Nuclear stretching

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(Received 19 October 1990)

In this paper, stretching of nuclei is microscopically interpreted via the fermion dynamical symmetry model. A four-parameter energy formula with stretching for the yrast band of deformed nuclei is derived. This formula can account for the states in the yrast band below backbending and with the ratio $R_4 = E(4^+)/E(2^+) > 0.32$ in the rare earths and the actinides. It is also shown that stretching can improve the $B(E2)$ fits in the rare earths as well. However, analyses of the fitted parameters reveal that the smooth increase of the moment of inertia and the enhancement of the $B(E2)$'s at low spins appear to be a more complicated phenomenon, thus suggesting the necessity to further microscopically understand these low-spin behaviors.

I. INTRODUCTION

Rotational bands and their electromagnetic transitions are fundamental manifestations of nuclear collectivity. The traditional approach to study them is via the geometrical model of Bohr and Mottelson (BM).[1] It is well known that nuclei are not rigid. This can be seen from the fact that as a nucleus spins faster (i.e., higher J), the moment of inertia, a quantity easily deduced from data, will generally increase as well. This effect is known as *stretching*.

In the past few decades, attempts were made to understand this effect. A notable one is by Scharff-Goldhaber and co-workers[2,3] in which this feature was treated by the variable moment-of-inertia model (VMI). An extension[4] was subsequently made (called GVMI) in order to widen the VMI range of validity to include vibrational, transitional, and rotational regions. Although the VMI and GVMI were successful in reproducing the energies of the yrast states below backbending, they are phenomenological and thus contain no microscopic information. Another approach which in spirit is similar to the VMI is carried out by Wu and Zheng.[5] These authors assumed a suitable potential energy with certain singularities, expanded the BM Hamiltonian up to cubic powers of $\sin 3\gamma$, and obtained a simple energy formula which is suitable for the yrast band of even-even rotational nuclei.

In the past decade and a half, the interacting boson model (IBM) has been one of the main tools for studying deformed nuclei. Motivated by the spherical shell model, it was constructed as a bosonic algebraic model and therefore deformation is not *a priori* assumed. Rotations in the IBM context[6,7] and associated with the dynamical group chain $U_6 \supset SU_3 \supset SO_3$, and the various bands are labeled by the SU_3 irreducible representations (irrep) $(\lambda\mu)$. Although the IBM SU_3 limit is successful in describing many aspects of deformed nuclei, it has limitations in handling two important physical aspects: the Pauli and stretching effects.

In the early 1980s, combining the advantages of the extended BM and IBM models, Moshinsky[7] proposed a hybrid model (HYB). Utilizing this framework, Partensky and Quesne[8] studied the shape alterations of nuclei with increase of angular momentum. By assuming that a d boson corresponds to a quadrupole phonon, the hybrid model establishes the link between the d-boson operators in the IBM with the collective coordinates of the geometrical model. Therefore, the d boson's creation and annihilation operators are related to the deformation parameters α_μ and their corresponding momenta π_μ as

$$d_\mu^\dagger = \sqrt{(1/2)}(\alpha_\mu - i\pi_\mu) ,$$
$$(-1)^\mu d_{-\mu} = \sqrt{(1/2)}(\alpha_\mu + i\pi_\mu) . \qquad (1)$$

Consequently, the expectation value of the square of the deformation β can be expressed in terms of the expectation value of the d-boson number operator $\hat{n}_d$,

$$\langle \beta^2 \rangle = \langle \alpha \cdot \alpha \rangle = \langle \hat{n}_d + \tfrac{5}{2} \rangle . \qquad (2a)$$

For a state with angular momentum L in the ground band [belonging to the SU_3 irrep$(2N,0)$] one has

$$\langle \beta^2 \rangle_L = \frac{1}{6(2N+1)}[L(L+1)+B] \ , \tag{2b}$$

where the IBM formula [Eq. (6.5) of Ref. 6] has been used. Here N is the total valence pair number, $N = N_\pi + N_\nu$, and

$$B = 8N^2 + 22N - 15 \ . \tag{3}$$

According to the BM model, the moment of inertia $\mathcal{J}_L$ is related to $\langle \beta^2 \rangle_L$ by

$$\mathcal{J}_L = 3B_2 \langle \beta^2 \rangle_L = \frac{B_2}{2(2N+1)}[L(L+1)+B] \ , \tag{4}$$

where B_2 is the inertia parameter of the BM Hamiltonian. In this way a one-free-parameter energy formula for the ground band is obtained:

$$E_L = \frac{1}{2\mathcal{J}_L} L(L+1) = \frac{A}{B+L(L+1)} L(L+1) \ . \tag{5}$$

Using such a formula, Bonatsos[9] was able to account for the gradual increase of the moment of inertia with angular momentum prior to backbending in the actinides and rare earths.

It should be pointed out that although the hybrid model can incorporate stretching, there is an inherent inconsistency in the theory. It can be seen as follows. Equation (2b) shows that in the hybrid model the deformation parameter β, and thus the intrinsic quadrupole moment, increases with the increasing angular momentum. However, the intrinsic quadrupole moment for the ground band belonging to the irrep$(2N,0)$ in the HYB or IBM can also be obtained through the calculation of the expectation values of the quadrupole operator with the result[6]

$$Q_0(\mathrm{HYB}) = Q_0(\mathrm{IBM}) = -\alpha_2 \sqrt{(2\pi/5)}(4N+3) \ . \tag{6}$$

Since N is half the valence nucleon number, which is, of course, independent of L (that is why there is no stretching effect in the IBM), the quadrupole moment Q_0 in (6) is independent of L. This contradicts the prediction of Eq. (2b).

The present work is to study the deformed rare earths and actinides by using the $Sp_6 \times \mathcal{SU}_2$ branch of the fermion dynamical symmetry model (FDSM).[10-12] The FDSM is a symmetry-dictated truncation spherical shell model, for which deformation is not an input. Furthermore, it resides entirely in the fermion space, and therefore the Pauli principle is fully taken into account. We would like to show in this paper that the stretching effect is naturally incorporated. In this approach one can derive a four-parameter formula for the states of the yrast band of even-even nuclei. We will also show that stretching will enhance the electromagnetic transition rates.

II. FDSM HAMILTONIAN

The essence of the FDSM (Refs. 11 and 12) is to select a truncated Hilbert space for the spherical shell model. It is assumed that the most important fermion pairs for low-lying spectra are the $\mathbf{S}$ and $\mathbf{D}$ pairs in the normal-parity levels, and the $\mathcal{S}$ pairs in the abnormal-parity level (the script $\mathcal{S}$ is used to differentiate it from the S pair in the normal-parity levels). For the symmetry Sp_6, the creation operators of the $\mathbf{S},\mathbf{D}$ pairs are expressed in terms of the creation operators $b^\dagger_{km_k i m_i}$ in the k-i basis as follows:[10-13]

$$\mathbf{S}^\dagger = \sum_i \sqrt{\Omega_{ki}/2}(b^\dagger_{ki} b^\dagger_{ki})^{00}_{00} \ ,$$
$$\mathbf{D}^\dagger_m = \sum_i \sqrt{\Omega_{ki}/2}(b^\dagger_{ki} b^\dagger_{ki})^{20}_{m0} \ , \tag{7a}$$

where $\Omega_{ki} = \frac{1}{2}(2k+1)(2i+1)$, and the pseudo orbital angular momentum $k=1$, while the $\mathcal{S}$ pair for the abnormal-parity level is

$$\mathcal{S}^\dagger = \sqrt{\Omega_0/2}(a^\dagger_{j_0} a^\dagger_{j_0})^0_0 \ , \tag{7b}$$

with $\Omega_0 = \frac{1}{2}(2j_0+1)$.

The $(\mathbf{S},\mathbf{D},\mathcal{S})$ subspace is chosen as the model space and is called the *heritage* $u=0$ subspace, where u is defined as the number of fermions which do not contribute to $\mathbf{S},\mathbf{D}$ or $\mathcal{S}$ pairs.

For an n-p system, we shall use the notations $\mathbf{S}(\sigma)^\dagger$, $\mathbf{D}(\sigma)^\dagger$, and $\mathcal{S}(\sigma)^\dagger$ with $\sigma = \pi, \nu$ to distinguish proton and neutron pairs. The most general two-body Hamiltonian in the $u=0$ subspace can be written as follows:

$$\mathbf{H}^{(0)}_{\mathrm{FDSM}} = \mathbf{H}^\pi(0) + \mathbf{H}^\nu(0) + \mathbf{H}^{\pi\nu}(0) \ , \tag{8a}$$

$$\mathbf{H}^\sigma(0) = \epsilon^\sigma_0 n^\sigma_0 + \epsilon^\sigma_1 \mathbf{n}^\sigma_1 + \mathbf{H}^\sigma_n + \mathcal{H}^\sigma_a + \mathbf{H}^\sigma_{na} \quad (\sigma = \pi, \nu) \ , \tag{8b}$$

$$\mathbf{H}^{\pi\nu}(0) = \mathcal{B}^{\pi\nu}_0 n^\pi_0 n^\nu_0 + b^{\pi\nu}_0 n^\pi_0 \mathbf{n}^\nu_1 + b^{\nu\pi}_0 n^\nu_0 \mathbf{n}^\pi_1 + \sum_{r=0}^{2} 2B^{\pi\nu}_r \mathbf{P}^r(\pi)\cdot\mathbf{P}^r(\nu) \ , \tag{8c}$$

where $\mathbf{H}^\pi(0)$ and $\mathbf{H}^\nu(0)$ represent the proton and neutron Hamiltonians, respectively, and $\mathbf{H}^{\pi\nu}(0)$, the n-p interaction; ϵ^σ_0 and n^σ_0 are the single-particle energy and particle-number operator for the abnormal-parity level, respectively; ϵ^σ_1 and $\mathbf{n}^\sigma_1 = 2\mathbf{P}^0(\sigma)$ are the average single-particle energy and particle-number operator for the normal-parity levels; the total number $\mathbf{n}^\sigma = n^\sigma_0 + \mathbf{n}^\sigma_1$.

