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**COMMENTS**


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**Comment on "Quasielastic electron scattering and Coulomb sum rule in  $^4\text{He}$ "**

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It is pointed out that center-of-mass correlations can explain most of the observed quenching in the structure function of  $^4\text{He}$ .

Recently the longitudinal response function of  $^4\text{He}$  has been measured in the region of momentum transfer,  $q$ , between 300 and 500 MeV/c.<sup>1</sup> By extrapolating their data to the high-energy region and by successively integrating over energy, the authors of Ref. 1 obtain an accurate measurement of the Coulomb sum rule in  $^4\text{He}$ . They compare the result obtained in this way with the static structure function  $S(q)$  calculated in Ref. 2. The calculation of Ref. 2, which is based on variational Monte Carlo wave functions, provides two different approximations for the structure function: (a) an uncorrelated approximation, given by

$$S_{uc}(q) = 1 - |F(q)|^2, \quad (1)$$

where  $F(q)$  is the (pointlike) elastic form factor; (b) a correlated structure function that is obtained by taking into account the effect of the short-range repulsion in the nucleon-nucleon interaction. These two approximations are shown in Fig. 1 by the dashed and solid curves, respectively.

The authors of Ref. 1 notice that the results of their extrapolations disagree with the uncorrelated calculation, while the correlated structure function is in better agreement with the  $^4\text{He}$  data. Thus they can claim that their data provide a first evidence of ground-state correlations in  $^4\text{He}$ .

Here we wish to point out that the somewhat trivial center-of-mass (c.m.) correlations do modify the uncorrelated structure function given by Eq. (1). Therefore their effect should be taken into account separately before any conclusion about the more interesting dynamical correlations can be drawn (see Ref. 3).

The variational calculations of Ref. 2 do include c.m. as well as short-range dynamical correlations. Unfor-

tunately in these calculations it is not possible to disentangle one kind of correlation from the other; however, it is reasonable to expect that the short-range correlations should affect the Coulomb sum rule mostly at momentum transfer larger than  $q \sim 2 \text{ fm}^{-1}$ .

A simple calculation in the framework of the harmonic oscillator shell model can help to understand the order of magnitude of effects involved. In this model the uncorrelated structure function (1) for  $s$ -shell nuclei is given by

$$S_{uc}(q) = 1 - \exp[-(1 - 1/A)\lambda], \quad (2)$$

where  $\lambda = (qb)^2/2$ ,  $b$  is the oscillator parameter, and  $A = 4$  in our case. It has been shown in Ref. 3 that the structure function corrected for c.m. motion is given by [cf. Eq. (4.25) of Ref. 3]

$$S(q) = S_{uc}(q) - M(q), \quad (3)$$

where  $S_{uc}(q)$  is given by Eq. (2) and

$$M(q) = \exp[-(1 - 1/A)\lambda] - \exp(-\lambda) \quad (4)$$

gives a measure of c.m. correlations.

If we take  $b = 1.38 \text{ fm}$  for the oscillator parameter, then we obtain the dashed curve in Fig. 1 for the uncorrelated structure function and the dot-dashed curve for the c.m. correlated structure function. Our uncorrelated function, given by Eq. (2), practically coincides with the uncorrelated result of Ref. 2 (this means that the square elastic form factors given by the two calculations are indistinguishable on the linear scale of Fig. 1). Our correlated result, given by Eq. (3), includes only c.m. correlations. If we compare it with the correlated result of Ref. 2 (solid line), which contains also short-range correlations, we see that they differ slightly only for  $q \gtrsim 2 \text{ fm}^{-1}$ .

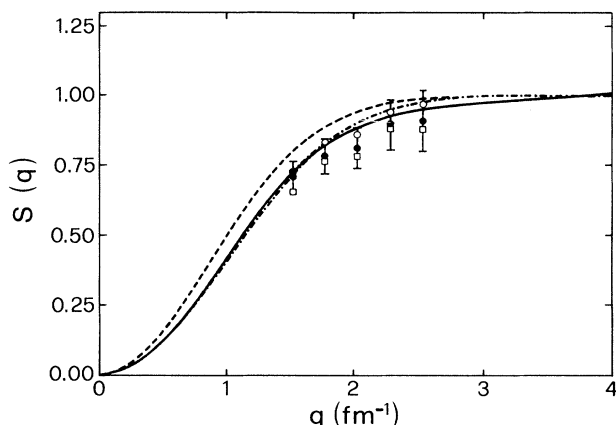


FIG. 1. Coulomb sum rule for  ${}^4\text{He}$ . The dashed curve shows the uncorrelated result both of Ref. 2 and of the present calculation. The solid curve displays the correlated calculation of Ref. 2, while the dot-dashed curve shows the present calculation which includes c.m. correlations, but no short-range correlations. The data are from Ref. 1.

