}Sr decay is roughly twice that for ^{102}Y decay. This, together with the assumption that the $A=102$ activity in the TRISTAN ion beam is mainly ^{102}Sr, leads to the conclusion that about half of the β^- decay of ^{102}Y goes directly to the ^{102}Zr ground state. Furthermore, due to the high β^--decay Q value, which is estimated to be 11.0(5) MeV from systematics,[23] and the weak level of $A=102$ activity observed at TRISTAN, it is likely that significant γ-ray intensity was not observed. Therefore the uncertainties are too large to warrant reporting β^- feedings and logft values based on the present data. Nevertheless, the available data indicate that both the 0^+ and 2^+ states of ^{102}Zr appear to have significant β^- feeding. This suggests a tentative conclusion that the low-spin isomer of ^{102}Y is likely to be limited to 0^-, $1^\pm$, or 2^-. Except for 0^-, which was ruled out in Ref. 21, these J^π values are consistent with those deduced from the decay[21] of ^{102}Sr.

C. ^{102}Zr from β^- decay of high-spin ^{102}Y

The γ rays observed in the JOSEF studies of the decay of ^{102}Y contain contributions from both the low- and high-spin isomers of ^{102}Y. The decays of the two isomers can be disentangled by using the intensity of the 743-keV γ ray, which is intense in the low-spin decay but is hardly seen at JOSEF. If it is assumed that the low-spin component of the intensity for a γ ray is proportional to the intensity of that γ ray relative to the 743-keV γ ray, then the decay scheme for the high-spin isomer of ^{102}Y can be deduced. The resulting γ-ray intensities are labeled "high spin" in Table I.

The decay scheme of the high-spin isomer of ^{102}Y is given in Fig. 4(b). Certain aspects of this decay scheme need to be discussed. The 1211-keV γ ray is not seen at JOSEF because of contaminants in the singles spectra; thus, its intensity given in Fig. 4(b) is deduced from the branching of the 1211-keV level [see Fig. 4(a)]. The relative ordering of the 159-, 579-, and 1091-keV cascade is not firm. The low intensity of the 159-keV γ ray suggests its position at the top of the cascade since internal conversion for this transition is not expected to account for the difference in the intensities of the 159- and 579-keV γ rays. The fact that the high-spin isomer of ^{102}Y populates the 6^+ and 4^+ states of ^{102}Zr strongly suggests that the spin of the isomer is too high for direct β^- feeding of the 0^+ ground state of ^{102}Zr.

With the assumption of no β^- feeding of the ^{102}Zr ground state by the high-spin ^{102}Y isomer, it is possible to deduce β^- feedings for the levels shown in Fig. 4(b). The logft values could also be deduced, using the estimated[23] β^--decay Q value of 11.0(5) MeV and the half-life[17] of 0.36(4) s. However, this procedure is not warranted for the following reason. The ratio of the γ strengths observed at JOSEF for the decays of ^{102}Y and ^{102}Zr should be the same as the ratio of their fission yields. The observed γ strengths of these isobars, however, show that only about one-third of the expected γ strength of the high-spin isomer of ^{102}Y was observed. This suggests that about two-thirds of the γ-ray intensity in the decay of the high-spin ^{102}Y isomer goes to high-lying levels that populate the low-lying levels of Fig. 4(b) via unobserved γ cascades. The level with the largest combined (i.e., direct β^- plus indirect γ cascade) feeding is the 4^+ level at 478 keV. Its combined feeding is 32(3)% of the high-spin isomer decays. All other levels have less than 20% combined feeding. There is little that can be deduced about the J^π of the high-spin ^{102}Y isomer. The mere fact that a high-spin isomer exists, however, suggests a spin difference between the two isomers of at least three. The observed large combined feeding of the 4^+ level is not unreasonable given the likely J^π values of 1^+ or 2^- for the low-spin isomer.

