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ARTICLES

Current conservation and interaction currents with relativistic separable interactions

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A method of minimal substitution is used to obtain the electromagnetic two-body interaction current operator associated with a relativistic separable interaction. The result is shown to satisfy a general restriction obtained from current conservation using the Ward-Takahashi identity within the Bethe-Salpeter formalism. It is shown that this two-body relativistic current operator can be reduced to an effective one-body form. Applications of this method to nuclear few-body systems, relativistic quark systems, and hadronic resonance processes are discussed. The numerical size of the interaction current for the pion charge form factor is estimated.

I. INTRODUCTION

It is well known that interaction (exchange) currents should be introduced, in addition to the one-body impulse current, to satisfy electromagnetic current conservation. The nature of the interaction currents required depends on the dynamics governing the system, and recently it has been learned how to construct such currents for two-body systems described by relativistic wave equations,[1–3] such as the Bethe-Salpeter or its reduced equations.[4] A relativistic treatment is essential at the large momentum transfers available today.

It is shown in Ref. 1 that the proper form of the relativistic two-body current operator can be derived in a quite general way using the Ward-Takahashi identities for the one-body current operators and the relativistic wave equations for the bound states. Furthermore, it was found that phenomenological strong and electromagnetic form factors can be introduced without violating gauge invariance. In Ref. 5, a schematic method for deriving the gauge-invariant interaction current for a general nonlocal interaction was introduced, and the method was used to calculate the photopion production amplitude from a nonlocal π-nucleon interaction Lagrangian.[6] The electromagnetic field was introduced through minimal substitution in the momentum dependence of the vertex function, and this provided a solid mathematical scheme for deriving the gauge-invariant amplitude corresponding to a vertex with an arbitrary functional form. In this paper, a phenomenological vertex function is introduced as part of a separable interaction, which permits us to treat the composite structure of hadrons.

The use of a separable Bethe-Salpeter kernel is an interesting and simple approach to the relativistic description of bound states. It is not only a mathematically convenient way to solve the relativistic two-body wave equation, but it also is a practical way to investigate the relativistic three-body problem.[7] Separable interactions also provide us with a simple description of mesons as an extended quark-antiquark ($q\bar{q}$) system. Here, quarks are confined by a very complicated many-gluon-exchange mechanism, which is clearly a nonlocal interaction. In fact, the instanton-induced nonlocal four-quark interaction can be expressed in a separable form.[8] Another example of the usefulness of a separable model is provided by the Nambu–Jona-Lasinio model[9] of $q\bar{q}$ mesons,[10] which incorporates the physics of a $q\bar{q}$ (superconducting) pair by using the simplest form of separable interaction.

Separable interactions are generally expressed in terms of products of vertex functions. It is therefore a simple task to apply the method developed in Ref. 5 to the derivation of the interaction current associated with the separable force.

In this paper, we derive the interaction current operators corresponding to a relativistic separable interaction by using the minimal substitution method.[5] While our procedure for obtaining the current gives a unique result, it is important to realize that additional terms could be added to the current which are separately gauge invariant, so that the interaction current we finally determine is not unique. It is, however, sufficient to insure that the original phenomenological separable interaction model can be applied to a study of electromagnetic interactions in a covariant, gauge-invariant manner. The results are shown to satisfy the general constraint[1] obtained from the use of the Ward-Takahashi identity and the Bethe-Salpeter equation. With an eye to applying the results to both relativistic few-nucleon systems and covariant constituent quark models of hadron structure, different forms of separable interaction are investigated. The matrix elements of interaction currents are evaluated with bound-state solutions of the Bethe-Salpeter equation. The results are reduced into a simple form, so that the physical significance of this new current operator becomes clear. In general, the gauge invariance of the matrix elements requires both the impulse and interaction terms. Finally, a numerical evaluation of the matrix elements is carried out for a simple separable model of the pion, which we regard as an illustration of the general method. The size of the interaction current contribution to the pion charge form factor is compared with the impulse contribution.

This paper is organized into five sections and two appendices. In Sec. II, we formulate the general constraints on relativistic two-body current operators through the principle of current conservation. The details of how the minimal substitution method yields the interaction current operator corresponding to a simple form of separable interaction are given. In Sec. III, the bound-state matrix element is evaluated and a simple interpretation of the new contribution is given. The illustrative numerical results for the pion charge form factor are given. We expand the method to more general form of separable interactions in Sec. IV, and summarize our results in Sec. V. A matrix representation useful for the description of the fermion-antifermion systems is used in this paper, but methods and conclusions are general, and apply to other systems, such as two-fermion or two-boson systems.

II. INTERACTION CURRENTS OF SEPARABLE INTERACTION

A. Bethe-Salpeter equation and conservation of electromagnetic current

We start with a general expression for the Bethe-Salpeter equation for fermion-antifermion scattering amplitudes and derive the wave equation for the vertex function of a bound state. The equation for the scattering amplitude [Fig. 1(a)] is given by

$$M_{\alpha\beta:\delta\gamma}(k',k;p)=V_{\alpha\beta:\delta\gamma}(k',k;p)+i\int\frac{d^4k''}{(2\pi)^4}V_{\alpha\beta:\epsilon\lambda}(k',k'';p)S_{\lambda\eta}(k''+p/2)S_{\zeta\epsilon}(k''-p/2)M_{\eta\zeta:\delta\gamma}(k'',k;p)\ , \qquad (2.1)$$

where k' (k) and p are the relative and center-of-mass four-momenta of the system, respectively, and $V(k',k;p)$ is the interaction kernel. The fermion propagator with the mass m is defined by $S(p)=i(\not p-m+i\epsilon)^{-1}$, and greek characters are used for the Dirac indices. The presence of a bound state implies a pole at the mass M_B in the M matrix,

$$M_{\alpha\beta:\delta\gamma}(k',k;p)=\frac{\Gamma_{\alpha\beta}(k';p)\bar{\Gamma}_{\delta\gamma}(k;p)}{p^2-M_B^2}+R_{\alpha\beta:\delta\gamma}(k,k';p)\ , \qquad (2.2)$$

where R is regular at $p^2=M_B^2$. Substituting Eq. (2.2) into Eq. (2.1) and evaluating the residue,

$$\lim_{p^2\to M_B^2}\ \text{Eq. (2.1)}\times(p^2-M_B^2)\ ,$$

gives the bound-state equation for the vertex function (Fig. 2),

$$\Gamma_{\alpha\beta}(k;p)=i\int\frac{d^4k''}{(2\pi)^4}V_{\alpha\beta:\epsilon\lambda}(k,k'';p)S_{\lambda\eta}(k''+p/2)\times\Gamma_{\eta\zeta}(k'';p)S_{\zeta\epsilon}(k''-p/2)\ . \qquad (2.3a)$$

Applying the same procedure to the integral equation depicted by Fig. 1(b), we have the equation for the conjugate state,

$$\bar{\Gamma}_{\delta\gamma}(k;p)=i\int\frac{d^4k''}{(2\pi)^4}S_{\zeta\epsilon}(k''-p/2)\bar{\Gamma}_{\epsilon\lambda}(k'';p)\times S_{\lambda\eta}(k''+p/2)V_{\eta\zeta:\delta\gamma}(k'',k;p)\ . \qquad (2.3b)$$

The conjugate vertex $\bar{\Gamma}(k;p)$ is defined by

$$\bar{\Gamma}(k;p)=-\gamma_0\Gamma^{\dagger}(k;p)\gamma_0\ ,$$

where γ^0 is the Dirac matrix. The vertex function and the wave function are related by

$$\Psi_{\alpha\beta}(k,p)=S_{\alpha\alpha'}(k+p/2)\Gamma_{\alpha'\beta'}(k;p)S_{\beta'\beta}(k-p/2)\ .$$

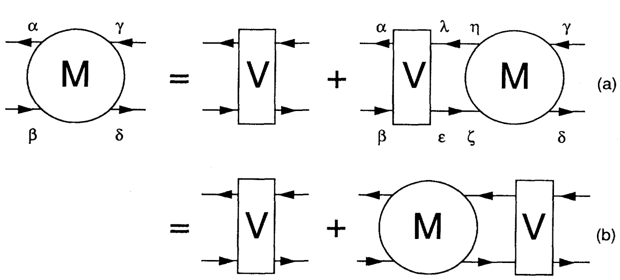

FIG. 1. The Bethe-Salpeter equation for fermion-antifermion scattering matrix $M(k',k;p)$, with a interaction kernel $V(k',k;p)$. The solid lines with arrows are fermions.

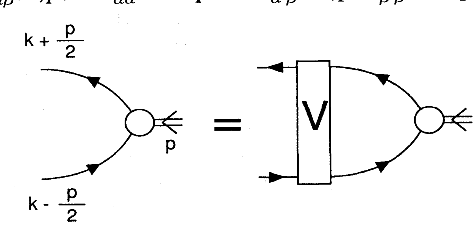

FIG. 2. The Bethe-Salpeter equation for a bound-state vertex function $[\Gamma(k;p)]$ indicated by the open circle.

