PHYSICAL REVIEW C VOLUME 43, NUMBER 6 JUNE 1991

Isovector content of N-N potentials and Pauli-forbidden states

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(Received 1 August 1990)

We study the isospin dependence of nucleon-nucleon potentials by examining solutions to the Schrödinger equation in paronic (Pauli-forbidden) states. We find a model dependence to the isovector content and discuss its origin. We observe a qualitative difference between meson-exchange-based models and those that are more phenomenological. This has consequences for a number of issues of current interest.

I. INTRODUCTION

An important issue in nuclear physics today is the extent to which interactions and processes in nuclear systems are adequately described in terms of hadrons—nucleons, mesons, and nucleonic excited states. Central elements in these descriptions are the nucleonic effective operators that arise when other degrees of freedom are eliminated and the system is described in terms of nucleons alone.

The operator that has received the most attention is the nucleon-nucleon potential. Dozens of models have been described in the past three decades. Meson-exchange models have recently reached new levels of sophistication and offer the possibility of developing a consistent theory of effective operators—electromagnetic and weak current operators and many-body forces as well as pair potentials.

In this paper we study the isospin dependence of a variety of nucleon-nucleon pair potentials for the purpose of understanding the amount of charge exchange built into a model at short distances. This should help us develop some intuition about the character of short-range processes built into the models and therefore to possible differences and sensitivities in other effective operators.

It is difficult to display the effect of the isospin dependence of a nucleon-nucleon potential for a number of reasons. It is now well known that the nucleon-nucleon potential itself is not well determined by the tenets of meson theory. Realistic potentials that give very similar results, both for on-shell scattering and in many-body calculations, may have matrix elements that differ by orders of magnitude. The on- and off-shell amplitudes, however, do seem to be reasonably well defined.[1] It therefore does not suffice to compare the coefficients of $\tau_1 \cdot \tau_2$ in the potential. One must look at the effect of the potential summed to all orders—the scattering amplitudes and the bound states.

It also does not suffice to consider the effect of the isospin dependence on similar states that differ by isospin. This is because, for two-nucleon states of good isospin, the available physical states are limited by the Pauli principle. One cannot change isospin alone; one must also change either the space or spin state as well. And since the nucleon-nucleon force depends strongly on both the space and spin states, the isospin dependence is obscured.

We have therefore chosen to look at the scattering and bound states produced by nucleon-nucleon potentials in Pauli-forbidden states. Although they are not physically accessible, they permit us to change only the isospin while keeping the space and spin states the same. We learn that realistic potentials that give very similar physical bound and scattering states produce significantly different Pauli-forbidden states. This clearly demonstrates substantial differences in the isospin content of the potentials and suggests that more substantial differences will be found between models when effective operators sensitive to short-range mesonic charge flow are calculated.[2,3]

Specifically, we observe that the isovector part of the potential dominates the short-range repulsion for most potentials whose short-range behavior is based on meson exchange, but more phenomenological potentials appear to have isoscalar cores.

We begin our study of the isospin dependence in Sec. II by discussing the use of Pauli-forbidden states. With this method, we can view the impact of isospin-dependent nonlocalities in the potential on the separation of isospin effects. When only local terms are included, the exchange of isoscalar mesons leads to isoscalar potentials and the exchange of isovector mesons leads to isovector potentials. When there are nonlocalities, the isoscalar mesons can affect the amount of apparent isovector potential present.

In Sec. III we consider three recent versions of the Bonn-meson-exchange potential, two of them in momentum space. Calculations in p space avoid some of the nonrelativistic approximations that are typically made to construct an r-space potential, and can go beyond one-meson exchange to incorporate more of the two-meson-exchange processes that are included in the full model.[4]

In Sec. IV we investigate the sensitivity of our result to the phenomenological strong-interaction form factors used in the Bonn models. To do this, we compare other published meson-exchange models that resemble the Bonn construction but differ in the functional form of the form factor applied at each vertex.

In Sec. V we report results obtained with phenomenological models of the N-N interaction. In these, fewer predetermined theoretical constructs have prejudiced the form of the $V(r)$ function ascribed to each of the various two-nucleon operators. We observe a consistently smaller degree of isospin-dependent effects obtained with these models.

II. METHOD

In this section, we motivate looking at the unphysical Pauli-forbidden states in order to extract isospin dependence.

We begin by decomposing the effective potential of a two-nucleon Schrödinger equation into isoscalar and isovector components:

$$W^{\text{eff}} = W^{\text{eff}}_{\text{is}} + W^{\text{eff}}_{\text{iv}} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \ . \tag{2.1}$$

The scattering amplitudes which depend on the potential to all orders can also be separated into isoscalar and isovector components, where each component receives contributions from both terms of Eq. (2.1). If we write a Lippmann-Schwinger equation for the scattering amplitude

$$T = W + WG_0 T \ , \tag{2.2}$$

where W is of the form of Eq. (2.1), then the solution for the scattering amplitude has the formal structure

$$T = \frac{1}{1 - WG_0} W \ , \tag{2.3}$$

which will lead to mixing of the isoscalar and isovector parts of Eq. (2.1) to form the amplitude in each isospin channel. As a result of this mixing, one cannot simply identify isoscalar and isovector parts of the scattering amplitude as arising from isoscalar and isovector meson exchange, respectively.[5] We note also that if a nonlocal potential is replaced by an effective local potential, that the transformation to local form will mix isoscalar and isovector parts. This means that the isoscalar and isovector parts of the effective local potential will each depend on both the isoscalar and isovector parts of the nonlocal potential. This is discussed in the Appendix.

Since it is only the amplitude which is reasonably determined and not the potential, we need to seek a method for displaying the isospin dependence.

One viable method would be to form some combination of partial waves that would isolate the effects of the $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ operator. Such a combination has been used[6] with P-wave phase shifts to separate the effects of the $T=1$ tensor and spin-orbit operators.

Unfortunately, this method cannot readily be used to isolate $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$. Although in S waves many operators (e.g., $\mathbf{L} \cdot \mathbf{S}$, S_{12}) have vanishing expectation value, too many (e.g., central $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$, $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$) are nonzero to form unique combinations of the two S waves that isolate just one of the operators.

A major contributor to the isospin dependence of a potential comes from the operator $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ which arises from exchange of vector-isovector mesons. We note that the expectation value of this operator does not change in the two S-wave channels, making its effect identical to that of a unit operator (central potential) and thereby foiling any comparison of the $^1S_0$ and $^3S_1$ partial waves to isolate isospin-dependent contributions. Much of the isospin dependence present in the potential is therefore "hidden" by the necessity of changing S as well as T to respect the Pauli principle.

Since some models possess up to fourteen[7,8] two-nucleon operators, a program of going beyond S waves to form linear combinations of phase shifts to identify $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ effects would require many partial waves, a requirement that seems prohibitive.

We propose to exhibit the isospin dependence by changing only the isospin quantum number, keeping fixed all other two-nucleon quantum numbers defining a partial wave. The new set of quantum numbers defines a partial wave that is Pauli forbidden for two nucleons. This method yields a well-defined procedure for viewing the isospin-channel effects. The physical potential in the partial wave of interest and the associated potential in the Pauli-forbidden partial-wave form precisely the two expectation values $(W^{\text{eff}}_{\text{is}} + W^{\text{eff}}_{\text{iv}})$ and $(W^{\text{eff}}_{\text{is}} - 3W^{\text{eff}}_{\text{iv}})$ of Eq. (2.1) in the two isospin channels. Linear combinations of these potentials can produce the separate components $W^{\text{eff}}_{\text{is}}$ and $W^{\text{eff}}_{\text{iv}}$. Furthermore, viewing the potentials, wave functions, and phase shifts in the Pauli-forbidden states gives dramatic visual demonstrations of the strength of the "hidden" isospin dependence of potential models.

The new, Pauli-forbidden quantum states are readily formed by changing only the isospinor component of the two-nucleon state. This has the effect of changing the isospin quantum number T that goes with the other quantum numbers summarized in the usual spectroscopic notation. We shall refer to the $^1S_0$, $T=0$ combination as the Pauli-forbidden $^1S_0$ channel, and the $^3S_1$, $T=1$ combination as the Pauli-forbidden $^3S_1$ channel. The changes we make are similar in spirit to the construction of a nucleon-antinucleon potential by changing the G parity[9] of each meson process in a nucleon-nucleon potential, although in the case considered here, the channels are not realized in nature and our purpose is to gain understanding of the structure of the potential.