IV. DISCUSSION

The new levels of interest for the structure of ^{102}Zr are those at 894 and 1211 keV with tentative spin-parity assignments of 0^+ and 2^+, respectively. These levels, which decay only to members of the yrast band, are referred to below as the 0_2^+ and 2_2^+ levels. Before discussing these levels further, however, we first discuss the yrast band in ^{102}Zr in order to determine whether mixing of coexistence shapes is indicated for ^{102}Zr as it is for ^{100}Zr. Only the four lowest members of the yrast band in ^{102}Zr are populated in the decay shown in Fig. 4(b), but other yrast band members are known: a 8_1^+ level at 1595 keV,[15,17] and a 10_1^+ level at 2352 keV.[24]

The yrast-band systematics presented in Fig. 5 reveal a smooth pattern for ^{102}Zr, with a linear relationship for the kinematic moments of inertia for the known yrast-band members. In contrast, the situation for ^{100}Zr shows that the lowest point deviates markedly from the linearity of the other points. This deviation reflects the mixing of the 0^+ states of the S and D bands discussed in Ref. 7.

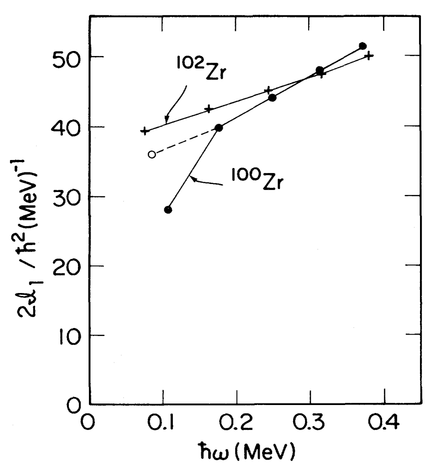

FIG. 5. Systematics of kinematic moments of inertia $[2\mathcal{J}_1/\hbar^2=(4I-2)/E_\gamma]$ as a function of rotational frequency $(\hbar\omega=E_\gamma/2)$ for ^{100}Zr and ^{102}Zr. The open point for ^{100}Zr was obtained after corrections for the mixing of the shape-coexistent 0^+ states (Ref. 7).

(In Ref. 7 the $E2$ transition between the 2_1^+ and 0_2^+ or 0_1^+ states was assumed to be due solely to the D-band component of these 0^+ states. The observed $E2$ strengths then determine the band mixing and thus the unperturbed energies of the S and D bands.) The mixing causes the 0_1^+ level to lie lower in energy than the 0^+ member of the D band; thus, it raises the energy of the 2_1^+ level. The energy difference between the 2_1^+ level (assumed[7] to be pure D band) and the 0^+ member of the D band can be used to "modify" the lowest point in Fig. 5. This gives the new point which restores, to a good approximation, the expected linearity. We note that the mixing in Ref. 7 was deduced independently of any moment of inertia systematics (but used the $E2$ branching from the 0_2^+ level), which increases our confidence in drawing inferences about mixing from curves like those in Fig. 5. Comparing the "mixing-modified" ^{100}Zr systematics with the ^{102}Zr systematics shows that no mixing correction is needed for the yrast band in ^{102}Zr. We further note that the yrast bands of ^{100}Zr and ^{102}Zr become nearly identical at higher spins. The yrast band in ^{102}Zr appears to be similar to the D band of ^{100}Zr, with both bands having deformations of ~ 0.4. These two bands are also similar in terms of their $2\mathcal{J}_1/\hbar^2$ values to those of ^{104}Zr, but differ significantly from the $2\mathcal{J}_1/\hbar^2$ values for ^{101}Zr and ^{103}Zr, as is discussed in Ref. 24.

