Relativistic kinetic equations with mesonic degrees of freedom

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We extend the same techniques found to be successful in condensed matter physics for deriving Boltzmann transport equations to the relativistic domain to facilitate the study of nonequilibrium phenomena where relativistic effects are important. Although the ensuing equations are discussed in the context of the relativistic heavy-ion system, the discussion is sufficiently general to cover other important physical situations such as those of astrophysical origin. Starting from an arbitrary local quantum field theory, we show how one may construct a Boltzmann transport equation and what the necessary assumptions that go into its construction are. To illustrate these techniques we derive a set of coupled Boltzmann equations with medium-dependent collision terms for relativistic fermions and spin-zero bosons interacting via a Yukawa coupling. We find that the dynamics may be described in terms of the fermion distribution function alone only under the very restrictive condition that the boson fields remain in local equilibrium throughout the collision process. Moreover, we find that one must go beyond the mass-shell constraint to find a physically reasonable boson production mechanism.

I. INTRODUCTION

In recent years, nonrelativistic Boltzmann-Uehling-Uhlenbeck (BUU) equations have been employed in the study of heavy-ion collisions with some success.[1] As one goes to increasingly higher energies, relativistic kinematics become necessary. To meet this need, relativistic Vlasov equations have been developed.[2] In addition, these equations have been extended through the *ad hoc* addition of collision terms using free-space nucleon-nucleon cross sections.[3-5] However, with few exceptions,[6,7] little work has been done to justify this naive extension of the collisionless Boltzmann equation. It is the purpose of this work to investigate whether or not such a picture is justified through the systematic construction of collision terms with medium dependence. It is also our aim to show how relativistic kinetic equations may be obtained from the underlying structure of relativistic quantum field theory (QFT). In our construction of collision terms, we utilize the same methods found to be successful in the nonrelativistic theory,[8,9] namely, the gradient expansion and the Kadanoff-Baym *Ansatz.*

Phenomenological hadronic field theories[10] provide a framework for incorporating the needs of special relativity and quantum mechanics. In addition, because they are based on quantum field theory, they allow a discussion of the creation and annihilation processes. As a starting point, we have adopted a quantum field theory consisting of spin-half fermions (nucleons) interacting with spin-zero scalar bosons (σ mesons) via a Yukawa coupling. Although this model cannot be taken seriously in describing nuclear systems, it does saturate and allows for a description of creation and annihilation of dynamical mesons, thus providing a reasonable starting point for the investigation of a relativistic quantum transport theory. This model has also been studied by Li *et al.*[6] However, in addition to ignoring retardation effects, their description does not allow for a discussion of boson creation and annihilation processes. Botermans and Malfliet[11] have investigated a similar model using a Dirac-Brueckner T-matrix approach but also failed to include retardation effects. This work goes beyond their investigations by including retardation effects and deriving a kinetic equation describing meson production. We want to emphasize that, in this work, we propose to study what elements of a complete relativistic transport theory may be important, and leave the more ambitious goal of using a "realistic" theory for later. At this point it is not even clear how one should calculate ground-state properties of nuclei with a realistic hadronic theory.[12]

The outline of the paper is as follows. In Sec. II we review the basic ideas upon which this work is founded. In particular, we review the nonequilibrium quantum field theory within the Schwinger-Keldysh closed-time-path framework[13-15] because, in addition to being essential to this work, we feel that the nonequilibrium theory is sufficiently unfamiliar to warrant such a review. We then show how the Schwinger-Dyson equations on the closed-time path may be used to easily obtain a "generalized" Boltzmann equation, also known as the "Kadanoff-Baym equation,"[9] and we show that, with an appropriate *Ansatz,* this equation may be reduced to the familiar form of a Boltzmann equation. Discussion of the validity of this "Kadanoff-Baym" *Ansatz*[9] will conclude Sec. II. In Sec. III of this paper, we apply the general recipe for constructing the quantum kinetic equations outlined in Sec. II to the simple Yukawa model. In doing so, we are able to derive a set of coupled Boltzmann equations complete with collision terms for the interacting fermion-boson system. We show that under the very restrictive condition of local equilibrium, the boson distribution function may be eliminated leaving a single Boltzmann equation for the fermion distribution function, thus making contact with other approaches. Finally, in the last section we conclude.