In Eq. (8b), $\mathbf{H}^\sigma_n$ is the Hamiltonian for the normal-parity levels,

$$\mathbf{H}^\sigma_n = G^\sigma_0 \mathbf{S}(\sigma)^\dagger \cdot \mathbf{S}(\sigma) + G^\sigma_2 \mathbf{D}(\sigma)^\dagger \cdot \mathbf{D}(\sigma) + \sum_{r=0}^{2} B^\sigma_r \mathbf{P}^r(\sigma)\cdot\mathbf{P}^r(\sigma) \ , \tag{9a}$$

$$\mathbf{P}^r_\mu(\sigma) = \sum_i \sqrt{\Omega^\sigma_{ki}/2}(b^{\sigma\dagger}_{ki} \tilde{b}^\sigma_{ki})^{r0}_{\mu 0} \ , \tag{9b}$$

while $\mathcal{H}^\sigma_a$ is the Hamiltonian for the abnormal-parity levels,

$$\mathcal{H}^\sigma_a = \mathcal{G}^\sigma_0 \mathcal{S}(\sigma)^\dagger \mathcal{S}(\sigma) + \frac{\mathcal{B}^\sigma_0}{4} n^{\sigma 2}_0 \ . \tag{9c}$$

The last term $\mathbf{H}^\sigma_{na}$ in (8b) is the interaction between the particles in the normal- and abnormal-parity levels

$$\mathbf{H}^{\sigma}_{na}=g^{\sigma}_{0}[\mathcal{S}(\sigma)^{\dagger}S(\sigma)+S(\sigma)^{\dagger}\mathcal{S}(\sigma)]+\frac{b^{\sigma}_{0}}{2}n^{\sigma}_{0}\mathbf{n}^{\sigma}_{1} . \tag{9d}$$

Note that we have used the nonscript (script) capital letters to denote the interaction strengths for the normal- (abnormal-) parity levels, and lower case letters for the coupling strengths between the pairs in the normal- and abnormal-parity levels.

For the $SU_3^{\pi+\nu}$ dynamical symmetry limit, the group chain is

$$(Sp_6^{\pi}\times Sp_6^{\nu}\supset SU_3^{\pi}\times SU_3^{\pi}\supset SU_3^{\pi+\nu}\supset SO_3^{\pi+\nu})\times \mathcal{SU}_2 .$$

In this case the pairing interactions $\mathbf{S}(\sigma)^{\dagger}\mathbf{S}(\sigma)$, $[\mathcal{S}(\sigma)^{\dagger}S(\sigma)+S(\sigma)^{\dagger}\mathcal{S}(\sigma)]$, and the proton (neutron) angular momentum $\mathbf{L}^{\pi^2}$ ($\mathbf{L}^{\nu^2}$) are symmetry breaking terms. By neglecting the off-diagonal part of these terms (details can be found in the Appendix), the FDSM Hamiltonian can be simplified as follows:

$$\mathbf{H}^{(0)}_{\text{FDSM}}=\mathbf{H}_0+\mathbf{H}_{SU_3}+\alpha\mathbf{L}^2 . \tag{10}$$

In Eq. (10), $\mathbf{H}_0$ is a quadratic function of $\mathbf{n}^{\sigma}_1$ and $\mathbf{n}^{\sigma}$ ($n^{\sigma}_0=\mathbf{n}^{\sigma}-\mathbf{n}^{\sigma}_1$),

$$\mathbf{H}_0=\sum_{\sigma=\pi,\nu}(a^{\sigma}\mathbf{N}_1^{\sigma^2}-2b^{\sigma}\mathbf{N}^{\sigma}_1+c^{\sigma})+d\mathbf{N}^{\pi}\mathbf{N}^{\nu} , \tag{11a}$$

$$b^{\sigma}=b^{\sigma}_1+b^{\sigma}_2\mathbf{N}^{\sigma}, \quad c^{\sigma}=c^{\sigma}_1\mathbf{N}^{\sigma}+c^{\sigma}_2\mathbf{N}^{\sigma^2} , \tag{11b}$$

where a^{σ}, b^{σ}_1, b^{σ}_2, c^{σ}_1, c^{σ}_2, and d are parameters, which are linear combinations of the interaction strengths in $\mathbf{H}^{\sigma}(0)$ and $\mathbf{H}^{\pi\nu}(0)$ in Eqs. (8b) and (8c), respectively.

The second term $\mathbf{H}_{SU_3}$ is the excitation energy of SU_3 representations,

$$\mathbf{H}_{SU_3}=\sum_{\sigma=\pi,\nu}(B^{\sigma}_2-G^{\sigma}_2-B^{\pi\nu}_2)\Delta\mathbf{C}^{\sigma}_{SU_3}+B^{\pi\nu}_2\Delta\mathbf{C}^{\pi+\nu}_{SU_3} , \tag{12a}$$

where

$$\Delta\mathbf{C}^{\sigma}_{SU_3}=\mathbf{C}^{\sigma}_{SU_3}-C_3(n^{\sigma}_1,0), \quad \Delta\mathbf{C}^{\pi+\nu}_{SU_3}=\mathbf{C}^{\pi+\nu}_{SU_3}-C_3(n_1,0) , \tag{12b}$$

and $\mathbf{C}^{\sigma}_{SU_3}$ ($\mathbf{C}^{\pi+\nu}_{SU_3}$) is the Casimir operator of SU_3^{σ} ($SU_3^{\pi+\nu}$) group. $C_3(n^{\sigma}_1,0)$ and $C_3(n_1,0)$ are eigenvalues for the SU_3 symmetric representation $(\lambda\mu)=(n^{\sigma}_1,0)$ and $(n_1,0)$, respectively. Generically,

$$C_3(\lambda\mu)=\tfrac{1}{2}(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu) . \tag{13}$$

For a given number of particles, say, n_1, it reaches its maximum value when $(\lambda\mu)=(n_1,0)$. Thus the symmetric representation is usually the ground state of fermion SU_3 symmetric representations, if the quadrupole-quadrupole interaction is attractive, unless $n_1>2\Omega_1/3$.

The last term in Eq. (10) is the collective rotational energy, where $\mathbf{L}$ is the total angular momentum, and α is related to the moment of inertia of a rotor ($\alpha=1/2\mathcal{I}$):

$$\alpha=\left[D+\frac{\delta_1}{n_1-1}\right], \quad D=\tfrac{3}{8}(B^{\pi\nu}_1-B^{\pi\nu}_2) , \tag{14a}$$

$$\begin{aligned}&\delta_1=(\delta^{\pi}_1+\delta^{\nu}_1)/2 ,\\ &\delta^{\sigma}_1=\tfrac{1}{4}(G^{\sigma}_2-G^{\sigma}_0)(\tilde{\Omega}^{\sigma}_1+1) \quad (\tilde{\Omega}_1=2\Omega_1/3) .\end{aligned} \tag{14b}$$

For reasonable residual force, monopole and quadrupole pairings as well as quadrupole-quadrupole force are attractive, and $|G_0|>|G_2|$, $|B_2|>|B_1|$, which will render the parameters D and δ_1 to be positive, as expected.

Note that in Eq. (14a), δ_1 is originated from the diagonal term of the pairing interaction (see Appendix). One sees that although for well-deformed nuclei the mixing of different SU_3 representations due to the off-diagonal term of pairing may be negligible, the diagonal term plays a very important role. Without pairing, $\alpha=D$ is a constant, which implies that a nucleus will behave like a rigid rotor. With pairing, the moment of inertia, first, will be reduced, because the pairing adds the term $\delta_1/(n_1-1)$ to α, and δ_1 is a positive number; second, it becomes n_1 dependent. In the following we will see that it is this n_1 dependence which causes nuclear stretching microscopically.

III. ENERGY FORMULA

In the $SU_3^{\pi+\nu}$ dynamical symmetry limit, we employ the irrep$|\mathcal{N}_0N_1(\lambda^{\pi}\mu^{\pi})(\lambda^{\nu}\mu^{\nu})(\lambda\mu)\kappa LM\rangle$ as the basis, where $\mathcal{N}_0=n_0/2$ and $N_1=n_1/2$ are pair numbers in the abnormal- and normal-parity levels, respectively, κ is the Vergados quantum number,[14] and L,M are angular momentum quantum numbers. The Hamiltonian $\mathbf{H}^{(0)}_{\text{FDSM}}$ [Eq. (10)] is diagonal in this basis. Furthermore, $\mathbf{H}_{SU_3}$ [Eq. (12a)] can be shown to be equivalent to the excitation energy of β-γ vibrational states [see Eq. (3.19) of Ref. 15],

$$\langle\mathbf{H}_{SU_3}\rangle=E(n_{\beta},n_{\gamma})\approx\hbar\omega(n_{\beta}+n_{\gamma}+K/2) , \tag{15a}$$

with

$$n_{\gamma}=(n_1-\lambda-2\mu)/3, \quad n_{\beta}=(\mu-\kappa)/2, \quad K=\kappa . \tag{15b}$$

Thus one obtains the energy formula

$$E_L=E_0(n^{\pi}_1,n^{\nu}_1)+E(n_{\beta},n_{\gamma})+\alpha L(L+1) , \tag{16a}$$

$$E_0(n^{\pi}_1,n^{\nu}_1)=\langle\mathbf{H}_0\rangle=\sum_{\sigma=\pi,\nu}[a^{\sigma}N_1^{\sigma^2}-2b^{\sigma}N^{\sigma}_1+c^{\sigma}]+dN^{\pi}N^{\nu} , \tag{16b}$$

For the ground band, i.e., $(\lambda,\mu)=(2N_1,0)$, $E(n_{\beta},n_{\gamma})=0$,

$$E_L=E_0(n^{\pi}_1,n^{\nu}_1)+\alpha L(L+1) . \tag{16c}$$

Until now, N_1 and N_0 are assumed to be fixed for a given nucleus. However, the interaction between the nucleons in the normal- and abnormal-parity levels, H^{σ}_{na}, will admix states with different N_1. A proper treatment of this admixture is beyond the scope of the present treatment. As an approximation, N_1 is treated here as a variational parameter and its values are determined from the equilibrium conditions

$$\frac{\partial E_L}{\partial N_1^\pi}=0, \quad \frac{\partial E_L}{\partial N_1^\nu}=0 . \tag{17}$$