If an external photon field couples to the bound state of the fermion-(a)–antifermion-($\bar{b}$) system, the impulse amplitudes (Fig. 3) can be expressed in terms of the solutions of Eqs. (2.3a) and (2.3b),

$$\langle J_a^\mu \rangle = -i \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\,\bar{\Psi}_f(k';p')(-ie_a\gamma^\mu) \times \Psi_i(k;p) S^{-1}(k-p/2)] \quad (2.4a)$$

and

$$\langle J_b^\mu \rangle = -i \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\,\bar{\Psi}_f(k'';p') S^{-1}(k+p/2) \times \Psi_i(k;p)(-ie_b\gamma^\mu)] \; , \quad (2.4b)$$

where $k'=k+q/2$ and $k''=k-q/2$, f and i denote final and initial bound states (which need not be the same), and $e_{a(b)}$ is the charge of the particle $a\,(b)$. The impulse form factor $[F_{\mathrm{RIA}}^{\pi}(q^2)]$ is given by

$$F_{\mathrm{RIA}}^{\pi}(q^2)(p+p')^\mu = \langle J_a^\mu \rangle + \langle J_b^\mu \rangle \; .$$

In Ref. 1 it is shown that electromagnetic current conservation requires the presence of an interaction current associated with the two-body interaction, and the proper form of the current operator is obtained in a very general way by using the wave equation and the Ward-Takahashi identity. Here, we briefly review the method and derive the restriction. With the use of the Ward-Takahashi identity

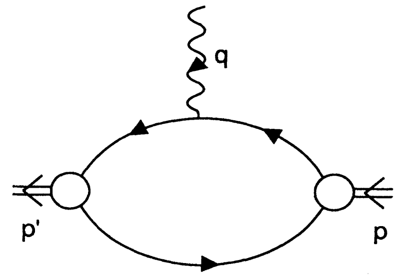

FIG. 3. An example of impulse diagram ($\langle J_a^\mu \rangle$) for the charge form factor, where a photon (wavy line) with the momentum q couples to the particle "a."

$$-i\not{q} = S^{-1}(k+p/2+q) - S^{-1}(k+p/2) \; , \quad (2.5)$$

the divergence of one-body current can be expressed as

$$\begin{aligned} q_\mu \langle J_a^\mu \rangle &= -ie_a \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[S^{-1}(k-p/2)\bar{\Psi}_f(k+q/2;p')S^{-1}(k+p/2+q)\Psi_i(k;p)] \\ &\quad + ie_a \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\bar{\Psi}_f(k+q/2;p')S^{-1}(k+p/2)\Psi_i(k;p)S^{-1}(k-p/2)] \\ &= e_a \int\int \frac{d^4k\,d^4k'}{(2\pi)^8} \bar{\Psi}_{f_{\alpha\beta}}(k';p+q)[V(k',k+q/2;p+q) - V(k'-q/2,k;p)]_{\beta\alpha:\delta\gamma} \Psi_{i_{\gamma\delta}}(k;p) \; , \end{aligned} \quad (2.6a)$$

where the last equality is derived by using the wave equation, Eqs. (2.3a) and (2.3b). Likewise,

$$q_\mu \langle J_b^\mu \rangle = e_b \int\int \frac{d^4k\,d^4k'}{(2\pi)^8} \bar{\Psi}_{f_{\alpha\beta}}(k';p+q)[V(k'+q/2,k;p) - V(k'k-q/2;p+q)]_{\beta\alpha:\delta\gamma} \Psi_{i_{\gamma\delta}}(k;p) \; . \quad (2.6b)$$

The divergence of the one-body current is not generally zero, and we rely on the presence of a two-body current to satisfy current conservation,

$$q_\mu \langle J_a^\mu + J_b^\mu + J_{\mathrm{int}}^\mu \rangle = 0 \; . \quad (2.7)$$

Thus, we obtain a general restriction to the form of the two-body interaction current, which we can express in terms of the two-body interaction. If the above relations are to hold for any bound-state wave function, the divergence of the current must satisfy the following operator equation:

$$q_\mu J_{\mathrm{int}}^\mu(k',k;[p,q]) = e_a[V(k'-q/2,k;p) - V(k',k+q/2;p+q)] + e_{\bar{b}}[V(k'+q/2,k;p) - V(k',k-q/2;p+q)] \; , \quad (2.8)$$

where $e_{\bar{b}} = -e_b$ is the charge of antiparticle $\bar{b}$.

B. Separable interactions and interaction currents

We now derive the interaction current by minimal substitution[5] of the photon field into a separable interaction. In coordinate space, separable interactions have the following nonlocal form (Fig. 4):

$$V_{\alpha\beta:\delta\gamma}(x_1',x_2':x_1,x_2) = \Delta_{\alpha\beta}(x_1',x_2')\bar{\Delta}_{\delta\gamma}(x_1,x_2) \; , \quad (2.9)$$

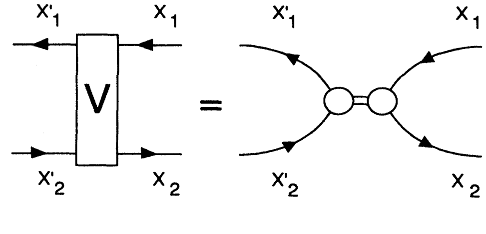

FIG. 4. A relativistic separable interaction as a product of two nonlocal vertices indicated by the open circles.

where we may write the vertex as a Fourier integral,

$$\Delta_{\alpha\beta}(x_1',x_2')=\int\int\frac{d^4k_1'd^4k_2'}{(2\pi)^8}\Delta_{\alpha\beta}(k_1',k_2')e^{ik_1'x_1'}e^{ik_2'x_2'} . \quad (2.10)$$

The vertex function $\Delta(k_1,k_2)$ $[=\Delta(k;p)]$, with $k\equiv(k_1-k_2)/2$ and $p\equiv k_1+k_2$, is generally a 4×4 matrix for the relativistic fermion-antifermion system, and can be expressed in terms of a sum of Dirac matrices. In momentum space, the most general form of the separable interaction could be expressed as

$$V_{\alpha\beta;\delta\gamma}(k',k;p)=\sum_{l',l}^{N}C_{l'l}\Delta^{l'}_{\alpha\beta}(k';p)\overline{\Delta}^{l}_{\delta\gamma}(k;p) , \quad (2.11)$$

where N is the rank of the interaction, and the channel couplings are $C_{l'l}$. For simplicity, we chose a rank-one interaction, since the generalization to cases of higher rank is apparent but tedious. Then,

$$V_{\alpha\beta;\delta\gamma}(k',k;p)=\Delta_{\alpha\beta}(k';p)\overline{\Delta}_{\delta\gamma}(k;p) , \quad (2.12)$$

where

$$\Delta_{\alpha\beta}(k';p)=f([k_1'-k_2']^2)\Omega_{\alpha\beta} ,$$

$$\overline{\Delta}_{\alpha\beta}(k;p)=f([k_1-k_2]^2)\overline{\Omega}_{\alpha\beta} .$$

The matrices Ω and $\overline{\Omega}$ are constant matrices, and $f([k_1'-k_2']^2)$ and $f([k_1-k_2]^2)$ are scalar functions of the relative momenta k' and k. (It will be clear after this simple derivation that the results for higher-rank interactions would give essentially the same conclusions. More general types of vertices will be explored later.)

The charges e_1 and e_2 are assigned to particle 1 and antiparticle 2, and the electromagnetic field is introduced through the minimal substitution of

$$\partial_i^\mu\to\partial_i^\mu+ie_iA^\mu(x_i)$$

for the momentum of each charged particle. In this paper we do not treat charge-exchange processes, but the ideas developed here could be extended to include such interactions. The photon field induces a modification in the vertex and a corresponding modification of the two-body interaction. Because the one-photon absorption or emission processes depend linearly on the photon field, the modification can be expressed in the following form:

$$\delta V(x_1',x_2';x_1,x_2)=\delta\Delta(x_1',x_2')\overline{\Delta}(x_1,x_2)+\Delta(x_1',x_2')\delta\overline{\Delta}(x_1,x_2) . \quad (2.13)$$

Here, $\delta\Delta(x_1',x_2')$ and $\delta\overline{\Delta}(x_1,x_2)$ are the modifications in the vertices $\Delta(x_1',x_2')$ and $\overline{\Delta}(x_1,x_2)$ induced by the photon field, and they are functions of the photon field operator. The two-body current operator for the one-photon absorption process can then be obtained by taking the matrix element of δV with the photon field

$$J^\mu_{\rm int}(x_1',x_2';x_1,x_2:q)=-(2\pi)^4\langle 0|\delta V|a^\dagger_\mu(q)\rangle ,$$

where $a^\dagger_\mu(q)$ is the creation operator of a photon with the momentum q and the polarization ϵ_μ.

To obtain the detailed results, we start with the power-series expansion,

$$f([k_1-k_2]^2)=\sum_n C_n[k_1-k_2]^{2n} ,$$

in the Fourier integral of the vertex function,

$$\Delta_{\alpha\beta}(x_1,x_2)=\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}f([k_1-k_2]^2)\Omega_{\alpha\beta}e^{ik_1x_1}e^{ik_2x_2} . \quad (2.14)$$

Here, C_n are the expansion coefficients. (It is not necessary to specify them because the resummation will be taken later.) This allows us to express Eq. (2.14) as

$$\Delta_{\alpha\beta}(x_1,x_2)=\sum_n C_nI^{(n)}(x_1,x_2)\Omega_{\alpha\beta} , \quad (2.15)$$

where

$$I^{(n)}(x_1,x_2)=\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}[k_1-k_2]^{2n}e^{ik_1x_1}e^{ik_2x_2}$$
$$=\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}(-)^n\left[\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right]^{2n}\times e^{ik_1x_1}e^{ik_2x_2} .$$

Through the minimal substitutions, $\partial_i^\mu\to\partial_i^\mu+ie_iA^\mu(x_i)$, the matrix element of the vertex correction defined by

$$\delta\Delta^\mu(x_1',x_2':q)\equiv\langle 0|\delta\Delta(x_1',x_2')|a^\dagger_\mu(q)\rangle$$

can be expressed as

$$\delta\Delta^\mu_{\alpha\beta}(x_1',x_2':q)=\sum_n C_n\Omega_{\alpha\beta}\langle 0|\delta I^{(n)}(x_1',x_2')|a^\dagger_\mu(q)\rangle , \quad (2.16a)$$

where

$$\delta I^{(n)}(x_1',x_2')=(-)^n\int\int\frac{d^4k_1'd^4k_2'}{(2\pi)^8}\left[\left\langle\!\!\left\langle\left[\frac{\partial}{\partial x_1'}-\frac{\partial}{\partial x_2'}\right]^{2n}\right\rangle\!\!\right\rangle-\left[\frac{\partial}{\partial x_1'}-\frac{\partial}{\partial x_2'}\right]^{2n}\right]e^{ik_1'x_1'}e^{ik_2'x_2'} .$$

Likewise,

$$\delta\overline{\Delta}^\mu_{\alpha\beta}(x_1,x_2:q)=\sum_n C_n\overline{\Omega}_{\alpha\beta}\langle 0|\delta I^{\dagger(n)}(x_1,x_2)|a^\dagger_\mu(q)\rangle , \quad (2.16b)$$

where

$$\delta I^{\dagger(n)}(x_1,x_2)=(-)^n\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\left[\left\langle\!\!\left\langle\left[\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right]^{2n}\right\rangle\!\!\right\rangle-\left[\frac{\partial}{\partial x_2}-\frac{\partial}{\partial x_2}\right]^{2n}\right]^{\dagger}e^{-ik_1x_1}e^{-ik_2x_2}\ .$$

