Nucleon self-energy in relativistic nuclear matter with pion ring series

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Nucleon self-energies from the pion-ring series are studied in the relativistic mean-field theory of linear σ model with the ω meson and the Δ. Near the Fermi surface of nuclear matter, the pion rings generate attractive scalar and vector potentials of 10–15 % of the nucleon mass. These strongly energy-dependent potentials cause the nucleons to have a significant probability to be in a collective N-hole or Δ-hole configuration.

I. INTRODUCTION

Since Yukawa conjectured a meson theory of nuclear force, the pion has been known as an important component of the nuclear force field. It is commonly believed that the one-pion exchange (OPE) generates the long-range nuclear force. However, the lowest-order OPE does not contribute to the binding energy in nuclear matter. On the other hand, the second-order OPE has a significant effect on the binding energy due to the OPE's strong tensor force.[1]

For the intermediate-range nuclear force, 2π exchanges (TPE), correlated or uncorrelated, are known to dominate. In nuclear matter, TPE's effects on the binding energy are expected to be even more important than the second-order OPE mainly because of the presence of the Δ isobars.

Pion propagation in the nuclear medium can generate a collective spin-isospin mode by coupling to the nucleon-hole and Δ-hole states.[2] Without the short-range correlations between baryons, the collective spin-isospin mode would develop into a so-called pion condensate even at normal densities.

If pion propagation in the nuclear medium is modified so as to excite collective modes, the effects of the uncorrelated 2π exchanges and the second-order OPE on the binding energy of nuclear matter should be enhanced correspondingly. In other words, the exchanged pions between nucleons can be rescattered by other nucleons, thus amplifying the attractive nuclear force. The combined effects of the uncorrelated TPE (plus the second-order OPE) and the modified pion propagation on the binding energy of nuclear matter are represented by the pion-ring series.

In our previous report,[3] we investigated the influence of the pion-ring series on the nuclear matter saturation properties based on the relativistic mean-field theory[4,5] of the linear σ model[6] with the vector meson (ω) and Δ. The role of Δ-hole excitations was emphasized along with the dependence on the πNN- and $\pi N\Delta$-form factors. One of the most important features of this approach is that the attraction from the scalar meson mean field is relatively small compared to the pionic attraction at normal nuclear matter densities. Thus the scalar field provides only a small shift of the effective Dirac mass of the nucleon from the free-space value. But pion effects cause such a shift, which we investigate here.

The shift of the Dirac mass of the nucleon is related to the single-particle energy and wave function of the nucleon. Clearly, these quantities are an important part of nuclear physics. Thus we wish to determine the effective mass (or proper nucleon self-energy). We do this by solving the Dyson equation for the nucleon propagator.

Note that the properties of the nucleon propagation in the presence of the nuclear pion field has not extensively studied in relativistic field theory models. There have been some related works,[7,8] but these are based on nonrelativistic formalisms. Here we attempt to study the single-particle nature of the nucleon in relativistic nuclear matter by calculating the nucleon self-energy in the presence of the pion-ring series.

II. FORMALISM

First, we briefly review the formalism concerning the single-particle nature of the nucleon in relativistic nuclear matter. In the rest frame, parity conservation, time-reversal invariance, and Hermiticity lead to the following general form of the proper nucleon self-energy:[4]

$$\Sigma_N(k)=\Sigma_N^s(k^0,|\mathbf{k}|)+\gamma^0\Sigma_N^0(k^0,|\mathbf{k}|)+\boldsymbol{\gamma}\cdot\mathbf{k}\Sigma_N^v(k^0,|\mathbf{k}|)\ . \quad (1)$$

Then the Dyson equation for the nucleon propagator can be solved formally as

$$G_N^{-1}(k)=G_N^{0-1}(k)-\Sigma_N(k)$$
$$=\gamma\cdot\tilde{k}-\tilde{M}\ , \quad (2)$$

where $\tilde{k}^{\mu}\equiv(k^0+\Sigma_N^0(k),\mathbf{k}(1+\Sigma_N^v(k))$ and $\tilde{M}\equiv M+\Sigma_N^s(k)$. The analytic structure of $G_N(k)$ in the complex k^0 plane must be specified to invert Eq. (2).

The analytic structure of the Green's function can be specified by the following Lehmann spectral representation:[9]

$$G_N(k^0,\mathbf{k})=\int_{-\infty}^{\infty}d\omega\frac{A(\omega,\mathbf{k})}{\omega-(k^0-\mu)(1+i\eta)}\ , \quad (3)$$

where μ is the chemical potential. Defining a function $\tilde{G}_N$, which analytically continues to G_N,

$$\tilde{G}_N(z,\mathbf{k})=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\frac{A(\omega,\mathbf{k})}{\omega-z}\ , \quad (4)$$

$$G_N(k^0,\mathbf{k})=\tilde{G}_N((k^0-\mu)(1+i\eta),\mathbf{k})\ , \quad (5)$$

the spectral function $A(\omega,\mathbf{k})$ is related to $\tilde{G}_N$ by

$$A(\omega,\mathbf{k})=-i[\tilde{G}_N(\omega+i\eta,\mathbf{k})-\tilde{G}_N(\omega-i\eta,\mathbf{k})]\ . \quad (6)$$

Then Eqs. (2) and (5) lead to

$$\tilde{G}_N^{-1}(k^0-\mu,\mathbf{k})=\gamma\cdot\tilde{k}-\tilde{M}\ , \quad (7)$$

which can be readily inverted because of its analyticity as

$$\tilde{G}_N(k^0-\mu,\mathbf{k})=\frac{\gamma\cdot\tilde{k}+\tilde{M}}{\tilde{k}^2-\tilde{M}^2}\ . \quad (8)$$