We are interested not only in the change in the potential, but in the effect of the potential to all orders. To display these effects, we calculate the bound states and phase shifts in these Pauli-forbidden channels.

The method of creating Pauli-forbidden two-nucleon states can be applied to any partial wave, but we focus primarily on the two $l=0$ partial waves since they are the most important for low-energy processes. All of the models discussed in this and the next two sections produce realistic deuteron parameters and fit low-energy (<300 MeV) phase shifts.

III. THE BONN MODELS

In this section we discuss the predictions of three of the Bonn-meson-exchange potentials in Pauli-forbidden states.

In the Bonn models,[4] the mesonic origins of the isospin-dependent contributions can be identified. Time-ordered perturbation theory is applied to an effective Lagrangian of baryons and mesons to generate the set of diagrams included in the model. Each meson-nucleon-nucleon vertex receives the form factor

$$F_n(k^2)=\left[\frac{\Lambda^2-\mu^2}{\Lambda^2+k^2}\right]^n , \tag{3.1}$$

where μ is the meson mass, Λ is a cutoff parameter, and k is the four-momentum carried by the meson.

The Bonn group uses the Blankenbecler-Sugar reduction of the four-dimensional Bethe-Salpeter equation to generate a three-dimensional equation of the Lippmann-Schwinger type. The potential is identified as the kernel of the three-dimensional scattering equation. The Blankenbecler-Sugar reduction guarantees that the potential has no dependence on energy transfer in the c.m. frame of the two nucleons, but it can depend on the total c.m. energy (E), the total three-momentum ($\mathbf{p}$) of the two nucleons in the c.m. frame, and the three-momentum transfer ($\mathbf{k}$) in the c.m. frame.

For ease of use in calculations, the Bonn group produced several simpler models based on further approximations to this structure. The simplest of these is the energy-independent r-space model, one-boson-exchange potential in coordinate space (OBEPR).[4] In this model, six one-meson-exchange terms define the potential. Each term is regularized by the form factor $F_1(k^2)$ at each meson-nucleon-nucleon vertex. The operator $\tau_1 \cdot \tau_2$ multiplies the contribution of the three isovector mesons (π,ρ,δ) to the potential. These contributions change their sign when we move from the potential in the normal channel to the potential in the Pauli-forbidden channel.

An ambiguity arises in constructing the Pauli-forbidden potentials for Bonn OBEP's since they choose the mass and coupling constant of the σ meson to be different in the two isospin channels. The σ in this Bonn model is therefore not purely isoscalar. Its isospin dependence can be viewed as a consequence of the effects (such as two-pion exchange, and processes involving isobar intermediate states) that the σ meson is understood to represent. The Bonn group has chosen to represent these effects by isospin-state-dependent parameters rather than by introducing separate isoscalar-scalar and isovector-scalar σ mesons. We use the Bonn $T=1$ σ parameters to construct the Pauli-forbidden $^3S_1$ potential, and the Bonn $T=0$ σ parameters to construct the Pauli-forbidden $^1S_0$ potential. We label the results obtained by using the reversed set of σ parameters as "unswitched sigma."

The resulting effective local potentials in the Pauli-forbidden $^1S_0$ and $^3S_1$ channels are shown in Fig. 1. We see that the familiar short-range repulsion (usually thought of as provided by the isoscalar ω meson) is no longer present. In fact, by dividing the mesons into the two categories, isoscalar and isovector, we find the sum of all isoscalar meson contributions to be attractive in the core region for S waves, with isovector mesons providing the repulsion. This information is of interest in meson models of the N-N interaction, since less sophisticated models (e.g., with only σ and ω mesons) have isoscalar core repulsion.

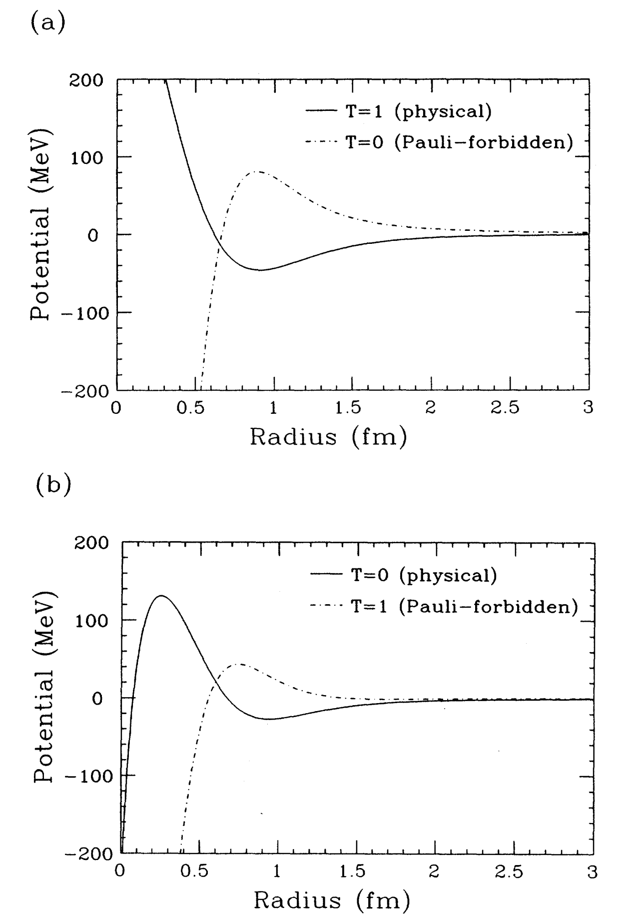


FIG. 1. A comparison of the OBEPR S-wave effective local potentials at the deuteron energy with their Pauli-forbidden counterparts. (a) $^1S_0$; (b) $^3S_1$, the Pauli-forbidden potentials of two newer r-space Bonn models (Ref. 21) behave similarly. The potentials possess a soft core characteristic (Ref. 37) of meson models. Since $V_{\rm local}$ is energy independent, the only energy dependence of $W^{\rm eff}$ comes from the k^2 term of Eq. (A3).

The net attraction at short range is sufficiently strong to enable the Pauli-forbidden $^3S_1$-$^3D_1$ system to support a bound state. The character of this state is very sensitive to how we treat the isospin dependence of the sigma. If the σ parameters are switched, we get a binding energy of 15 MeV and rms radius 0.7 fm. These become 220 MeV and 0.3 fm if the σ parameters are unswitched. The D-state probability is small, at 0.3% and 0.03%, respectively. We note that in the intermediate-range (0.6–1.5 fm) region of the potential, the usual attraction has turned into a repulsion. The phase shifts in the Pauli-forbidden S states using OBEPR are graphed in Fig. 2.

The behavior of OBEPR serves as a useful reference to compare with the behavior of more sophisticated meson-exchange models. We have already noted that the r-space σ meson masks some of the more complicated, isospin-dependent processes that are explicitly included in p-space Bonn models. Another reason to compare OBEPR with p-space Bonn models is to judge the validity

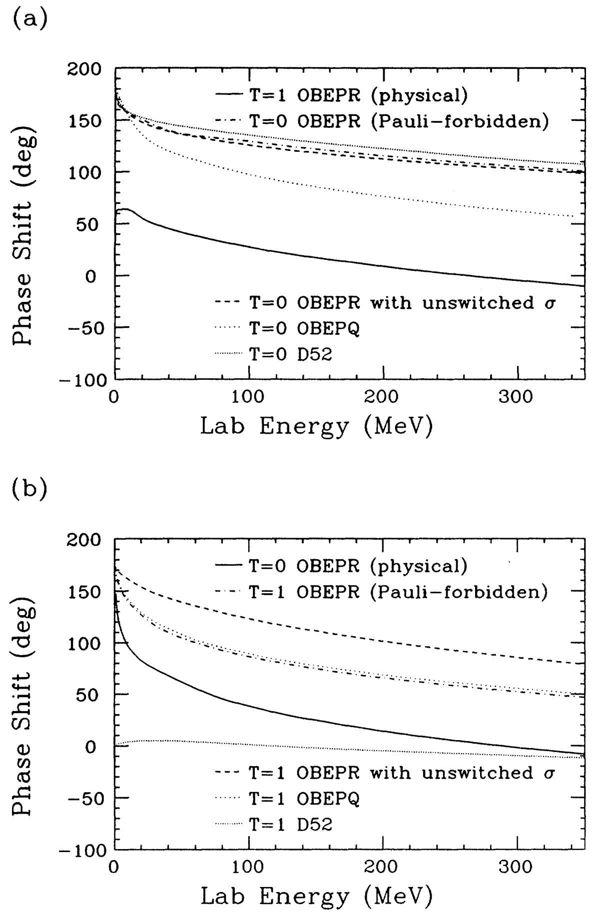


FIG. 2. Phase shifts for the Bonn potentials. The solid and dot-dashed lines correspond to the potentials graphed in Fig. 1. The dashed line uses the unswitched σ parameters in OBEPR. (a) ${}^1S_0$; (b) ${}^3S_1$.

of one of the nonrelativistic approximations that is made in converting from p-space to r-space.