Here, the double angular bracket is meant to be the minimal substitution

$$\left\langle\!\!\left\langle\left[\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right]^{2}\right\rangle\!\!\right\rangle\equiv\left[\left[\frac{\partial}{\partial x_1}+ie_1A(x_1)\right]-\left[\frac{\partial}{\partial x_2}+ie_2A(x_2)\right]\right]^2, \tag{2.17}$$

for which we introduce the following notations:

$$\left\langle\!\!\left\langle\left[\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right]^{2}\right\rangle\!\!\right\rangle=\Box_{12}+iZ+W\ , \tag{2.18}$$

where

$$\Box_{12}=\left[\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right]^2,$$

$$Z=e_1\frac{\partial A^\mu(x_1)}{\partial x_1^\mu}+e_2\frac{\partial A^\mu(x_2)}{\partial x_2^\mu}+2[e_1A^\mu(x_1)-e_2A^\mu(x_2)]\left[\frac{\partial}{\partial x_1^\mu}-\frac{\partial}{\partial x_2^\mu}\right],$$

and

$$W=[ie_1A_\mu(x_1)-ie_2A_\mu(x_2)]^2\ .$$

It should be understood that the differential operators at the last term of Z operate on $e^{ik_1'x_1'}e^{ik_2'x_2'}$ or $e^{-ik_1x_1}e^{-ik_2x_2}$ in Eqs. (2.16a) and (2.16b). The photon field is quantized and has the usual plane-wave expansion,

$$A_\mu(x)=\int\frac{d^4q}{(2\pi)^4}[a_\mu(q)e^{-iqx}+a_\mu^\dagger(q)e^{iqx}]\ .$$

With this notation, the inside of the square bracket in Eq. (2.16) can be expressed as

$$\begin{aligned}\left\langle\!\!\left\langle\left[\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right]^{2n}\right\rangle\!\!\right\rangle-\left[\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right]^{2n}&=(\Box_{12}+iZ+W)^n-(\Box_{12})^n\\&=(\Box_{12}+iZ+W)(\Box_{12}+iZ+W)\cdots(\Box_{12}+iZ+W)-(\Box_{12})^n\\&=(\Box_{12})^{n-1}(iZ)+(\Box_{12})^{n-2}(iZ)(\Box_{12})+\cdots+\Box_{12}(iZ)(\Box_{12})^{n-2}+(iZ)(\Box_{12})^{n-1}\\&=\sum_{M=0}^{n-1}(\Box_{12})^{n-1-M}(iZ)(\Box_{12})^n\ .\end{aligned} \tag{2.19}$$

Note that the third equality is valid for one-photon absorption or emission processes, where $\langle 0|W|a_\nu^\dagger(q)\rangle$ and $\langle 0|Z^N|a_\nu^\dagger(q)\rangle(N\geq 2)$ do not contribute. Then we find a compact expression

$$\delta I^{(n)}(x_1',x_2')=(-)^n\int\int\frac{d^4k_1'd^4k_2'}{(2\pi)^8}\sum_{M=0}^{n-1}(\Box_{12})^{n-1-M}(iZ)(\Box_{12})^Me^{ik_1'x_1'}e^{ik_2'x_2'}\ , \tag{2.20a}$$

$$\delta I^{\dagger(n)}(x_1,x_2)=(-)^n\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\sum_{M=0}^{n-1}(\Box_{12})^{n-1-M}(iZ)^*(\Box_{12})^Me^{-ik_1x_1}e^{-ik_2x_2}\ . \tag{2.20b}$$

The matrix elements,

$$\langle 0|\delta I^{(n)}(x_1,x_2)|a_\nu^\dagger(q)\rangle$$

and

$$\langle 0|\delta I^{\dagger(n)}(x_1',x_2')|a_\nu^\dagger(q)\rangle$$

appearing in Eqs. (2.16), are easily evaluated by using Eqs. (A4) in Appendix A. By using an identity,

$$\sum_{M=0}^{n-1}(a)^{n-1-M}(b)^M=\frac{a^n-b^n}{a-b}\ ,$$

and making the resummation with $f(X)=\sum_n C_nX^n$, we finally arrive at the following expressions for the vertex corrections:

$$\delta\Delta^{\mu}_{\alpha\beta}(x'_1,x'_2:q)=\int\int\frac{d^4k'_1d^4k'_2}{(2\pi)^{12}}\Omega_{\alpha\beta}\sum_n C_n u^{\mu}(n)$$

$$=-\int\int\frac{d^4k'_1d^4k'_2}{(2\pi)^{12}}\Omega_{\alpha\beta}\left[e_1\frac{(2k'_1-2k'_2-q)^{\mu}}{(2k'_1-2k'_2-q)\cdot q}[f([k'_1-k'_2-q]^2)-f([k'_1-k'_2]^2)]e^{ik'_1x'_1}e^{ik'_2x'_2}e^{-iqx'_1}\right.$$
$$\left.+e_2\frac{(2k'_1-2k'_2+q)^{\mu}}{(2k'_1-2k'_2+q)\cdot q}[f([k'_1-k'_2+q]^2)-f([k'_1-k'_2]^2)]e^{ik'_1x'_1}e^{ik'_2x'_2}e^{-iqx'_2}\right] \quad (2.21a)$$

and

$$\delta\bar{\Delta}^{\mu}_{\delta\gamma}(x_1,x_2:q)=\int\int\frac{d^4k_1d^4k_2}{(2\pi)^{12}}\bar{\Omega}_{\delta\gamma}\sum_n C_n w^{*\mu}(n)$$

$$=\int\int\frac{d^4k_1d^4k_2}{(2\pi)^{12}}\bar{\Omega}_{\delta\gamma}\left[e_1\frac{(2k_1-2k_2+q)^{\mu}}{(2k_1-2k_2+q)\cdot q}[f([k_1-k_2+q]^2)-f([k_1-k_2]^2)]e^{-ik_1x_1}e^{-ik_2x_2}e^{-iqx_1}\right.$$
$$+e_2\frac{(2k_1-2k_2-q)^{\mu}}{(2k_1-2k_2-q)\cdot q}$$
$$\left.\times[f([k_1-k_2-q]^2)-f([k_1-k_2]^2)]e^{-ik_1x_1}e^{-ik_2x_2}e^{-iqx_2}\right], \quad (2.21b)$$

where $u^{\mu}(n)$ and $w^{\mu}(n)$ appearing in the intermediate expressions are defined in Appendix A. The momentum-space representation of the interaction current operator (Fig. 5) is obtained from the products of the vertex (Δ) with the vertex correction ($\delta\Delta$), so that the kinematical variables satisfy momentum conservation $k'_1+k'_2=k_1+k_2+q$:

$$J^{\mu}_{\rm int}(k'_1,k'_2;k_1,k_2:q)=\Delta_{\alpha\beta}(k'_1,k'_2)\delta\bar{\Delta}^{\mu}_{\delta\gamma}(k_1,k_2:q)+\delta\Delta^{\mu}_{\alpha\beta}(k'_1,k'_2:q)\bar{\Delta}_{\delta\gamma}(k_1,k_2)\,, \quad (2.22)$$

where the vertex corrections $\delta\Delta^{\mu}_{\alpha\beta}(k'_1,k'_2:q)$ [Fig. 6(a)] and $\delta\bar{\Delta}^{\mu}_{\delta\gamma}(k_1,k_2:q)$ [Fig. 6(b)] are given by

$$\delta\Delta^{\mu}_{\alpha\beta}(k'_1,k'_2:q)=\Omega_{\alpha\beta}\left[e_1\frac{K'^{\mu}_{-}}{K'_{-}\cdot q}[f(S'^2_{-})-f(S'^2)]+e_2\frac{K'^{\mu}_{+}}{K'_{+}\cdot q}[f(S'^2_{+})-f(S'^2)]\right] \quad (2.23a)$$

and

$$\delta\bar{\Delta}^{\mu}_{\alpha\beta}(k_1,k_2:q)=-\bar{\Omega}_{\alpha\beta}\left[e_1\frac{K^{\mu}_{+}}{K_{+}\cdot q}[f(S^2_{+})-f(S^2)]+e_2\frac{K^{\mu}_{-}}{K_{-}\cdot q}[f(S^2_{-})-f(S^2)]\right]. \quad (2.23b)$$

Here $S^{(\prime)}\equiv k^{(\prime)}_1-k^{(\prime)}_2$, $S^{(\prime)}_{\pm}\equiv k^{(\prime)}_1-k^{(\prime)}_2\pm q$, and $K^{(\prime)}_{\pm}\equiv 2k^{(\prime)}_1-2k^{(\prime)}_2\pm q$. The matrix element of the interaction current is evaluated by using the solution of the Bethe-Salpeter equation $[\Psi(k;P)]$,

$$\langle J^{\mu}_{\rm int}\rangle=\int\int\frac{d^4k\,d^4k'}{(2\pi)^8}\bar{\Psi}_f(k';p+q)J^{\mu}_{\rm int}(k',k:[p,q])\Psi_i(k;p)\,, \quad (2.24)$$

where the interaction current operator is expressed in terms of the interaction

$$J^{\mu}_{\rm int}(k',k:[p,q])=-e_1\frac{K^{\mu}_{+}}{K_{+}\cdot q}[V(k',k+q/2)-V(k',k)]+e_1\frac{K'^{\mu}_{-}}{K'_{-}\cdot q}[V(k'-q/2,k)-V(k',k)]$$
$$-e_2\frac{K^{\mu}_{-}}{K_{-}\cdot q}[V(k',k-q/2)-V(k',k)]+e_2\frac{K'^{\mu}_{+}}{K'_{+}\cdot q}[V(k'+q/2,k)-V(k',k)]\,. \quad (2.25)$$

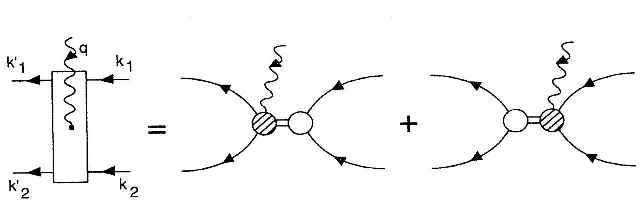

FIG. 5. A two-body interaction current operator expressed in terms of the product of a nonlocal vertex and a vertex modification (hatched circle).