We next consider the 894-keV 0_2^+ and 1211-keV 2_2^+ levels of ^{102}Zr. The energy of the 0_2^+ is almost three times higher in ^{102}Zr than the value of 331 keV for the corresponding state in ^{100}Zr. The relevant question is as follows: Do either of these levels resemble the S-band levels of ^{100}Zr, or are they members of β- or γ-vibrational excitations of the deformed core? First we note that the 2_2^+ and 0_2^+ levels lie too close in energy to both be members of a band similar to the S band of ^{102}Zr. The 2_2^+ and 0_2^+ levels could both belong to the same β band, however, since the dynamic deformation calculation by Kumar *et al.*[25] gave a β band with an 840-keV 0^+ level and a 1207-keV 2^+ level. This calculation also gave a γ band with its 2^+ bandhead at 823 keV, but there is no analogous 2^+ level observed in the present study. The calculated β and γ bands both have β values that are nearly identical to the $\simeq 0.4$ deformation of the yrast band. Unfortunately, there are no lifetime data on the 0_2^+ or 2_2^+ levels that could determine whether they belong to β- or γ-vibrational bands. We note that the 2_2^+ level has the γ-decay pattern expected for a γ-vibrational 2^+ state if we assume that the transition $2_2^+\rightarrow 2_1^+$ is pure $E2$, since this gives a value of 1.42 ± 0.20 for the ratio $B(E2,2_2^+\rightarrow 2_1^+)/B(E2,2_2^+\rightarrow 0_1^+)$, in good agreement with the Alaga rule value of 10/7. It is thus quite plausible that the level at 1211 keV is the 2^+ member of the γ-vibrational band. (We further note that the 2_2^+ level does not have the γ-decay pattern of a β-vibrational 2^+ state, for which the analogous Alaga rule value is 5017.)

The possibility remains that the 0_2^+ level is not the head of a β band but is instead the head of a band similar to the S band of ^{100}Zr, which has an rms deformation of magnitude $\simeq 0.2$ (instead of a value of $\simeq 0.4$ expected for a β band). The feasibility of this depends upon whether the 0_2^+ energy of 894 keV is reasonable for shape coexistence of "spherical" and deformed 0^+ states in ^{102}Zr. Calculations[26–28] of 0^+ states for the Zr isotopes have been made in order to determine how the energy difference between normal 0^+ and intruder 0^+ states depends upon N. The normal 0^+ state is assumed to have an intact $Z=40$ subshell whereas the intruder 0^+ state has proton particle-hole excitations across the $Z=40$ subshell. The intruder 0^+ state falls below the normal 0^+ state for ^{100}Zr; thus, the normal 0^+ state models the "spherical" or S-band 0^+ state and the intruder 0^+ state models the deformed D-band 0^+ state. According to the calculations of Etchegoyen, Federmann, and Vergini[26] the energy difference $E(0_D)-E(0_S)$ is -0.19 MeV for ^{100}Zr and -0.57 MeV for ^{102}Zr, in good agreement with the experimental value of -0.33 for ^{100}Zr and the present value of -0.89 MeV for ^{102}Zr. They[26] conclude that the simultaneous polarization of protons and neutrons to the spin-orbit partner orbits (in particular $1g_{7/2}$ and $1g_{9/2}$) seems to play a crucial role in obtaining the rotational bands in the Zr isotopes.

Heyde *et al.*[27] calculated the excitation energy for low-lying 0^+ intruder states as a function of N for major closed shells ($Z=50, Z=82$) and subshells ($Z=40, Z=64$). For Zr, two-particle–two-hole (2p-2h) excitations across the $Z=40$ subshell separated intruder from normal states. The intruder excitation energy, or $E(0_D)-E(0_S)$, includes single-particle energies, pairing, and two p-n interaction terms ΔE_M and ΔE_Q. The monopole p-n term ΔE_M for $Z=40$ was calculated for N between 56 and 82 using a residual δ interaction and the quadrupole term ΔE_Q was calculated using an SU(3) expression[28] for two choices of the neutron shell closure. Figure 6 gives the results of the calculation[27] together with experimental values for $N=56$–62. The excellent agreement between experiment and the dashed curve for deformed ^{100}Zr and ^{102}Zr implies that the $N=56$ subshell does not remain closed once deformation sets in at $N=60$. The experimental points for ^{96}Zr and ^{98}Zr lie within the hatched region, suggesting an "intermediate" situation for these nuclei. In other words, the energy gap associated with the $N=56$ subshell decreases with N and is no longer significant after the onset of deformation at $N=60$.