Hence we get the analytic structure of the nucleon propagator:

$$G_N(k^0,\mathbf{k})=\frac{\gamma\cdot\tilde{k}+\tilde{M}}{\tilde{k}^2-\tilde{M}^2+i\eta\epsilon(\tilde{k}^0)\epsilon(k^0-\mu)}\ , \quad (9)$$

where $\epsilon(x)=\theta(x)-\theta(-x)=\mathrm{sgn}(x)$

For noninteracting nuclear matter of density $\rho=(2/3\pi^2)k_F^3$, $\mu=(k_F^2+M^2)^{1/2}$, and

$$G_N^0(k)=\frac{\gamma\cdot k+M}{k^2-M^2+i\eta\epsilon(k^0)\epsilon(k^0-\mu)}$$
$$=(\gamma\cdot k+M)\left[\frac{1}{k^2-M^2+i\eta}+2\pi i\delta(k^2-M^2)\theta(k^0)\theta(k_F-|\mathbf{k}|)\right]\ . \quad (10)$$

In general, $\Sigma_N(k)$ is complex because of various excitation modes (e.g., two nucleon–one hole or one nucleon–two hole) that alter the Fermi distribution of the noninteracting ground state. The nucleon-nucleon correlations modify the Fermi distribution further.

To make a connection with the shell-model picture, one usually assumes the quasiparticle property $|\mathrm{Re}\Sigma_N|\gg|\mathrm{Im}\Sigma_N|$. In the limit $\mathrm{Im}\Sigma_N\to 0$, the Dyson equation for the nucleon has the solution

$$G_N(k)=[\gamma\cdot\tilde{k}+\tilde{M}(k)]\left[\frac{1}{\tilde{k}^2-\tilde{M}^2(k)+i\eta}+2\pi i\delta(\tilde{k}^2-\tilde{M}^2)\theta(\tilde{k}^0)\theta(\mu-k^0)\right]$$
$$\equiv G_N^F(k)+G_N^D(k)\ . \quad (11)$$

The single-particle spectrum is the zeros of the argument in the delta function. There are two regular zeros:

$$E_k^{(\pm)}=[\pm\tilde{E}(k)-\Sigma_N^0(k)]\Big|_{k^0=E_k^{(\pm)}}\ , \quad (12)$$

with $\tilde{E}(k)=\{[M+\Sigma_N^s(k)]^2+\mathbf{k}^2[1+\Sigma_N^v(k)]^2\}^{1/2}$. The spectroscopic factor $Z_k^{(\pm)}$ is given by

$$(Z_k^{(\pm)})^{-1}=\left[1+\frac{\partial\Sigma_N^0}{\partial k^0}\mp\frac{\tilde{M}(k)}{\tilde{E}(k)}\,\frac{\partial\Sigma_N^s}{\partial k^0}\mp\frac{\tilde{\mathbf{k}}^2}{\tilde{E}(k)}\,\frac{\partial\Sigma_N^v}{\partial k^0}\right]\Bigg|_{k^0=E_k^{(\pm)}}\ . \quad (13)$$

The Lehman representation requires that $0\le Z_k<1$. $Z_k=1$ would mean that a particle propagates freely with no coupling to excitation modes.

Now we may rewrite the nucleon propagator as

$$G_N(k)=[\gamma\cdot\tilde{k}+\tilde{M}(k)]\left[\frac{1}{\tilde{k}^2-\tilde{M}^2(k)+i\eta}+\pi i\frac{Z_k^{(+)}}{\tilde{E}(k)}\delta(k^0-E_k^{(+)})\theta(\mu-k^0)\right]\ . \quad (14)$$

We will make use of this representation of the nucleon propagator in our calculations with further assumptions on the momentum dependence of Σ and Z_k and the distribution function $\theta(\mu-k^0)$.

III. NUCLEON SELF-ENERGY

We derive the nucleon self-energy from the entire pion-ring series including the lowest-order exchange term. It is convenient to separate the contribution of the intermediate pion-nucleon propagation from that of the intermediate pion-delta propagation (Fig. 1):

$$\Sigma_N^\pi(k)=\Sigma_N^{\pi N}(k)+\Sigma_N^{\pi\Delta}(k)\ . \tag{15}$$

Feynman rules lead us to

$$\Sigma_N^{\pi N}(k)=3i\left[\frac{g_\pi}{2M^*}\right]^2\int\frac{dq}{(2\pi)^4}\gamma^5\gamma\cdot qG_N^*(k-q)\gamma^5\gamma\cdot q\Gamma^2(q)D_\pi^0(q)\Pi(q)D_\pi(q)+\Sigma_{N,\mathrm{ex}}^{\pi N}\ ,$$

$$\Sigma_N^{\pi\Delta}(k)=2i\left[\frac{g_\Delta}{2M^*}\right]^2\int\frac{d^4q}{(2\pi)^4}(q^\mu-\xi\gamma\cdot q\gamma^\mu)D_{\mu\nu}^\Delta(k-q)(q^\nu-\xi\gamma^\nu\gamma\cdot q)\Gamma^2(q)D_\pi^0(q)\Pi(q)D_\pi(q)\ .$$

Here $G_N^*(k-q)$ is the nucleon propagator in the mean-field approximation (MFA):

$$[G_N^*(k)]^{-1}=\gamma\cdot k^*-M^*\ , \tag{16}$$

where M^* and k^* are defined by

$$M^*\equiv M-g_\pi\phi_0,\quad k^{*\mu}\equiv(k^0-g_\omega\omega_0,\mathbf{k})\ , \tag{17}$$

with ϕ_0 and ω_0 being the mean fields of the scalar and the vector mesons, respectively. And $D_\pi^0(q)$ $[D_\pi(q)]$ is the free [full] pion propagator and $\Pi(q)$ is the proper pion self-energy containing the effects of the short-range baryon-baryon correlations through the Landau-Migdal parameter g':

$$\Pi(q)=\frac{\Pi^0(q)}{1+(g'/q^2)\Pi^0(q)}\ , \tag{18}$$

with $\Pi^0(q)=\Pi_N^0(q)+\Pi_\Delta^0(q)$ (see Appendix). $\Gamma(q)$ is the modification of the πNN or $\pi N\Delta$ vertex due to g' effects (Fig. 2):

$$\Gamma(q)=\left[1+\frac{g'}{q^2}\Pi^0(q)\right]^{-1}\ . \tag{19}$$

The parameter ξ in $\Sigma_N^{\pi\Delta}$ is to consider the effects of the off-mass shell Δ.[10]

