Tests of microscopic calculations of multiple band structures and large deformations in ^{68}Ge and ^{72}Se

L. Chaturvedi,* X. Zhao, A. V. Ramayya, J. H. Hamilton, J. Kormicki,[†] S. Zhu,[‡] C. Girit,[§] H. Xie, W.-B. Gao, and Y.-R. Jiang
Physics Department, Vanderbilt University, Nashville, Tennessee 37235

A. Petrovici,** K. W. Schmid, and Amand Faessler
Institut für Theoretische Physik, Universität Tübingen, Germany

N. R. Johnson, C. Baktash, I. Y. Lee, F. K. McGowan, M. L. Halbert, M. A. Riley,[††] and J. H. McNeill
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

M. O. Kortelahti[‡‡]
Physics Department, Louisiana State University, Baton Rouge, Louisiana 37830

J. D. Cole
Idaho Nuclear Engineering Laboratory, Idaho Falls, Idaho 43415

R. B. Piercey
Department of Physics, Mississippi State University, Starkville, Mississippi 39762

H. Q. Jin
Department of Physics, University of Tennessee, Knoxville, Tennessee 37996
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High-spin states in ^{68}Ge and ^{72}Se have been studied by in-beam, gamma-ray spectroscopy. In ^{68}Ge the three known bands beginning at 8^+ are found to exhibit crossing transitions and other new positive-parity bands are discovered to feed into them. In ^{72}Se only a single positive-parity yrast band is seen to high spin. Detailed, more realistic microscopic model calculations have been carried out for ^{68}Ge and ^{72}Se. There is striking agreement between the calculations and the new data for ^{68}Ge, including a predicted new band with superdeformation, $\beta \sim 0.42$. The calculations for ^{72}Se predict similar multiple band structures that are not seen experimentally.

The multiplicity of structures and their rapid changes with N and Z make the $A=70-80$ region an important testing ground of nuclear models.[1] The coexistence of overlapping bands built on different nuclear shapes was discovered[2] in $^{72}_{34}$Se$_{38}$ where the low-lying excited band has $\beta \sim 0.4$ and the ground band $\beta \sim 0.14$. The coexistence is related to the competition between the shell gaps at very large prolate superdeformation, $\beta \sim 0.4-0.45$, for $Z=N=38$ and the spherical gaps at 40 (see Ref. 1). This gives rise to a new island of ground-state superdeformation, $\beta \sim 0.4-0.45$ and the order of 3:2 axis ratios, first seen in ^{74,76}Kr (Ref. 3). Similar superdeformations and axis ratios were then discovered for excited superdeformed bands in the $A=132$ region (e.g., Ref. 4) and more recently in $^{191-194}$Hg (Ref. 5). In sharp contrast, the near spherical $^{68}_{32}$Ge$_{68}$ triple forks at 8^+ into three bands with no crossing transitions observed earlier.[6] The g-factor measurements in ^{68}Ge support the interpretation that the two lowest 8^+ bands are aligned neutron configurations, and the third band is a continuation of the ground state.[7]

One of the important challenges in nuclear physics is to develop more microscopic models which can predict the excited energy levels of nuclei. With the various shape coexisting structures seen in the mass-70 region, these nuclei provide stringent challenges for microscopic theories. Recently, a microscopic investigation of the low-spin states in ^{68}Ge and ^{72}Se has been made by using a self-consistent description of the nuclear excitation by a Hartree-Fock-Bogoliubov-based theory with spin and number projection before the variation, the excited VAMP approach (variation after mean-field projection in realistic model space[8]). These calculations predict a much more complex level structure for ^{68}Ge including strong $M1$ transition crossings between the three known bands beginning at 8^+ and between new bands. A similar complex band structure is predicted in ^{72}Se, but such complex structures were not observed in recent work.[9]