This approximation is an expansion of the nucleon energies E_i $(i=1,2)$ up to second order in momenta:

$$E_i = m_N + \frac{p_i^2}{2m_N} + \cdots \ . \tag{3.2}$$

With the change of coordinates

$$\mathbf{k} = \mathbf{p}_1 - \mathbf{p}_2, \quad \mathbf{p} = \tfrac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2) \ , \tag{3.3}$$

terms quadratic in $\mathbf{p}$ [of the form indicated in Eq. (A1)] appear in all scalar (σ,δ) and vector (ω,ρ) meson contributions to the resulting r-space potential. Since these terms appear as a consequence of the nonrelativistic expansion of Eq. (3.2), one could use the influence of these p^2 terms in the r-space potential to judge the validity of this expansion.

In Fig. 3(a) we plot the OBEPR ${}^3S_1$ W^{eff} of Eq. (A3) and the V_{local} of Eq. (A1). We see that for low-energy processes, the presence of the p^2 terms has a minor effect on the potential, but Fig. 3(b) shows that their presence significantly influences the shape of the potential probed at higher energy. From Fig. 3(b), one would expect the absence of the p^2 terms to significantly affect the potential seen at a c.m. energy near 200 MeV. We observe a change in the physical ${}^3S_1$ phase shift from $-13.8°$ to $-9.6°$ at this energy when the p^2 terms are dropped.

The energy-independent one-boson-exchange potential in momentum-space (OBEPQ) model[4] is similar to the OBEPR model, with the same six mesons $(\pi,\rho,\omega,\sigma,\eta,\delta)$ represented by one-meson-exchange forms. As in OBEPR, the exponent n of the form factor $F_n(k^2)$ is 1 for all mesons except the ρ, for which $n=2$. The σ mass and coupling constant are again isospin-channel dependent.

With the OBEPQ potential, the Pauli-forbidden ${}^3S_1$-${}^3D_1$ system supports a bound state at 20 MeV, 260 MeV if the σ parameters are unswitched. Again, the bound state is almost entirely S wave; the D-state proba-

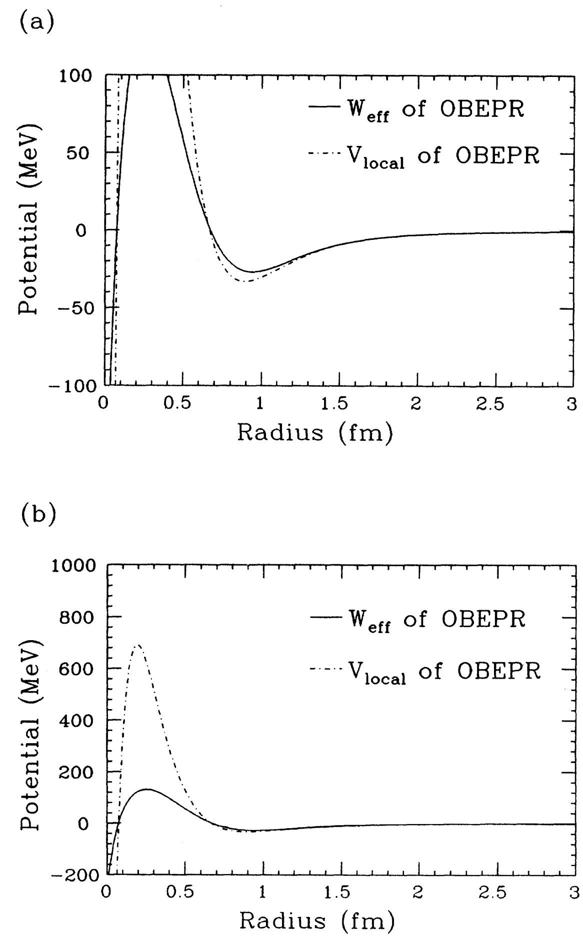


FIG. 3. A comparison between the full OBEPR effective local ${}^3S_1$ potential at the deuteron energy and that obtained by removing the p^2 terms, without any other changes to the model. Differences are small on the scale of (a). The effect of removing the p^2 terms on the deuteron is to weaken its binding energy to a value of 0.8 MeV. The effect of the p^2 terms is seen to be more appreciable in (b), where the same curves are graphed on a larger scale.

bilities are 0.35% and 0.33%, respectively. The phase shifts of the Pauli-forbidden $^1S_0$ and $^3S_1$ channels are graphed in Fig. 2.

A more sophisticated Bonn model is D52. This is an energy-dependent p-space model[10] containing five form-factored one-meson-exchange terms. (The η is omitted.) Also included in D52 are a subset (the iterative diagrams of Ref. 4) of time-ordered box diagrams in which 2π or $\pi\rho$ exchange occurs, with an intermediate baryon state of $N\Delta$ or $\Delta\Delta$. Crossed box and stretched box diagrams are not included. Each meson-baryon-baryon vertex has a form factor $F_1(k^2)$. The σ parameters in D52 are independent of isospin channel, a feature which is also shared by the full model.[4] Figure 2 shows the S-wave phase shifts of D52.

The 2π and $\pi\rho$ exchanges with delta intermediate states contribute to the isospin dependence of the model. The isospin structure of 2π and $\pi\rho$ exchange diagrams with NN, $N\Delta$, or $\Delta\Delta$ intermediate states, can be collapsed to the form $A+B\tau_1\cdot\tau_2$, where A and B are constants determined by manipulations of isospinor states[11] and the use of projection operators[12] in isospin space.

The Pauli-forbidden $^3S_1$-$^3D_1$ system fails to support a bound state in the D52 model. However, the existence of a bound state is not the only measure of attraction. The positive phase shift between laboratory energies of 0 and 116 MeV (see Fig. 2) gives evidence for attraction in the D52 Pauli-forbidden $^3S_1$ system. This behavior is qualitatively similar to the normal $^1S_0$ phase shift. The differences between the D52 and OBE models, as represented in Fig. 2(b), can be interpreted as indicative of the additional isospin-dependent processes present in D52 but absent in the OBE models.

In the OBEPR model, which produces a potential that is phase-shift equivalent to a local potential, the existence of a low-energy bound state and the Levinson theorem[13] enable one to deduce that the phase shift goes to π, rather than 0, at zero energy.[14] Since solving for the phase shifts involves solving for a quantity $S=\exp(2i\delta_l)$, our phase-shift codes produce phases that cannot be *a priori* known within an additive multiple of π. Hence, we need our knowledge of the existence of bound states in order to determine δ_l to $\pm n\pi$. Of course, such modifications to the phase produce no effect on physical observables, such as the cross section. We fix the low-energy phase of OBEPR by finding the number of bound states numerically. We adopt the usual convention of setting the phase to zero in the infinite-energy limit.

The nonlocal nature of OBEPQ and D52 prevents Levinson's theorem from being applicable. Nonetheless, we observe the phase approaches an integral multiple of π. To compare these phases with those of local models, we use our knowledge of the number of bound states to set the low-energy phase to correspond to the behavior of a local potential with the same number of bound states.

The isospin structure of the D52 model and the full Bonn model differ due to the fact that different sets of diagrams are included in each model. Absent in D52 but present in the full model are the noniterative (i.e., crossed box and stretched box) diagrams of 2π exchange with $N\Delta$ or $\Delta\Delta$ intermediate states. Analyses by many groups[12,15–18] suggest that the sum of all (iterative and noniterative) 2π exchange contributions provides a largely isoscalar interaction (as in term A above), with near cancellation of the isovector contributions (represented by term B). Without the noniterative contributions to produce this cancellation, the D52 model can be suspected of having an overly large isovector 2π exchange contribution.