There is also $\Sigma_{N,\mathrm{ex}}^{\pi N}$, the exchange diagram contribution neglecting correlation effects:

$$\Sigma_{N,\mathrm{ex}}^{\pi N}(k)=3\left[\frac{g_\pi}{2M^*}\right]^2\int\frac{d^3q}{(2\pi)^3}\,\frac{\theta(k_F-|\mathbf{k}-\mathbf{q}|)}{2E_{k-q}^*}[(\gamma\cdot k^*+M^*)q^2-(k^{*2}-M^{*2})\gamma\cdot q]D_\pi^0(q)|_{q^0=k^{*0}-E_{k-q}^*}\ . \tag{20}$$

Note that the nucleon self-energy is diagonal in isospin space for symmetric nuclear matter, i.e., $\Sigma_{N,ab}=\Sigma_N\delta_{ab}$.

Simplifying the γ-matrix algebra, we obtain

$$\begin{aligned}\Sigma_N^{\pi N}(k)=&\,3i\left[\frac{g_\pi}{2M^*}\right]^2\int\frac{d^4q}{(2\pi)^4}\left[\frac{-(\gamma\cdot k^*+M^*)q^2+(k^{*2}-M^{*2})\gamma\cdot q}{(k^*-q)^2-M^{*2}}-\gamma\cdot q\right]\Gamma(q)^2D_\pi^0(q)\Pi(q)D_\pi(q)\\&+3\left[\frac{g_\pi}{2M^*}\right]^2\int\frac{d^3q}{(2\pi)^3}\,\frac{\theta(k_F-|\mathbf{k}-\mathbf{q}|)}{2E_{k-q}^*}[(\gamma\cdot k^*+M^*)q^2-(k^{*2}-M^{*2})\gamma\cdot q]\\&\times\Gamma(q)^2D_\pi^0(q)\Pi(q)D_\pi(q)|_{q^0=k^{0*}-E_{k-q}^*}+\Sigma_{N,\mathrm{ex}}^{\pi N}\end{aligned} \tag{21}$$

$$\Sigma_N^{\pi\Delta}(k)=2i\left[\frac{g_\Delta}{2M^*}\right]^2\int\frac{d^4q}{(2\pi)^4}\left\{\frac{\Lambda_{01}(k,q)}{(k^*-q)^2-M_\Delta^2}+\Lambda_{02}(k,q)+\Lambda_1(k,q)\xi+\Lambda_2(k,q)\xi^2\right\}\Gamma(q)^2D_\pi^0(q)\Pi(q)D_\pi(q)\ , \tag{22}$$

where $\Lambda(k,q)$'s are defined as

$$\Lambda_{01}(k,q)=\frac{2}{3M_\Delta^2}[(k^*\cdot q)^2-k^{*2}q^2](-\gamma\cdot q+\gamma\cdot k^*+M_\Delta)\ ,$$

$$\Lambda_{02}(k,q)=\frac{2}{3M_\Delta^2}q^2(-\gamma\cdot q+\gamma\cdot k^*+M_\Delta)\ ,$$

$$\Lambda_1(k,q)=\frac{2}{3M_\Delta}q^2+\frac{4}{3M_\Delta^2}(q^2-k^*\cdot q)\gamma\cdot q\ ,$$

$$\Lambda_2(k,q)=-\frac{4}{3M_\Delta}q^2-\frac{2}{3M_\Delta^2}[q^2\gamma\cdot k^*+(q^2-2k^*\cdot q)\gamma\cdot q]\ .$$

Remembering that $\Pi(q)$ is analytic in the region $\mathrm{Re}q^0\cdot\mathrm{Im}q^0>0$ of the complex q^0 plane, we can make use of Wick rotation to perform the q^0 integral. For the q^0 integral of $\Sigma_N^{\pi N}$, we use

$$i\int_{-\infty}^{\infty} dq^0 \frac{f(q^0)}{(k^{0*}-q^0)^2-E_{k-q}^{*2}+i\eta}$$
$$=-\int_{-\infty}^{\infty} d\nu \frac{f(i\nu)}{(k^{*0}-i\nu)^2-E_{k-q}^{*2}}+\frac{\pi}{E_{k-q}^*}[\theta(k^{*0}-E_{k-q}^*)f(k^{*0}-E_{k-q}^*)-\theta(-k^{*0}-E_{k-q}^*)f(k^{*0}+E_{k-q}^*)] \ . \quad (23)$$

A similar relation applies to the q^0 integral of $\Sigma_N^{\pi\Delta}$, where additional nonpole terms are straightforward to compute. Also, the angular integral is simplified by rotational invariance:

$$\int \frac{d^3q}{(2\pi)^3} g(\mathbf{k},\mathbf{q})\gamma\cdot\mathbf{q}=\frac{\gamma\cdot\mathbf{k}}{|\mathbf{k}|^2}\int \frac{d^3q}{(2\pi)^3} g(\mathbf{k},\mathbf{q})\mathbf{q}\cdot\mathbf{k} \ ,$$

where $g(\mathbf{k},\mathbf{q})$ is a scalar.