We have reinvestigated both ^{68}Ge and ^{72}Se with the spin spectrometer and the close-packed Ge ball at HHIRF (Holifield Heavy Ion Research Facility) to study their high-spin structures which provide important tests of the new microscopic calculations. In ^{68}Ge new bands and mixing of bands are observed, but not in ^{72}Se. The simultaneous and independent microscopic calculations for ^{68}Ge and ^{72}Se (Ref. 8) based on the excited VAMP, which has been used with much success in heavier nuclei,[10] are extended in this paper in a new, improved excited few determinant VAMP approach which has been developed recently and applied to *sd*-shell nuclei.[11] At high spins some changes in the level patterns are observed between the two approaches. Still, in both nuclei, additional multiple band structures, with considerable mixing of the bands and *M*1 crossing transitions, are predicted, including a new superdeformed band with $\beta \sim 0.42$ in ^{68}Ge. Our new ^{68}Ge data are in striking agreement with the complex band structures and multiple shape coexistence predicted for ^{68}Ge, including the band-crossing transitions and the new band with superdeformation. On the other hand, we analyzed the ^{72}Se data to try to find evidence for the multiple band structures predicted for ^{72}Se and found no evidence for additional positive-parity side bands at higher spin (the observed 0_2^+-2_2^+ members of the deformed bands are not germane to this paper).

In the present experiment to enhance the high-spin states in ^{68}Ge, a target enriched in ^{46}Ti to ~85% was bombarded with a 68-MeV ^{25}Mg beam from the HHIRF tandem. By using the ^{46}Ti(^{25}Mg,2*pn*) reaction, population of the high-spin states of ^{68}Ge was enhanced because the ~40$\hbar$ units of angular momentum brought into the compound system is higher than in earlier work.[6] A stack of three targets of total thickness 0.777 mg/cm^2 was used. About 2×10^8 double or higher-fold coincidence events were recorded with 19 Compton-suppressed Ge detectors in the spin spectrometer. The ^{72}Se was studied in the reaction 95-MeV ^{28}Si on ^{47}Ti with 20 Compton-suppressed Ge detectors in the close geometry γ-ray spectrometer (compact ball) at HHIRF with 2×10^8 γ-γ coincidences recorded.

In ^{68}Ge, 19 new levels were observed including two new bands. The new band beginning with the 12^+ 7561-keV level goes up to the highest spin observed. New crossing transitions between the known bands and the new bands also are observed. More importantly, the new band beginning at 7561 keV has a larger moment of inertia than the previously known ones as can be seen by comparing the $14^+ \rightarrow 12^+$ energies in the different bands. Only the positive-parity bands are shown in Fig. 1.

On the other hand, careful searches revealed no new band structures in ^{72}Se (Fig. 2). Note in Fig. 2 the experimental 0^+ ground state and 2_1^+ state are members of the ground-state band with small oblate deformation. The observed 0_2^+ and 2_2^+ deformed states are not shown. There is mixing of the near spherical oblate and very large prolate deformed bands in the 4^+ and 6^+ levels. Discrete γ-ray transitions were observed above 18^+ up to 26^+. The high-spin states in ^{72}Se are dominated by a single cascade as shown in Fig. 2. The negative-parity bands, which are only seen at lower spins, are not shown. The static and dynamic moments of inertia of ^{72}Se are remarkably constant and large, approaching the rigid-body value at higher spins.