The full Bonn model has a more complex structure and we have not carried out our calculation using it. One should note that a superficial analysis of the additional diagrams[4,19] contained in the full model cannot yield a definitive statement about its isospin dependence since the parameters are refit to the scattering data.

To summarize the results of this section, we have compared the isospin dependence of p-space Bonn models to that of the energy-independent, r-space model OBEPR. The energy-independent, p-space model OBEPQ avoids some of the approximations made in constructing an r-space model from p-space amplitudes. The energy-dependent D52 model possesses a sophistication approaching that of the full Bonn model. We observe in Fig. 2 some similarities in the Pauli forbidden states calculated with these three models. Most of them support one bound state in each channel. All show some attraction in terms of a positive phase shift. None of the Pauli-forbidden phase-shift curves follows closely the normal phase-shift behavior over the entire energy range of the graphs. We interpret these similarities as showing that the isovector nature of the short-range region is a feature that persists when some of the shortcomings of the r-space model are improved upon.

IV. FORM-FACTOR DEPENDENCE

Isospin-dependent processes such as isovector meson exchange play an important role in shaping the intermediate- and short-range regions of the Bonn models. It is instructive to determine the sensitivity of this behavior to the way in which the short-range region is treated, since the choice of vertex functions in the Bonn models [Eq. (3.1)] is not strongly theoretically motivated.

In this section we consider whether the special choice of form factor selected by the Bonn group plays an important role in determining the relative strengths of the core components. We do this by investigating a variety of other meson-exchange models that make different choices.

Although the form factors in a meson-exchange model are of reasonably short range, they have a structure which modifies the leading terms in an expansion in powers of k. Therefore, the long-range behavior of the function in coordinate space is also affected. To show this, we note that the low k behavior of the usual p-space Yukawa form

$$g^2\left[\frac{1}{k^2+\mu^2}\right]=\left[\frac{g}{\mu}\right]^2\left[1-\left[\frac{k}{\mu}\right]^2+\left[\frac{k}{\mu}\right]^4-\cdots\right] \tag{4.1}$$

is transformed by the presence of two $F_n(k^2)$ factors to

$$g^2\left[\frac{\Lambda^2-\mu^2}{\Lambda^2+k^2}\right]^{2n}\left[\frac{1}{k^2+\mu^2}\right] = \left[\frac{g}{\mu}\right]^2\left[1-\left[\frac{\mu}{\Lambda}\right]^2\right]^{2n} \times\left\{1-\left[\frac{k}{\mu}\right]^2\left[1+2n\left[\frac{\mu}{\Lambda}\right]^2\right]+O(k^4)\right\} . \quad (4.2)$$

The coefficients of both 1 and k^2 in Eqs. (4.1) and (4.2) cannot be made to agree by simply readjusting g, n, and Λ if the meson mass μ is not changed.

Except for the pion, the ratio $(\mu/\Lambda)^2$ is often sizable in meson models. Therefore, when the mass of the meson is fixed, the presence of the form factor affects even the low k behavior of the meson-exchange potential function.

The form factors affect the short-range behavior in a well-known way. For example, the r-space equivalent to Eq. (4.2), for $n=\frac{1}{2}$, is obtained[20,21] by replacing the Yukawa form

$$V(\mu,r)=\frac{g^2e^{-\mu r}}{4\pi r} \quad (4.3)$$

by the function

$$V(\mu,r)-V(\Lambda,r) . \quad (4.4)$$

Higher powers of n in Eq. (4.2) produce more structure. For example, the use of $n=1$ is identical in r-space to the replacement of $V(\mu,r)$ by[4,22,23]

$$V(\mu,r)-V(\Lambda,r)+\left[\frac{\Lambda^2-\mu^2}{2\Lambda}\right]\frac{d}{d\Lambda}V(\Lambda,r) . \quad (4.5)$$

For $n=\frac{3}{2}$, the corresponding r-space potential has first, second, and third derivatives of V with respect to Λ built in.[24] It is a general result[24,25] that a higher derivative appears with each integral increase of the exponent $2n$. These additional terms have a range set by Λ instead of μ, and therefore primarily modify the short-range region.

With these $F_n(k^2)$ forms influencing both the long- and short-range behavior of each meson contribution, it is reasonable to question the sensitivity of the results of the preceding section to the presence of these vertex functions in the model. We study the effect of using different values of n in the $F_n(k^2)$ form by looking at other meson-exchange models. In order for our study to be meaningful, we must compare only models that produce reasonably good fits to the phase-shift data.

One possible way to produce such models is to try to adjust the cutoff parameter Λ and the coupling constant g to preserve the long-range behavior of each meson contribution in OBEPR when we change the exponent $2n$ by ±1. The two functional forms

$$g^2\left[\frac{\Lambda^2-\mu^2}{\Lambda^2+k^2}\right]^{2n}\left[\frac{1}{k^2+\mu^2}\right] \quad (4.6)$$

and

$$(g')^2\left[\frac{(\Lambda')^2-\mu^2}{(\Lambda')^2+k^2}\right]^{2m}\left[\frac{1}{k^2+\mu^2}\right] , \quad (4.7)$$

for an arbitrary integer m, can be made to agree in their behavior to order k^2 if we choose

$$\Lambda'=\left[\frac{2m}{2n}\right]^{1/2}\Lambda \quad (4.8)$$

and

$$(g')^2=\frac{[1-(\mu/\Lambda)^2]^{2n}}{[1-(\mu/\Lambda')^2]^{2m}}(g^2) . \quad (4.9)$$

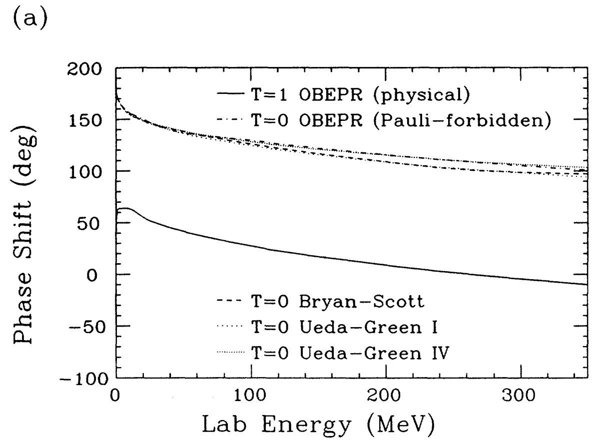
However, such a modification to OBEPR, for $2m=2n\pm1$, produces potentials with unrealistic deuteron properties and phase shifts. We conclude that this modification is an insufficient way to compensate for the n dependence of the individual meson contributions to the potential. The failure of this construction confirms that the $F_n(k^2)$ forms are significant components of the model, even at long range.

Our method is to compare Pauli-forbidden S waves with other meson-exchange models that use vertex functions of the type in Eq. (3.1). Each of these models fits phase-shift and deuteron data, but with different exponents, n, used in the $F_n(k^2)$ form. To compare with the OBEPR model, which uses $n=1$ for all one-meson-exchange contributions to the potential, we look at the Bryan-Scott model[26,27] ($n=\frac{1}{2}$ for all mesons), the Ueda-Green-I model[22,28] ($n=1$ for all mesons), and the Ueda-Green-IV model[24] ($n=2$ for all mesons).

These one-boson-exchange models are similar in construction to OBEPR. The one-meson-exchange form represents the potential term of the nonrelativistic Lippmann-Schwinger equation. This three-dimensional integral equation has the same form[4] as the equation generated from the Blankenbecler-Sugar method used in the Bonn models. Aside from the differences in form factors, the one-meson-exchange expressions for these potentials differ from the OBEPR expressions in the numerical coefficients of the p^2 terms, a difference that arises from the "minimal relativity"[4,26,29] prescription and is the only vestige of the relativistic starting point of OBEPR. The Bryan-Scott potential uses, in addition to the familiar π, η, ω, and ρ mesons, a scalar-isoscalar meson and a scalar-isovector meson. Both Ueda-Green models use the π, η, ω, and ρ mesons, two scalar-isoscalar mesons, and a scalar-isovector meson.

In all these models, we observe a substantial attraction at the origin in the Pauli-forbidden S waves, with graphs of these potentials similar to Fig. 1. Although two of these models produce a bound state in the Pauli-forbidden ${}^3S_1$-${}^3D_1$ system, the existence of a bound state is not a well-controlled probe, since the potential needs to be extremely attractive for such a bound state to form. We comment in Sec. VI on the nature of these bound states, which are localized at short range. We resort to phase shifts as a more meaningful probe of the isospin-dependent modifications. These are shown in Fig. 4.