Note: For the most part, we follow the conventions of Bjorken and Drell[16] setting $g_{\mu\nu}=\text{diag}(1,-1,-1,-1)$ and $px \equiv p\cdot x = p_\mu x^\mu$. Also, unless explicitly indicated, we work with natural units $\hbar=c=1$.

II. BASIC FORMALISM

In this section we review the essential ideas central to this work. We will keep the discussion sufficiently general that the ideas expressed here may be applied to any quantum field theory.

A. Nonequilibrium quantum field theory

We choose to work within the Schwinger-Keldysh framework[13–15] rather than that of the more complex thermofield dynamics[17] because we feel that the closed-time-path method is more closely related to the methods of ordinary QFT. To illustrate the salient features of the formalism, we will concentrate on a QFT with a scalar bosonic field Φ. The generalization to more complex field theories is reasonably straightforward.

The basic problem of quantum field theory is to solve the *Heisenberg* equations of motion

$$\frac{d}{dt}Q(t)=i[H-j(t)\cdot\Phi(t),Q(t)] \tag{2.1}$$

for a *Heisenberg* operator Q. For later convenience the Heisenberg field operator Φ has been coupled to an external space-time-dependent c-number source $j(t)$ with compact support. Furthermore, the Hamiltonian H is assumed to be time independent. Here, and in most of this section, we have adopted the convention of suppressing the spatial dependence of Φ and we use the shorthand notation $j(t)\cdot\Phi(t)$ for

$$\int d^3x\, j(t,\mathbf{x})\Phi(t,\mathbf{x}) \ ,$$

suppressing spatial integrations.

In the Heisenberg picture, observables are calculated by taking expectation values of the appropriate operators with respect to the initial state of the system. In general, the initial state is a *mixed* state and must be described by a density matrix $\rho(t_0)$, t_0 being the initial time. The expectation value of a Heisenberg operator $Q(t)$ is given by

$$\langle Q(t)\rangle=\text{Tr}[\rho(t_0)Q(t)] \ . \tag{2.2}$$

To facilitate the construction of Green functions, it is convenient to work in a *sourceless* representation defined by

$$Q'(t)=U(t,t_0)Q(t)U^\dagger(t,t_0) \ , \tag{2.3}$$

where the *unitary* operator

$$U(t_1,t_2)\equiv\begin{cases}\mathcal{T}\exp\left[i\int_{t_2}^{t_1}dt\, j(t)\cdot\Phi'(t)\right] \\ \text{for } t_1\geq t_2 \ , \\ \tilde{\mathcal{T}}\exp\left[i\int_{t_1}^{t_2}dt\, j(t)\cdot\Phi'(t)\right] \\ \text{for } t_1\leq t_2\end{cases} \tag{2.4}$$

satisfies the usual group property $U(t_1,t_3)U(t_3,t_2)=U(t_1,t_2)$. Here $\mathcal{T}$ is the usual time ordering operator while $\tilde{\mathcal{T}}$ is an *anti*time ordering operator and Φ' is the field operator in the sourceless representation. It is important to note that, for $t>t_0$,

$$U^\dagger(t,t_0)=U(t_0,t)$$

describes *backward* time evolution from t to t_0. The sourceless representation should not be confused with the more familiar interaction picture representation which is commonly used for deriving Feynman rules. As will be illustrated in Sec. III, the sourceless representation is ideally suited for deriving Schwinger-Dyson equations; these are *nonperturbative* equations satisfied by the Green's functions, whereas the Feynman rules are used in connection with perturbation theory.

The transformed operator $Q'(t)$ satisfies the *sourceless* field equation

$$\frac{d}{dt}Q'(t)=i[H',Q'(t)] \ , \tag{2.5}$$

where the transformed Hamiltonian $H'=U(t,t_0)HU^\dagger(t,t_0)$ is time independent by virtue of $[H',H']=0$.