In a theoretical description of the complex experimen-

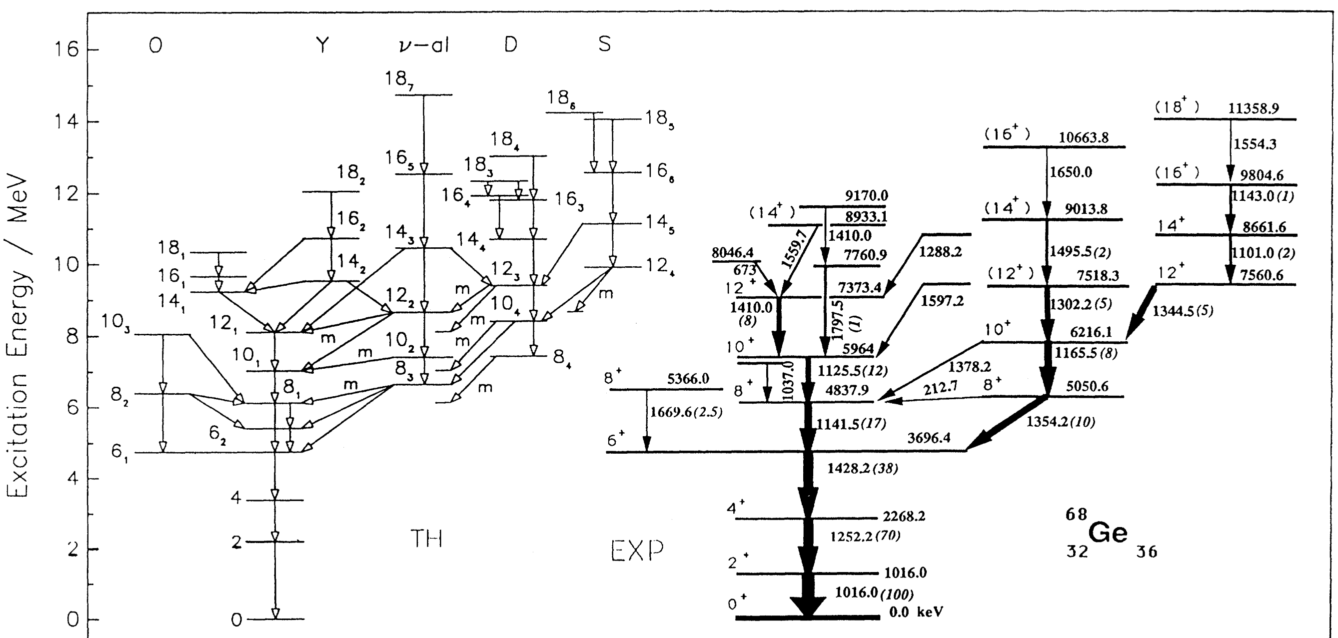

FIG. 1. Positive-parity levels in ^{68}Ge. On the left are the levels calculated in the FED VAMP approach. The symbols 0, γ, ν-al, *D*, and *S* used to identify the bands are described in the text. Excited combinations of the oblate ground band is on the left and a new superdeformed prolate ($\beta \sim 0.42$) band is shown on the right. The experimental levels reported here are shown on the right-hand side of the picture. The experimental bands are ordered (oblate on the left, superdeformed on the right) to match up with the calculated levels on the left side and matching the observed properties (lifetimes, magnetic moments, and moments of inertia) of each band with the calculated properties of each band. The *D* band in the calculations may be the new band which feeds the 5963-keV 10^+ level.

tal bands encountered in the ^{68}Ge nucleus, one needs a model in which all the essential degrees of freedom are accounted for in a completely microscopic fashion and not put in case by case. These should include the coexistence of different shapes and other collective phenomena as well as the various alignments and other single-particle excitations. One way to construct such a model is the use of variational techniques, as is done in the various approaches of the so-called VAMP (variation after mean-field projection in realistic model spaces) family (see Refs. 10 and 11 and references therein). The basic ideas behind this approach may be summarized as follows.

In the VAMP approach (see Ref. 10) to describe the yrast state of a particular symmetry, e.g., the lowest $I^{\pi}=8^{+}$ state, one starts with a general Hartree-Fock-Bogoliubov (HFB) vacuum projected onto the desired symmetry quantum numbers as a trial wave function and extracts the underlying quasiparticle transformation via a variational calculation directly from the chosen effective many-body Hamiltonian. This approach yields the optimal approximation to this state which can be reached by a single symmetry-projected HFB determinant. Already in this simplest approach drastic changes of the structure with increasing spin can be described.