As with OBEPR, all these potentials are phase-shift equivalent to effective local potentials. The transformation function $[1+2\lambda\omega(r)]$ that relates the true wave function $R_l(r)$ to the solution $u_l(r)/r$ of the local potential has no nodes. We therefore can use Levinson's theorem



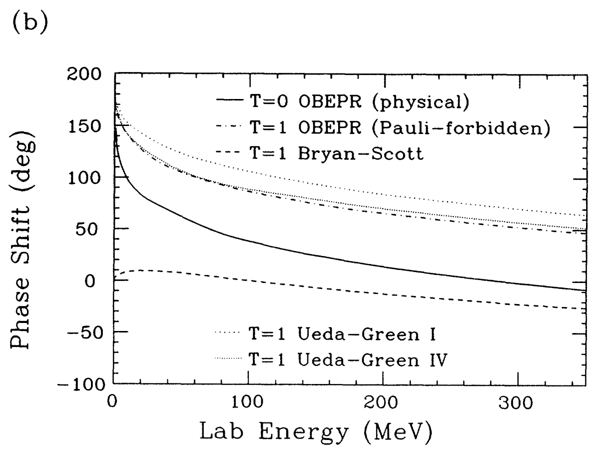


FIG. 4. Phase shifts for the meson-exchange models used to study the dependence of the OBEPR results on the exponent of the strong interaction form factor. (a) $^1S_0$; (b) $^3S_1$.

for local potentials to determine the low-energy phase given by these meson-exchange models. The Pauli-forbidden $^1S_0$ phase shifts in Fig. 4 are all seen to resemble the OBEPR behavior.

In the Pauli-forbidden $^3S_1$ partial wave, the Bryan-Scott model (with phase shift positive at low energy, crossing zero at laboratory energies of 0 and 102 MeV) resembles the D52 model, while the Ueda-Green models resemble OBEPR. The OBEPR and the Ueda-Green models both have $^3S_1$ (Pauli-forbidden) bound states while the D52 and Bryan-Scott models do not. The lack of a Pauli-forbidden bound state in these latter two models can be understood from their composition.

No Pauli-forbidden bound-state forms in the Bryan-Scott model due to the character of the regularization of the tensor force. The smaller degree of screening in the Bryan-Scott model gives it a much larger tensor force than the rest of the OBEP models studied here. The uncoupled expectation values of the Bryan-Scott potential in the Pauli-forbidden $^3S_1$ and $^3D_1$ partial waves are very similar to those of OBEPR and the Ueda-Green models. The expectation value of the tensor potential of models with $n \geq 1$ in $F_n(k^2)$ can be shown to be sufficiently regularized to approach 0 as $r \to 0$. This is not true for models with $n = \frac{1}{2}$, as in the Bryan-Scott potential. Here the $V(r)$ function multiplying the tensor operator diverges as $r \to 0$. The large tensor potential strongly mixes the Pauli-forbidden $^3S_1$ component of the wave function with the D state which eliminates the bound state.

A Pauli-forbidden bound state fails to form in the D52 model due to the effects of the constituent two-meson exchanges. Strong cancellation occurs in the isovector components of the D52 model between the contributions of the one- and two-meson-exchange diagrams. The one-meson-exchange contributions produce a strong isovector core interaction, but this is offset by the two-meson-exchange contributions.

We conclude that all of these meson-exchange models, regularized by forms of Eq. (3.1), exhibit phase-shift behavior similar to the Bonn models in the Pauli-forbidden channels. This behavior is therefore not unique to the choice of exponent that is used in OBEPR, but rather is a more general feature characterizing the structure of meson models using such cutoffs. The constituent isovector processes (due primarily to the π and ρ one-meson exchanges) in these models emerge as important contributors to the physical S-wave interaction.

V. RESULTS WITH PHENOMENOLOGICAL MODELS

The results of the previous two sections reveal a strong isovector component of the meson-exchange N-N interaction at intermediate and short ranges. Potential models which are only partially based on meson-exchange ideas often turn out to have their core region dominated by isoscalar interactions. In this section we study the predictions in the Pauli-forbidden states produced by four potentials which contain more phenomenological freedom than meson-exchange potentials.

Although the strong interaction can depend on the total isospin of the two-body system, this isospin dependence is often thought of as small. One can cite the similarity of the $^1S_0$ $(T=1)$ and $^3S_1$ $(T=0)$ phase shifts (and related potentials) as supporting evidence for the fact that a realistic pair potential at low energy should be weakly dependent on isospin. In effective interactions constructed to fit N-N scattering amplitudes, the functional forms chosen to represent the bare N-N interaction lead to the dominance of the central (scalar-isoscalar) component of the N-N t matrix for projectile laboratory energies up to 800 MeV.[30]

Groups who model the two-nucleon interaction often argue that isospin-dependent contributions are typically suppressed compared to isoscalar contributions. Dispersion theoretic arguments can be made to show how the effects of ρ mesons and isobars at short range can be modeled by an effective ω meson.[12] Similar methods show how an effective σ meson emerges in S waves as an equivalent description of the effects of correlated two-pion exchange.[17]

These arguments are not meant to definitively rule out alternative ways of simplifying the meson and baryon degrees of freedom present in the N-N interaction. However, in these reductions of the number of degrees of freedom, the isoscalar mesons σ and ω emerge as the dominant contributors to a typical meson model of the N-N interaction, consistent with the notion of the minor role of isospin-dependent effects.

Contractions in isospin space also result in a reduction of isovector effects. Even though intermediate states involve isospin multiplets (e.g., the nucleon or delta) and involve the exchange of isovector mesons (e.g., the pion and rho), integrating out these degrees of freedom to obtain an interaction between two nucleons is a process that generates an isoscalar term.[11] For example, the isospin sums for the time-ordered box diagrams (in the terminology of the Bonn group) of two-pion exchange with intermediate nucleon states produce[4] the operator $\frac{2}{3}+\frac{2}{3}\tau_1\cdot\tau_2$, where τ_1 and τ_2 act on the external nucleon states. Similarly, the time-ordered crossed box processes of two-pion exchange with nucleon intermediate states produce[4] the operator $\frac{2}{3}-\frac{2}{3}\tau_1\cdot\tau_2$. The structure of these isospin operators is the same for similar-looking Feynman diagrams.

A simplification, made in the full Bonn model, further suppresses the role of these isospin operators. In treating $\pi\rho$ exchange involving one or two Δ intermediate states, the sum of box and crossed box diagrams combines $\tau_1\cdot\tau_2$ contributions of opposite sign. The approximation is made[4,19] that these contributions nearly cancel, producing a dominantly isoscalar interaction.

We note that the functional forms of meson-exchange models are fixed prior to the fit to phase-shift data. In a phenomenological potential model, one has the freedom to add any term to the potential that improves the phase-shift fit. One can trade off short-range central effects against the spin-isospin term $V(r)\sigma_1\cdot\sigma_2\tau_1\cdot\tau_2$ in S waves and thereby get less isospin dependence than is dictated by an underlying meson-exchange structure.

We consider three local r-space phenomenological potentials: Reid v8,[31] Urbana v14,[7] and Argonne v14.[8] Each of the models is to be used in the nonrelativistic Lippmann-Schwinger equation. All three potentials become positive definite in the Pauli-forbidden S waves, with Fig. 5 showing typical behavior. The lack of a bound state for such potentials enables us to deduce from Levinson's theorem the limiting value of the phase shifts at zero energy. Figure 6 shows the phase shifts in the Pauli-forbidden S waves.

The fourth phenomenological model in our study is the (nonlocal) Paris potential.[32] Although it contains contributions from π, ω, and 2π meson exchange processes[33] (the latter contains a ρ-like resonant structure[34]), these contributions are supplemented by phenomenological core contributions.[33,34]

Figure 6 shows that the isospin dependence of these four phenomenological models is similar. Comparing these results to Figs. 2 and 4 shows that a distinction arises between meson-exchange and phenomenological potentials, not between local and nonlocal ones.

With the Paris, OBEPQ, and OBEPR potential models, we notice substantial positive phase shifts in the Pauli-forbidden $^3P_1$ partial wave, and smaller positive phase shifts in the Pauli-forbidden $^1P_1$ partial wave.