The correlation function $\langle\Phi(t_1)\Phi(t_2)\rangle$ may be expressed in the sourceless representation as

$$\langle\Phi(t_1)\Phi(t_2)\rangle = \langle U^\dagger(t_1,t_0)\Phi'(t_1)U(t_1,t_2)\Phi'(t_2)U(t_2,t_0)\rangle \ , \tag{2.6}$$

or by functional differentiation

$$\langle\Phi(t_1)\Phi(t_2)\rangle=\left\langle\frac{\delta U^\dagger(\infty,t_0)}{i\delta j(t_1)}\frac{\delta U(\infty,t_0)}{i\delta j(t_2)}\right\rangle \ . \tag{2.7}$$

Using a trick originally due to Schwinger,[13] we write

$$\langle\Phi(t_1)\Phi(t_2)\rangle=\frac{1}{i^2}\frac{\delta^2}{\delta j_-(t_1)\delta j_+(t_2)}\Bigg|_{j_+=j_-=j}\left\langle\left[\tilde{\mathcal{T}}\exp\left[i\int_\infty^{t_0}-j_-\Phi'\right]\right]\left[\mathcal{T}\exp\left[i\int_{t_0}^\infty j_+\Phi'\right]\right]\right\rangle \ , \tag{2.8}$$

where we have introduced two sources, j_+ and $-j_-$, allowing the functional derivatives to be pulled out of the expectation value. We have explicitly indicated here that the sources j_+ and j_- are to be identified with the *physical* source j *after* functional differentiation. This expression may be further simplified by introducing a time contour $\mathcal{C}$ which runs from $t=t_0$ to $+\infty$ and back to $t=t_0$.[15,18] This contour, shown in Fig. 1, is known in the

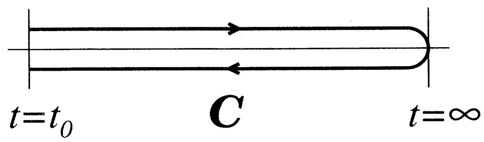

FIG. 1. The closed-time-path contour. The contour starts at $t=t_0$ and extends to $t=\infty$ and then comes back to $t=t_0$.

literature as the *closed time path*.[15] In addition, we define a contour source $j_\mathcal{C}$ which takes on the value $j_+(t)$ or $-j_-(t)$ for t on the forward or backward branch of $\mathcal{C}$, respectively. Ordering along the contour will be denoted by $\mathcal{T}_\mathcal{C}$. With these definitions we may write

$$\langle \Phi(t_1)\Phi(t_2)\rangle = \frac{1}{i^2}\frac{\delta^2}{\delta j_-(t_1)\delta j_+(t_2)} Z[j_\mathcal{C}] \, , \qquad (2.9)$$

where

$$Z[j_\mathcal{C}] = \left\langle \mathcal{T}_\mathcal{C} \exp\left[i \int_\mathcal{C} j_\mathcal{C}\Phi' \right] \right\rangle \qquad (2.10)$$

is a contour generating functional allowing for the construction of *contour* Green's functions. Contour Green's functions are defined as expectation values of *contour-ordered* products of the field operators which may be obtained through functional differentiation of the generating functional, e.g.,

$$i\Delta_\mathcal{C}(x,y) \equiv \langle \mathcal{T}_\mathcal{C}\Phi(x)\Phi(y)\rangle = \frac{1}{i^2}\frac{1}{Z[j_\mathcal{C}]}\frac{\delta^2}{\delta j_\mathcal{C}(x)\delta j_\mathcal{C}(y)} Z[j_\mathcal{C}] \, , \qquad (2.11)$$

with higher-order Green's functions defined analogously. Here, functional differentiation has been extended to the contour by writing

$$\frac{\delta j_\mathcal{C}(y)}{\delta j_\mathcal{C}(x)} = \delta_\mathcal{C}(x-y) \, , \qquad (2.12)$$

where the contour delta function is a delta distribution on the contour.

We mention that it is possible to write a path-integral representation of the contour generating functional. Indeed, Calzetta and Hu[19] have formulated a closed-time-path QFT in this manner and derived a Boltzmann equation from a two-loop approximation to the path integral. Of course, collision terms do not occur at the one-loop level, so one is forced to go to at least two loops to see collisions. However, there is no justification for the use of the loop expansion in strongly coupled theories,[12] so we do not take this approach here.