Excited states with the same symmetry, e.g., the first excited 8^{+} state, can be calculated in complete analogy. Since this variational calculation is completely independent from the one performed for the yrast state, the resulting structure for the first excited state may be completely different from that of the latter. Finally, the residual interaction between all these configurations is diagonalized. This is the excited VAMP approach.[10]

However, by introducing only one additional determinant for each new state to be considered, this approach is still a type of mean-field approximation and may not always be sufficient, since the residual interaction between the lowest m configurations does not necessarily account for the dominant correlations on top of each of the various excited VAMP solutions. Often,[10] the dominant correlations, for example, to the yrast solution are related to rather high-lying configurations. Within the excited VAMP approximation such could only be accounted for if a rather high m number of configurations would be constructed. An improved variational scheme which incorporates such correlations in a systematic way, no matter where in energy they occur, is done in the new excited FED (from few determinant) VAMP approach.[11] Starting with the VAMP solution which is not taken out of the variational space, one instead looks first for a second symmetry-projected determinant correlating this solution in a variational calculation. Then, a third determinant is constructed, and so forth, up to n_1 configurations. The resulting correlated yrast state is then eliminated from the variational space, and the procedure is repeated to construct the first excited state with the same symmetry, and so on, through the m lowest states (each now being a linear combination of several configurations). Finally, as in excited VAMP, the now

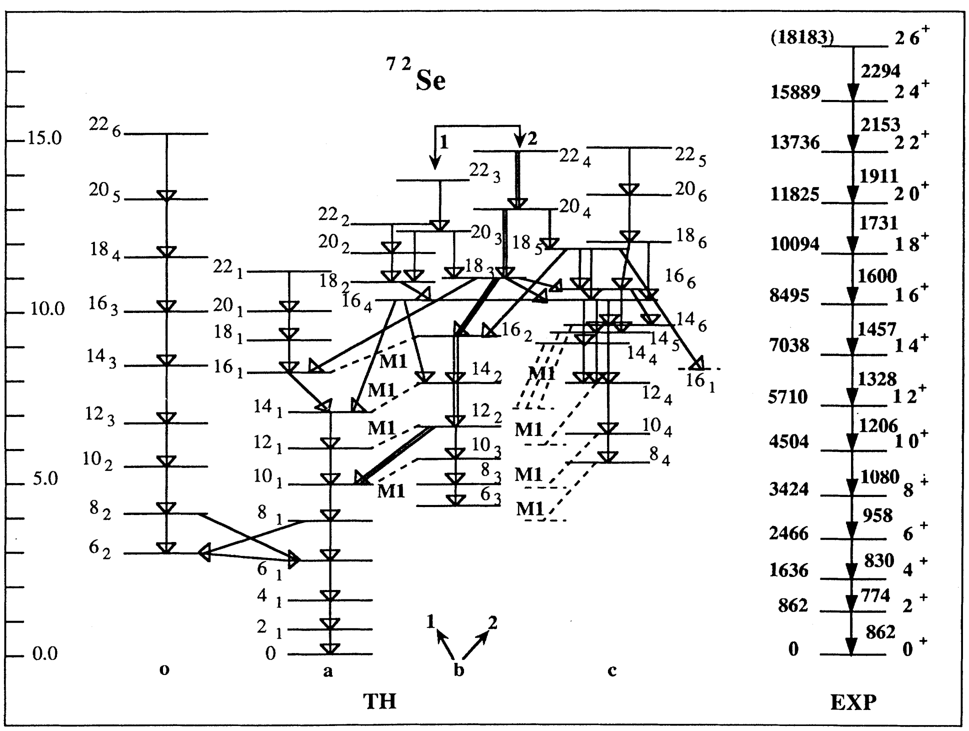

FIG. 2. The positive-parity levels of ^{72}Se are shown. The excited FED VAMP calculated levels are shown on the left with the oblate band with small deformation on the far left. A series of very fast $E2$ transitions connecting bands with the large deformation are shown by double arrows. Fast $M1$ transitions connecting the bands are shown. Except for the 937-keV, 1317-keV, 0_2^+, and 2_2^+ members of the excited deformed band, only a single yrast band is observed with positive parity. The yrast level starts out as an oblate band for the ground and 2_1^+ states and the prolate band with large deformation dominates at above 4^+.

much smaller residual interaction is diagonalized. This procedure has the big advantage that the dominant correlations on top of the projected mean-field solutions are accounted for in each state separately.