From (16) and (17), one gets

$$N_1=\left[\frac{b^\pi}{a^\pi}+\frac{b^\nu}{a^\nu}\right]+\frac{\delta_1}{(2N_1-1)^2}(1/a^\pi+1/a^\nu)L(L+1)$$
$$\cong N_1^{(0)}+FL(L+1), \quad \text{valid for low } L , \tag{18a}$$

where $N_1^{(0)}$ is the value of N_1 for the ground state $L=0$, and F is a quantity depending on $N_1^{(0)}$ but not on L,

$$N_1^{(0)}=N_1^{(0\pi)}+N_1^{(0\nu)}=\left[\frac{b^\pi}{a^\pi}+\frac{b^\nu}{a^\nu}\right], \tag{18b}$$

$$F=\frac{\delta_1}{a(2N_1^{(0)}-1)^2}, \quad 1/a=(1/a^\pi+1/a^\nu) . \tag{18c}$$

According to (16b), the coefficients a^σ and b^σ must be positive in order for a minimum to exist for the curve E_L vs N_1. Besides, as we have discussed before, the coefficient δ_1 is positive for a reasonable residual force. Therefore, the coefficient F in (18a) is positive. We immediately see that the number of valence pairs N_1 in the normal-parity levels increases with the increasing angular momentum L. According to Eq. (14a), the increase of N_1 naturally leads to the decrease of α and, thus, the increase of the moment of inertia, and also to the deformation of the nucleus, as we shall see later [Eq. (22b)]. This is precisely the nuclear stretching.

Substituting Eq. (18a) into Eq. (14a), one gets the energy formula from Eq. (16c):

$$\Delta E_L=(E_L-E_{\text{g.s.}})=\left[D+\frac{A}{L(L+1)+B}\right]L(L+1) , \tag{19a}$$

where $E_{\text{g.s.}}$ is the ground-state energy:

$$E_{\text{g.s.}}=E_0(n_1^{(0)\pi},n_1^{(0)\nu}) ,$$
$$n_1^{(0)\sigma}=2N_1^{(0)\sigma} \quad (\sigma=\pi,\nu) , \tag{19b}$$

$$A=\frac{\delta_1}{2F}=A'(2N_1^{(0)}-1)^2, \quad A'=\frac{a}{2} , \tag{19c}$$

$$B=\frac{2N_1^{(0)}-1}{2F}=B'(2N_1^{(0)}-1)^3, \quad B'=\frac{a}{2\delta_1} . \tag{19d}$$

From (19a) and using Eq. (4-19) of Ref. 1, we obtain the moment of inertia:

$$\mathcal{I}_L=\frac{1}{2}\left[D+\frac{AB}{[L(L+1)+B]^2}\right]^{-1} . \tag{20}$$

Equation (19a) or (20) contains three parameters D, A', and B'. From Eq. (20) we see that an increase of the angular momentum L naturally leads to an increase of the moment of inertia $\mathcal{I}_L$.

IV. $B(E2)$ FORMULA

The quadrupole operator is defined by

$$\hat{Q}_0^2=-\alpha_2\sqrt{(16\pi/5)}P_0^2, \quad P_0^2=P_0^2(\pi)+P_0^2(\nu) \tag{21}$$

where α_2 is the effective charge. Using (A.1c) in Ref. 13, we have

$$Q_L((2N_1,0)\chi=0)=\alpha_2\sqrt{(2\pi/5)}(4N_1+3)\frac{L}{2L+3} . \tag{22a}$$

Thus the intrinsic quadrupole moment predicted by the FDSM is

$$Q_0(\text{FDSM})=\alpha_2\sqrt{(2\pi/5)}(4N_1+3) . \tag{22b}$$

Equation (22b) shows that the deformation is proportional to the nucleon number in the normal-parity levels. Note that Eq. (22b) has the same form as (6) of the IBM except that N is changed to N_1. As we will see, this "slight" modification has profound effect.

The $B(E2)$ transition rate for the ground band is [see (A.2b) in Ref. 12]

$$B(E2,(2N_1,0)L+2\rightarrow L)$$
$$=\tfrac{3}{4}(\alpha_2)^2\frac{(L+1)(L+2)}{(2L+3)(2L+5)}(2N_1-L)(2N_1+L+3) . \tag{23}$$

Equation (23) has the same form as Eq. (4.9) of Ref. 6, except that the integer N in the latter is now replaced by N_1. To take into account the stretching effect, the quantity N_1 in Eq. (23) is to be calculated according to (18a).

V. RESULTS AND DISCUSSIONS

Using the formalism discussed above, we have analyzed the experimental data [energy levels and $B(E2)$'s] for the actinides and rare earths. It should be mentioned that the actinides have protons and neutrons occupying shells 7 and 8, respectively, thus having $\mathrm{Sp}_6^\pi\times\mathrm{Sp}_6^\nu$ symmetry. The rare earths have protons and neutrons in the shells 6 and 7, respectively, thus $\mathrm{SO}_8^\pi\times\mathrm{Sp}_6^\nu$ symmetry. The former has a coupled $\mathrm{SU}_3^{\pi+\nu}$ dynamical symmetry, while the latter does not. However, Wu and Vallieres[16] have demonstrated, using their $\mathrm{SO}_8\times\mathrm{Sp}_6$ code, that for a deformed nucleus, if the n-p quadrupole-quadrupole interaction is strong, the energy spectra and $B(E2)$ transition rates of the ground band of a $\mathrm{SO}_8^\pi\times\mathrm{Sp}_6^\nu$ system behave very much like an $\mathrm{Sp}_6^\pi\times\mathrm{Sp}_6^\nu$ system. Thus, in this paper, we shall assume that the $\mathrm{SU}_3^{\pi+\nu}$ formulas [Eqs. (19) and (23)] can be applied to both actinides and rare earths.

The $B(E2)$'s and the energy levels before the backbend for rare earths and the energy levels for the actinides are fitted by the least-square method. From Eq. (20) one sees that when pairing vanishes ($A=0$) the moment of inertia is $1/2D$, which corresponds to the rigid-rotor moment of inertia, i.e., $\mathcal{I}_{\text{rig}}=1/2D$. Thus the parameter D should decrease with increasing N_1 since $\mathcal{I}_{\text{rig}}$ increases as mass number increases. To take this into account in a simple manner, it is assumed that for a given Z, the parameter D

TABLE I. Effective charges α_2 for the rare earths.

Nucleus	^{154}Sm	^{156}Gd	^{158}Gd	^{160}Dy	^{162}Dy	^{164}Dy	^{164}Er	^{166}Er	^{168}Er	^{170}Er	
α_2	0.188	0.185	0.185	0.172	0.168	0.167	0.180	0.171	0.164	0.164	
Nucleus	^{166}Yb	^{170}Yb	^{172}Yb	^{174}Yb	^{176}Yb	^{170}Hf	^{178}Hf	^{180}Hf	^{182}W	^{184}W	^{186}W
α_2	0.186	0.169	0.180	0.172	0.158	0.183	0.168	0.170	0.169	0.168	0.263

for different isotopes varies linearly with $N_1^{(0)}$:

$$D = D_1 + D_2 N_1^{(0)} , \tag{24a}$$

where D_1 *and* D_2 are parameters. Thus, instead of three parameters for each nucleus, we have four parameters $D1$, $D2$, A', and B' for each set of nuclei with the same Z.

The values of $N_1^{(0)}$ can be determined by Eqs. (18b) and (11b):

$$N_1^{(0)\sigma} = \frac{b_1^\sigma}{a^\sigma} + \frac{b_2^\sigma}{a^\sigma} N^\sigma . \tag{24b}$$

Empirically it has been found[17] that a universal formula can be applied to both actinide and rare-earth nuclei and for both protons and neutrons:

$$N_1^{(0)\sigma} = 0.75 + 0.5 N^\sigma, \quad \sigma = \pi, \nu , \tag{24c}$$

where N^π and N^ν are the number of valence proton and neutron pairs, respectively. If $N_1^{(0)\sigma} > \Omega_1^\sigma/2$, the above distribution is assumed to be the same except that $N_1^{(0)\sigma}$ (N^σ) is now the number of normal-parity hole pairs (total number of hole pairs).

For nucleons in Sp_6 shells (the valence nucleons of actinides and the rare-earth valence neutrons), there is a dynamical Pauli effect that when $N_1^{(0)\sigma} > \Omega_1^\sigma/3$, the symmetric SU_3 representation $(2N_1^{(0)\sigma},0)$ is Pauli forbidden and the nuclei have to be in less symmetrical states with higher energies. This effect tends to decrease the increment of $N_1^{(0)\sigma}$. To take this into account, we shall assume that the slope of Eq. (24b) is reduced from 0.5 to 0.25 when $N_1^{(0)\sigma} > \Omega_1^\sigma/3$, i.e.,

$$N_1^{(0)\sigma} = \frac{\Omega_1^\sigma}{3} + 0.25 \left[N^\sigma - \frac{2\Omega_1^\sigma}{3} + 1.5 \right] , \tag{24d}$$

for $\Omega_1^\sigma/3 < N_1^{(0)\sigma} < \Omega_1^\sigma/2$. The number 1.5 in (24b) is added to ensure the continuity of $N_1^{(0)\sigma}$. Of course, the slope reduction cannot be easily justified on physical grounds. Fortunately, our results are insensitive to such variations.

Strictly speaking, when $N_1^{(0)\sigma} > \Omega_1^\sigma/3$, not only is here uncertainty for the N_1 distribution, but also some changes must be introduced for the matrix elements given by Eq. (A14), the energy formula given by Eq. (19), and the $B(E2)$ formula given by Eq. (23). This is because the SU_3^σ representation is no longer $(2N_1^{(0)\sigma},0)$. According to Ref. 12, there are in principle no technical difficulties in computing such matrix elements for arbitrary SU_3 irreps. However, in the interest of clearly exhibiting the physics and that such details do not alter the essential features of our microscopic stretching mechanism, and using the fact that most well deformed nuclei satisfy $N_1^{(0)\sigma} < \Omega_1^\sigma/3$, we have in this paper employed only the symmetric irreps for all deformed actinides and rare earths. Hence the terminology "ground band" is synonymous here with the term "yrast band."