Given the field equations for Φ, we can derive a Dyson equation for the contour Green's function $\Delta_\mathcal{C}$ using functional techniques which are made particularly simple in the sourceless representation. Since such manipulations are standard,[21] they will be omitted here. The only new feature present in the contour Dyson equations is, as the reader may have guessed, that time integrations have been replaced by contour integrations. Thus, a prototypical Dyson equation is of the form

$$\Delta_\mathcal{C}(x,y) = \Delta^0_\mathcal{C}(x,y) + \int_\mathcal{C} dz \int_\mathcal{C} dz' \Delta^0_\mathcal{C}(x,z)\Pi_\mathcal{C}(z,z')\Delta_\mathcal{C}(z',y) \, . \qquad (2.13)$$

Before leaving this section we show what role the backward flowing part of the contour plays in the ordinary vacuum theory. The vacuum theory results when we take the density matrix to be $\rho(t_0) = |0\rangle\langle 0|$. Here, $|0\rangle$ is the *exact* vacuum of the system at time t_0 and we will take $t_0 = -\infty$. Furthermore, in accordance with the usual postulates of ordinary QFT, we make the assumptions that the vacuum $|0\rangle$ is nondegenerate and the source $j(x)$ is adiabatically switched on and off in the past and future, respectively. With these two assumptions, the conditions for the quantum-mechanical *adiabatic theorem*[20] are fulfilled. Concentrating on the backward time piece, we may write

$$\langle 0|U(-\infty,t) = \sum_n \langle 0|U(-\infty,\infty)|n\rangle\langle n|U(\infty,t) = \langle 0|U(-\infty,\infty)|0\rangle\langle 0|U(\infty,t) \, , \qquad (2.14)$$

where we have introduced a complete set of states $|n\rangle$ and exploited the group property of the evolution operator U. The second equality follows because the adiabatic theorem guarantees that only the vacuum state $|0\rangle$ contributes to the sum since an adiabatic perturbation cannot induce transitions between different states. In addition, the adiabatic theorem implies that $\langle 0|U(-\infty,\infty)|0\rangle$ is just a phase and so by complex conjugation we can write

$$\langle 0|U(-\infty,t) = \frac{\langle 0|U(\infty,t)}{\langle 0|U(\infty,-\infty)|0\rangle} \, . \qquad (2.15)$$

In this form, we see that the backward flowing part of the contour manifests itself in the vacuum theory by *canceling* disconnected vacuum to vacuum graphs.

At this point the reader should be convinced that the structure of the nonequilibrium quantum field theory is formally identical to the ordinary vacuum theory. This concludes our discussion of closed-time-path quantum field theory.

B. Generalized Boltzmann equation

Although contour functions are particularly suited for formal manipulations, they are not convenient for practical calculations. Instead, we find it useful to use a matrix representation of the contour functions. Contour Green's functions have a natural matrix representation defined by

$$[i\Delta_\mathcal{C}(x,y)]_{a,b} = \frac{1}{i^2}\frac{\delta^2}{\delta j_a(x)\delta j_b(y)} Z[j_\mathcal{C}] \, , \quad a,b \in \{+,-\} \, . \qquad (2.16)$$

With the definitions

$$i\Delta_F(x,y) \equiv \langle \mathcal{T}\Phi(x)\Phi(y)\rangle \, , \qquad (2.17)$$

$$i\Delta_{\bar{F}}(x,y) \equiv \langle \tilde{\mathcal{T}}\Phi(x)\Phi(y)\rangle \, , \qquad (2.18)$$

$$i\Delta^{>}(x,y)\equiv\langle\Phi(x)\Phi(y)\rangle\ , \tag{2.19}$$

$$i\Delta^{<}(x,y)\equiv\langle\Phi(y)\Phi(x)\rangle\ , \tag{2.20}$$

we can write

$$\Delta_{\mathscr{C}}(x,y)=\begin{bmatrix}\Delta_F(x,y) & \Delta^{<}(x,y)\\ \Delta^{>}(x,y) & \Delta_{\bar F}(x,y)\end{bmatrix}\ . \tag{2.21}$$

Since any contour two-point function $A_{\mathscr{C}}$ will have a matrix representation, we define a *canonical* form for $A_{\mathscr{C}}$ as

$$A_{\mathscr{C}}(x,y)=\begin{bmatrix}A_F(x,y) & A^{<}(x,y)\\ A^{>}(x,y) & A_{\bar F}(x,y)\end{bmatrix}\ . \tag{2.22}$$

It is important to realize that, in the matrix representation, the matrix elements of the contour functions are defined on the real axis and not the contour.