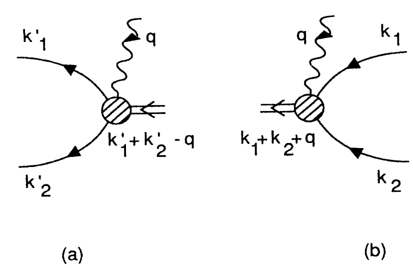

FIG. 6. The vertex modification ($\delta\Delta$) induced by the substitution of photon field into the vertex (Δ).

The divergence of the interaction current is given by

$$q_\mu J^\mu_{\rm int}(k',k:[p,q])=e_1[V(k'-q/2,k)-V(k',k+q/2)]+e_2[V(k'+q/2,k)-V(k',k-q/2)]\ . \qquad (2.26)$$

Here, e_1 and e_2 are the charges of the particle and antiparticle. We observe that the interaction current derived here, by the minimal substitution method, satisfies the general constraint [Eq. (2.8)], obtained from current conservation.

Because terms linear in the photon field enter into the final results only, it is clear that the conclusion given by Eqs. (2.25) and (2.26) is still valid for the case of higher-rank separable interactions. In Sec. IV, we extend the present result to separable interactions having momentum dependences in the matrix Ω, but continue to restrict ourselves to the rank-one separable interaction.

III. MATRIX ELEMENTS OF THE INTERACTION CURRENT

A. Theoretical reduction

In this section, we evaluate the matrix elements of the interaction current operator formulated in the previous section, by using bound-state solutions of the Bethe-Salpeter equation. We use the simplest type of separable interaction employed in the previous section,

$$V_{\alpha\beta:\delta\gamma}(k',k)=gf([k_1'-k_2']^2)f([k_1-k_2]^2)\Omega_{\alpha\beta}\overline{\Omega}_{\delta\gamma}\ , \qquad (3.1)$$

where Ω is a constant matrix and g is the coupling constant, and there is no dependence on the total momentum p. In this case, the Bethe-Salpeter vertex function does not depend on p, so we use the notation, $\Gamma(k)\equiv\Gamma(k;p)$. The solution for the bound-state vertex function, obtained from the wave equation Eq. (2.3a), has the form

$$\Gamma(k)=\mathcal{N}f([k_1-k_2]^2)\Omega\ ,$$

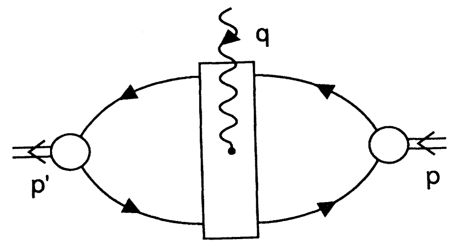

FIG. 7. Matrix element of the interaction current operator, where the open circles are bound-state vertex functions.

where $\mathcal{N}$ is the normalization constant. Note that $V(k',k)=(g/\mathcal{N}^2)\Gamma(k')\overline{\Gamma}(k)$. The eigenvalue conditions is

$$1=i\frac{g}{\mathcal{N}^2}\int\frac{d^4k}{(2\pi)^4}\{\overline{\Gamma}(k)S(k+p/2)\Gamma(k)S(k-p/2)\}\ . \qquad (3.2)$$

(Throughout this section the curly brackets are meant to be the traces of the Dirac matrices and matrices describing the other degrees of freedom which enter the problem. For example, in the quark model the traces of color and flavor matrices are taken.)

It is possible for the eigenvalue equation (3.2) to have more than one solution, corresponding to the existence of excited states. In order to treat the most general case, we will assume at least two solutions to exist with masses $p^2=\mu^2$ and $p'^2=\mu'^2$.

The Bethe-Salpeter equation and the interaction current operators are covariant, and the scalar matrix elements should not depend on the frame in which they are evaluated. We chose the Breit frame. The four-momenta of initial and final states are given by $p=(P-q)/2$ and $p'=(P+q)/2$ with their respective masses μ and μ', and the expression for the current, Eq. (2.25), becomes

$$J^\mu_{\rm int}(k',k:q)=-e_1\left[\frac{\xi^\mu}{\xi\cdot q}[V(k',k+q/2)-V(k',k)]-\frac{\xi'^\mu}{\xi'\cdot q}[V(k'-q/2,k)-V(k',k)]\right]+(e_2\ \text{term})\ , \qquad (3.3)$$

where $\xi_\mu=(4k+q)_\mu/(4k+q)\cdot q$ and $\xi'_\mu=(4k'-q)_\mu/(4k'-q)\cdot q$. Hereafter we drop the e_2 term for simplicity. The matrix element of the current operator describing the transition from a bound state of mass μ to one of mass μ', both described by the same Γ, is given by

$$\langle J^\mu_{\rm int}(q)\rangle=\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}\{J^\mu_{\rm int}\}\ , \qquad (3.4)$$

where

$$\{J^\mu_{\rm int}\}=[S(k'-p'/2)\overline{\Gamma}(k')S(k'+p'/2)]_{\beta\alpha}J^\mu_{\rm int}(k',k:q)_{\alpha\beta:\delta\gamma}[S(k+p/2)\Gamma(k)S(k-p/2)]_{\gamma\delta}\ .$$

See Fig. 7. The use of the Bethe-Salpeter equation considerably simplifies the matrix element, giving

$$\langle J^\mu_{\rm int}(q)\rangle = ie_1\int \frac{d^4k}{(2\pi)^4}\frac{(k+q/4)^\mu}{(k+q/4)\cdot q}\{[\bar\Gamma(k+q/2)-\Gamma(k)]S(k+(P-q)/4)\Gamma(k)S(k-(P-q)/4)\}$$
$$-ie_1\int \frac{d^4k}{(2\pi)^4}\frac{(k-q/4)^\mu}{(k-q/4)\cdot q}\{S(k-(P+q)/4)\bar\Gamma(k)S(k+(P+q)/4)[\Gamma(k-q/2)-\Gamma(k)]\}\ . \qquad (3.5)$$

The interaction current can be reduced further by shifting the integral variable, $k\to k+(P-q)/4$ in the first and $k\to k+(P+q)/4$ in the second terms. We get

$$\langle J^\mu_{\rm int}(q)\rangle = ie_1\int \frac{d^4k}{(2\pi)^4}\frac{(k+P/4)^\mu}{(k+P/4)\cdot q}$$
$$\times(\{[\bar\Gamma(k+(P+q)/4)-\bar\Gamma(k+(P-q)/4)]S(k+(P-q)/2)\Gamma(k+(P-q)/4)S(k)\}$$
$$-\{\bar\Gamma(k+(P+q)/4)S(k+(P+q)/2)[\Gamma(k+(P-q)/4)-\Gamma(k+(P+q)/4)]S(k)\})\ . \qquad (3.6)$$

In the Breit frame, the kinematical variables are given by

$$\begin{aligned} p^\mu &= \tfrac{1}{2}(P-q)^\mu=(E,-\alpha\mathbf{q})\ ,\\ p'^\mu &= \tfrac{1}{2}(P+q)^\mu=(E,[1-\alpha]\mathbf{q})\ ,\\ q^\mu &= (0,\mathbf{q})\ ,\\ P^\mu &= (2E,[1-2\alpha]\mathbf{q})\ , \end{aligned} \qquad (3.7)$$

where α satisfies

$$E=(\mu^2+\alpha^2\mathbf{q}^2)^{1/2}=[\mu'^2+(1-\alpha)^2\mathbf{q}^2]^{1/2}\ ,$$

and it is convenient to introduce the orthogonal four-vectors q^μ and $\eta^\mu\equiv P^\mu-(P\cdot q/q^2)q^\mu$. Using the fact that the d^4k integration will turn k^μ into a multiple of q^μ and P^μ, we can project the four-vector $(k+P/4)^\mu$ in Eq. (3.6) onto these vectors:

$$(k+P/4)^\mu=(k+P/4)\cdot\eta\frac{\eta^\mu}{\eta^2}+(k+P/4)\cdot q\frac{q^\mu}{q^2}\ . \qquad (3.8)$$

This separates the matrix element of the interaction current into two terms, one proportional to q^μ and the other proportional to η^μ,

$$\langle J^\mu_{\rm int}\rangle=\langle J^\mu_{\rm int}\rangle_q+\langle J^\mu_{\rm int}\rangle_\eta\ .$$

Using the eigenvalue condition Eq. (3.2), the second and fourth terms in the q^μ part cancel in Eq. (3.6), and the q^μ part can be written

$$\langle J^\mu_{\rm int}(q)\rangle_q=ie_1\int\frac{d^4k}{(2\pi)^4}\frac{q^\mu}{q^2}\{\bar\Gamma(k+(P+q)/4)[S(k+(P-q)/2)-S(k+(P+q)/2)]\Gamma(k+(P-q)/4)S(k)\}$$
$$=ie_1\int\frac{d^4k}{(2\pi)^4}\frac{q^\mu}{q^2}\{\bar\Gamma(k+p'/2)S(k+p')(-i\not{q})S(k+p)\Gamma(k+p/2)S(k)\}\ , \qquad (3.9)$$

where the identity

$$S(A)-S(B)=S(B)[S^{-1}(B)-S^{-1}(A)]S(A)$$

is used to obtain the last equality. Note that this matrix element, when combined with the impulse process, gives a new relativistic impulse approximation (RIA) with the photon-fermion vertex γ^μ replaced by $\gamma^\mu-\not{q}q^\mu/q^2$,

$$\langle J^\mu_{\rm RIA}(q)\rangle_G=-e_1\int\frac{d^4k}{(2\pi)^4}\{\bar\Gamma(k+p'/2)S(k+p')(\gamma^\mu-\not{q}q^\mu/q^2)S(k+p)\Gamma(k+p/2)S(k)\}\ . \qquad (3.10)$$

Note that this new form of the RIA now explicitly conserves current, even if $\mu\neq\mu'$. To summarize: the sum of the original relativistic impulse approximation and q^μ part of interaction current

$$[\langle J^\mu_{\rm RIA}(q)\rangle+\langle J^\mu_{\rm int}(q)\rangle_q=\langle J^\mu_{\rm RIA}(q)\rangle_G]$$

is equivalent to the result which would be obtained using an effective photon-fermion vertex $\gamma^\mu-\not{q}q^\mu/q^2$ (Fig. 8) in the impulse diagram. Furthermore, $q_\mu(\gamma^\mu-\not{q}q^\mu/q^2)=0$ identically, so that this new RIA is gauge invariant.