A. $B(E2)$ values

For the rare earths, we first fit the $B(E2)$ values in order to obtain the parameter B' by (18a) and (19d). The

TABLE II. Parameters for the formulas (20) and (24d): D_1, D_2, A' (in keV), and B' for rare earths and actinides.

Nuclei	D_1	D_2	A'	B'
$_{60}$Nd	10.6701	−0.3544	6.8725	0.1484
$_{62}$Sm	10.2513	−0.4144	13.4980	0.1673
$_{64}$Gd	11.8212	−0.6948	17.5078	0.1604
$_{66}$Dy	10.1063	−0.7672	29.2785	0.1656
$_{68}$Er	10.1812	−0.6855	29.2031	0.1685
$_{70}$Yb	12.7529	−0.8731	25.1005	0.1642
$_{72}$Hf	10.5153	−0.6070	26.8036	0.1689
$_{74}$W	11.8283	−1.0921	37.4509	0.1677
$_{90}$Th	4.2208	−0.0987	20.4028	0.3118
$_{92}$U	3.3707	−0.0143	14.7546	0.2171
$_{94}$Pu	3.5935	−0.0028	13.9980	0.2042
$_{96}$Cm	3.5919	−0.0075	11.6860	0.1521
$_{98}$Cf	3.5919	−0.0075	11.3455	0.1437
$_{100}$Fm	3.9919	−0.0064	12.3455	0.1437

effective charge α_2 is determined from the experimental $B(E2,2\rightarrow 0)$ value. The parameters α_2 and B' are listed in Tables I and II, respectively.

In Fig. 1 typical results for rare earths are presented (^{152}Sm, ^{156}Gd, ^{160}Dy, ^{166}Er, ^{166}Yb, ^{170}Hf, and ^{186}W). The experimental data are taken from Ref. 19. It is seen that for not too large values of L, the stretching effect does improve the fit between the experimental and theoretical

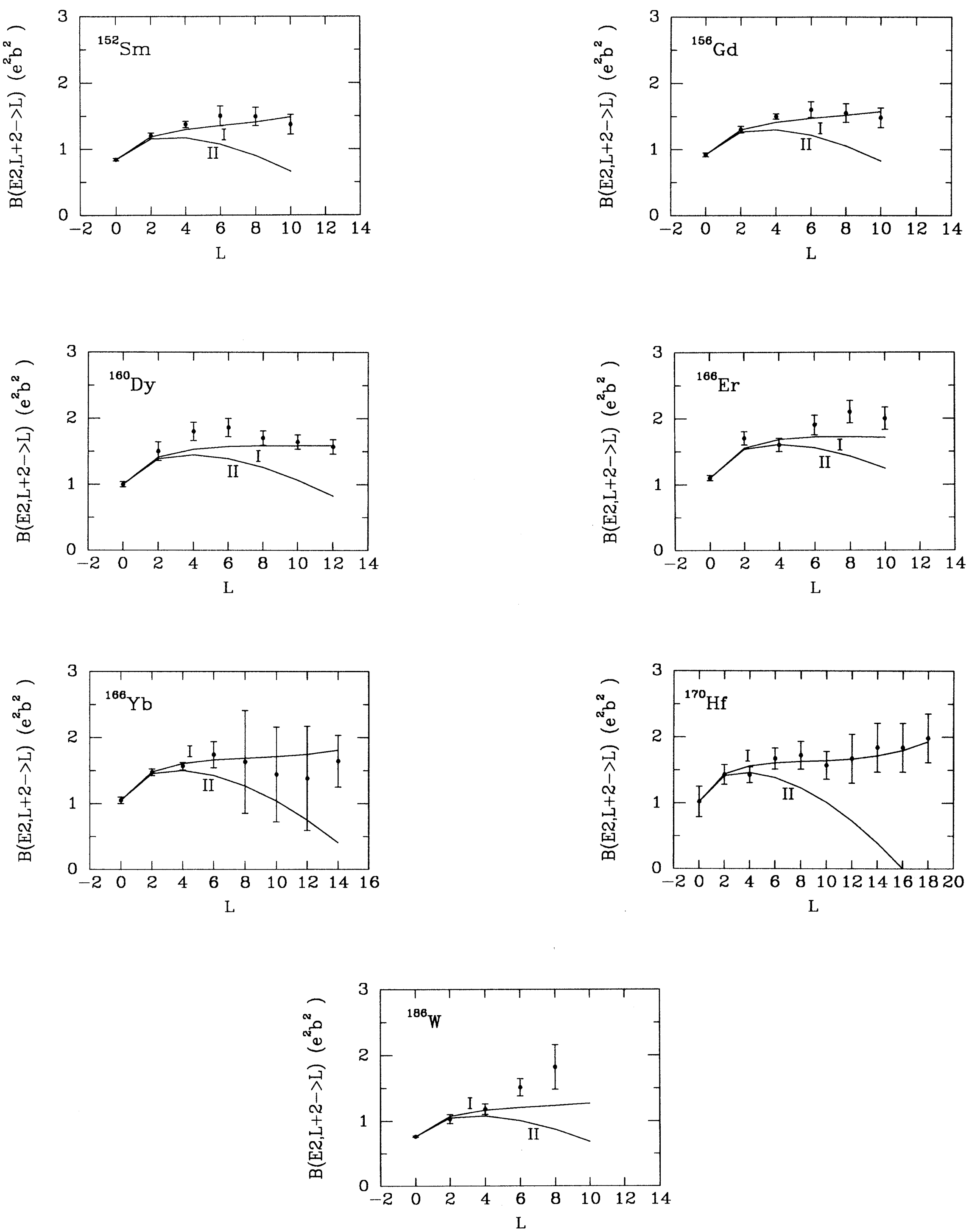

FIG. 1. $B(E2)$ values for ^{152}Sm, ^{158}Gd, ^{160}Dy, ^{166}Er, ^{166}Yb, ^{170}Hf, and ^{186}W. The curve I (curve II) represents the result for which the stretching effect has (has not) been taken into account.

results. Without it, N_1 is fixed for a given nucleus, and Eq. (23) shows that the $B(E2)$ value will increase first with L, but will soon decrease with L, and finally become zero when $L_1 = 2N_1$ (i.e., when all pairs in the normal-parity levels become the D pairs), as indicated by curve II. It should be mentioned that this rapid vanishing of the $B(E2)$'s is because our study is limited to the heritage zero space. We have shown recently that by considering the nonzero heritage space,[18] one can account for the so-called loss of collectivity and band termination.

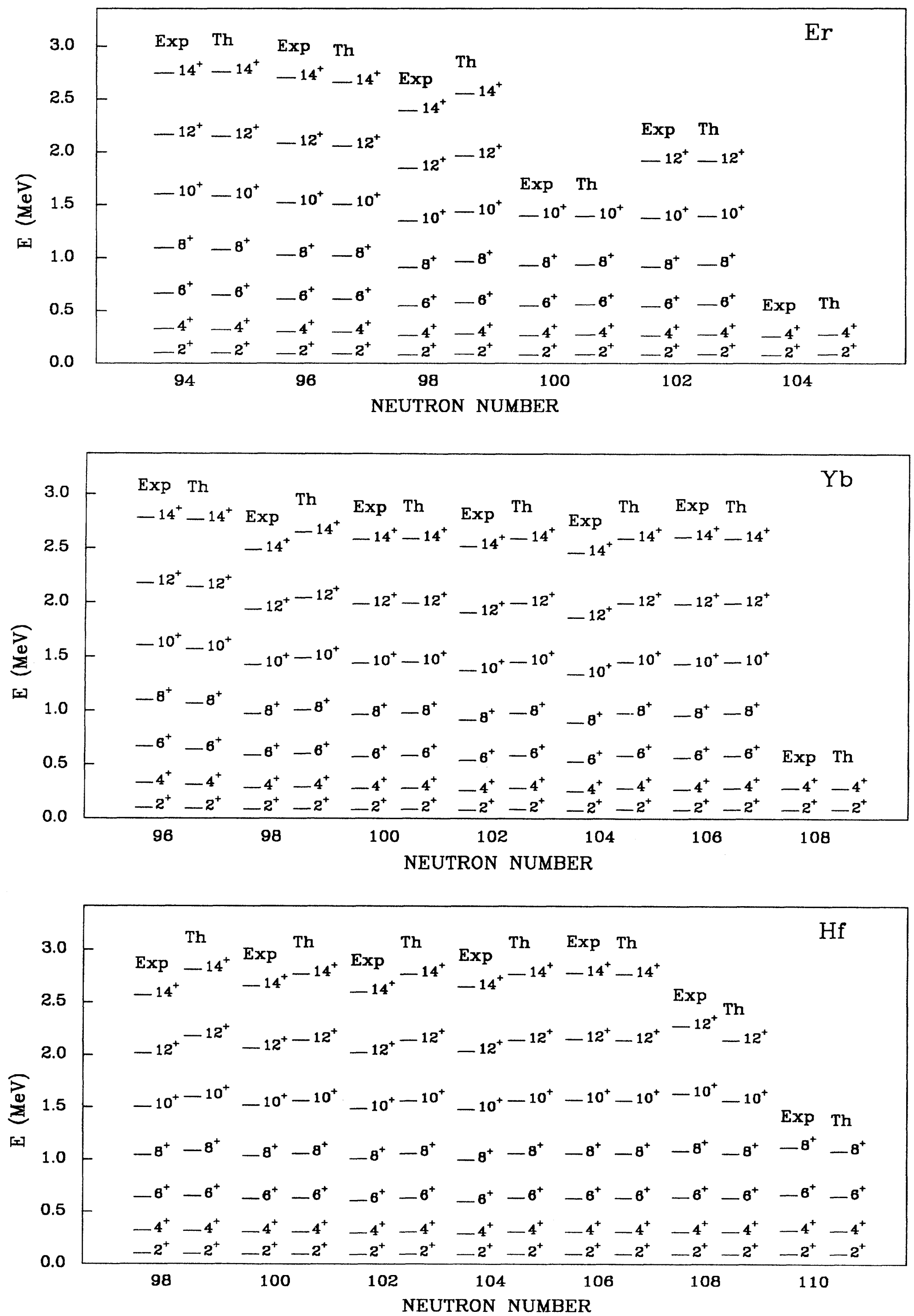

FIG. 2. Experimental and FDSM predicted spectra for the ground bands of the isotopes of Er, Yb, and Hf.