This simple calculation suggests that, up to values of  $q \sim 2 \text{ fm}^{-1}$ , most of the difference between the uncorrelated and the correlated structure functions is actually due to the c.m. correlations. In fact a comparison of the three curves plotted in Fig. 1 shows that the effect of short-range correlations on the inelastic Coulomb sum rule is not of the order of the difference between the solid and the dashed curve, but rather of the order of the difference between the solid and the dot-dashed curve. Thus, in order to reveal the effect of genuine dynamical correlations, the data should be able to distinguish between the solid and the dot-dashed curves in Fig. 1. Unfortunately, this seems to be still far from the present experimental possibilities.

Clearly we are not implying that harmonic oscillator wave functions are sufficiently accurate to reproduce all the measured properties of  ${}^4\text{He}$ . For example it is well known that they cannot reproduce the diffraction structure that is observed in the elastic cross section for  $q \gtrsim 3 \text{ fm}^{-1}$ . The correlated wave functions instead give an elastic form factor in excellent agreement with experiment up to large values of momentum transfer.<sup>4</sup> However,

it is generally agreed that this fact cannot be interpreted as compelling evidence for short-range correlations because elastic scattering experiments cannot possibly measure the two-body correlation function. Indeed this is the very reason why recent investigations of correlation effects in nuclei have concentrated on the inelastic sum rule  $S(q)$ , which in principle can provide such information.

In a recent paper<sup>5</sup> Beck has analyzed electron scattering data in the  $A = 3$  system by extracting an experimental proton-proton density  $\rho_{pp}(q)$  and by comparing it with the calculations of Ref. 2. The overall agreement between data and calculations is satisfactory. We can ask ourselves if this procedure gives an unambiguous identification of short-range correlation effects. In terms of the quantities defined here the proton-proton density plotted by Beck reads

$$\rho_{pp}(q) = Z[ZF^2(q) + S(q) - 1]. \quad (5)$$

Compared to  $S(q)$ , this quantity has the advantage that it is not affected by c.m. correlations because c.m. effects on  $S(q)$  are exactly compensated by c.m. effects on  $ZF^2(q)$  (see Ref. 3); however,  $\rho_{pp}(q)$  is a somewhat hybrid quantity in the sense that it combines information from inelastic scattering [ $S(q)$ ] with information from elastic scattering [ $F(q)$ ]. In view of the fact that, as mentioned before,  $F(q)$  cannot *a priori* contain information on the two-body correlation function, in our opinion  $\rho_{pp}(q)$  is less suitable to unambiguously expose the effect of two-body correlations than the inelastic sum rule  $S(q)$ .

The simple harmonic oscillator estimate of c.m. correlation, which we have discussed above for  ${}^4\text{He}$ , can be trivially extended to  ${}^3\text{He}$ . The effects of c.m. correlations on the inelastic sum rule for  ${}^3\text{He}$  are similar to those shown in Fig. 1 for  ${}^4\text{He}$ . Thus it is our belief that, also in the case of  ${}^3\text{He}$ , the experimental data<sup>6</sup> on the inelastic Coulomb sum rule are not accurate enough to pinpoint the effect of genuine dynamical correlations.

Before concluding we want to stress that we are not questioning the legitimacy of correlated wave functions and of their use in the interpretation of electron scattering experiments, but only what we consider to be the incorrect attribution to dynamical correlations of effects that can be explained by the somewhat trivial c.m. correlations.

<sup>1</sup>K. F. von Reden *et al.*, Phys. Rev. C **41**, 1084 (1990).

<sup>2</sup>R. Schiavilla, D. S. Lewart, V. R. Pandharipande, S. C. Pieper, R. B. Wiringa, and S. Fantoni, Nucl. Phys. A**473**, 267 (1987); R. Schiavilla, V. R. Pandharipande, and A. Fabrocini, Phys. Rev. C **40**, 1484 (1989).

<sup>3</sup>A. Dellafiore and M. Traini, Nucl. Phys. A**344**, 509 (1980).

<sup>4</sup>R. Schiavilla, V. R. Pandharipande, and D. O. Riska, Phys. Rev. C **41**, 309 (1990).

<sup>5</sup>D. H. Beck, Phys. Rev. Lett. **64**, 268 (1990).

<sup>6</sup>K. Dow *et al.*, Phys. Rev. Lett. **61**, 1706 (1988).