Return to the η^μ part of the interaction current. We will write this as

$$\langle J^\mu_{\rm int}(q)\rangle_\eta=i\int\frac{d^4k}{(2\pi)^4}[\{\bar J^\mu_\Gamma(q)S(k+p)\Gamma(k+p/2)S(k)\}-\{S(k)\bar\Gamma(k+p'/2)S(k+p')J^\mu_\Gamma(q)\}]\ . \qquad (3.11)$$

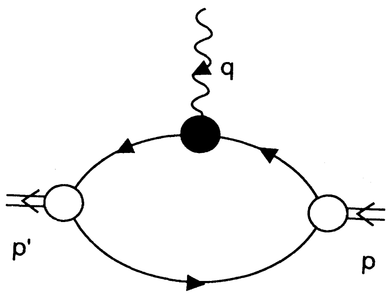

FIG. 8. The matrix element of the effective photon-fermion vertex, $\gamma^\mu - \not{q} q^\mu / q^2$, indicated by a solid circle.

where the effective "vertex currents" $[\bar{J}^\mu_\Gamma(q)$ and $J^\mu_\Gamma(q)]$, which can be used in the single-loop diagrams (Fig. 9), are given by

$$\bar{J}^\mu_\Gamma(q) = e_1 \frac{(k+\eta/4)\cdot\eta}{\eta^2} \times \eta^\mu \left[\frac{\bar{\Gamma}(k+p'/2) - \bar{\Gamma}(k+p/2)}{(k+P/4)\cdot q} \right], \quad (3.12a)$$

and

$$J^\mu_\Gamma(q) = e_1 \frac{(k+\eta/4)\cdot\eta}{\eta^2} \times \eta^\mu \left[\frac{\Gamma(k+p/2) - \Gamma(k+p'/2)}{(k+P/4)\cdot q} \right]. \quad (3.12b)$$

From the form of this expression, the η^μ part of the interaction current can be considered to be a new contact interaction which arises because of the structure of the bound state. Because $q_\mu \eta^\mu = 0$, this new interaction current is gauge invariant by itself. Furthermore, if $\mu = \mu'$, $\eta^\mu = P^\mu$, and it can be seen that $\langle J^\mu_{\rm int}(q) \rangle_\eta = 0$ when $q = 0$. Hence, the interaction current does not contribute to the charge form factor $F(q^2)$ at $q^2 = 0$. This allows us to determine the normalization of the wave function, $\mathcal{N}^2$, from the normalization of the RIA process alone. [To see that $\langle J^\mu_{\rm int}(0) \rangle_\eta = 0$, note that $\mu = \mu'$ implies that Eqs. (3.12a) and (3.12b) have equal magnitude but opposite sign, and hence the two terms in Eq. (3.11) are identical when $q = 0$ or $p = p'$. Changing the integration over the time component k_0 to $k_0' - P_0/4$ shows that the integrand is odd in k_0', and hence the current is zero.]

B. Example: The pion charge form factor

Here, we apply the formalism developed in the previous sections to a simple, spin-zero, physical system. (The formalism can be extended to higher spin systems, if desired.) We introduce a simple model of the pion as a relativistic quark-antiquark system interacting via a separable interaction. We have found that such a separable model can give a phenomenological soft pion wave function which successfully describes the pion form factor and weak decay.[11] In this paper, we use another separable model which is, in some respect, simpler.[12] In this section, we calculate the matrix elements of the interaction current contribution to the pion charge form factor, and compare the results with the RIA.

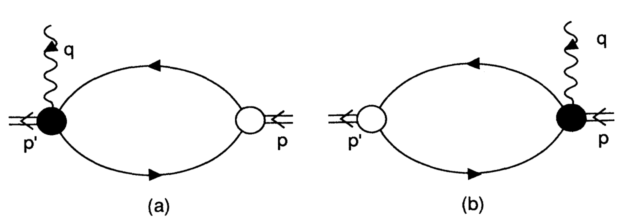

FIG. 9. Matrix element of the η part of interaction current. The solid circle is meant to be the effective "vertex current" defined by Eqs. (3.12a) and (3.12b).

Our model is motivated by our search to identify an interaction that renders the Bethe-Salpeter equation analytically solvable. We chose the following interaction:

$$V_{\alpha\beta:\delta\gamma}(k',k) = g \frac{\gamma^5_{\alpha\beta}}{D(k'^2)} \frac{\gamma^5_{\delta\gamma}}{D(k^2)}, \quad (3.13)$$

where $D(k^2) = k^2 - \Lambda^2$ with $\Lambda \sim$ few hundred MeV, a typical hadronic mass scale. This separable potential is taken to be an effective interaction between constituent quarks forming a meson, and the Bethe-Salpeter equation is solved for a quark mass in the range $m = 200-400$ MeV. Our model can be regarded as a covariant generalization of the Nambu–Jona-Lasinio model; the form factor $1/D(k^2)$ cuts off all loop integrals in a covariant fashion. In addition, the pion vertex functions which emerge from this model are covariant generalizations of Hulthen-like functions, familiar from nuclear physics applications. The parameter Λ is clearly related to the size of system. The details of the calculation are given in Ref. 12, including the effect of vector meson dominance. Here, we focus on the evaluation of the interaction current.

The bound-state solution for the $\pi q \bar{q}$ vertex is simply

$$\Gamma(k) = \frac{\mathcal{N}\gamma^5}{D(k^2)}, \quad (3.14)$$

where $\mathcal{N}$ is the normalization determined from the normalization of charge. This simple vertex actually gives a pion wave function which contains all of the four invariant functions[13] necessary to describe the wave function of a pseudoscalar meson. In addition, the invariant functions now depend on both k^2 and $k \cdot p$.

The normalization $\mathcal{N}$ can be obtained by calculating the RIA amplitude only, $F_{\rm RIA}(0) = 1$, because the interaction current does not renormalize the charge. Introducing the color and flavor wave function, the weak pion decay constant is given by

$$f_\pi = \frac{-i4}{\sqrt{2}} \sqrt{n_c}\, \mathcal{N} \int \frac{d^4k}{(2\pi)^4} \times \frac{m}{D(k^2)([k-p/2]^2 - m^2)([k+p/2]^2 - m^2)}, \quad (3.15)$$

where $n_c = 3$ is the number of colors. The RIA to the charge form factor of the π^+ is given by

$$F^{\pi}_{\rm RIA}(q^2)[p+p']^{\mu}=i\mathcal{N}^2\int\frac{d^4k}{(2\pi)^4}\frac{{\rm Tr}[\gamma^5(\not p'+\not k+m)\gamma^{\mu}(\not p+\not k+m)\gamma^5(\not k+m)]}{D([k+p'/2]^2)D([k+p/2]^2)([k+p']^2-m^2)([k+p]^2-m^2)(k^2-m^2)} . \quad (3.16)$$

From the symmetric structure under the interchange of variables $p \leftrightarrow p'$, the right-hand side of Eq. (3.16) is only proportional to $[p+p']^{\mu}$, reflecting the fact that the pions in the initial and final states are real with equal mass μ. Therefore, the q^{μ}-dependent part of the interaction current, which can be obtained by replacing γ^{μ} by $\not q q^{\mu}/q^2$ in the impulse diagram, does not contribute. The other, η^{μ} part, interaction current contribution is not zero, and we calculate it.

Since the initial and final states are physical pion states, $p^2=p'^2=\mu^2=\mu'^2$, it follows that $\eta^{\mu}=P^{\mu}$ and $q\cdot P=0$. The effective "vertex currents" defined by Eqs. (3.12a) and (3.12b) become

$$J^{\mu}_{\Gamma}(q)=e_1\frac{([k+P/4]\cdot P)}{P^2}P^{\mu}\mathcal{N}\gamma^5\left[\frac{1}{([k+p/2]^2-\Lambda^2)}-\frac{1}{([k+p'/2]^2-\Lambda^2)}\right]$$

$$=e_1\mathcal{N}\left[\frac{k\cdot P}{P^2}+\frac{1}{4}\right]\frac{P^{\mu}\gamma^5}{([k+p/2]^2-\Lambda^2)([k+p'/2]^2-\Lambda^2)} , \quad (3.17a)$$

and

$$\bar{J}^{\mu}_{\Gamma}(q)=-e_1\mathcal{N}^*\left[\frac{k\cdot P}{P^2}+\frac{1}{4}\right]\frac{P^{\mu}\gamma^5}{([k+p/2]^2-\Lambda^2)([k+p'/2]^2-\Lambda^2)} . \quad (3.17b)$$

Inserting into Eq. (3.11), we get the correction to the charge form factor

$$F^{\pi}_{\rm int}(q^2)=\frac{i\mathcal{N}^2}{P^2}\int\frac{d^4k}{(2\pi)^4}\frac{4k\cdot P+P^2}{([k+p/2]^2-\Lambda^2)([k+p'/2]^2-\Lambda^2)(k^2-m^2)}$$
$$\times\left[\frac{m^2+\mu^2/4-(k+p/2)^2}{([k+p/2]^2-\Lambda^2)([k+p]^2-m^2)}+\frac{m^2+\mu^2/4-(k+p'/2)^2}{([k+p'/2]^2-\Lambda^2)([k+p']^2-m^2)}\right] . \quad (3.18)$$

The numerical results for the interaction current correction $[F_{\rm int}(Q^2)]$ and the impulse form factor $[F_{\rm RIA}(Q^2)]$ are presented in Figs. 10 and 11. Model parameters used in the three sets of results are listed in Table I, along with the results of the pion charge radius ($r^{\rm exp}_{\pi}=0.66$ fm) and weak decay constant ($f^{\rm exp}_{\pi}=93$ MeV). The effect of the interaction current on the charge radius is about a percent, as is seen from the slope of the results in Fig. 10. In Figs. 12(a) and 12(b), we observe that the interaction current contributes a significant amount to the charge form factor in the large-momentum-transfer region. This reduces the total magnitude of the soft form factor $[F_{\rm RIA}(Q^2)+F_{\rm int}(Q^2)]$.