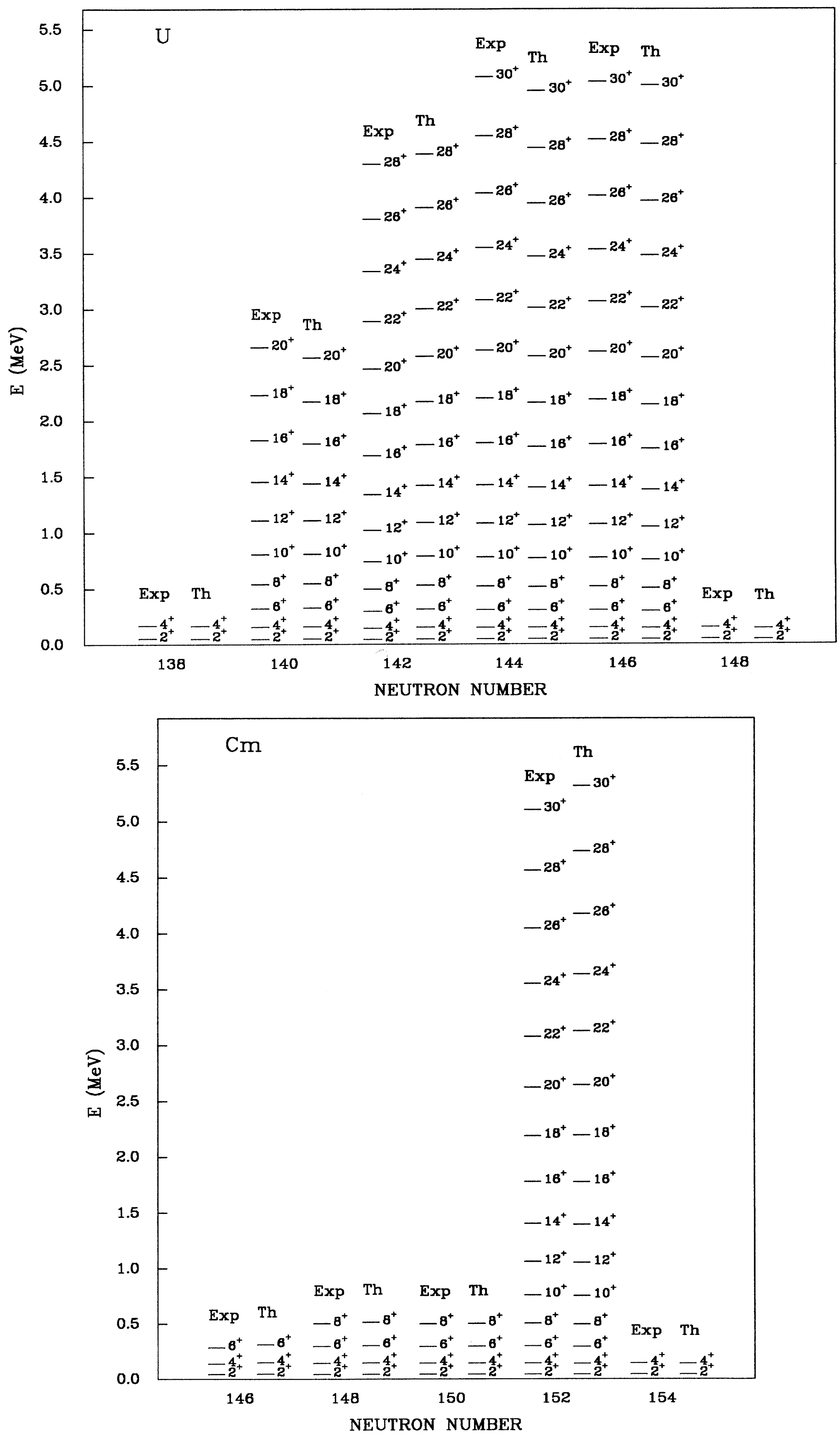

FIG. 3. Experimental and FDSM predicted spectra for the ground bands of the isotopes of U and Cm.

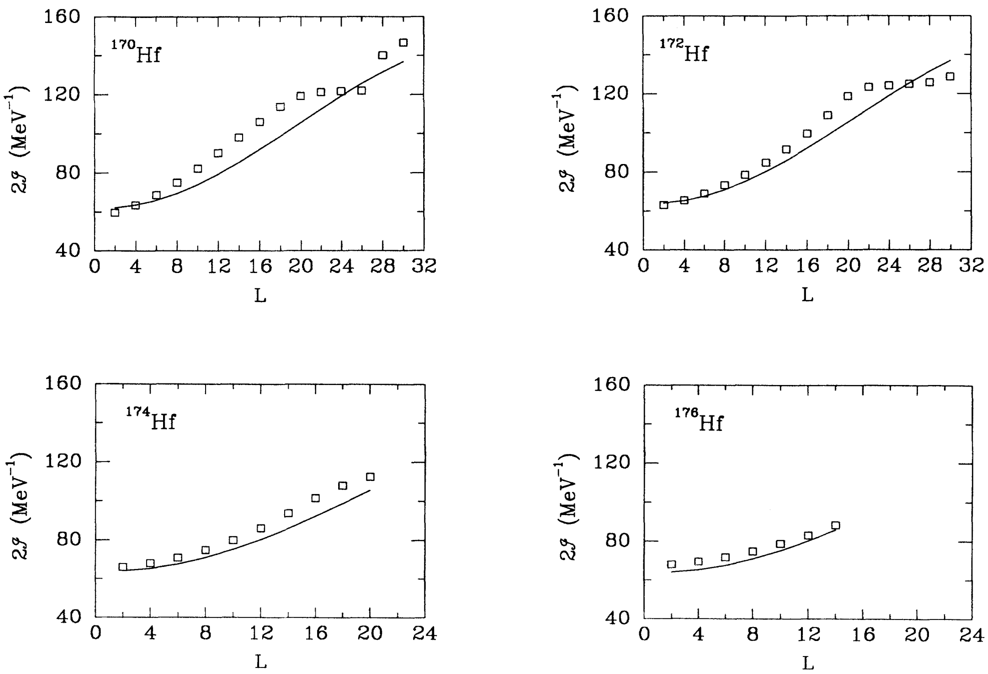

FIG. 4. Moment of inertia $\mathcal{J}$ vs angular momentum L for $^{170-176}$Hf. The open boxes are data (same as Figs. 5 and 6).

Such additional ingredients of the theory are not included so that we may clearly focus on the problem of stretching in low spins. With stretching, N_1 will increase with L, and the $B(E2)$ strengths will continue to increaes with L over a broader range.

B. Spectra

For rare earths the parameter B' is determined from F obtained from the $B(E2)$ fit by Eq. (19d), and the parameters D_1, D_2, and A' are determined by the spectra fit,

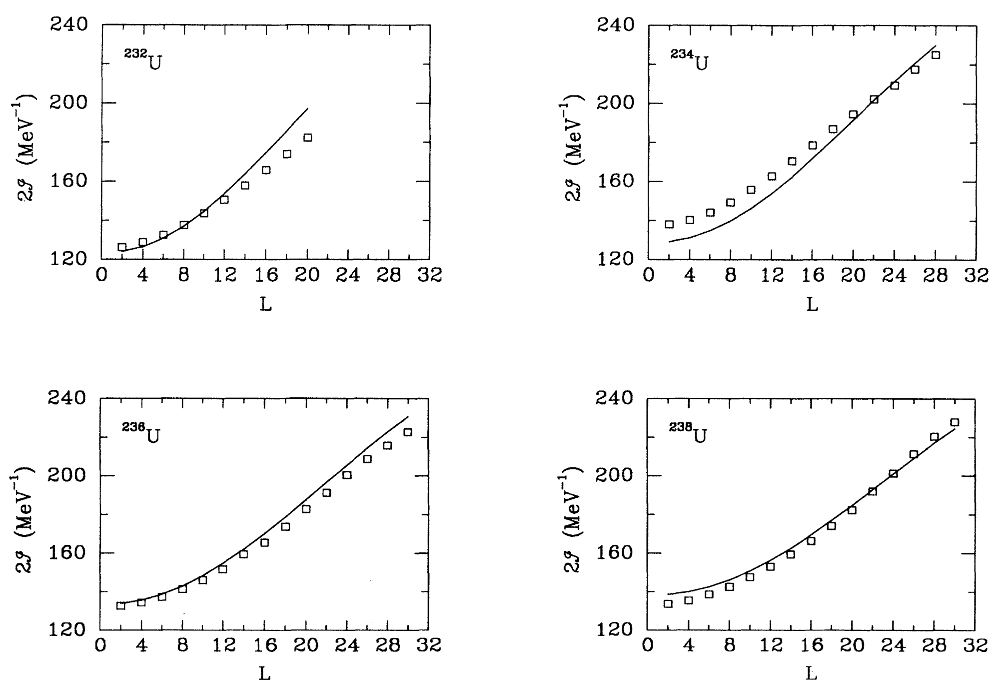

FIG. 5. Moment of inertia $\mathcal{J}$ vs angular momentum L for $^{232-238}$U.

while for actinides, all the four parameters D_1, D_2, A', and B' are determined from the spectra fit. The parameters are listed in Table II. We have fitted the energy spectra for deformed rare earths (eight group of isotopes: Nd, Sm, Gd, Dy, Er, Yb, Hf, and W) and deformed actinides (six groups of isotopes: Th, U, Pu, Cm, Cf, and Fm). Experimental data are taken from Refs. 20–29. Typical results are given in Figs. 2 and 3. Good agreement for the rare earths and very good agreement for the actinides between theory and experiment are obtained. In some cases the spin goes up to 28 for the actinides and 12 for the rare earths, which is very close to the region where the onset of backbending or backbending has been observed or expected. From Table II we also see that the coefficients D_1 are nearly constant, while the coefficients D_2 are small negative numbers, as they should be. The better fit for the actinides than the rare earths is not surprising, since the $SO_8 \times Sp_6$ symmetry for the latter is approximated by $Sp_6 \times Sp_6$ in this study.