Then the nucleon self-energy is reduced to the following two-dimensional integral which we need to evaluate numerically:

$$\Sigma_N^{\pi N}(k)=-3\left[\frac{g_\pi}{2M^*}\right]^2\int \frac{d|\mathbf{q}|d\nu}{(2\pi)^3}|\mathbf{q}|^2\Gamma^2(q)D_\pi^0(q)\Pi(q)D_\pi(q)$$
$$\times[-(\gamma\cdot k^*+M^*)q^2I_1^N-(k^{*2}-M^{*2})(\gamma^0\nu I_2^N+\gamma\cdot\mathbf{k}I_3^N)]|_{q^0=i\nu}$$
$$+3\left[\frac{g_\pi}{2M^*}\right]^2\int \frac{d|\mathbf{q}|d(\hat{q}\cdot\hat{k})}{(2\pi)^2 2E_{k-q}^*}|\mathbf{q}|^2\Gamma^2(q)D_\pi^0(q)\Pi(q)D_\pi(q)$$
$$\times\left[(\gamma\cdot k^*+M^*)q^2-(k^{*2}-M^{*2})\left[\gamma^0q^0-\gamma\cdot\mathbf{k}\frac{\mathbf{k}\cdot\mathbf{q}}{|\mathbf{k}|^2}\right]\right]\Bigg|_{q^0=k^{*0}-E_{k-q}^*}$$
$$\times[\theta(k^F-|\mathbf{k}-\mathbf{q}|)-\theta(k^{*0}-E_{k-q}^*)]+\Sigma_{N,\text{ex}}^{\pi N}(k) \ , \quad (24)$$

$$\Sigma_N^{\pi\Delta}(k)=-2\left[\frac{g_\Delta}{2M^*}\right]^2\int \frac{d|\mathbf{q}|d\nu}{(2\pi)^3}|\mathbf{q}|^2\Gamma^2(q)D_\pi^0(q)\Pi(q)D_\pi(q)$$
$$\times\frac{2}{3M_\Delta^2}\Bigg[\left[\frac{1}{4}(k^{*2}+q^2-M_\Delta^2)^2-k^{*2}q^2\right][(\gamma\cdot k^*+M_\Delta)I_1^\Delta+\gamma^0\nu I_2^\Delta+\gamma\cdot\mathbf{k}I_3^\Delta]$$
$$-\frac{1}{2}(M_\Delta^2-k^{*2}-q^2)(\gamma\cdot k^*+M_\Delta)+2q^2M_\Delta(1+\xi-2\xi^2)$$
$$+2q^2\gamma\cdot k^*(1-\xi^2)+[4(\xi^2-\xi)+1]\left[-\gamma^0k^{*0}\nu^2+\frac{2}{3}\gamma\cdot\mathbf{k}|\mathbf{q}|^2\right]\Bigg]\Bigg|_{q^0=i\nu} \ . \quad (25)$$

Here I^N's and I^Δ's are the angular integrals of the pole terms on the imaginary q^0 axis:

$$I_1^N=\frac{1}{2|\mathbf{q}||\mathbf{k}|}\left[\ln\left[\frac{(k^{*0}-E_{|\mathbf{k}|-|\mathbf{q}|}^*)^2+\nu^2}{(k^{*0}-E_{|\mathbf{k}|+|\mathbf{q}|}^*)^2+\nu^2}\right]+\ln\left[\frac{(k^{*0}+E_{|\mathbf{k}|-|\mathbf{q}|}^*)^2+\nu^2}{(k^{*0}+E_{|\mathbf{k}|+|\mathbf{q}|}^*)^2+\nu^2}\right]\right] ,$$

$$I_2^N=\frac{1}{|\mathbf{q}||\mathbf{k}|}\left[\tan^{-1}\left[\frac{k^{*0}-E_{|\mathbf{k}|-|\mathbf{q}|}^*}{\nu}\right]-\tan^{-1}\left[\frac{k^{*0}-E_{|\mathbf{k}|+|\mathbf{q}|}^*}{\nu}\right]\right.$$
$$\left.+\tan^{-1}\left[\frac{k^{*0}+E_{|\mathbf{k}|-|\mathbf{q}|}^*}{\nu}\right]-\tan^{-1}\left[\frac{k^{*0}+E_{|\mathbf{k}|+|\mathbf{q}|}^*}{\nu}\right]\right] ,$$

$$I_3^N=\frac{1}{|\mathbf{k}|^2}\left[2+\frac{1}{2}(|\mathbf{k}|^2+|\mathbf{q}|^2+M^{*2}-k^{*02}+\nu^2)I_1^N-k^{*0}\nu^2I_2^N\right] ,$$

where $E_{|\mathbf{k}|\pm|\mathbf{q}|}^*=[M^{*2}+(|\mathbf{k}|\pm|\mathbf{q}|^2]^{1/2}$ and I_i^Δ's are the same as I_i^N's with the replacement $M^*\to M_\Delta$.

The integration over $|\mathbf{q}|$ and ν can be done by using polar coordinates. The imaginary part of the nucleon self-energy comes from the second integral of (24) only because $\Pi^0(q)$ is real on the imaginary q^0 axis. It is apparent that $\mathrm{Im}\Sigma_N$ changes sign at $k^{*0}=(k_F^2+M^{*2})^{1/2}$ so that the chemical potential is still the same as the MFA result $\mu=(k_F^2+M^{*2})^{1/2}+g_\omega\omega_0$. This is an artifact of using the MFA nucleon propagator in the evaluation of the nucleon self-energy. In general, the chemical potential may be determined from baryon number conservation.

Note that the Lorentz structure of the nucleon self-energy from the pion-ring series is of the general form (1):

$$\Sigma_N^\pi=\Sigma_N^{\pi,s}+\gamma^0\Sigma_N^{\pi,0}-\boldsymbol{\gamma}\cdot\mathbf{k}\Sigma_N^{\pi,v} . \tag{26}$$

The total nucleon self-energy is the sum of the self-energy of the MFA and the above self-energy from the pion-ring series:

$$\Sigma_N(k)=G_N^{-1}(k)-G_N^{0-1}(k) , \tag{27}$$

$$=G_N^{-1}(k)-G_N^{*-1}(k)+G_N^{*-1}(k)-G_N^{0-1}(k) , \tag{28}$$

$$=\Sigma_N^\pi(k)+\Sigma_N^*(k) , \tag{29}$$

where $\Sigma_N^*(k)=-g_\pi\phi_0+\gamma^0 g_\omega\omega_0$.

