Nonlocality of the optical potential and the adiabatic approximation

Luciano Canton* and Yukap Hahn
Department of Physics, University of Connecticut, Hillside Road, Storrs, Connecticut 06269

Giorgio Cattapan
Dipartimento di Fisica, Universitá di Padova, and Istituto Nazionale di Fisica Nucleare, Via Marzolo 8, Padova I-35131, Italy
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The nonlocality property of the elastic channel optical potential is studied using a schematic coupled-channel system. It is explicitly shown that, consistent with the usual adiabatic approximation, the nonlocality is reduced as the scattering energy approaches the elastic threshold. The residual nonlocality found in the data may be partly due to the artificial truncation of the channels, and strong indications are found that a more complete treatment which includes a large number of channels would lead to better locality of the optical potential at low energy. For optical potentials corresponding to the inelastic channels, we find that the adiabaticity and the consequent locality is not well satisfied.

It is well known that the nucleon-nucleus optical potential (OP) yields the elastic component of the quantum-mechanical wave function as the solution of an effective single-channel elastic Schrödinger equation. Much effort has been devoted (in quite a few fields, ranging from molecular to atomic and nuclear physics) to understanding the properties of this potential, which, in principle, is an energy-dependent, nonlocal, and non-Hermitian operator. In this Brief Report we are interested in the nonlocal properties of the OP at very low scattering energy. The reason for this interest is in the fact that, from these properties, one can ascertain the validity of the so-called adiabatic approximation, which allows one to approximate the actual nonlocal OP by a local form in the low-energy scattering region.

The method we use to evaluate the OP has been extensively explained in Refs. 1 and 2. We write the OP for the elastic channel as

$$U(E)=V_{11}+\Delta U(E)\ , \tag{1}$$

where the first term represents the static potential $V_{11}(\mathbf{R})\equiv\langle 1|V|1\rangle$, while the second term contains the dynamical contributions due to the inelastic channels and is therefore called the dynamic polarization potential (DPP). In coordinate space it is[3]

$$\Delta U(\mathbf{R},\mathbf{R}';E)=\sum_{c,c'=2} V_{1c}(\mathbf{R})G_{cc'}^{(Q)}(\mathbf{R},\mathbf{R}';E)V_{c'1}(\mathbf{R}')\ , \tag{2}$$

where V_{1c} and V_{c1} represent the coupling potentials for the elastic-inelastic and inelastic-elastic transitions, respectively, and $G_{cc'}^{(Q)}$ is the full multichannel Green's function referring to the space Q of inelastic channels ($Q\equiv 2,\ldots,\infty$). The nontrivial part in the calculation of the OP consists in the evaluation of the quantity $G_{cc'}^{(Q)}$, which is the solution of a coupled-channel integral equation of Lippmann-Schwinger type,

$$G_{cc'}^{(Q)}=G_{0c}^{(Q)}\delta_{cc'}+\sum_{c''=2}^{Q} G_{0c}^{(Q)}V_{cc''}G_{c''c'}^{(Q)}\ . \tag{3}$$

Here $G_{0c}^{(Q)}$ is the free Green's function in channel c with outgoing boundary conditions when the incoming energy is above the c-channel threshold or with bound-state-type (decaying) boundary conditions when the energy E is below the threshold of that particular channel.

We now examine the DPP defined in the Q space orthogonal to the elastic channel, referring particularly to the approximations generally assumed in the literature. In a slightly more explicit form[4] we write

$$\Delta U(\mathbf{R},\mathbf{R}';E)=\sum_{cc'=2}^{Q} V_{1c}(\mathbf{R})\left[\frac{1}{E-K(\mathbf{R})-h(\mathbf{r})-V(\mathbf{r},\mathbf{R})+i\epsilon}\right]_{cc'}V_{c'1}(\mathbf{R}')\ , \tag{4}$$

where it is assumed that the tridimensional internal coordinate $\mathbf{r}$ is integrated over. In Eq. (4), K is the projectile kinetic energy, h is the target Hamiltonian, and V is the interaction between the target and the projectile. We neglected here the possible exchange effect to simplify the discussion; it can be incorporated if necessary.[5]