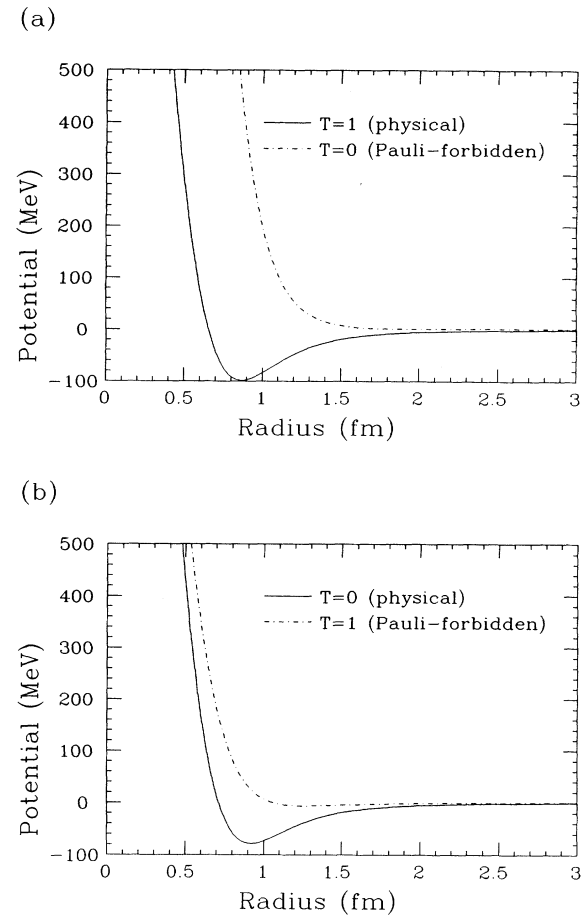


FIG. 5. Typical behavior of a phenomenological potential model (Urbana v14) in the Pauli-forbidden S waves. (a) $^1S_0$; (b) $^3S_1$.

However, they have no bound states.

We conclude that phenomenological potentials display behavior differing from that of meson-exchange models in the Pauli-forbidden S waves. In terms of two-nucleon operators, this difference can be identified with the larger magnitude of $\sigma_1\cdot\sigma_2\tau_1\cdot\tau_2$ contributions in meson-exchange models than in phenomenological models.

VI. ANALYSIS AND CONCLUSIONS

We have demonstrated that evaluating the effect of a nucleon-nucleon potential in Pauli-forbidden states demonstrates in a clear way the true isospin dependence of the model. We document these behaviors by looking at the phase shifts and bound states in the Pauli-forbidden channels. In this section we summarize our findings and discuss their implication for a variety of physically interesting questions. We also present suggestions for further work.

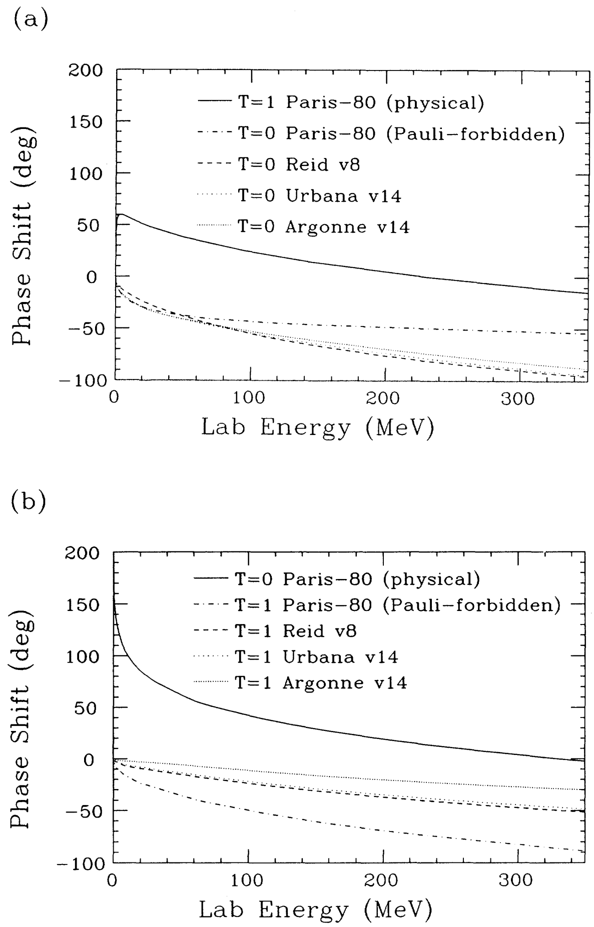


FIG. 6. Phase shifts for the phenomenological models. (a) ${}^1S_0$; (b) ${}^3S_1$.

A. Summary

We have learned that potentials with a phenomenological interior, such as Paris, and those with a meson based interior, such as Bonn, can have substantially different isospin content, despite very similar results for all physical two-body observables.

In particular, we have observed that simply identifying the strength of the isovector part of the potential is not an adequate way of comparing the isovector components of different potentials. First, if the potential has an isospin-dependent nonlocality, the isovector part of the effective local potential depends on both the isoscalar and isovector inputs. Second, the potential itself is poorly determined and direct comparisons of potentials are not instructive. Since it is the effect of the potential to all orders that is constrained via fits to the two-body data, it is only in the potential's effect to all orders where a comparison is meaningful.

We have therefore extracted the isospin dependence by comparing the predictions of different nucleon-nucleon potentials in Pauli-forbidden states. This clarifies the isospin dependence since in getting to the forbidden states from the physical ones only the isospin is changed; the spin and spatial states are kept constant.

We have considered three forms of the Bonn potential: OBEPR, a one-boson-exchange potential in coordinate space; OBEPQ, a one-boson-exchange potential in momentum space; and D52, an energy-dependent momentum-space potential containing the effect of Δ boxes. Three other one-boson-exchange potentials were considered in order to study the effect of the choice of form factor: two Ueda-Green potentials and the Bryan-Scott potential. Four phenomenological potentials were analyzed: Paris, Argonne v14, Urbana v14, and Reid v8.

After formally eliminating nonlocalities, we observe that when we switch from the physical ${}^1S_0$ to the Pauli-forbidden ${}^1S_0$ state, the interior of the meson-exchange potentials switches from strong repulsion to strong attraction. The phenomenological potentials retain their interior core. This indicates that short-range repulsion in meson potentials tends to be isovector in character, a feature not reproduced by the phenomenological ones.

The meson-exchange potentials all have a bound state more deeply bound than the deuteron in the Pauli-forbidden ${}^1S_0$ state; none of the phenomenological potentials do. In the ${}^3S_1$ Pauli-forbidden state, all of the meson-exchange potentials have a deeply bound state except D52 and the Bryan-Scott potential. None of the phenomenological potentials has a bound state. The phase shifts reflect these facts via the Levinson theorem.

All three Bonn models show a significant role for isovector mesons in shaping the short-range behavior. Other meson models that fit phase shifts using exponents of $n=\frac{1}{2}$, 1, and 2 in the strong-interaction vertex function $F_n(k^2)$ also show this behavior. In contrast, more phenomenological models with greater freedom of form result in a less dominant role for the isovector terms in the interior region.

Our study has a number of implications for consideration of phenomena of physical interest. A recent study[35] has shown that the Bonn representation of the ρ-meson contribution is consistent with the functional form of the $N\bar{N}$-$\pi\pi$ helicity amplitudes. The calculations discussed below compare in a similar way the representation of meson degrees of freedom in a derived NN potential and in other processes.

B. Ingredients of meson models

One cannot decide from our results alone whether the isovector core of most meson-exchange models should be taken to represent the correct short-range behavior in a meson-motivated model. The isovector core behavior of meson models is a consequence of the functional forms [such as the $F_n(k^2)$ form factors] used to construct and constrain the short-range behavior of meson-exchange potentials. As shown in Sec. IV, the strong-interaction form factors significantly influence the potential form outside the range of the cutoff due to the mild falloff with k^2.

In a typical meson-exchange potential there are large cancellations between individual meson contributions, so the total potential is much weaker than the size of each meson component. As a result of these large cancellations, the model becomes sensitive to the long-range behavior of the form factor.

We need to understand whether the leading isovector core in these models (arising mostly from single π and ρ exchange) is valid. Two studies shed some light on this issue.