IV. INPUT PARAMETERS

We evaluate the nucleon self-energy Σ_N^π explicitly with the parameters determined by the nuclear matter saturation properties. This provides an idea of how the dynamics of the nucleon changes in the presence of the pionic medium polarization.

First, we discuss how the input parameters are determined. The energy density from the pion-ring series[3,11,12] is

$$\mathcal{E}_\pi^{\text{ring}}=-\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\{\ln[1-D_\pi^0(k)\Pi(k)]+D_\pi^0(k)\Pi(k)\}+\mathcal{E}_\pi^{\text{ex}} . \tag{30}$$

We add this pion-ring energy density to the mean-field energy density ($\mathcal{E}^{\text{MFA}}$ of Ref. 4). Then the total energy density is minimized with respect to the scalar mean field or M^*. Phenomenological parameters g' and Λ (see Appendix) are taken as conventional values $g'\approx 0.7$ and $\Lambda\approx 1$ GeV.[2,13,14] However, the dependence on the cutoff Λ is rather delicate, and $\Lambda\approx 1$ GeV is favored by the stability and the incompressibility of nuclear matter. The parameter ξ has been suggested to be $1\geq\xi\geq 0$ from theoretical or phenomenological considerations.[10] The other parameters in the theory, m_ϕ and g_ω, are adjusted to the saturation properties of nuclear matter: The binding energy per nucleon has a maximum value of 15.76 MeV at $k_F=1.3\ \text{fm}^{-1}$.

For the case of $\xi=0$, $g'=0.7$, and $\Lambda=1.15$ MeV, which reproduces the density dependence of the binding energy very well with the resulting incompressibility being around 200 MeV, we have (at $k_F=1.3\ \text{fm}^{-1}$) $M^*=0.95M$, $m_\phi=2815$ MeV, and $g_\omega=8.844$. These are the values of the input parameters we use to calculate the nucleon self-energies. Figure 3 shows the k_F dependence of the binding energy of nuclear matter. In Fig. 4 the pion-ring energy and the scalar mean-field energy are shown separately. The saturation of nuclear matter is achieved adding the ω mean-field energy to the short-dashed curve of Fig. 4.

Next, we mention that our method to include the short-range correlation effects in calculating the energy density of nuclear matter differs from the nonrelativistic formalism. In the nonrelativistic approach, the pion propagator is replaced by the one-pion-exchange potential in momentum space. The short-range correlation effects may be included in different ways, one of which is to replace the one-pion-exchange potential by the effective particle-hole interaction $V_{\text{eff}}^\pi(k)$ in the pionic channel[2] (diagonal in isospin space for symmetric nuclear matter):

$$V_{\text{eff}}^\pi(k)=\frac{1}{k^2-m_\pi^2}+\frac{g'}{|\mathbf{k}|^2} . \tag{31}$$

Then substituting the nonrelativistic pion self-energy $\Pi_{\text{NR}}^0(k)$ for $\Pi(k)$ in (30) leads to the conventional ring series which might be divergent due to the zero-momentum repulsion if $D_\pi^0(k)\Pi(k)<-1$.[12]

In our approach, we consider that the ω mean field simulates part of the repulsive short-range baryon-baryon interaction. The influence of the short-range repulsion on the pion propagation is also included in an analogy with the nonrelativistic treatment of the pion dynamics. Then our pion-ring energy reflects the effects of the short-range repulsion as follows:

$$\ln[1-D_\pi^0(k)\Pi(k)]+D_\pi^0(k)\Pi(k)=\ln\left[1-\left[D_\pi^0(k)-\frac{g'}{k^2}\right]\Pi^0(k)\right]+\left[D_\pi^0(k)-\frac{g'}{k^2}\right]\Pi^0(k)$$
$$-\left[\ln\left[1+\frac{g'}{k^2}\Pi^0(k)\right]-\frac{g'}{k^2}\Pi^0(k)\right]+D_\pi^0(k)\Pi^0(k)\frac{-(g'/k^2)\Pi^0(k)}{1+(g'/k^2)\Pi^0(k)} . \tag{32}$$

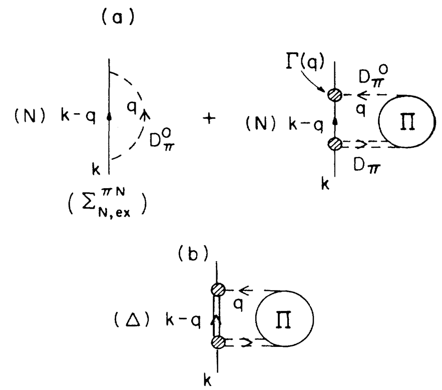

FIG. 1. Nucleon self-energy diagrams. (a) Contribution of the intermediate π-N: $\Sigma_N^{\pi N}$. This includes the lowest-order exchange contribution $\Sigma_{N,\mathrm{ex}}^{\pi N}$. (b) Contribution of the intermediate π-Δ: $\Sigma_N^{\pi\Delta}$.

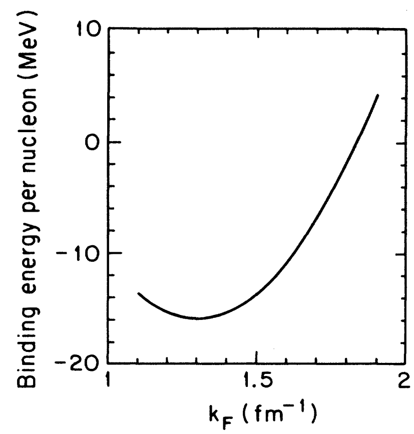

FIG. 3. k_F dependence of the binding energy of nuclear matter.

If we identify $D_\pi^0(k)-g'/k^2$ as V_π^{eff}, then the difference between our pion-ring energy and the conventional pion-ring energy is that the diagrams with fewer than two pion propagators (Fig. 5) are not included explicitly except the exchange diagram. In the presence of the mean field of ω mesons, including these diagrams would be wrong since they are caused by the ω-meson effects already included.