We also find it convenient to define retarded and advanced two-point functions as

$$A_R(x,y)\equiv\theta(x_0-y_0)[A^{>}(x,y)-A^{<}(x,y)] \tag{2.23}$$

and

$$A_A(x,y)\equiv-\theta(y_0-x_0)[A^{>}(x,y)-A^{<}(x,y)]\ , \tag{2.24}$$

respectively. It is important to note that the definitions of A_R and A_A, Eqs. (2.23) and (2.24), hold for fermions as well as bosons. We also *assume* that the *Feynman* functions $A_{F,\bar F}$ have the decomposition

$$A_{F,\bar F}(x,y)=A^{>,<}(x,y)\theta(x_0-y_0)+A^{<,>}(x,y)\theta(y_0-x_0)\ , \tag{2.25}$$

indicating that the elements of $A_{\mathscr{C}}$, Eq. (2.22), are not independent. Although the two-point Green's functions can always be decomposed in this way, the self-energy cannot if it includes a mean-field contribution, since the mean-field contributes a piece proportional to $\delta(x-y)$. Such a contribution only has support for $x_0=y_0$, whereas Eq. (2.25) is undefined at $x_0=y_0$. Thus, we will always separate out the mean field from the self-energy by absorbing it into the free propagator. This will be illustrated in more detail later. For an alternative procedure, the reader is referred to Ref. 8.

By considering the *contraction* $D_{\mathscr{C}}$ of two contour functions, $A_{\mathscr{C}}$ and $B_{\mathscr{C}}$, i.e.,

$$D_{\mathscr{C}}(x,y)=\int_{\mathscr{C}}dz\ A_{\mathscr{C}}(x,z)B_{\mathscr{C}}(z,y)\ , \tag{2.26}$$

we can derive a convenient set of rules, the so-called *Langreth-Wilkins* rules,[22] for continuing from the contour to the real time axis. Using the canonical matrix form for contour two-point functions, Eq. (2.22), we find in the limit $t_0\to-\infty$,

$$D^{>,<}(x,y)=\int dz[A^{>,<}(x,z)B_A(z,y)+A_R(x,z)B^{>,<}(z,y)] \tag{2.27}$$

and

$$D_{R,A}(x,y)=\int dz\ A_{R,A}(x,z)B_{R,A}(z,y)\ . \tag{2.28}$$

These are the first two Langreth-Wilkins rules. The other two rules are useful for reducing *antiparallel* products of contour functions

$$iD_{\mathscr{C}}(x,y)=iA_{\mathscr{C}}(x,y)iB_{\mathscr{C}}(y,x)\ . \tag{2.29}$$

Indeed, as the reader can readily show,

$$iD^{>,<}(x,y)=iA^{>,<}(x,y)iB^{<,>}(y,x) \tag{2.30}$$

and

$$D_{R,A}(x,y)=A_{R,A}(x,y)iB^{<}(y,x)+iA^{<}(x,y)B_{A,R}(y,x)\ . \tag{2.31}$$

The antiparallel Langreth-Wilkins rules are particularly useful for continuing contour self-energies to the real axis. Here we mention that these rules are valid only in the limit as t_0 goes to infinity. This limit corresponds to the neglect of initial correlations. However, we do not consider this a serious limitation since neglecting initial correlations is tantamount to the assumption of molecular chaos and we believe that this assumption is an integral part of the whole Boltzmann transport philosophy.