The sensitivity of the potential to the contribution of the δ meson provides one way to judge how well-controlled meson-exchange models are. Removing the δ from the model should produce a new model that resembles the original, since the δ is the heaviest meson of the model, with a relatively small coupling constant. If a convergent series of meson-exchange processes can be written down, then one can think of the processes excluded from the model as constituting a "remainder term," which should have an effect which is hopefully smaller than, or perhaps of comparable strength to, the weakest included process. In this context, the changes in phase shifts resulting from removing the δ are indicative of the degree to which such a convergence has been reached.

Removing the δ from OBEPR causes a nonzero phase shift of $-4.5°$ at the laboratory energy (264 MeV) where the phase shift goes to zero in $^1S_0$ $(T=1)$, and causes a nonzero phase shift of $+10.5°$ at the laboratory energy (288 MeV) where the phase shift goes to zero in $^3S_1$ $(T=0)$. These effects are small compared to the largest phase shifts produced by the potential, but they are not insignificant.

A slow convergence suggests that more processes contribute to meson-exchange models than are represented by the functional forms used for the potential. These functional forms must then necessarily model more meson-mediated processes than they are intended to represent. Therefore, the functional forms chosen can be an overly constraining feature of the model.

One way to explore this dependence on the fundamental forms used is to form Pauli-forbidden S waves using meson-exchange models that use other functional forms for the cutoffs. For example, the Nijmegen potential uses exponential form factors[36] to regularize the behavior of one-meson-exchange forms, a Stony Brook model[37] uses eikonal form factors, and the potential of de Tourreil *et al.*[38] uses a step-like cutoff

$$F(r)=\frac{(\Lambda r)^{20}}{1.0+(\Lambda r)^{20}}, \quad \text{with } \Lambda=1.2\ \text{fm}^{-1}\ , \tag{6.1}$$

in an expression of the form

$$V(r)=V_{\text{theo}}(r)[F(r)]+V_{\text{pheno}}(r)[1-F(r)]\ . \tag{6.2}$$

The Paris potential uses similar cutoff expressions to terminate the influence of the cutoff beyond 1 fm, thereby allowing the individual meson contributions to assume nearly the full value of their one-meson-exchange forms.

A second probe of meson-exchange potentials is a study[39] that investigates the consequence of freeing the meson masses to determine the mass values that would minimize χ^2 in the phase-shift fit. Except for the pion mass, the meson masses migrated to values lower than their accepted values, a result that was taken as evidence for the need to supplement one-meson-exchange forms with expressions for multi-uncorrelated meson-exchange effects.

We quote this study to point out that the functional forms and fixed input parameters of OBEP models provide some constraint on the goodness of fit to phase-shift data. It is therefore not unreasonable to explore whether it is the assumed functional forms of meson models that lead them to have large isovector core contributions.

C. Elastic electromagnetic form factors

In a consistent meson-exchange model of all effective operators, calculations of elastic electromagnetic form factors at sufficiently large momentum transfer should be sensitive to the isovector content of the N-N potential. Since the isovector contributions of meson-exchange models come from the charge-carrying degrees of freedom, these degrees of freedom should be represented in the electromagnetic current operator, J_μ.

To identify the contributions of isovector processes, one would like to see a calculation in which the same underlying processes defining the N-N interaction (and hence the nonrelativistic potential and ground-state wave functions) also determine the form of J_μ. A meson model represents one way to generate both the nuclear states and the associated effective operator J_μ.

The consistency of such a calculation cannot be achieved with phenomenological models. Nevertheless, some calculations using phenomenological potentials include efforts to match the form of J_μ with the potential. As an example, a treatment of the mesonic effects in the electrodisintegration of the deuteron with the Paris potential[40] uses in J_μ the same strong-interaction form factors that appear in the potential. Such consistency is not always practiced in the state of the art theoretical treatments[41,42] of delta and meson-exchange current contributions to charge and magnetic elastic form factors of ^{3}He.

We hope that such consistency can place limits on the degree to which deltas and isovector mesons contribute to the core region of the N-N interaction, perhaps ruling out models containing overly large (or small) isovector meson contributions. Similar calls[40,42] for such consistency express this aim of improving our understanding of the N-N potential. Such a consistent calculation serves to test the simplifying assumption that is made[3] in some electromagnetic form-factor calculations that the short-range repulsion between nucleons is isoscalar in character.

These calculations show[41] a great sensitivity of the size of meson-exchange current contributions to the value of the πNN vertex cutoff. We look forward to improvements in our knowledge of meson-exchange current contributions, from experiments at Continuous Electron Beam Accelerator Facility (CEBAF) and from consistent calculations of the type we are proposing. These meson-exchange currents could be used to explore the present discrepancy that different analyses[43] place on the πNN vertex cutoff.

D. Charge-symmetry breaking

Recently, experiments at TRIUMF and Indiana University Cyclotron Facility (IUCF)[44] have measured observables in neutron-proton scattering which are sensitive to the small charge-symmetry-breaking part of the neutron-proton interaction. The effective operator that produces this effect in a meson-nucleon-delta model depends on some of the same parameters that the two-nucleon potential does. A recent calculation[2] achieves consistency by using for the meson coupling constants in the charge-symmetry-breaking (CSB) interaction operator the same values that are used in the meson potential model, OBEPR. With this potential defining the nuclear states, matrix elements using the charge-symmetry-breaking operator have matched the states with the appropriate effective operators that should be used with these states. The authors also carry out calculations using phenomenological potential models which they acknowledge lack this type of matching. This group finds a substantial difference in the CSB predicted by OBEPR and phenomenological potentials.

This consistency issue has been explored further. One analysis[45] has used in the fit to low-energy N-N scattering data a potential incorporating the dominant charge-symmetry-breaking effect, that of the n-p mass difference. The associated T matrix can be used to calculate the CSB analyzing power. This calculation of matrix elements is consistent because the CS and CSB pieces of the total interaction are treated on the same footing.

A current challenge is to describe the experimental difference in proton and neutron analyzing powers with a calculation using a meson potential model. These calculations of the isospin-dependent, short-range charge-symmetry-breaking interactions probe the short-range isovector content of the potential model. As we have demonstrated here, phenomenological and meson-exchange models tend to have substantially different short-range isovector contents. If information is to be extracted from CSB experiments, it is therefore critical that calculations maintain a consistency between the potential and the CSB operator.

E. Models of the N-Δ and Δ-Δ interaction

Another situation where the effects reported here are relevant is in modeling the N-Δ (and Δ-Δ) interactions. An N-Δ interaction can be constructed[46] from a meson-exchange model of the N-N interaction by using a valence quark model of the baryons to relate nucleon-meson couplings to delta-meson couplings. The sets of quantum numbers that are analogous to those that represent Pauli-forbidden partial waves are allowed for the N-Δ system. The meson composition of the N-N interaction then has consequences for the form of the N-Δ interaction; in fact, behavior similar to that of the paronic S waves shown in Fig. 1 has been seen in at least one N-Δ potential model.[47] This attractive behavior is not a feature shared by all N-Δ models.[48]

Constraints on these behaviors can come from demanding reasonable ground-state observables and (elastic) electromagnetic form factors from a Faddeev calculation of tritium or ^{3}He that incorporates the delta channels in a coupled channel approach.[49,41] Even at relatively low (<2.9 fm^{-1}) momentum transfers, the discrepant situation between theoretical and experimental determinations of the isovector charge and isoscalar magnetic form factors of the trinuclear systems is not fully resolved.[50] Better treatments of N-Δ and Δ-Δ contributions to the two-body interaction look promising as ways to understand the ^{3}He (Ref. 42) and deuteron[51] form factors and threshold deuteron electrodisintegration[51] for $q>5-6$ fm^{-1}. Such calculations can be thought of as testing the composition of a meson model of the N-N potential.

F. Direct (p,n) reactions and Gamow-Teller transitions

Because of the change of charge state, the effective interaction operator that drives intermediate-energy (p,n) direct reactions on nuclei selects the isovector part of the p-n interaction.

In the distorted-wave impulse approximation (DWIA) analysis of (p,n) reactions, it has been argued[52] that at sufficiently high energies (>80 MeV) the free N-N interaction can be used to represent the effective interaction. In the DWIA, the (p,n) cross section at 0° and at zero momentum transfer depends upon the volume integrals of the isospin $(\tau_1\cdot\tau_2)$ and spin-isospin $(\sigma_1\cdot\sigma_2\tau_1\cdot\tau_2)$ components of the two-body potential. These volume integrals are coefficients of the Fermi and Gamow-Teller matrix elements.