V. RESULTS

Now we discuss our numerical results. We are especially interested in the quasiparticle properties. Thus it is informative to look into the energy dependence of Σ_N^π near the Fermi surface [i.e., at $k_0 \approx \mu = (k_F^2 + M^{*2})^{1/2} + g_\omega \omega_0$].

From Eqs. (24) and (25), it is apparent that the Lorentz structure of the nucleon self-energy at $k_0 \approx \mu$ is given by $(\gamma \cdot k^* + M^*)$ for $\Sigma_N^{\pi N}$ and mostly by $(\gamma \cdot k^* + M_\Delta)$ for $\Sigma_N^{\pi\Delta}$. This implies that both the scalar part $\Sigma_N^{\pi,s}$ and the vector part $\Sigma_N^{\pi,0}$ are attractive and $|\Sigma_N^{\pi,0}| > |\Sigma_N^{\pi,s}|$. Thus, combined with Σ_N^*, the total real nucleon self-energy becomes roughly $\Sigma_N \approx 50\gamma^0 - 100$ MeV at $k_F = 1.3$ fm^{-1}.

In Fig. 6(a), $|\mathrm{Re}\Sigma_N^{\pi,s}(k_0,k_F)|$ is shown as a function of k_0 for $k_F = 1.3$ fm^{-1}. Note that the derivative of $|\mathrm{Re}\Sigma_N^{\pi,s}(k_0,k_F)|$ with respect to k_0 has a minimum at $k_0 = \mu$, which is a consequence of the nucleon propagator's analytic structure and a well-known result in the dynamic shell model.[15] However, the k_0 dependence of $\mathrm{Re}\Sigma_N^\pi$ is so strong that the spectroscopic factor or quasiparticle strength Z_{k_F} (<0.5 for $k_F = 1.3$ fm^{-1}) becomes too small. It should be recalled that most calculations in the conventional approach result in $Z_{k_F} \approx 0.7$ for $k_F \approx 1.3$ fm^{-1}.[16] The small Z_{k_F} means that a nucleon (N) in the nuclear medium has a significant chance to be in a NN-hole or $N\Delta$-hole state. Therefore, the strong energy dependence of $\mathrm{Re}\Sigma_N^\pi$ makes it necessary to keep $\mathrm{Im}\Sigma_N^\pi$, which might be important for realistic calculations of the observables. It is worthwhile to mention that most of the energy dependence comes from $\Sigma_N^{\pi,N}$ as is known in the nonrelativistic calculations.[7]

Figure 6(b) shows the result for $|\mathrm{Im}\Sigma_N^\pi(k_0,k_F)|$. The imaginary part of the nucleon self-energy vanishes at $k_0 = \mu$, if the nucleus is stable. At $k_0 \approx \mu$ it behaves as

$$\mathrm{Im}\Sigma_N^\pi(k_0,|\mathbf{k}|) \approx \alpha_k (k_0-\mu)^2 \epsilon(\mu-k_0) \ . \tag{33}$$

Here α_k measures the spreading of the quasiparticle spec-

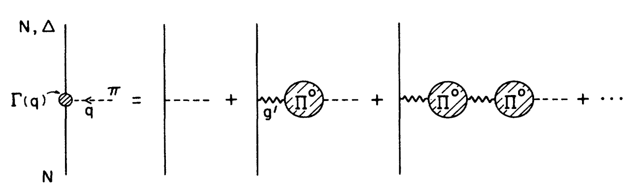

FIG. 2. Modification of the form factor due to g' correlations.

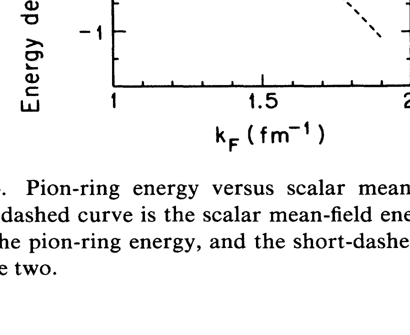

FIG. 4. Pion-ring energy versus scalar mean-field energy: The long-dashed curve is the scalar mean-field energy, the solid curve is the pion-ring energy, and the short-dashed curve is the sum of the two.

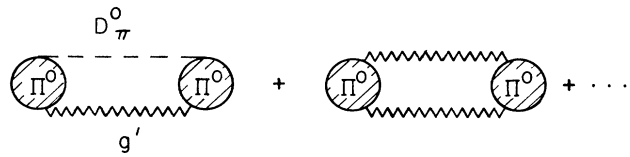

FIG. 5. Diagrams for repulsive short-range interaction energy. These diagrams, present in the nonrelativistic approach, are absent in our approach because they are included in terms of the ω mean-field energy.

trum (or decay rate of the quasiparticle). Again, the intermediate Δ propagation does not contribute to the imaginary part of the nucleon self-energy.

VI. DISCUSSION

Now we discuss the limitations and possible improvements of our approach. First, we should mention that the above nucleon self-energy was obtained by replacing the full nucleon propagator with the MFA nucleon propagator. Although the mean field is determined by a variational method, it does not include the complete pionic dressing of the nucleon propagator.

The strength of the self-energy from the pion exchanges is larger than 10% of the nucleon mass for $k_F \geq 1.3$ fm^{-1}. Thus the nucleons move in rather strong pion-exchange-induced potentials. These potentials are not as strong as those of the Walecka model. More importantly, the self-energy is strongly energy-dependent, which means that the quasiparticle approximation may not be valid at high densities. Then it may be necessary to keep $\mathrm{Im}\Sigma_N$ in the calculation of the pion and nucleon self-energies.