C. Moment of inertia

In Figs. 4 and 5, typical moments of inertia $\mathcal{J}_L$ for the Hf and U isotopes are plotted as functions of L. The increase of $\mathcal{J}_L$ with increasing L is obvious and is in agreement with Ref. 9.

D. Comparison between several models

Figure 6 shows a comparison between seven energy formulas for fitting the energy levels of typical rare-earth and actinide nuclei (^{168}Yb and ^{238}U). The curves denoted as FDSM are the present calculations, HYB the hybrid model, WZ Wu-Zheng's, *AB*(*ABC*)→two- (three-) term formula in the expansion in terms of the power of $L(L+1)$, and VMI (GVMI)→the variable moment of inertia model (the generalized VMI). Figure 7 shows a comparison between the six formulas for the moment of inertia $\mathcal{J}_L$ as a function of the angular momentum L for ^{166}Er, ^{168}Yb, ^{238}U, and ^{248}Cm.

From Figs. 6 and 7, we see that the FDSM curves give a better account of the data than the two- or three-term formulas, about the same as the VMI and slightly inferior to the formula of WZ. However, it should be emphasized that all the former fits, except those by the HYB, *are done for each nucleus individually* (i.e., *each nucleus has its own set of parameters*). If such a flexibility is allowed and used in the present approach, then we can obviously obtain excellent fits to the data for all the rotational levels with the ratio $R(4)=E(4)/E(2)>3.2$.

E. Microscopic mechanism for the stretching

The physical reason for stretching can be easily understood with our formalism. In order to generate rotation,

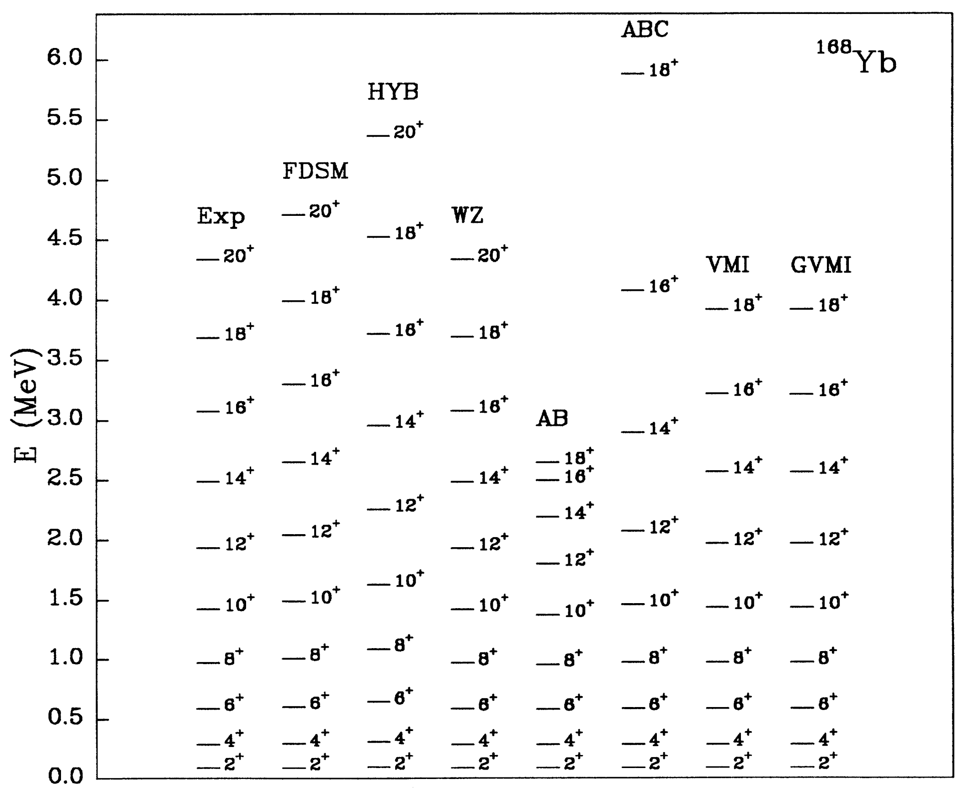

FIG. 6. Comparison of the several models in their predictions for the spectra of ^{168}Yb.

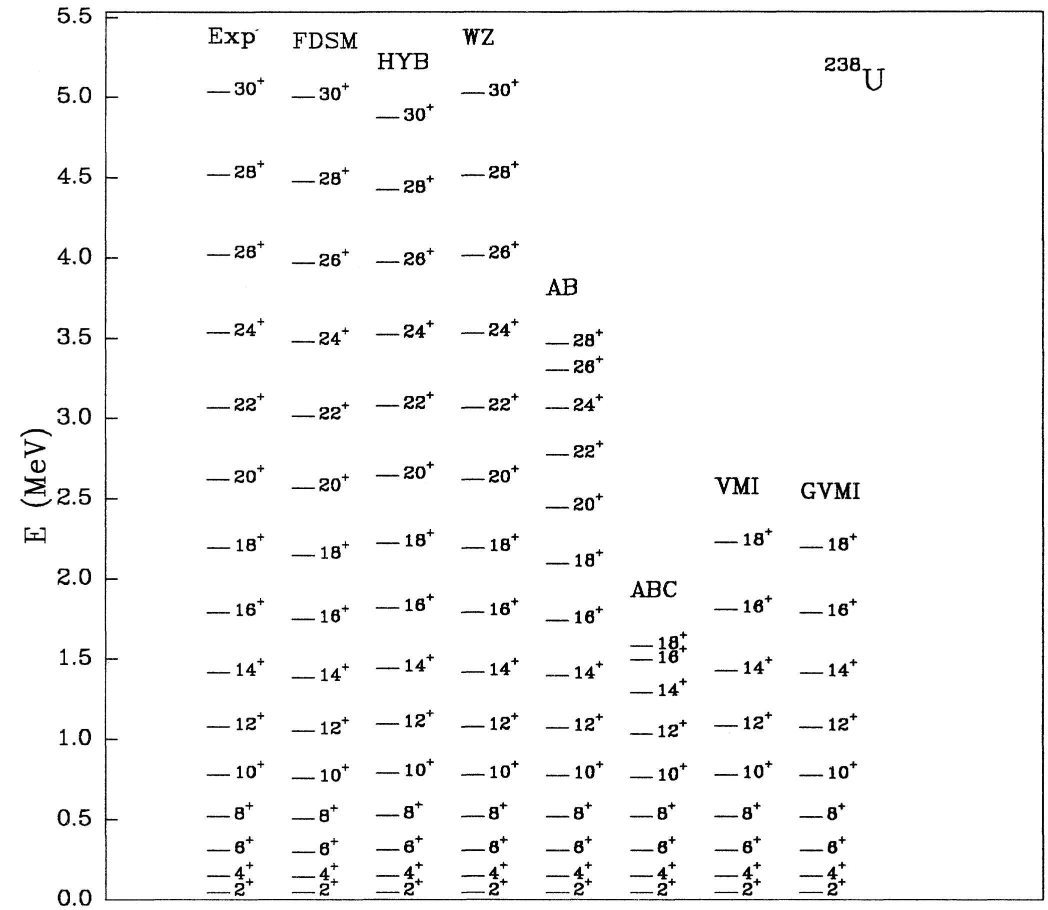

FIG. 6. (*Continued*).

one needs to shift nucleons from the S to D state, i.e., reducing the number of S pairs and thus the effectiveness of the pairing. Therefore, rotation is in favor of the quadrupole forces and consequently in favor of large deformation, which is realized by shifting nucleons from the abnormal- to normal-parity levels, i.e., by increasing n_1, since the deformation is proportional to N_1, as shown in (22b). This is in contrast to the Coriolis antipairing (CAP) effect[12] where the increase of the moment of inertia is realized by breaking pairs without changing n_1.

Another interesting fact worth mentioning is that from Eq. (14) we see that stretching will disappear either when there is no pairing: $G_0=G_2=0$, or when the number of nucleons becomes infinite, $N_1 \to \infty$. When there is no pairing or when $N_1 \to \infty$, a nucleus behaves like a rigid body. For a given pairing strength, the smaller the N_1 value, the larger the F [Eq. (18c)], thus the softer the nucleus. This is also in agreement with the empirical situation. It shows that *the stretching effect is microscopically due to the superfluidity of the nuclear matter and the finiteness of the particle number.* This of course is a well-known generic feature in nuclear-structure physics.[30]

If there is no distinction between the normal- and abnormal-parity levels, and N_1 in Eq. (22) is interpreted as the total number of valence nucleon pairs, N, as in HYB or IBM, then the nuclear deformation will not change. The widely observed stretching effect indicates the correctness of the assumption that the valence nucleons in the normal-parity levels are mainly responsible for nuclear deformation.