(a) At medium energies and high excitations, $V(\mathbf{r},\mathbf{R})$ may be neglected in the denominator of ΔU, and we obtain the usual second Born-type result

$$\Delta U^{B}(\mathbf{R},\mathbf{R}';E)=\sum_{c=2}^{Q} V_{1c}(\mathbf{R})\left[\frac{1}{E-K(\mathbf{R})-h(\mathbf{r})+i\epsilon}\right]_{c}V_{c1}(\mathbf{R}')\ . \tag{5}$$

Note that ΔU^B corresponds to the zero-order iteration of $V^{(Q)}$, and the exact second-order Born amplitude is given by the second-order iteration of V_{11} and the first-order iteration of ΔU^B. The solution of the elastic channel equation with $V_{11}+\Delta U^B$ gives much more than the second Born amplitude. Obviously, this potential is nonlocal in the variables $\mathbf{R}$ and $\mathbf{R}'$, and this is caused by the inverse of the differential operator $K(\mathbf{R})$.

(b) At high energies, the impulse approximation may be valid where h^Q is neglected in $G^{(Q)}$ as

$$\Delta U^{\text{imp}}(\mathbf{R},\mathbf{R}';E)=\sum_{cc'=2}^{Q} V_{1c}(\mathbf{R})\left[\frac{1}{E_1-K(\mathbf{R})-V(\mathbf{r},\mathbf{R})+i\epsilon}\right]_{cc'} V_{c'1}(\mathbf{R}') , \tag{6}$$

with $E_1=E-e_1$, where e_1 is the binding energy of the target in its ground state and E_1 is the relative kinetic energy of the projectile in channel 1.

Obviously, this is an approximation where the internal motions of the target nucleons are assumed to be unimportant compared with the projectile energy (K) and the relative projectile-target interaction (V). In other words, the target particles are considered frozen in the Q-space configuration during the collision. This is the approximation studied by Johnson and Soper[6] and by Amakawa *et al.*[7] They called the approximation corresponding to (6) "adiabatic", which is perhaps a misnomer. Like (5), the form (6) is still strongly nonlocal in $\mathbf{R}$.

(c) Finally, at low energies we may neglect the relative motion of the projectile, as represented by K, and set[4]

$$\Delta U^{\text{adia}}(\mathbf{R},\mathbf{R}';E)=\sum_{cc'=2}^{Q} V_{1c}(\mathbf{R})\left[\frac{\delta(\mathbf{R}-\mathbf{R}')}{e_1-h(\mathbf{r})-V(\mathbf{r},\mathbf{R})+i\epsilon}\right]_{cc'} V_{c'1}(\mathbf{R}) . \tag{7}$$

The most important property of ΔU^{adia} is its local character. This is a consequence of the presence of c numbers only (with respect to $\mathbf{R}$) in the denominator of Eq. (7). Approximation (7) is physically reasonable since the target system is completely distorted when the projectile is slowly moving. (In fact, the distortion is overestimated in ΔU^{adia}.) This is the form that we focus our attention on below, with particular emphasis on the nonlocality of ΔU, and comparing it with ΔU^{adia}. That is, if the adiabatic approximation to ΔU is valid at low enough energies, then we expect ΔU to become local as we approach the elastic threshold. Before studying this problem numerically, we make the following three remarks.

(i) At very low energies, with E below the lowest inelastic threshold, it was shown that[8] $\Delta U<0$ and that any variational approximation to $G_{cc'}^{(Q)}$ provides a bound $\Delta U\le\Delta U_t<0$. Therefore, the resulting phase shift satisfies the following inequalities:

$$\tan\delta(V_{11})\le\tan\delta(V_{11}+\Delta U_t)\le\tan\delta(V_{11}+\Delta U) . \tag{8}$$

Multichannel extensions of the above bound are also possible. These inequalities (which allow one to determine the best trial ΔU_t) hold only if there are no resonances—i.e., states of QHQ—below E. In the model used in the present paper, there are residual states in the Q space below the elastic-scattering threshold so that ΔU generally comes out to be positive.