We have applied this new method to the high-spin states in ^{68}Ge. As described above, correlated wave functions for the lowest three 0^+, 2^+, and 4^+, the lowest four 6^+, the lowest five 8^+, 10^+, 12^+, and 14^+, the lowest six 16^+, and the lowest seven 18^+ levels were constructed. These states are grouped into several bands in Fig. 1 based entirely on the $B(E2)$'s of the transitions between them. Since the model space and the effective interaction were not changed, the results can be directly compared to those obtained within the more restricted excited VAMP approach.[8]

It turns out that the essential qualitative features of the calculated spectrum, e.g., the occurrence of many shape coexisting bands at comparable excitation energies and the complexity of the resulting decay patterns, were not influenced by the additional correlations. However, some changes do occur at the higher spins compared to the excited VAMP calculation,[8] and a new band with superdeformation, $\beta \sim 0.42$, labeled S in Fig. 1, now appears. Because of the additional correlations, the 0^+ ground state gains considerably more energy than all the higher-spin states, so the $2_1^+ \rightarrow 0_1^+$ transition energy is considerably larger than observed experimentally.

The new excited FED VAMP results are compared with our new experimental data for the high-spin, positive-parity bands in ^{68}Ge by renormalizing the theoretical spectrum to the experimental 2_1^+ level as shown in Fig. 1. The main features are summarized below. The experimental bands are ordered from left to right according to the theoretical predictions in Fig. 1.

Up to angular momentum 6^+ the calculated yrast levels are almost pure oblate states. The theoretical g factor for the 6_1^+ is 0.37 while the experimental[6] value is 0.4. The oblate band then continues where the 8_2^+ (with an oblate component of about 85%) most probably is the 8_3^+ state of the experimental spectrum. The 10_3^+ state has a pure oblate nature to form an oblate band, labeled by "0" in Fig. 1, that continues, as indicated by the older excited VAMP results,[7] up to rather high angular momenta. These oblate states are at considerably higher excitation energy than the other calculated bands and therefore are more likely to be weakly populated. This presumably is the reason that no continuation of this band above the 8_3^+ level is observed experimentally.

Besides this oblate structure, four other distinct bands are obtained in the calculations. They are all prolate deformed but differ in the magnitude of their quadrupole and hexadecapole moments as well as in their pairing properties. They also display different alignments. The band labeled as "ν-al" is characterized by an almost empty $0g_{9/2}$ proton level while the neutrons in the same shell-model orbit contribute to it a considerable portion of its total angular momentum. This strong neutron alignment is reflected in the small g factors of the members of this band: -0.03 for the theoretical 8_3^+ level which likely corresponds to the experimental 8_2^+ level, which has a measured[5] g factor of -0.28 ± 0.14, and 0.12, 0.07, 0.01, 0.07, and 0.08 for the 10_2^+, 12_2^+, 14_3^+, 16_5^+, and 18_7^+, respectively. The two bands labeled as "D" and "S" have almost the same quadrupole deformation on the neutron side; however, on the proton side, the latter band is considerably more deformed. This is evident from the intrinsic quadrupole moments ($\beta_2 \sim 0.42$ as compared to ~ 0.34) of the leading configuration as well as from the $B(E2)$ values, which above spin 12 are about twice as large in the "S" band as in the "D" band. Note, the predicted new "S" band with a large moment of inertia agrees quite well with the new experimental band shown on the right. The "D" band may be associated with the experimental band beginning at 7761 keV. Finally, the $18_2^+ \rightarrow 16_2^+ \rightarrow 14_2^+$ sequence, labeled as "Y," is probably associated with the experimental yrast spectrum at these spin values, since the calculated $18_1^+ \rightarrow 16_1^+ \rightarrow 14_1^+$ decay path is rather slow (mainly because of the small transition energies) and, thus, unlikely to be seen in the data.