Using Eqs. (2.27) and (2.28) for continuing the contraction of contour functions to the real axis, the contour Dyson equation (2.13) may be expressed as

$$\Delta_0^{-1}\Delta^{>,<}=\Pi^{>,<}\Delta_A+\Pi_R\Delta^{>,<}\ , \tag{2.32}$$

$$\Delta_{R,A}=\Delta^0_{R,A}+\Delta^0_{R,A}\Pi_{R,A}\Delta_{R,A}\ . \tag{2.33}$$

Here we have used the fact that the greater and lesser components of the contour delta function $\delta_{\mathscr{C}}$ are zero, i.e., $\delta^{>,<}(x-y)=0$. Subtracting Eq. (2.32) and its adjoint, we obtain the generalized Boltzmann equation

$$[\Delta^{>,<},\Delta_0^{-1}-\tfrac{1}{2}(\Pi_R+\Pi_A)]+[\Pi^{>,<},\tfrac{1}{2}(\Delta_R+\Delta_A)]=\tfrac{1}{2}\{\Pi^{<},\Delta^{>}\}-\tfrac{1}{2}\{\Pi^{>},\Delta^{<}\}\ . \tag{2.34}$$

We shall see that, upon using a semiclassical expansion of the generalized Boltzmann equation, the first term on the left-hand side corresponds to the familiar drift term of a classical Boltzmann equation while the second term is a quantum-mechanical correction to the drift. The drift term contains contributions from other particles in the medium through the self-consistent Hartree-Fock potential set up by the particles in the background. The right-hand side corresponds to the collision term—the first term provides scattering into a phase-space element and the second term provides scattering out. However, this interpretation is valid only in a semiclassical limit. Furthermore, the generalized Boltzmann equation above does not have the single-time structure that one expects a Boltzmann equation to have. In the next section, we describe an *Ansatz,* due to Kadanoff and Baym, which relates the double-time function $\Delta^{<}(x,y)$ to a single-time function that has the interpretation of a phase-space density in a semiclassical approximation.

In this section, we have derived the generalized Boltzmann equation directly from the contour Dyson equation, Eq. (2.13). Continuation from the contour to the real axis was facilitated through the use of the Langreth-Wilkins rules. We emphasize that these rules, as written here, are independent of the statistics of the fields.

C. Wigner transform, gradient expansion, and the Kadanoff-Baym *Ansatz*

In ordinary vacuum quantum field theory, Green's functions are translationally invariant and calculations are simplified when working in a Fourier-transform representation. However, for the general nonequilibrium situation, the Green's functions depend on $r=x-y$ as well as $R=\frac{1}{2}(x+y)$. The Wigner transform is a generalization of the Fourier transform and is realized by Fourier transforming over the difference coordinate r. Hence, we define the Wigner transform of a two-point function $\Delta(x,y)$ as

$$\Delta(R,p)\equiv\int_{-\infty}^{\infty}dr\, e^{ipr/\hbar}\Delta\left[R+\frac{r}{2},R-\frac{r}{2}\right] . \quad (2.35)$$

Obviously, the Wigner transform reduces to the Fourier transform for translationally invariant systems.

If the variation with respect to the sum coordinate R is small on a scale set by the variations of the relative coordinate r, then it is permissible to perform a *gradient expansion* in derivatives with respect to R. It is straightforward to show that, if

$$C(x,y)=\int dz\, A(x,z)B(z,y) , \quad (2.36)$$

then

$$C(R,p)=\exp\left[\frac{i\hbar}{2}(\partial_p^A\cdot\partial_R^B-\partial_R^A\cdot\partial_p^B)\right]A(R,p)B(R,p) . \quad (2.37)$$

Upon expansion and neglecting derivatives of higher order than the first, we find

$$C(R,p)=A(R,p)B(R,p)+\frac{i\hbar}{2}[A(R,p),B(R,p)]_{\rm PB}+O(\hbar^2) , \quad (2.38)$$

where the generalized Poisson bracket (PB) is defined

$$[A,B]_{\rm PB}\equiv\partial_p A\cdot\partial_R B-\partial_R A\cdot\partial_p B . \quad (2.39)$$

It is also obvious that the gradient expansion is, in some sense, an expansion in powers of $\hbar$. This is because the gradient expansion is valid if $\Delta R\,\Delta p\gg\hbar$, where ΔR is on the order of the scale set by a typical variation with respect to R and $\Delta p\sim\hbar/\Delta r$ sets the scale at which the function varies with p. For this reason, it is also a *semiclassical* expansion. Hence, one expects such an approximation to hold only for systems which are close to equilibrium. For a discussion of this point in the context of heavy-ion collisions, the reader is referred to the last section of the present paper.