The energy-dependent volume integrals have been extracted from 0° $A(p,n)A'$ cross-section data using the DWIA,[53] for incident proton energies between 5 and 200 MeV. A realistic meson model can reproduce[54] their behavior. Phenomenological effective interactions have been used[30] to produce the same behavior, but do not make connection with low-energy phase-shift data. Our work suggests that such phenomenological potential models not having a strong isovector core might have difficulty describing the magnitude and energy dependence of these volume integrals. However, the problem of estimating effects due to medium modifications clouds this issue somewhat.

G. Tests of the fundamental symmetry embodied in the Pauli principle

Although we have presented the Pauli-violating states of the the two-nucleon system as an heuristic (if unphysical) device for clarifying the isovector content of various model potentials, there has been some speculation in the literature that the Pauli principle may be an approximate rather than an exact symmetry. If this were the case, then it would be appropriate to ask what form the potential takes for two nucleons occupying a totally symmetric quantum state.

A Pauli principle violation is permitted in a theory proposed[55] by Greenberg and Mohapatra. Their commutation relations allow a system of identical fermions, dubbed "parons," to occupy with small probability a totally symmetric quantum state. Experiments were initiated and conducted[56] for signatures of paronic electrons in atomic systems, but no proposals of detecting paronic nuclear systems were made.

Greenberg and Mohapatra have since retracted[57] their theory because the norms of their quantum states are not positive definite. However, the absence of a valid quantum field-theoretic mechanism does not prevent violations of the exclusion principle from being posed and sought within the context of quantum mechanics.[56] One can still speculate whether some other theory would allow fermions to occupy symmetric states. The predicted N-N potential in these states would then be critical in deciding where to look for such violations.

Our results show that our understanding of nucleon-nucleon potential models from nucleon-meson-delta models is not yet sufficiently stable to offer unambiguous statements about the nucleon-nucleon interaction in paronic states. Our prediction of the existence of a paronic two-nucleon bound system is model dependent.

Most Bonn and other meson-exchange models in this study produced such bound states, with wave functions peaked and localized at small (less than 1 fm) separation between the two nucleons. This result is unphysical. The existence of a short-range repulsion between nucleons is used to justify an expansion in the range of the mesons exchanged. The near-completeness of the processes included in the Bonn model is reached only if the exchanged mass of the virtual particle(s) is comparable to the range of the repulsive core. For a state with significant probability at short range, this expansion in the range of exchanged mesons would need to include many more short-range processes; the types of processes represented in the Bonn model are incomplete in representing the physics at such short separation.

We note from Fig. 1 that the paronic interaction is repulsive in to a radius of 1 fm, discouraging two separated parons from forming a bound state. Without an attractive two-body force, it seems unlikely for nucleosynthesis to incorporate parons into nuclei of more than two nucleons.

If we assume that paronic bound states exist and have the properties found using the Bonn OBE potentials, a number of factors would make them detectable. First, of course, the diproton would be doubly charged. Even for the n-p paronic state its difference in binding energy from that of the normal deuteron would lead to a substantial isotope shift. A $^1S_0$ paronic bound state would lack a nuclear magnetic moment, leading to the absence of hyperfine structure. Paronic nuclear systems, which could perhaps be created in the early Universe or in collisions, could be detected by such signatures. However, the long-range repulsion reduces significantly the probability of finding such states even if they are possible in principle.

ACKNOWLEDGMENTS

We thank O. W. Greenberg for suggesting the problem of violations of the Pauli principle in nuclear systems. We gratefully acknowledge Ch. Elster, T. Cheon, and R. Wiringa for the use of their computer codes. This work was supported in part by U.S. Department of Energy Grant No. DE-FG05-87ER-40322.

APPENDIX: ISOSPIN DECOMPOSITION OF THE EFFECTIVE LOCAL POTENTIAL

When a nonlocal potential is replaced by an effective local potential, the isospin dependence in the original nonlocal form gets mixed when the local form is separated into parts of good isospin.

To see this, we briefly consider the treatment[58,26] of p^2 terms in r-space potentials. Momentum-dependent forms are readily generated in meson-exchange models, and have been used phenomenologically with good success. If the nonlocalities are expanded in momentum space, the first-order nonlocality takes the form

$$v(r,p)=V_{\rm local}(r)+(\lambda/M)[p^2\omega(r)+\omega(r)p^2]\ , \tag{A1}$$

where M is twice the reduced mass of two nucleons and p is the relative momentum operator.

If $R_l(r)$ solves the Schrödinger equation with the nonlocal potential $v(r,p)$, we may transform to an effective local equation by introducing

$$u_l(r)=r[1+2\lambda\omega(r)]R_l(r)\ . \tag{A2}$$

The nonlocal Schrödinger equation for $R_l(r)$ then becomes a local Schrödinger equation for $u_l(r)$. This Schrödinger equation contains an effective local potential given by

$$W^{\rm eff}(k,r)=\left[\frac{1}{1+2\lambda\omega(r)}\right]\times\left[\frac{M}{\hbar^2}V_{\rm local}(r)-2\lambda k^2\omega(r)+\frac{\lambda^2\omega'^2(r)}{1+2\lambda\omega(r)}\right]\ . \tag{A3}$$

Since the function $\omega(r)$ is of short range, the functions $R_l(r)$ and $u_l(r)/r$ have the same asymptotic behavior. Hence, phase shifts calculated for $u_l(r)$ are the same as the true phase shifts.

The first factor in brackets in Eq. (A3) shows the presence of nonlinear coupling. Through this factor isoscalar mesons can contribute to the isospin dependence of $W^{\rm eff}$.

To exhibit the coupling, consider the decomposition in Eq. (2.1) of the effective local potential into isoscalar and isovector components. To see how isoscalar parts in the original form (A1) contribute to the isovector component of Eq. (2.1), we note that $\omega(r)$ has an isovector part. Therefore, we can write the isospin structure of Eq. (A3) as

$$\left[\frac{1}{1+C+D\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}\right](A+B\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ , \tag{A4}$$

with

$$2\lambda\omega(r)=C+D\tau_1\cdot\tau_2\ , \tag{A5a}$$

$$\frac{M}{\hbar^2}V_{\mathrm{local}}(r)-2\lambda k^2\omega(r)=A+B\tau_1\cdot\tau_2\ , \tag{A5b}$$

$$A=A_{\mathrm{local}}+A_{\mathrm{nonlocal}}k^2\ , \tag{A5c}$$

$$B=B_{\mathrm{local}}+B_{\mathrm{nonlocal}}k^2\ . \tag{A5d}$$

We have dropped the last term of Eq. (A3), since it is quite small in practice. In a meson model, the $\lambda\omega(r)$ in Eq. (A3) and V_{local} each contain contributions from both isoscalar and isovector mesons. We represent isoscalar contributions by A and C, and isovector contributions by B and D.

Since $\tau_1\cdot\tau_2$ is diagonal in a basis of good total isospin, any function of this operator will be diagonal in isospin space. This enables us to treat $\tau_1\cdot\tau_2$ as a c number in forming the expectation value of the W^{eff} of Eq. (A4) in each isospin channel. Solving for $W^{\mathrm{eff}}_{\mathrm{is}}$ and $W^{\mathrm{eff}}_{\mathrm{iv}}$ yields

$$W^{\mathrm{eff}}_{\mathrm{is}}=A-CA-3DB+O(C^2,CD,D^2)\ , \tag{A6a}$$

$$W^{\mathrm{eff}}_{\mathrm{iv}}=B-CB-D(A-2B)+O(C^2,CD,D^2)\ . \tag{A6b}$$

We have only kept leading terms in C and D since they are both small compared to 1.

We see from Eq. (A6b) that isoscalar contributions, represented by terms A and C, contribute to $W^{\mathrm{eff}}_{\mathrm{iv}}$. The terms C and D, the coefficients of the p^2 terms of Eq. (A1), provide the mixing. Terms like this are generated by the one-meson exchanges of all scalar and vector mesons in a meson model. Hence, a nonlocality of the form in Eq. (A1) enables isoscalar meson contributions to enter $W^{\mathrm{eff}}_{\mathrm{iv}}$.

This feature prevents a simple separation of the potential into an isovector part arising from exchange of isovector mesons and an isoscalar part arising from the exchange of isoscalar mesons. These p^2 terms can be significant in shaping the core region (see Fig. 3).

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