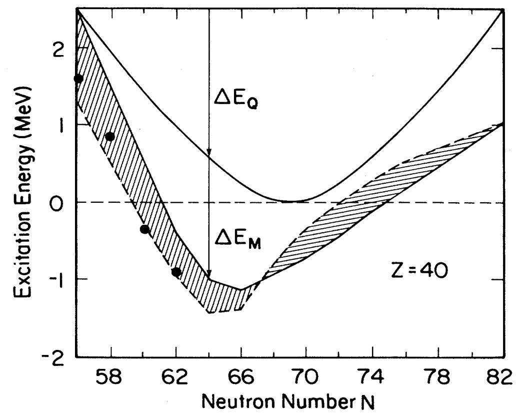

FIG. 6. The excitation energy for the lowest 2p-2h 0^+ intruder state as a function of N. The thick, full line is the value of $\Delta-\Delta E_M-\Delta E_Q$ obtained for a neutron shell of 56–82. The Δ value of 2.5 MeV was deduced for ^{96}Zr.[27] This neutron shell and the corresponding ΔE_Q curve shown above result from assuming the $2d_{5/2}$ neutron subshell is intact for all values of N. The dashed curve results when calculating ΔE_Q for a $N=50$–82 shell. The hatched region gives the possible range for the intruder 0^+ state for the two extremes $N=56$ and $N=50$ for the neutron closed shell. The solid dots for $N=56$–62 are the experimental values. (This figure is Fig. 16 of Ref. 27 with the experimental points added.)

In conclusion, the 0^+ state at 894 keV could be interpreted either as an essentially "spherical" state[26,27] or as the bandhead of a β-vibrational excitation of the highly deformed core.[25] In addition, the 2^+ state at 1211 keV could belong either to such a β band or be the bandhead of a γ band. Additional experiments, involving either location of other members of β and γ bands, or the determination of the lifetimes of these states, could remove the ambiguity in the character of these nonyrast states.

ACKNOWLEDGMENTS

The authors acknowledge K. Heyde and R. F. Casten for useful discussions. This work was supported by the U.S. Department of Energy under Contract Nos. W-7405-ENG-82 and DE-AC02-76CH00016.

*Present address: National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824.

[1] *Nuclear Structure of the Zirconium Region,* edited by J. Eberth, R. A. Meyer, and K. Sistemich (Springer-Verlag, Berlin, 1988).

[2] P. Federman and S. Pittel, Phys. Rev. C **20**, 820 (1979).

[3] R. Bengtsson, P. Möller, J. R. Nix, and J. Zhang, Phys. Scr. **29**, 402 (1984).

[4] F. Schussler, J. A. Pinston, E. Monnand, A. Moussa, G. Jung, E. Koglin, B. Pfeiffer, R. V. F. Janssens, and J. Van Klinken, Nucl. Phys. **A339**, 415 (1980).

[5] K. Becker, G. Jung, K. H. Korbas, H. Wollnik, and B. Pfeiffer, Z. Phys. A **319**, 193 (1984).

[6] F. K. Wohn, J. C. Hill, C. B. Howard, K. Sistemich, R. F. Petry, R. L. Gill, H. Mach, and A. Piotrowski, Phys. Rev. C **33**, 677 (1986).

[7] H. Mach, M. Moszyński, R. L. Gill, F. K. Wohn, J. A. Winger, J. C. Hill, G. Molnár, and K. Sistemich, Phys. Lett. B **230**, 21 (1989).

[8] H. Ohm, M. Liang, G. Molnár, and K. Sistemich, Z. Phys. A **334**, 519 (1989).