IV. A GENERALIZATION

In Sec. II, the interaction current was derived for a very simple form of separable interaction, where the ma-

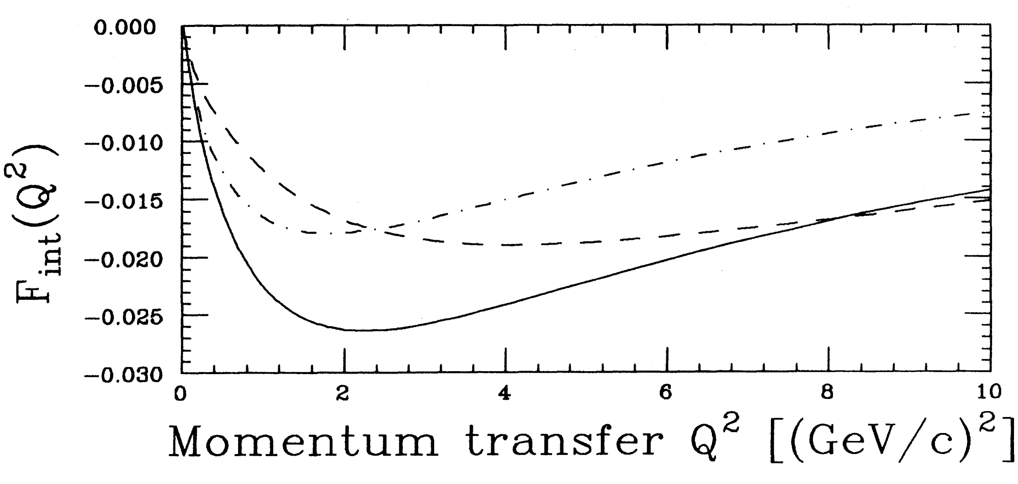

FIG. 10. The contribution of the interaction current in the charge form factor of pion. Here, $Q^2=-q^2>0$. The meaning of three lines is m (quark mass) = 300 MeV and $\Lambda=500$ MeV (solid line), $m=300$ MeV and $\Lambda=750$ MeV (dashed line), and $m=200$ MeV and $\Lambda=500$ MeV (dot-dashed line).

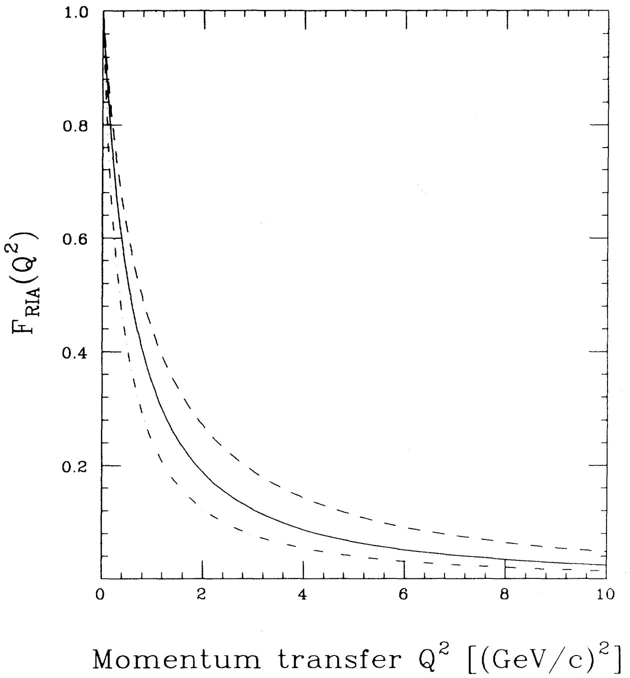

FIG. 11. The impulse form factor $F_{\rm RIA}(Q^2)$. The meaning of lines is the same as Fig. 10.

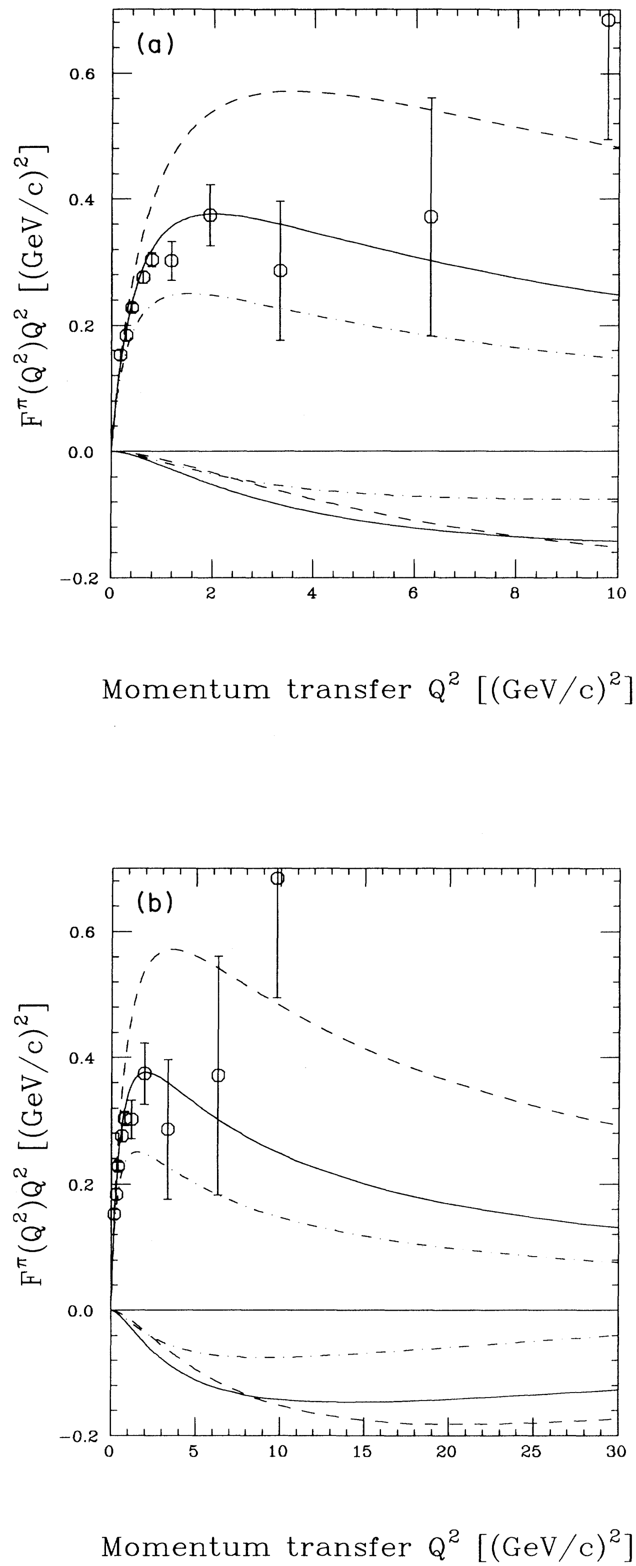

FIG. 12. Each contribution of the impulse $[F_{\rm RIA}(Q^2)]$ and interaction $[F_{\rm int}(Q^2)]$ form factors multiplied by Q^2. The meaning of lines is the same as Fig. 10. The experimental data are taken from Ref. 14.

trix in the vertex was assumed to be independent of momenta associated with the charged particles. Here, we apply the same method to a more general form of separable interaction which has a simple momentum dependence. It will be shown that our result for the interaction current satisfies the general restriction from current conservation, Eq. (2.8).

We use a rank-one separable interaction,

$$V_{\alpha\beta;\delta\gamma}(k',k;p)=\Delta_{\alpha\beta}(k_1',k_2')\bar{\Delta}_{\delta\gamma}(k_1,k_2) , \tag{4.1}$$

which is a product of two vertices conserving the total momentum of the initial and final states,

$$p'=k_1'+k_2'=p=k_1+k_2 .$$

We introduce the momentum dependence in the following form:

$$\begin{aligned}\Delta(k;p)&=\sum_j f_j([k_1-k_2]^2)\Omega_j(k_1,k_2)\\ &=f_0([k_1-k_2]^2)\mathcal{O}+2f_1([k_1-k_2]^2)\mathcal{O}\not k\\ &\quad+f_2([k_1-k_2]^2)\mathcal{O}\not p\\ &\quad+f_3([k_1-k_2]^2)\mathcal{O}(\not k\not p-\not p\not k) ,\end{aligned} \tag{4.2}$$

where $k=(k_1-k_2)/2$ and $p=k_1+k_2$, and $\mathcal{O}$ is a con-

TABLE I. Quark mass (m) and cutoff mass (Λ) used in the model of pion, and the numerical results of static observables, r_π (pion charge radius) and f_π (weak decay constant).

m (MeV)	Λ (MeV)	r_π (fm)	f_π (MeV)
300.0	500.0	0.64	108.0
300.0	750.0	0.55	123.0
200.0	500.0	0.84	81.9

stant matrix to be chosen depending upon the specific modeling of the interaction. The results obtained with this model can be immediately applied to any interaction of this general form. For example,

$$V(k',k:p)=f_2([k_1'-k_2']^2)f_2([k_1-k_2]^2)\mathcal{O}p\!\!\!/'\bar{\mathcal{O}}p\!\!\!/$$

or

$$V(k',k:P)=\sum_j f_j([k_1'-k_2']^2)f_j([k_1-k_2]^2)\times\Omega_j(k_1',k_2')\bar{\Omega}_j(k_1,k_2)\ .$$

With the notation introduced in Sec. II, $S^{(\prime)}\equiv k_1^{(\prime)}-k_2^{(\prime)}$, $S_\pm^{(\prime)}\equiv k_1^{(\prime)}-k_2^{(\prime)}\pm q$, and $K_\pm^{(\prime)}=2k_1^{(\prime)}-2k_2^{(\prime)}\pm q$, the modification of the vertex, Eq. (4.2), caused by the introduction of photon field is given by

$$\delta\Delta_{\alpha\beta}(x_1,x_2)=\sum_j\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}[\langle\langle f_j(S^2)\Omega_j(k_1,k_2)\rangle\rangle-f_j(S^2)\Omega_j(k_1,k_2)]e^{ik_1x_1}e^{ik_2x_2}$$
$$=\sum_j\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\{[\langle\langle f_j(S^2)\rangle\rangle-f_j(S^2)]\Omega_j(k_1,k_2)+f_j(S^2)[\langle\langle\Omega_j(k_1,k_2)\rangle\rangle-\Omega_j(k_1,k_2)]\}e^{ik_1x_1}e^{ik_2x_2}\ . \tag{4.3}$$