The net result is that self-consistency corrections seem to be important. To be self-consistent, the input propagators appearing in the nucleon self-energy diagrams should be equivalent to the output propagators resulting from self-energy calculations. Carrying out the fully self-consistent procedure would require an extremely elaborate calculation, because treating the important k^μ dependence requires the determination of $\Sigma_N(k)$ at each k^μ. There are also problems associated with avoiding overcounting. Moreover, the phenomenological parameters (g', Λ) are not well understood yet. This is a necessary step before making detailed self-consistency calculations.

Thus we limit our discussion to the qualitative aspects of self-consistency. A commonly used approximation is to neglect the k^μ dependence entirely. This may be reasonable to calculate the correlation effects due to ω exchange,[17] but is less accurate when one calculates the strongly energy-dependent pionic correlations.

The important issue is how to treat the energy dependence more consistently in a practical way. In this regard, we follow the procedure in Ref. 7 and the representation (14) in the quasiparticle limit is informative. If we use this nucleon propagator in the calculation of the pion self-energy, the main effects would be the suppression due to the spectroscopic factor Z_k (<1) and enhancement due to the $\tilde{M}$ $(<M^*)$. Thus one may expect the simple iteration with constant values for Σ_N and Σ_k would be working. Here $\Sigma_N \approx \Sigma_N^s + \gamma^0 \Sigma_N^0$. Guessing the input values for Σ_N^s, Σ_N^0, and Z_k, we evaluate the pion self-energy and then the nucleon self-energy at $|\mathbf{k}| = k_F$, where the quasiparticle limit is exact. When the output values for Σ_N^s, Σ_N^0, and Z_k coincide with the input values, the self-consistency is achieved.

However, so far as we have checked, the above iteration procedure does not converge for $k_F > 1.4$ fm^{-1}. The nonconvergence of the direct iteration method has already been observed in the self-consistent Hartree-Fock calculations for high densities $(\rho \geq 2\rho_0)$[9,18]. In our case, the main reason for the failure of the iteration method seems to be the oversimplifying of the momentum distribution of the nucleons.

While the momentum distribution of nucleons is a Fermi sphere in the mean-field approximation, it is modified by the interaction through pion exchanges. In the quasiparticle limit, the momentum distribution is still a step function $\theta(\mu - E_k^{(+)})$, which is an artifact of the assumption $\mathrm{Im}\Sigma_N \rightarrow 0$. Keeping the imaginary part of the nucleon self-energy results in the leak out of the momentum distribution above $|\mathbf{k}| = k_F$. Thus $Z_k < 1$ becomes consistent with the baryon number conservation. In other words, the energy dependence of the nucleon self-energy requires keeping $\mathrm{Im}\Sigma_N$ to get the correct momentum distribution of nucleons. Hence there is definitely more work to be done in this direction.

Also, it would be important to investigate the effects of the correlated 2π exchanges, which are difficult to compute explicitly. For instance, with the idea that π-π correlations in the $J=0$, $T=0$ channel might be strongly enhanced in the nuclear medium, Schuck, Chanfray, and Nörenberg[19] studied nuclear matter saturation by considering a two-pion pair-coherent state as an alternative to the σ meson of the relativistic mean-field theory. In-

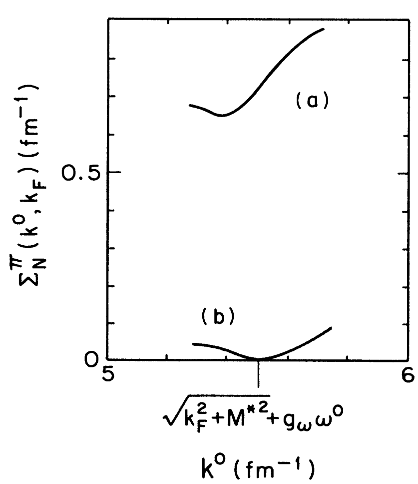

FIG 6. Nucleon self-energy at the Fermi surface. (a) The real part of the nucleon self-energy: $|\mathrm{Re}\Sigma_N^\pi(k^0, k_F)|$. (b) The imaginary part of the nucleon self-energy: $|\mathrm{Im}\Sigma_N^\pi(k^0, k_F)|$.

terestingly, however, they found that the attractive $\pi\pi$ interaction does not play the major role in nuclear matter binding properties. In this model, nuclear matter saturation is achieved mainly due to the strong density dependence of screening effects[8] in the nucleon-hole and Δ-hole excitations. But the important ω repulsion is not included explicitly. Nevertheless, their result seems to indicate the importance of summing the nucleon-hole and Δ-hole excitations to all orders (this corresponds to the pion-ring series) and emphasizes the effects of the short-range correlations between baryons.

Finally, there is an ongoing issue about pionic enhancement in nuclei. An interesting concept is the pion excess per nucleon, which is suggestive of overall pionic enhancement in nuclei. Nonrelativistic calculations agree with the presence of roughly one excess pion per ten nucleons.[20–22] Since more virtual pions may be associated with the increase of sea quarks (correspondingly, antiquarks) in nuclei, the idea of nuclear pion excess has been rather successful in explaining the slight enhancement of the nuclear structure functions at low Bjorken x.[22,23] However, the recent E772 experiment at Fermilab claims no nuclear enhancement of the antiquark distribution per nucleon.[24] The details of the nuclear pion excess in our formalism and whether or not the idea of nuclear pion excess is compatible with this experiment will be discussed elsewhere.

APPENDIX: ANALYTIC EXPRESSION OF THE PION SELF-ENERGY

The effective πNN and $\pi N\Delta$ interactions in the mean-field background are as follows[3] (see Ref. 10 for the discussion of the parameter ξ):

$$\mathcal{L}_{\pi NN}=-\frac{g_\pi}{2M^*}\bar{N}\gamma^5\gamma^\mu\tau\cdot\partial_\mu\pi N+\mathcal{O}(\pi^3)\ ,$$

$$\mathcal{L}_{\pi N\Delta}=\left[\frac{f_\Delta}{m_\pi}\right]\left[\frac{M}{M^*}\right]\bar{\Delta}_\mu(g^{\mu\nu}-\gamma^\mu\gamma^\nu\xi)\mathbf{T}\cdot\partial_\nu\pi N+\mathcal{O}(\bar{\pi}^3)+\text{H.c.}$$

The lowest-order pion self-energies $\Pi^0_N(q)$ and $\Pi^0_\Delta(q)$ including the form factor $F(q)=\exp(q^2/\Lambda^2)$ can be expressed in analytic forms for certain kinematical regions.