F. Are there other mechanisms causing the moment of inertia to vary with L?

There are of course other physical mechanisms which can cause the moment of inertia to alter with L. Two mechanisms readily come to mind. One is the mixing due to pairing of the β or γ band with the ground band and the other is the mixing of the $u\neq0$ with the $u=0$ states. For well-deformed nuclei, neglecting the mixing due to pairing may be a reasonable approximation. However, the mixing of the $u\neq0$ states, which does not break the SU_3 symmetry, can have a significant effect on the moment of inertia. After all, it is intuitively clear that sufficiently strong rotational motion can either break the $\mathcal{S}$ pairs in the abnormal-parity level ($v_0\neq0$) or the S,D pairs in the normal-parity levels ($u_1\neq0$). We have previously shown[31] that the broken pairs in the abnormal-

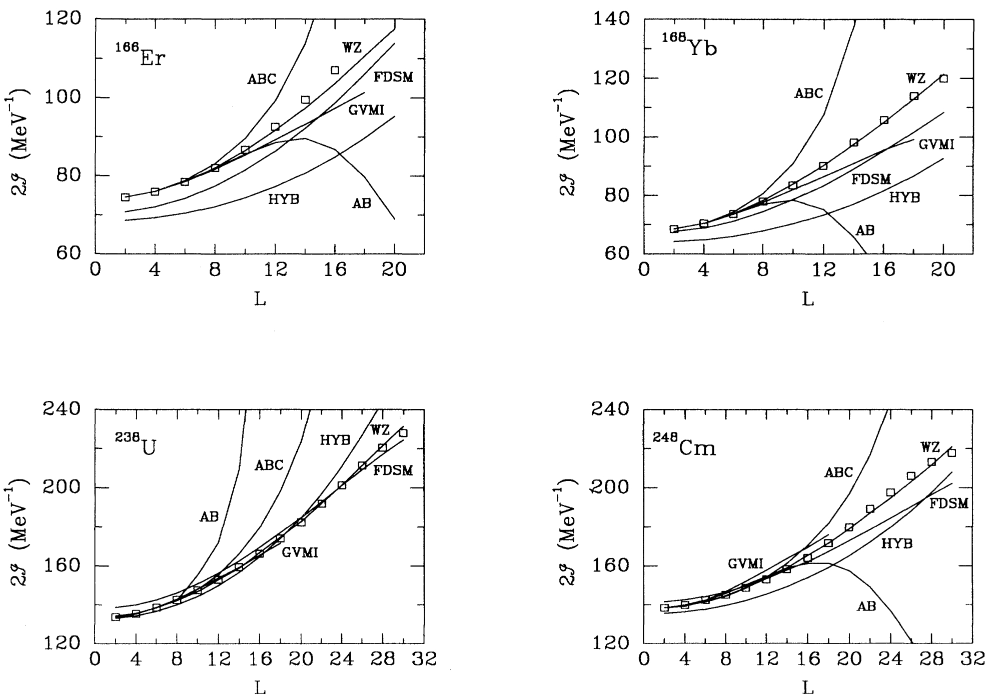

FIG. 7. Comparison of the several models in their predictions for the $\mathcal{J}$-vs-L curve.

parity levels are primarily responsible for backbending and may affect the smooth increase of the momenta of inertia at low spins as well (below the crossing or backbending point). On the other hand, the broken pairs in the normal-parity levels, which correspond to the CAP effect in the geometrical model language, will primarily affect the low-spin moment of inertia via strong mixing with the ground band.

A subtle point has emerged in the present studies which requires some discussion here. Even though the CAP effect (and other effects as well) is not explicitly considered here, the data appear to be well fitted with the formula given by Eq. (19a) (Figs. 2 and 3). We must emphasize that this *does not* mean that the CAP effect plays an insignificant role in stretching the nucleus. On the contrary, its importance can be succinctly seen as follows. Note that the required energy to change N_1 from $N^{(0)}$ to $N^{(0)}\pm 1$ is $2A'$ [note that $2A'=a$; see Eqs. (19c) and (11a)]. This means that since there is no known excited 0^+ state below 600 keV, there is *de facto* a constraint on A': It cannot be less than 300 keV. However, according to Table II, in order to fit the data, $A'=7$–37 keV, which is obviously too small by an order of magnitude compared with the energy constraint lower bound. Clearly, there must exist other physical mechanisms to effectively reduce A'. The mixing of $u\neq 0$ states (i.e., the CAP effect and the coupling between abnormal-broken pairs and the ground band) is most likely to play such a role since its effect is to increase the moment of inertia which, to some extent, is equivalent to reducing A'.

For the $B(E2)$'s, the mixing of $u\neq 0$ states also plays an important role. In fact, in previous studies[18,31] by just taking this effect into consideration we were able to account for the $B(E2)$'s behavior, from low to high spins. We have also shown that the CAP band crossing is in fact responsible for the $B(E2)$ fluctuations in the low-spin region of actinides.[31] On the contrary, here we have only included stretching, and it appears to fit the low-spin $B(E2)$ equally well. Hence, as in the study of the moment of inertia, other physical mechanisms may also be masked by the parameters. This is what we will next discuss.

The L dependence of the $B(E2)$'s depends on a (dimensionless) parameter B' [see Eqs. (23), (18a), (18c), and (19d)]. One knows well that the pairing gap energy $G_0^\sigma \Omega_1^\sigma$ is of the order of 2 MeV. Thus, with Eq. (14b), one can easily estimate that δ_1 should be around $\frac{1}{3}$ MeV. We also know that a is of the order of 2 MeV. Then, according to Eq. (19d), the parameter B' must be around 3. However, according to Table II, $B'=0.14$–0.31, which is also an order of magnitude too small. This suggests that stretching cannot be the sole effect in enhancing the $B(E2)$'s. Mixing the $u\neq 0$ states will also have the same effect. Note that the angular momentum L in the $B(E2)$ formula (23) is actually the "core" angular momentum (i.e., the angular momentum of S-D pairs in the normal-parity levels); only for the $u=0$ case is L equal to the total angular momentum J. When there is an alignment I due to the broken pair, the core angular momentum for the yrast band will be reduced, $L=J-I$. As one can see from Eq. (23), reducing L and keeping N_1 constant is equivalent to increase N_1 and keeping L constant; both will lead to an increase of the $B(E2)$. Thus taking the broken pairs ($u\neq 0$) into account is tantamount to increasing N_1 or, equivalently, reducing B' [see Eqs. (19d) and (18a)].

It is thus seen that near the SU_3 limit of the FDSM, by using the pairing force as a perturbation, we can obtain a microscopic explanation for the stretching effect of nuclei. Based on the stretching effect, an energy formula and $B(E2)$ formula for the yrast band of the even-even nuclei can be obtained and fit to the experimental data rather well. However, an analysis of the fitted parameters suggested that the increase of the moment of inertia and the $B(E2)$'s in the low-spin region appears to be a more complicated process than just a stretching of the nucleus. A more detailed study including both stretching and $u\neq 0$ mixing is clearly called for. This work is now in progress.

ACKNOWLEDGMENTS

It is a pleasure to thank M. W. Guidry, Z.-P. Li, F. Wang, S. Pittel, and B. R. Barrett for numerous illuminating discussions. Funding for this research is by the Natural Science Foundation of the Chinese Education Commision, Academia Sinica, the United States National Science Foundation, and the Department of Energy.

APPENDIX: FDSM HAMILTONIAN IN SU_3 SYMMETRY LIMIT

The most general two-body Hamiltonian in the $u=0$ subspace [Eq. (8)] can be rewritten in the following form:

$$\mathbf{H}^{(0)}_{\mathrm{FDSM}}=\mathbf{H}^0+\mathbf{H}_{\mathrm{SU}_3}+\mathbf{H}_{\mathrm{br}}+\tfrac{3}{8}(B_1^{\pi\nu}-B_2^{\pi\nu})\mathbf{L}^2\ , \tag{A1}$$

where $\mathbf{L}$ is the total angular momentum, $\mathbf{H}_{\mathrm{SU}_3}$ the excitation energy coming from the SU_3 Casimir operator, $\mathbf{H}_{\mathrm{br}}$ the SU_3 symmetry breaking Hamiltonian, and $\mathbf{H}^0$ the remaining part of the Hamiltonian:

$$\mathbf{H}^0=\sum_{\sigma=\pi,\nu}\Big\{\epsilon_0^\sigma \mathscr{n}_0^\sigma+\epsilon_1^\sigma\mathbf{n}_1^\sigma+\mathscr{G}_0^\sigma\mathscr{S}(\sigma)^\dagger\mathscr{S}(\sigma)+\frac{\mathscr{B}_0^\sigma}{4}\mathscr{n}_0^{\sigma^2}+\frac{b_0^\sigma}{2}\mathscr{n}_0^\sigma\mathbf{n}_1^\sigma+\frac{B_1^\sigma}{4}n_1^{\sigma^2}+G_2^\sigma[\mathbf{C}_{\mathrm{Sp}_6}^\sigma-S_0^\sigma(S_0^\sigma-6)]$$
$$+(B_2^\sigma-G_2^\sigma-B_2^{\pi\nu})C_3(n_1^\sigma,0)\Big\}+B_2^{\pi\nu}C_3(n_1,0)+\mathscr{B}_0^{\pi\nu}\mathscr{n}_0^\pi\mathscr{n}_0^\nu+b_0^{\pi\nu}\mathscr{n}_0^\pi\mathbf{n}_1^\nu+b_0^{\nu\pi}\mathscr{n}_0^\nu\mathbf{n}_1^\pi+B_0^{\nu\pi}/2\mathbf{n}_1^\pi\mathbf{n}_1^\nu\ , \tag{A2}$$

$$\mathbf{H}_{\mathrm{SU}_3}=\sum_{\sigma=\pi,\nu}(B_2^\sigma-G_2^\sigma-B_2^{\pi\nu})\Delta\mathbf{C}_{\mathrm{SU}_3}^\sigma+B_2^{\pi\nu}\Delta\mathbf{C}_{\mathrm{SU}_3}^{\pi+\nu}\ , \tag{A3}$$

$$\mathbf{H}_{\mathrm{br}}=\sum_{\sigma=\pi,\nu}\{(G_0^\sigma-G_2^\sigma)\mathbf{S}(\sigma)^\dagger\mathbf{S}(\sigma)+\tfrac{3}{8}[(B_1^\sigma-B_2^\sigma)-(B_1^{\pi\nu}-B_2^{\pi\nu})]\mathbf{L}^{\sigma^2}+g_0^\sigma[\mathscr{S}(\sigma)^\dagger S(\sigma)+S(\sigma)^\dagger\mathscr{S}(\sigma)]\}\ . \tag{A4}$$