(ii) The adiabatic approximation introduced above is slightly different from the ones[9] used in a more rigorous expansion procedure where

$$\Delta\tilde{U}^{\text{adia}}(\mathbf{R},\mathbf{R}';E)=\sum_{cc'=2}^{Q} V_{1c}(\mathbf{R})\left[\frac{\delta(\mathbf{R}-\mathbf{R}')}{\delta_{cc'}\hat{e}_1(\mathbf{R})-h(\mathbf{r})-V(\mathbf{r},\mathbf{R})+i\epsilon}\right]_{cc'} V_{c'1}(\mathbf{R}) , \tag{9}$$

where $\hat{e}_c(\mathbf{R})$ are the adiabatic channel potential energies which obviously depend on the location $\mathbf{R}$ of the projectile. An explicit expression for $\hat{e}_c(\mathbf{R})$ was given earlier[10] where it was shown that the highly nonlinear relation

$$\hat{e}_1(\mathbf{R})-e_1=V_{11}+\Delta\tilde{U}^{\text{adia}}$$

is valid.

(iii) In the Faddeev formulation,[11] the scattering of the incoming particle (say, 1) off a two-body bound state (2,3) is mediated by the couplings of the elastic channel "1" to the open rearrangement channels. It is straightforward to obtain an operator which plays the role of elastic channel "potential" U_{11}^F by solving the equations for the standard Faddeev components ψ^1, ψ^2, ψ^3 with respect to ψ^1, which asymptotically contains the elastic outgoing wave. In the resulting equation we have, for U_{11}^F, the terms $V_{23}G_2V_{13}$ and $V_{23}G_3V_{12}$ (neglecting the higher orders), where V_{ij} is the interaction between particles i and j and G_i represents the i-channel resolvent operator. In general, the operator U_{11}^F is totally nonlocal owing to the couplings of ψ^1 to the rearrangement channels.[12] In the adiabatic limit, however, even this complicated three-body operator becomes local in the channel coordinate $\mathbf{R}_1$. Indeed, rewriting, for example, G_2 in terms of the pair Hamiltonian h_1 and of the projectile kinetic operator K_1, instead of the corresponding quantities h_2 and K_2, one has

$$\begin{aligned} G_2&=(E-K_2-h_2)^{-1}\\ &=(E-K_1-h_1+V_{32}-V_{13})^{-1}\\ &\simeq(e_1-h_1+V_{23}-V_{13})^{-1} , \end{aligned} \tag{10}$$

which is local in $\mathbf{R}_1$.

We numerically solve Eq. (3) for $G_{cc'}^{(Q)}$ in momentum space by expanding the original local interactions $V_{cc'}$ into a complete set of Sturmian functions. The chosen expansion basis contains as many as 90 functions. It has already been shown[13] that this set is sufficiently large in order to obtain a correct expansion of a local potential, provided that the original interaction one deals with is assumed to be local. As shown in Refs. 1 and 2, this method leads to a finite-rank expansion for the DPP whose coefficients can be easily calculated by matrix-inversion methods. This allows one to evaluate the OP with full inclusion of coupling effects, i.e., without the need of approximate optical propagators employed in more phenomenological studies.[14–16] A method which is equivalent to the one used here has been developed previously[17,18] for the study of open-channel effects. We have analyzed the optical potential using the same schematic coupled-channel model used in Refs. 17 and 18. This model describes the inelastic-scattering process of two spinless particles, with target excitations of the monopole type. The diagonal potentials are Gaussian wells, 50 MeV deep and 5 fm large. The coupling potentials are surface-peaked Woods-Saxon derivatives, with the radius at 5 fm, 12 MeV deep, and with thickness parameter $a=0.5$ fm. We have considered up to six coupled channels with inelastic excitation thresholds at 3.00, 4.98, 8.04, 12.34, and 17.70 MeV. This model schematically describes the inelastic scattering of nucleons off a medium heavy target ($A=60$). In Fig. 1 we show the s-wave component of the DPP as a function of the absolute values of $\mathbf{R}$ and $\mathbf{R}'$ for decreasing values of E. The complete set of six channels has been considered in this calculation, and the center-of-mass elastic energy ranges from the top to the bottom of Fig. 1 in the following sequence: 0.80, 0.40, 0.20, and 0.10 MeV. As is clearly shown by the contour plots, at 0.80 MeV, two repulsive peaks are present and are well separated in configuration space away from the diagonal $R=R'$. Gradually these two peaks approach the $R=R'$ line as E is decreased, indicating that the potential becomes more local. Indeed, the trend of the potential going towards the scattering threshold can be reinterpreted introducing the nonlocal factorization of Frahn and Lemmer; as a matter of fact, once the coordinates $s=R-R'$ and $r=(R+R')/2$ are defined, ΔU_{11} acquires a bell-type shape in the s variable so that it can be reproduced by a Perey-Buck potential. This is a clear indication that the structure of the potential is becoming more and more local as the energy approaches the scattering threshold. It is worth noting here that a Frahn-Lemmer behavior for the real part of the DPP has also been found in Ref. 15 away from the elastic threshold (in the energy region between 10 and 50 MeV). In that paper, a microscopic nuclear-structure approach has been employed to analyze the nonlocal structure and the energy dependence of the optical potential, with inclusion of the collective excitations of the target through random-phase approximation (RPA) states.