All these prolate bands discussed in the last section have one common feature: their members become strongly mixed as soon as spin values as low as 12^+ or 14^+ are reached. Consequently, there are many competing decay branches for the various 14^+, 12^+, 10^+, and even 8^+ states. Thus, the decay does not only run via stretched $E2$ transitions within the various bands but also via a few strong $\Delta I = 0$, $M1$ transitions (indicated by "m" in the figure) which mainly feed the yrast band at these angular momenta. So, for example, the calculated 8_1^+ is linked to the 8_3^+ and 8_4^+ by $B(M1)$ values of 9.94 and by $B(M1)$ values of 9.94 and 0.046, the 10_1^+ to the 10_2^+ and the 10_4^+ by values of 4.21 and 0.47, and the 12_1^+ to the 12_2^+ and 12_3^+ by values of 0.82 and 0.64 (all in μ_N^2, where μ_N is the nuclear magneton), respectively. Such crossing transitions are now seen in the experimental data, too. Similar large $B(M1)$ transitions are also predicted in between some of the higher states. The theoretical results are strongly supported by the observation of not only several 8^+, but also several energetically bunched 10^+, 12^+, and 14^+ states in the new data with cross transitions and the new "S" band with much larger deformation.

Similar new theoretical calculations were carried out for ^{72}Se, and the results are shown in Fig. 2. The complex positive-parity-energy level spectrum looks rather similar to ^{68}Ge and is in sharp contrast to the single yrast band seen to high spin. Any crossing transition to other side bands seen in the calculations has experimental branching intensities less than 10% of the observed cascade feedings. However, a detailed look at the calculations reveals that there is a path of predicted very fast cascade $E2$ transitions (noted in Fig. 2) which cross from one band to another with large deformation, with only slow (weak) transitions to other states. It is possible that the experimental yrast band seen to high spins is a composite of several bands all with quite large deformation and linked by a very fast $E2$ path, as indicated in Fig. 2. However, the regularity of the energies of the yrast band would suggest a single band structure above the 4^+ level.

In summary, the detailed agreement between the new microscopic excited FED VAMP calculations and the new experimental structures observed in ^{68}Ge are quite striking for such complex multiple shape coexisting structures. This agreement in such a complex nucleus provides strong encouragement for the development of realistic microscopic models. On the other hand, the new calculations indicate similar complex structures for ^{72}Se, but in sharp contrast to ^{68}Ge the ^{72}Se experimental high-spin states are surprisingly dominated by a single band. This strong confirmation of the microscopic calculations in ^{68}Ge and marked differences in ^{72}Se are a clear challenge. It is possible that this difference is related to the fact that $^{72}_{34}$Se$_{38}$ is much closer to the new island of ground-state deformation centered around $N=Z=38$ (see Ref. 1) and that the dominance of these reinforcing proton and neutron shell gaps at 38 is not included in the calculations yet.

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*Present address: Physics Dept., Banaras Hindu Univ., Varanasi-221005, India.
†On leave from the Institute of Nuclear Physics, Cracow, Poland.
‡Present address: Tsinghua University, Beijing, People's Republic of China.
§Present address: Dept. of Physics, Princeton University, Princeton, NJ 08544.
**Present address: Institute of Atomic Physics, Bucharest, Romania.
††Present address: Oliver Lodge Laboratory, Univ. of Liverpool, Liverpool LG9 3BX, England.
‡‡Present address: Department of Physics, University of Jyväskylä, Jyväskylä Finland.
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