[9] G. Lhersonneau, H. Gabelmann, N. Kaffrell, K.-L. Kratz, B. Pfeiffer, and the ISOLDE Collaboration, Z. Phys. A **332**, 243 (1989).

[10] H. Mach, F. K. Wohn, M. Moszyński, R. L. Gill, and R. F. Casten, Phys. Rev. C **41**, 1141 (1990); F. K. Wohn, H. Mach, M. Moszyński, R. L. Gill, and R. F. Casten, Nucl. Phys. **A507**, 141c (1990).

[11] G. Lhersonneau, H. Gabelmann, N. Kaffrell, K.-L. Kratz, B.

Pfeiffer, and K. Heyde, Z. Phys. A **337**, 143 (1990).
[12]F. Tondeur, Nucl. Phys. **A359**, 278 (1981).
[13]D. Galeriu, D. Bucurescu, and M. Ivaşcu, J. Phys. G **12**, 329 (1986).
[14]E. Cheifetz, J. B. Wilhelmy, R. C. Jared, and S. G. Thompson, Phys. Rev. C **4**, 1913 (1971).
[15]T. A. Khan, D. Hofmann, and F. Horsch, Nucl. Phys. **A205**, 488 (1973).
[16]P. L. Reeder, R. A. Warner, R. L. Gill, and A. Piotrowski, in *Proceedings of the Specialists Meeting of Delayed Neutron Properties, University of Birmingham, 1986,* edited by D. R. Weaver (Birmingham, England, 1986), p. 37.
[17]K. Shizuma, J. C. Hill, H. Lawin, M. Shaanan, H. A. Selic, and K. Sistemich, Phys. Rev. C **27**, 2869 (1983).
[18]J. C. Hill, F. K. Wohn, J. A. Winger, D. D. Schwellenbach, R. L. Gill, and K. Sistemich, Bull. Am. Phys. Soc. **33**, 1584 (1988).
[19]A. Piotrowski, R. L. Gill, and D. C. McDonald, Nucl. Instrum. Methods **26B**, 249 (1987).
[20]A. C. Wahl, At. Data Nucl. Data Tables **39**, 1 (1988).
[21]J. C. Hill, J. A. Winger, F. K. Wohn, R. F. Petry, J. D. Goulden, R. L. Gill, A. Piotrowski, and H. Mach, Phys. Rev. C **33**, 1727 (1986).
[22]K. Sistemich, J. W. Grüter, H. Lawin, J. Eidens, R. Fabbri, T. A. Kahn, W. D. Lauppe, G. Sadler, H. A. Selic, M. Shaanan, and P. Armbruster, Nucl. Instrum. Methods **130**, 491 (1975).
[23]A. H. Wapstra, G. Audi, and R. Hoekstra, At. Data Nucl. Data Tables **29**, 281 (1988).
[24]M. A. C. Hotchkis, J. L. Durell, J. B. Fitzgerald, A. S. Mowbray, W. R. Phillips, I. Ahmad, M. P. Carpenter, R. V. F. Janssens, T. L. Khoo, E. F. Moore, L. R. Mors, Ph. Benet, and D. Ye, Phys. Rev. Lett. **64**, 3123 (1990).
[25]K. Kumar, B. Remaud, P. Auger, J. S. Vaagen, A. C. Rester, R. Foucher, and J. H. Hamilton, Phys. Rev. C **16**, 1235 (1977).
[26]A. Etchegoyen, P. Federman, and E. G. Vernini, Phys. Rev. C **39**, 1130 (1989).
[27]K. Heyde, J. Jolie, J. Moreau, J. Ryckebusch, M. Waroquier, P. Van Duppen, M. Huyse, and J. L. Wood, Nucl. Phys. **A466**, 189 (1987).
[28]K. Heyde, P. Van Isacker, R. F. Casten, and J. L. Wood, Phys. Lett. **155B**, 303 (1985).