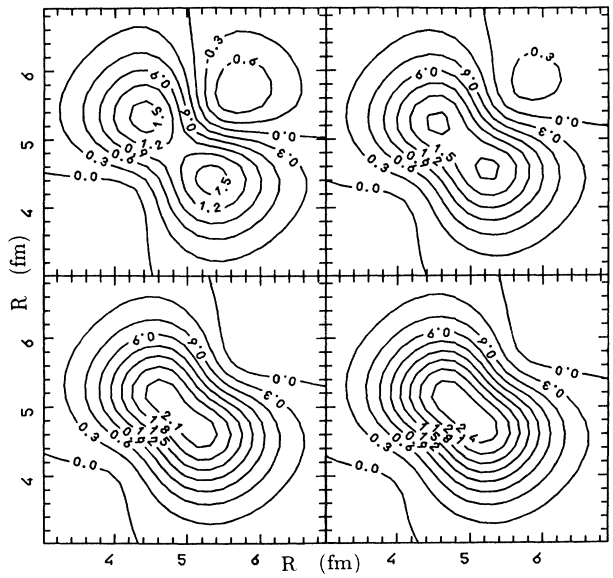

FIG. 1. The calculated s-wave component of the dynamic polarization potential $\Delta U(R,R';E)$ as a function of the absolute values of $\mathbf{R}$ and $\mathbf{R}'$ for decreasing energies of the incoming projectile. Below inelastic thresholds, the DPP is real. Five inelastic channels have been taken into account. Quotes are in MeV/fm.

It is important to understand how the nonlocality may be affected by the number of channels involved in a given calculation. Therefore, we have calculated the DPP in the same conditions as the previous figure, but with 4 channels instead of 6. In going to lower energy, the two nonlocal peaks remain well separated outside the diagonal and do not change their shape even when the energy is much lower than the ones used in Fig. 1. This result indicates that the adiabatic limit in the OP depends sensitively on the number of channels retained, and suggests that the locality is more completely satisfied when the number of channels becomes large; this is similar to the Fourier transform of $\delta(\mathbf{R}-\mathbf{R}')$ in (7) by a finite number of terms. Obviously, the more spread the result is, the poorer the approximation. Therefore, at low energies, the approximate locality of the DPP as suggested by the adiabaticity condition appears to be valid when the scattering problem is treated correctly by including a large number of channels in the coupled equations. The truncation of (2) to a finite number of channels may hence produce a spurious nonlocality. Conversely, the nonlocality of the DPP in a given approximation may be used to test the goodness of the DPP at low energies. That is, the more local the DPP is, the better the approximation used is, provided that the interaction potential is weak enough to satisfy the adiabaticity.

It is also of interest to examine the nonlocality of the optical potential ΔU_{22} corresponding to the first excited channel. Unlike in ΔU_{11}, ΔU_{22} now contains the contribution of the elastic channel 1 which is open. By induction, we expect that ΔU_{22} would behave exactly as ΔU_{11} if channel 1 were not the open elastic channel but just one of the higher excited states. Therefore, any deviation in the behavior of ΔU_{22} from that of ΔU_{11} must be attributed to the role played by channel 1. Figure 2 shows the real part of ΔU_{22} as a function of R and R' as the energy approaches the first inelastic threshold in the following