In deriving Eqs. (A1)–(A4), the following relations have been used to convert quadrupole pairing $\mathbf{D}(\sigma)^\dagger\cdot\mathbf{D}(\sigma)$ and multipole operators $\mathbf{P}^r(\sigma)\cdot\mathbf{P}^r(\sigma')$ into monopole pairing $\mathbf{S}(\sigma)^\dagger\cdot\mathbf{S}(\sigma)$, SU_3 Casimir operators $\mathbf{C}_{\mathrm{SU}_3}^\sigma$ and $\mathbf{C}_{\mathrm{SU}_3}^{\pi+\nu}$, number operators $\mathbf{n}_1^\sigma$ and $\mathbf{n}_1$, and angular momenta $\mathbf{L}^\sigma$ and $\mathbf{L}$:

$$\mathbf{D}(\sigma)^\dagger\mathbf{D}(\sigma)=\mathbf{C}_{\mathrm{Sp}_6}^\sigma-S_0^\sigma(S_0^\sigma-6)-\mathbf{S}(\sigma)^\dagger\mathbf{S}(\sigma)-\sum_{r=1}^{2}\mathbf{P}^r(\sigma)\cdot\mathbf{P}^r(\sigma)\ , \tag{A5}$$

$$2\mathbf{P}^2(\pi)\cdot\mathbf{P}^2(\nu)=\mathbf{P}^2\cdot\mathbf{P}^2-\mathbf{P}^2(\pi)\cdot\mathbf{P}^2(\pi)-\mathbf{P}^2(\nu)\cdot\mathbf{P}^2(\nu)\ , \tag{A6}$$

$$\mathbf{P}^2(\sigma)\cdot\mathbf{P}^2(\sigma)=\mathbf{C}_{\mathrm{SU}_3}^\sigma-\mathbf{P}^1(\sigma)\cdot\mathbf{P}^1(\sigma),\quad \mathbf{P}^2\cdot\mathbf{P}^2=\mathbf{C}_{\mathrm{SU}_3}^{\pi+\nu}-\mathbf{P}^1\cdot\mathbf{P}^1\ , \tag{A7}$$

$$\mathbf{P}^1(\sigma)=\sqrt{(3/8)}\mathbf{L}^\sigma,\quad 2\mathbf{P}^0(\sigma)=\mathbf{n}_1^\sigma,\quad \mathbf{P}^1=\sqrt{(3/8)}\mathbf{L},\quad 2\mathbf{P}^0=\mathbf{n}_1\ . \tag{A8}$$

Note that the expectation values of Sp_6 Casimir operator $\mathbf{C}_{\mathrm{Sp}_6}$ and $\mathscr{S}^\dagger\mathscr{S}$ in the $u=0$ space are[11]

$$\langle u=0|\mathbf{C}_{\mathrm{Sp}_6}^\sigma|u=0\rangle=\tfrac{1}{4}\Omega_1^\sigma(\Omega_1^\sigma+12)\ , \tag{A9}$$

$$\langle u=0|\mathscr{S}^\dagger\mathscr{S}|u=0\rangle=\tfrac{1}{4}\mathscr{n}_0(2\Omega_0-\mathscr{n}_0+2)\ , \tag{A10}$$

and

$$S_0^\sigma=\tfrac{1}{2}(\mathbf{n}_1^\sigma-\Omega_1^\sigma)\ . \tag{A11}$$

Therefore, $\mathbf{H}^0$ is a quadratic function of number operators $\mathbf{n}_1^\sigma$ and $\mathbf{n}^\sigma$ ($\mathscr{n}_0^\sigma=\mathbf{n}^\sigma-\mathbf{n}_1^\sigma$).

In the SU_3 symmetry limit, we neglect the off-diagonal terms and only consider the diagonal term of $\mathbf{H}_{\mathrm{br}}$ denoted as $\langle\mathbf{H}_{\mathrm{br}}\rangle$. Thus the term $[\mathscr{S}(\sigma)^\dagger S(\sigma)+S(\sigma)^\dagger\mathscr{S}(\sigma)]$ is omitted since it has only off-diagonal terms. We employ the irrep$|\mathscr{N}_0N_1(\lambda^\pi\mu^\pi)(\lambda^\nu\mu^\nu)(\lambda\mu)\kappa LM\rangle$ as the basis, where $\mathscr{N}_0=\mathscr{n}_0/2$ and $N_1=n_1/2$ are pair numbers in the abnormal- and normal-parity levels, respectively, κ is the Vergados quantum number,[14] and L,M are angular momentum quantum numbers. Using (6.4) and (6.5) in Ref. 12 and the SU_3 Racah coefficients, it can be shown that

$$\langle \mathcal{N}_0 N_1(\mathbf{n}_1^\pi,0)(\mathbf{n}_1^\nu,0)(\mathbf{n}_1,0)\kappa LM|\mathbf{S}(\sigma)^\dagger \mathbf{S}(\sigma)|\mathcal{N}_0 N_1(\mathbf{n}_1^\pi,0)(\mathbf{n}_1^\nu,0)(\mathbf{n}_1,0)\kappa LM\rangle$$
$$\equiv \langle \mathbf{S}(\sigma)^\dagger \mathbf{S}(\sigma)\rangle = \frac{n_1^\sigma(\tilde\Omega_1^\sigma - n_1^\sigma+2)[n_1(n_1+1)-L(L+1)]}{4n_1(n_1-1)}, \quad \tilde\Omega_1^\sigma = \frac{2\Omega_1^\sigma}{3}, \tag{A12}$$

$$\langle \mathcal{N}_0 N_1(\mathbf{n}_1^\pi,0)(\mathbf{n}_1^\nu,0)(\mathbf{n}_1,0)\kappa LM|\mathbf{L}^{\sigma^2}|\mathcal{N}_0 N_1(\mathbf{n}_1^\pi,0)(\mathbf{n}_1^\nu,0)(\mathbf{n}_1,0)\kappa LM\rangle = \frac{n_1^\sigma(n_1^\sigma-1)}{n_1(n_1-1)}L(L+1), \tag{A13}$$

and one can obtain

$$\langle \mathbf{H}_{\rm br}\rangle = V_p(L=0) + \sum_{\sigma=\pi,\nu}\left[-\delta^\sigma + \frac{\delta_1^\sigma}{n_1^\sigma-1}\right]\frac{n_1^\sigma(n_1^\sigma-1)}{n_1(n_1-1)}L(L+1), \tag{A14}$$

where $V_p(L=0)$ is the pairing interaction for $L=0$ in the normal-parity level,

$$V_p(L=0) = \sum_{\sigma=\pi,\nu}\tfrac{1}{4}(G_0^\sigma - G_2^\sigma)\frac{(n_1+1)}{(n_1-1)}n_1^\sigma(\tilde\Omega_1^\sigma - n_1^\sigma+2)$$
$$\approx \sum_{\sigma=\pi,\nu}\tfrac{1}{4}(G_0^\sigma - G_2^\sigma)n_1^\sigma(\tilde\Omega_1^\sigma - n_1^\sigma+2) \tag{A15}$$

and

$$\delta_1^\sigma = \tfrac{1}{4}(G_2^\sigma - G_0^\sigma)(\tilde\Omega_1^\sigma+1), \tag{A16}$$

$$\delta^\sigma = \tfrac{1}{4}(G_2^\sigma - G_0^\sigma) - \tfrac{3}{8}[(B_1^\sigma - B_2^\sigma) - (B_1^{\pi\nu} - B_2^{\pi\nu})]. \tag{A17}$$

Note that $\mathbf{H}^0$ [Eq. (A2)] and $V_p(L=0)$ are both quadratic functions of n_1^π and n_1^ν. Therefore, they can be combined into a single quadratic function $\mathbf{H}_0$:

$$\mathbf{H}_0 = \sum_{\sigma=\pi,\nu}(a^\sigma N_1^{\sigma^2} - 2b_1^\sigma N_1^\sigma - 2b_2^\sigma N_1^\sigma N^\sigma + c_1^\sigma N^\sigma + c_2^\sigma N^{\sigma^2}) + dN^\pi N^\nu. \tag{A18}$$

In Eq. (A18) the part for identical particles is the most general one, where a^σ, b_1^σ, b_2^σ, c_1^σ, and c_2^σ are parameters, which are linear combinations of the interaction strengths in $\mathbf{H}^0$ and $V_p(L=0)$ [(A2) and (A15)]. For the *n-p* monopole-monopole interaction, we have assumed that only $d\mathbf{N}^\pi\mathbf{N}^\nu$ is important, where d is a parameter. The fact that empirically the ground-state proton- (neutron-) number distribution among normal and abnormal levels does not sensitively depend on the neutron (proton) number[17] seems to provide a justification for this simplification.

The second term on the right side of (A14) can be combined with the rotor term [the last term in (A1)] and provides a correction to the moment of inertia. Thus Eq. (A1) can be written as

$$\mathbf{H}_{\rm FDSM}^{(0)} = \mathbf{H}_0 + \mathbf{H}_{SU_3} + \alpha\mathbf{L}^2, \tag{A19}$$

where

$$\alpha = \sum_{\sigma=\pi,\nu}\left[-\delta^\sigma + \frac{\delta_1^\sigma}{n_1^\sigma-1}\right]\frac{n_1^\sigma(n_1^\sigma-1)}{n_1(n_1-1)} + \tfrac{3}{8}(B_1^{\pi\nu} - B_2^{\pi\nu}). \tag{A20}$$

From (A16) and (A17), one see that $\delta_1^\sigma \gg \delta^\sigma$, because it contains a large factor $(\tilde\Omega_1^\sigma+1)$. Assuming that the parameter δ^σ is negligible and δ_1^σ for protons and neutrons are roughly the same, i.e., $\delta_1^\pi \approx \delta_1^\nu \equiv \delta_1$, then Eq. (A20) can be further simplified:

$$\alpha = \left[D + \frac{\delta_1}{n_1-1}\right], \quad D = \tfrac{3}{8}(B_1^{\pi\nu} - B_2^{\pi\nu}), \quad \delta_1 = (\delta_1^\pi + \delta_1^\nu)/2. \tag{A21}$$

Equations (A19), (A18), (A3), and (A21) are the Eqs. (10), (11), (12), and (14), respectively, in Sec. II.

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