Here, again, the linear dependence on the photon field, which is a characteristic aspect of one-photon processes, is used in the last equality. In this respect, the evaluation of two-photon interaction currents appearing in two-photon processes, such as Compton scattering, would not be trivial at all. Note that the momentum of charged particle 1(2), $k_{1(2)}$, is replaced by $(1/i)\partial/\partial x_{1(2)}$ in $f_j([k_1-k_2]^2)$ and $\Omega(k_1,k_2)$. Using Eqs. (B1)–(B4) in Appendix B, we can derive the following expression for the second term in Eq. (4.3):

$$\sum_j\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}f_j(S^2)[\langle\langle\Omega_j(k_1,k_2)\rangle\rangle-\Omega_j(k_1,k_2)]e^{ik_1x_1}e^{ik_2x_2}$$
$$=\sum_j\int\int\frac{d^4k_1d^4k_2d^4q}{(2\pi)^{12}}e^{ik_1x_1}e^{ik_2x_2}[a_\mu(q)\,\mathbf{A}_j^\mu(k_1,k_2,q)+a_\mu^\dagger(q)\,\mathbf{A}_j^\mu(k_1,k_2,-q)]\ , \tag{4.4}$$

where

$$\mathbf{A}_0^\mu(k_1,k_2,q)=0\ ,$$
$$\begin{Bmatrix}A_1^\mu(k_1,k_2,q)\\A_2^\mu(k_1,k_2,q)\end{Bmatrix}=\begin{Bmatrix}f_1\\f_2\end{Bmatrix}(S_-^2)e_1\mathcal{O}\gamma^\mu e^{-iqx_1}+\begin{Bmatrix}-f_1\\+f_2\end{Bmatrix}(S_+^2)e_2\mathcal{O}\gamma^\mu e^{-iqx_2}\ ,$$
$$\mathbf{A}_3^\mu(k_1,k_2,q)=f_3(S_-^2)e_1\mathcal{O}(\gamma^\mu k\!\!\!/_2-k\!\!\!/_2\gamma^\mu)e^{-iqx_1}-f_3(S_+^2)e_2\mathcal{O}(\gamma^\mu k\!\!\!/_1-k\!\!\!/_1\gamma^\mu)e^{-iqx_2}\ .$$

The first term in Eq. (4.3), which we obtained in Sec. II, can be written

$$\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\sum_j[\langle\langle f_j(S^2)\rangle\rangle-f_j(S^2)]\Omega_j(k_1,k_2)e^{ik_1x_1}e^{ik_2x_2}$$
$$=\int\int\frac{d^4k_1d^4k_2d^4q}{(2\pi)^{12}}e^{ik_1x_1}e^{ik_2x_2}[a_\mu(q)\mathbf{C}^\mu(k_1,k_2,q)+a_\mu^\dagger(q)\mathbf{C}^\mu(k_1,k_2,-q)]\ , \tag{4.5}$$

where

$$\mathbf{C}^\mu(k_1,k_2,q)=-e_1\frac{K_-^\mu}{K_-\cdot q}\sum_j[f_j(S_-^2)-f_j(S^2)]\Omega_j(k_1,k_2)e^{-iqx_1}-e_2\frac{K_+^\mu}{K_+\cdot q}\sum_j[f_j(S_+^2)-f_j(S^2)]\Omega_j(k_1,k_2)e^{-iqx_2}\ .$$

The vertex modifications are obtained by taking the matrix elements of Eqs. (4.4) and (4.5) for the one-photon absorption process. The interaction current is then the sum of products of vertex and vertex modification terms which satisfy momentum conservation $(k_1+k_2+q=k_1'+k_2')$

$$J^\mu(k_1',k_2';k_1,k_2:q)=\delta\Delta^\mu(k_1',k_2':q)\bar{\Delta}(k_1,k_2)+\Delta(k_1',k_2')\delta\bar{\Delta}(k_1,k_2:q)\ , \tag{4.6}$$

where the vertex modifications are

$$\delta\Delta^{\mu}(k_1',k_2':q)=+e_1\left[\frac{K'^{\mu}_{-}}{K'_{-}\cdot q}\sum_j[f_j(S'^2_{-})-f_j(S'^2)]\Omega_j(k_1',k_2')-f_1(S'^2_{-})\mathcal{O}\gamma^{\mu}-f_2(S'^2_{-})\mathcal{O}\gamma^{\mu}-f_3(S'^2_{-})\mathcal{O}(\gamma^{\mu}\not{k}_2'-\not{k}_2'\gamma^{\mu})\right]$$

$$+e_2\left[\frac{K'^{\mu}_{-}}{K'_{-}\cdot q}\sum_j[f_j(S'^2_{+})-f_j(S'^2)]\Omega_j(k_1',k_2')\right.$$

$$\left.+f_1(S'^2_{+})\mathcal{O}\gamma^{\mu}-f_2(S'^2_{+})\mathcal{O}\gamma^{\mu}+f_3(S'^2_{+})\mathcal{O}(\gamma^{\mu}\not{k}_1'-\not{k}_1'\gamma^{\mu})\right],$$

and

$$\delta\overline{\Delta}^{\mu}(k_1,k_2:q)=-e_1\left[\frac{K^{\mu}_{+}}{K_{+}\cdot q}\sum_j[f_j(S^2_{+})-f_j(S^2)]\overline{\Omega}_j(k_1,k_2)+f_1(S^2_{+})\gamma^{\mu}\overline{\mathcal{O}}+f_2(S^2_{+})\gamma^{\mu}\overline{\mathcal{O}}+f_3(S^2_{+})(\not{k}_2\gamma^{\mu}-\gamma^{\mu}\not{k}_2)\overline{\mathcal{O}}\right]$$

$$-e_2\left[\frac{K^{\mu}_{-}}{K_{-}\cdot q}\sum_j[f_j(S^2_{-})-f_j(S^2)]\overline{\Omega}_j(k_1,k_2)\right.$$

$$\left.-f_1(S^2_{-})\gamma^{\mu}\overline{\mathcal{O}}+f_2(S^2_{-})\gamma^{\mu}\overline{\mathcal{O}}-f_3(S^2_{-})(\not{k}_1\gamma^{\mu}-\gamma^{\mu}\not{k}_1)\overline{\mathcal{O}}\right].$$

Note that

$$\sum_j f_j\Omega_j(k_1,k_2)-f_1\mathcal{O}\not{q}-f_2\mathcal{O}\not{q}-f_3\mathcal{O}(\not{q}\not{k}_2-\not{k}_2\not{q})=\sum_j f_j\Omega_j(k_1-q,k_2)$$

and

$$\sum_j f_j\Omega_j(k_1,k_2)+f_1\mathcal{O}\not{q}-f_2\mathcal{O}\not{q}+f_3\mathcal{O}(\not{q}\not{k}_1-\not{k}_1\not{q})=\sum_j f_j\Omega_j(k_1,k_2-q)\ .$$

With the use of these identities we can express the divergence of the current in a very simple form:

$$q_{\mu}\delta\Delta^{\mu}(k_1',k_2':q)=+e_1\sum_j[f_j(S'^2_{-})\Omega_j(k_1'-q,k_2')-f_j(S'^2)\Omega_j(k_1',k_2')]$$

$$+e_2\sum_j[f_j(S'^2_{+})\Omega_j(k_1',k_2'-q)-f_j(S'^2)\Omega_j(k_1',k_2')]$$

$$=+e_1[\Delta(k_1'-q,k_2')-\Delta(k_1',k_2')]+e_2[\Delta(k_1',k_2'-q)-\Delta(k_1',k_2')] \tag{4.7a}$$

and

$$q_{\mu}\delta\overline{\Delta}^{\mu}(k_1,k_2:q)=-e_1[\overline{\Delta}(k_1+q,k_2)-\overline{\Delta}(k_1,k_2)]-e_2[\overline{\Delta}(k_1,k_2+q)-\overline{\Delta}(k_1,k_2)]\ . \tag{4.7b}$$

Finally, we observe the divergence of the interaction current to have the desired form

$$q_{\mu}J^{\mu}(k_1',k_2';k_1,k_2:q)=+e_1\Delta(k_1'-q,k_2')\overline{\Delta}(k_1,k_2)+e_2\Delta(k_1',k_2'-q)\overline{\Delta}(k_1,k_2)$$

$$-e_1\Delta(k_1',k_2')\overline{\Delta}(k_1+q,k_2)-e_2\Delta(k_1',k_2')\overline{\Delta}(k_1,k_2+q)$$

$$=+e_1[V(k'-q/2,k:p)-V(k',k+q/2:p+q)]+e_2[V(k'+q/2,k:p)-V(k',k-q/2:p+q)]\ . \tag{4.8}$$

V. SUMMARY

The present study uses the minimal substitution method to obtain the interaction currents associated with a relativistic separable interaction. We start with the Bethe-Salpeter equation for a fermion-antifermion system (for definiteness only) with a general form of the interaction kernel. Using the wave equation of a bound state and the Ward-Takahashi identity for the divergence of the one-body current, we demonstrate that bound-state matrix elements of the total current are conserved, provided there exists a two-body interaction current. This interaction current must satisfy the general constraint Eq. (2.8), which can be expressed in terms of the interaction kernel.

The specific form of the interaction current is then constructed for a simple separable interaction. Bound-state matrix elements of this interaction current are evaluated, and it is shown that the two-body current operator can be reduced into an effective one-body operator, which contains two terms. One term is a photon-fermion vertex correction, Eq. (3.9) or (3.10), and the other is a new interaction term involving the bound-state vertex, Eq.

(3.12). It is shown that the interaction current does not contribute to the charge form factor at $q=0$ for this simple form of the interaction, so that the normalization constant of the wave function is obtained from the charge normalization of the impulse process alone.

As an example, the interaction current contribution to the pion charge form factor is calculated in a model in which the pion is a bound state of a quark and antiquark interacting through a separable interaction,

$$V(k',k)\sim F(k')F(k)\gamma^5\gamma^5 .$$

The result shows that the interaction current contribution is significant, especially in the large-momentum-transfer region. However, the interaction current does not contribute significantly to the charge radius.