Performing the angular intergration, we have

$$\Pi^0_N(q)=-\left[\frac{g_\pi}{2M^*}\right]^2 q^2F(q)^2M^{*2}\frac{1}{|\mathbf{q}|\pi^2}I_N(q,k_F,M^*)\ ,$$

where

$$I_N(q,k_F,M^*)=\int_0^{k_F}d|\mathbf{k}|\frac{|\mathbf{k}|}{E^*_k}\ln\left[\frac{(q^2+2|\mathbf{k}||\mathbf{q}|^2-4q^{02}E^{*2}_k}{(q^2-2|\mathbf{k}||\mathbf{q}|^2-4q^{02}E^{*2}_k}\right]$$

and with $g_\Delta/2M\equiv f_\Delta/m_\pi$,

$$\begin{aligned}\Pi^0_\Delta(q)=&-\frac{4}{9}\left[\frac{g_\Delta}{2M^*}\right]^2F(q)^2\frac{1}{M^2_\Delta\pi^2}\\&\times\left\{\left[q^2M^{*2}-\frac{(M^2_\Delta-M^{*2}-q^2)^2}{4}\right]\left[\frac{(M^*+M_\Delta)^2-q^2}{4|\mathbf{q}|}\right]I_\Delta(q,k_F,M^*)\right.\\&+\left[\frac{1}{2}(q^2+M^{*2}-M^2_\Delta)[(M^*+M_\Delta)^2-q^2]-2M^{*2}|\mathbf{q}|^2\right.\\&\left.+8M^{*2}q^{02}(\xi-\xi^2)+4M^{*2}q^2(\xi^2-1)+4M^*M_\Delta q^2(2\xi^2-\xi-1)\right]I_1(k_F,M^*)\\&\left.+\left[2\left[q^{02}+\frac{|\mathbf{q}|^2}{3}\right](4\xi-4\xi^2-1)\right]I_2(k_F,M^*)\right\}\ ,\end{aligned}$$

where

$$I_\Delta(q,k_F,M^*)=\int_0^{k_F}d|\mathbf{k}|\frac{|\mathbf{k}|}{E^*_k}\ln\left[\frac{(q^2+M^{*2}-M^2_\Delta+2|\mathbf{k}||\mathbf{q}|)^2-4q^{02}E^{*2}_k}{(q^2+M^{*2}-M^2_\Delta-2|\mathbf{k}||\mathbf{q}|)^2-4q^{02}E^{*2}_k}\right]\ ,$$

$$I_1(k_F,M^*)=\int_0^{k_F}d|\mathbf{k}|\frac{|\mathbf{k}|^2}{E^*_k}=\frac{k_FE^*_{k_F}}{2}-\frac{M^{*2}}{2}\ln\left[\frac{k_F+E^*_{k_F}}{M^*}\right]\ ,$$

$$I_2(k_F,M^*)=\int_0^{k_F} d|\mathbf{k}|\frac{|\mathbf{k}|^4}{E_k^{*2}}$$

$$=\frac{k_F^3E_{k_F}^*}{4}-\frac{3}{8}M^*k_FE_{k_F}^*+\frac{3}{8}M^{*4}\ln\left[\frac{k_F+E_{k_F}^*}{M^*}\right].$$

For $q^{02}(1-4M^{*2}/q^2)\geq 0$, $I_N(q,k_F,M^*)$ can be evaluated analytically:

$$I_N(q,k_F,M^*)=\sum_{\pm}\left[(E_{k_F}^*-E_{\alpha_\pm}^*)\ln\left[\frac{k_F-\alpha_\pm}{k_F+\alpha_\pm}\right]+E_{\alpha_\pm}^*\ln\left[\frac{E_{\alpha_\pm}^*E_{k_F}^*+M^{*2}+\alpha_\pm k_F}{E_{\alpha_\pm}^*E_{k_F}^*+M^{*2}-\alpha_\pm k_F}\right]-2\alpha_\pm\ln\left[\frac{k_F+\alpha_\pm}{M^*}\right]\right],$$

where

$$\alpha_\pm=\frac{1}{2}\left\{|\mathbf{q}|\pm\left[q^{02}\left[1-\frac{4M^{*2}}{q^2}\right]\right]^{1/2}\right\}.$$

For $q^{02}\{[1+(M^{*2}-M_\Delta^2)/q^2]^2-4M^{*2}/q^2\}\geq 0$, $I_\Delta(q,k_F,M^*)$ can be evaluated analytically:

$$I_\Delta(q,k_F,M^*)=\sum_{\pm}\left[(E_{k_F}^*-E_{\beta_\pm}^*)\ln\left[\frac{k_F-\beta_\pm}{k_F+\beta_\pm}\right]+E_{\beta_\pm}^*\ln\left[\frac{E_{\beta_\pm}^*E_{k_F}^*+M^{*2}+\beta_\pm k_F}{E_{\beta_\pm}^*E_{k_F}^*+M^{*2}-\beta_\pm k_F}\right]-2\beta_\pm\ln\left[\frac{k_F+\beta_\pm}{M^*}\right]\right],$$

where

$$\beta_\pm=\frac{1}{2}\left[|\mathbf{q}|\left[1+\frac{M^{*2}-M_\Delta^2}{q^2}\right]\pm\left\{q^{02}\left[\left[1+\frac{M^{*2}-M_\Delta^2}{q^2}\right]^2-\frac{4M^{*2}}{q^2}\right]\right\}^{1/2}\right].$$

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