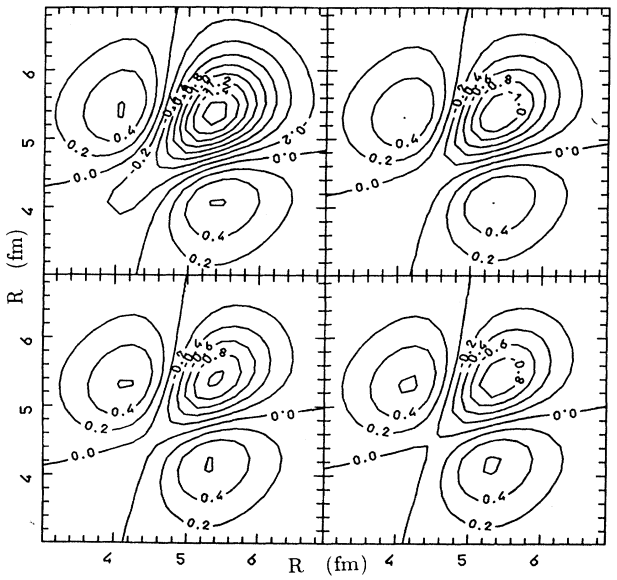

FIG. 2. The real part of $\Delta U_{22}(R,R';E)$, which refers to the first inelastic channel, for energies approaching the first inelastic threshold.

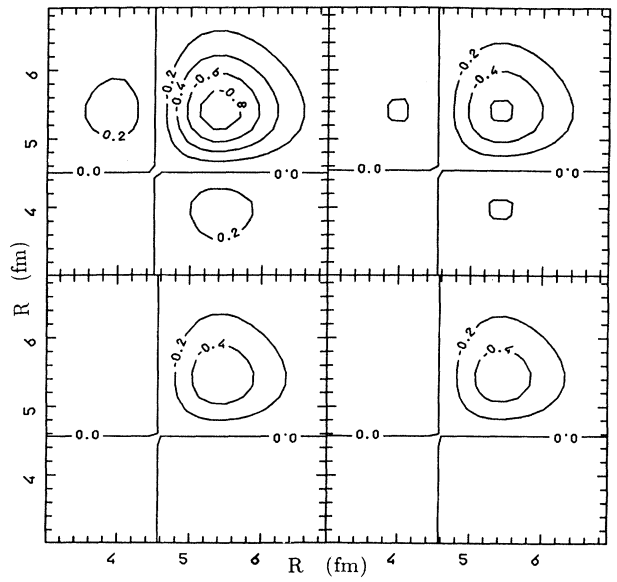

FIG. 3. The corresponding imaginary part of $\Delta U_{22}(R,R';E)$.

sequence: 3.8 MeV (top, left), 3.4 MeV (top, right), 3.2 MeV (bottom, left), and 3.1 MeV (bottom, right). The second inelastic threshold has been moved from 4.98 to 5.98 MeV in order to shift up to 4.53 a narrow compound resonance which occurred originally at 3.53 MeV and was perturbing our analysis. It is shown in Fig. 2 that the two nonlocal repulsive peaks persist as E approaches e_2, contrary to the case of Fig. 1 for ΔU_{11}. On the other hand, the imaginary part of ΔU_{22} decreases rapidly as $E \rightarrow e_2$ but still remains nonlocal in the same energy range, as is shown in Fig. 3. There are, indeed, indications that the imaginary part of ΔU_{22} maintains a separable rank-one character, as indicated in the figure by the persistent occurrence of the zeroes of $\mathrm{Im}\Delta U_{22}$ along the two perpendicular straight lines at $R=4.5$ fm and $R'=4.5$ fm.

In conclusion, we have shown in an explicit *model* calculation that the nonlocality of the elastic OP is reduced as the collision energy is reduced, in accordance with the adiabaticity assumption. However, the approach to locality is interrupted when the number of channels adopted in the evaluation of $G^{(Q)}$ is not sufficient. This surprising result may, in turn, be used, in some cases, as a criterion for the convergence of the finite-rank approximation for ΔU. It also suggests that, at low energies, the representation of the scattering wave function by an adiabatic basis set $\{\phi_c(r,R)\}$ and $\hat{e}_c(R)$ may be preferable. The channel coupling is then mediated by the kinetic-energy operator $K(R)$, and the resulting optical potential should be only mildly nonlocal. Contrary to ΔU_{11}, however, for higher OP's the nonlocality seems to remain strong. One should then reexamine the conventional distorted wave procedure for stripping and excitations, in which both the incoming elastic and outgoing inelastic wave functions are distorted by a local optical potential.

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*Permanent address: Istituto Nazionale di Fisica Nucleare, via Marzolo 8, Padova I-35131, Italy.

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