Many applications of the formalism in this paper come to mind: For example, Tjon and Rupp[7] obtain relativistic wave functions of two- and three-nucleon systems by using a separable two-nucleon interaction in the Bethe-Salpeter equation. The calculation of the charge form factors, including the effect of the interaction current, is quite possible and interesting. Baryon resonance production, such as $\gamma N\to\Delta\to\pi N$, can be studied with a separable interaction model, which incorporates nonlocal aspects[15] associated with the propagation of a resonance particle and its extended structure. Lastly, employing the separable interaction technique presented in this paper to study $N\bar{N}$ physics is quite possible.

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APPENDIX A

By applying the plane-wave expansion of the photon field,

$$A_\mu(x)=\int\frac{d^4q}{(2\pi)^4}a_\mu(q)e^{-iqx}+a_\mu^\dagger(q)e^{iqx} ,$$

in the expression

$$Z=e_1\frac{\partial A^\mu(x_1)}{\partial x_1^\mu}+e_2\frac{\partial A^\mu(x_2)}{\partial x_2^\mu}+2\{e_1A^\mu(x_1)-e_2A^\mu(x_2)\}\left[\frac{\partial}{\partial x_1^\mu}-\frac{\partial}{\partial x_2^\mu}\right] ,$$

we get the following useful formula:

$$iZe^{ik_1x_1}e^{ik_2x_2}=\int\frac{d^4q}{(2\pi)^4}[a_\mu(q)\mathbf{F}^\mu(k_1,k_2,x_1,x_2)+a_\mu^\dagger(q)\mathbf{G}^\mu(k_1,k_2,x_1,x_2)] , \tag{A1}$$

where

$$\mathbf{F}^\mu(k_1,k_2,x_1,x_2)=e_1(q+2k_2-2k_1)^\mu e^{ik_1x_1}e^{ik_2x_2}e^{-iqx_1}+e_2(q+2k_1-2k_2)^\mu e^{ik_1x_1}e^{ik_2x_2}e^{-iqx_2} ,$$

and

$$\mathbf{G}^\mu(k_1,k_2,x_1,x_2)=e_1(-q+2k_2-2k_1)^\mu e^{ik_1x_1}e^{ik_2x_2}e^{iqx_1}+e_2(-q+2k_1-2k_2)^\mu e^{ik_1x_1}e^{ik_2x_2}e^{iqx_2} .$$

Applying a d'Alembertian operator to $\mathbf{F}^\mu$ and $\mathbf{G}^\mu$, we get

$$\begin{aligned}(\Box_{12})^N\mathbf{F}^\mu(k_1,k_2,x_1,x_2)=&\ e_1(q+2k_2-2k_1)^\mu(-[k_1-k_2-q]^2)^N e^{ik_1x_1}e^{ik_2x_2}e^{-iqx_1}\\&+e_2(q+2k_1-2k_2)^\mu(-[k_1-k_2+q]^2)^N e^{ik_1x_1}e^{ik_2x_2}e^{-iqx_2} ,\end{aligned} \tag{A2a}$$

and

$$\begin{aligned}(\Box_{12})^N\mathbf{G}^\mu(k_1,k_2,x_1,x_2)=&\ e_1(-q+2k_2-2k_1)^\mu(-[k_1-k_2+q]^2)^N e^{ik_1x_1}e^{ik_2x_2}e^{iqx_1}\\&+e_2(-q+2k_1-2k_2)^\mu(-[k_1-k_2-q]^2)^N e^{ik_1x_1}e^{ik_2x_2}e^{iqx_2} .\end{aligned} \tag{A2b}$$

Therefore, Eq. (1.21a) can be expressed as

$$\begin{aligned}\delta I^n(x_1,x_2)&=(-)^n\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\sum_{M=0}^{n-1}(\Box_{12})^{n-1-M}(iZ)(\Box_{12})^M e^{ik_1x_1}e^{ik_2x_2}\\&=(-)^n\sum_{M=0}^{n-1}\int\int\int\frac{d^4k_1d^4k_2d^4q}{(2\pi)^{12}}[-(k_1-k_2)^2]^M\\&\qquad\times[a_\mu(q)(\Box_{12})^{n-1-M}\mathbf{F}^\mu(k_1,k_2,x_1,x_2)+a_\mu^\dagger(q)(\Box_{12})^{n-1-M}\mathbf{G}^\mu(k_1,k_2,x_1,x_2)] .\end{aligned} \tag{A3}$$

With an algebraic identity,

$$\sum_{M=0}^{n-1} a^{n-1-M} b^{M} = \frac{a^n - b^n}{a-b} ,$$

we arrive at the following compact expressions:

$$\delta I^n(x_1,x_2) = \int\int\int \frac{d^4k_1 d^4k_2 d^4q}{(2\pi)^{12}} [a_\mu(q) u^\mu(n) + a_\mu^\dagger(q) w^\mu(n)] , \quad \text{(A4a)}$$

$$\delta I^{\dagger n}(x_1,x_2) = \int\int\int \frac{d^4k_1 d^4k_2 d^4q}{(2\pi)^{12}} [a_\mu^\dagger(q) u^{\dagger\mu}(n) + a_\mu(q) w^{\dagger\mu}(n)] , \quad \text{(A4b)}$$

where

$$u^\mu(n) = e_1(2k_1 - 2k_2 - q)^\mu \frac{[k_1-k_2-q]^{2n} - [k_1-k_2]^{2n}}{[k_1-k_2-q]^2 - [k_1-k_2]^2} e^{ik_1x_1} e^{ik_2x_2} e^{-iqx_1}$$
$$+ e_2(2k_2 - 2k_1 - q)^\mu \frac{[k_1-k_2+q]^{2n} - [k_1-k_2]^{2n}}{[k_1-k_2+q]^2 - [k_1-k_2]^2} e^{ik_1x_1} e^{ik_2x_2} e^{-iqx_2}$$

and

$$w^\mu(n) = e_1(2k_1 - 2k_2 + q)^\mu \frac{[k_1-k_2+q]^{2n} - [k_1-k_2]^{2n}}{[k_1-k_2+q]^2 - [k_1-k_2]^2} e^{ik_1x_1} e^{ik_2x_2} e^{iqx_1}$$
$$+ e_2(2k_2 - 2k_1 + q)^\mu \frac{[k_1-k_2-q]^{2n} - [k_1-k_2]^{2n}}{[k_1-k_2-q]^2 - [k_1-k_2]^2} e^{ik_1x_1} e^{ik_2x_2} e^{iqx_2} .$$

APPENDIX B

This appendix presents the useful formulas used in Sec. IV. The external photon field induces corrections in the matrix part of the vertices,

$$\delta\Omega_j = \int\int \frac{d^4k_1 d^4k_2}{(2\pi)^8} [\langle\langle \Omega_j(k_1,k_2) \rangle\rangle - \Omega_j(k_1,k_2)] e^{ik_1x_1} e^{ik_2x_2} , \quad \text{(B1)}$$

where the matrices $\Omega_j(k_1,k_2)$ are defined by

$$\Omega_j(k_1,k_2) = \begin{cases} 2\not{k} & (j=1) , \\ \not{p} & (j=2) , \\ (\not{k}\not{p} - \not{p}\not{k}) & (j=3) . \end{cases}$$

The double angular bracket means the minimal substitution of photon field in the momentum of each charged particle, and $k=(k_1-k_2)/2$ and $p=k_1+k_2$. Then we have

$$\begin{Bmatrix} \delta\Omega_1 \\ \delta\Omega_2 \end{Bmatrix} = \int\int\int \frac{d^4k_1 d^4k_2 d^4}{(2\pi)^{12}} e^{ik_1x_1} e^{ik_2x_2} [a_\mu(q)\gamma^\mu(e_1 e^{-iqx_1} \mp e_2 e^{-iqx_2}) + a_\mu^\dagger(q)\gamma^\mu(e_1 e^{iqx_1} \mp e_2 e^{iqx_2})] \quad \text{(B2a)}$$

and

$$\delta\Omega_3 = \int\int\int \frac{d^4k_1 d^4k_2 d^4q}{(2\pi)^{12}} e^{ik_1x_1} e^{ik_2x_2} [a_\mu(q)(e_1[\gamma^\mu \not{k}_2 - \not{k}_2\gamma^\mu] e^{-iqx_1} - e_2[\gamma^\mu \not{k}_1 - \not{k}_1 \gamma^\mu] e^{-iqx_2})$$
$$+ a_\mu^\dagger(q)(e_1[\gamma^\mu \not{k}_2 - \not{k}_2\gamma^\mu] e^{+iqx_1} - e_2[\gamma^\mu \not{k}_1 - \not{k}_1\gamma^\mu] e^{+iqx_2})] . \quad \text{(B2b)}$$

Introduction of a form factor results in the following expressions:

$$\int\int\int \frac{d^4k_1 d^4k_2 d^4q}{(2\pi)^{12}} f([k_1-k_2]^2)(\langle\langle [\not{k}_1 - \not{k}_2] \rangle\rangle - [\not{k}_1 - \not{k}_2]) e^{ik_1x_1} e^{ik_2x_2}$$
$$= \int\int\int \frac{d^4k_1 d^4k_2 d^4q}{(2\pi)^{12}} e^{ik_1x_1} e^{ik_2x_2} \{a_\mu(q)\gamma^\mu[e_1 f(S_-^2) e^{-iqx_1} - e_2 f(S_+^2) e^{-iqx_2}] + a_\mu^\dagger(q)\gamma^\mu(q \to -q)\} \quad \text{(B3)}$$

and

$$\int\int\int \frac{d^4k_1 d^4k_2 d^4q}{(2\pi)^{12}} f([k_1-k_2]^2)(\langle\langle [\not{k}_1\not{k}_2-\not{k}_2\not{k}_1]\rangle\rangle - [\not{k}_1\not{k}_2-\not{k}_2\not{k}_1])e^{ik_1x_1}e^{ik_2x_2}$$

$$=\int\int\int \frac{d^4k_1 d^4k_2 d^4q}{(2\pi)^{12}} e^{ik_1x_1}e^{ik_2x_2}$$

$$\times(a_\mu(q)\{e_1 f(S_-^2)[\gamma^\mu\not{k}_2-\not{k}_2\gamma^\mu]e^{-iqx_1}-e_2 f(S_+^2)[\gamma^\mu\not{k}_1-\not{k}_1\gamma^\mu]e^{-iqx_2}\}+a_\mu^\dagger(q)(q\to -q)) , \quad \text{(B4)}$$

where $S_\pm = k_1 - k_2 \pm q$.

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