To motivate the Kadanoff-Baym *Ansatz*,[8,9] we consider the case in which the quantum fields are in equilibrium. For this case, the density matrix is given by

$$\rho(t_0)=\exp(-\beta H)/{\rm Tr}\exp(-\beta H) .$$

In equilibrium, the Green's functions $\Delta^{>,<}(x,y)$ defined in Eqs. (2.19) and (2.20) are related through the well-known Kubo-Martin-Schwinger[23] (KMS) boundary condition. For bosons, this is expressed by the statement

$$\Delta^{>}(\mathbf{x},\mathbf{y},x_0-y_0-i\beta)=\Delta^{<}(\mathbf{x},\mathbf{y},x_0-y_0) , \quad (2.40)$$

which follows trivially from the cyclic property of the trace. Here, we have analytically continued $\Delta^{>,<}(x,y)$ into the complex-time plane. In the Wigner transform representation, the KMS condition is expressed in the form

$$e^{-\beta p_0}\Delta^{>}(\mathbf{R},p)=\Delta^{<}(\mathbf{R},p) , \quad (2.41)$$

where we note that, in equilibrium, the Wigner-transformed Green's functions are time independent since only the difference x_0-y_0 enters. Using the identity $\Delta_R-\Delta_A=\Delta^{>}-\Delta^{<}$, which follows from the definitions for $\Delta_{R,A}$, Eqs. (2.23) and (2.24), together with the KMS condition, Eq. (2.41), we find

$$\Delta^{<}(\mathbf{R},p)=f_{\rm BE}(p_0)[\Delta_R(\mathbf{R},p)-\Delta_A(\mathbf{R},p)] , \quad (2.42)$$

where

$$f_{\rm BE}(p_0)=\frac{1}{e^{\beta p_0}-1} \quad (2.43)$$

is the Bose-Einstein (BE) distribution function. Thus, in equilibrium, $\Delta^{<}(R,p)$ is related to the particle distribution function.

Kadanoff and Baym assumed Eq. (2.42) to hold locally for the nonequilibrium case, i.e.,

$$\Delta^{<}(R,p)=f(R,p)[\Delta_R(R,p)-\Delta_A(R,p)] , \quad (2.44)$$

where $f(R,p)$ plays the role of a nonequilibrium quasiparticle distribution function. Note that, unlike the equilibrium case, it depends upon R and $\mathbf{p}$ as well as p_0.

The *spectral function* $\Delta_R-\Delta_A$ has the formal solution

$$\Delta_R-\Delta_A=\frac{2i\,{\rm Im}\Pi_R}{(\Delta_0^{-1}-{\rm Re}\Pi_R)^2+({\rm Im}\Pi_R)^2}+O(\hbar^2) , \quad (2.45)$$

which follows from a gradient expansion and Wigner transform of the Dyson equations (2.33) and from the fact that $\Delta_R(R,p)=\Delta_A^*(R,p)$. In the limit of weak scattering, ${\rm Im}\Pi_R$ is small and the spectral function is sharply peaked about some value of $p_0=E_{\rm p}(R)$. If the width of the peak is sufficiently narrow, the excitations of the system may be described in terms of *quasiparticles* and the distribution function becomes a distribution function for the quasiparticles. In the limit of the vanishing width, the spectral function has a delta-function-like behavior and we may write

$$\Delta^{<}(\mathbf{R},p)=2if(R,\mathbf{p},E_{\rm p}){\rm Im}\Delta_R(R,p) , \quad (2.46)$$

replacing p_0 in the argument of the distribution function f by $E_{\rm p}$. Thus, $\Delta^{<}$ can be eliminated from the

Boltzmann equation in favor of the time-dependent distribution function $f(R,\mathbf{p},E_{\mathrm{p}})$ defined on the phase space $(\mathbf{R},\mathbf{p})$. This is the essence of the Kadanoff-Baym *Ansatz*.[9] We want to emphasize that this is an *Ansatz* and not an approximation; it is strictly valid only for equilibrium. As such, it can only be justified *a posteriori*. Throughout this paper, we use "*Ansatz*" in this fashion.

To summarize, under certain conditions, the generalized Boltzmann equation can be reduced to a Boltzmann equation for a quasiparticle distribution function. The essential ingredients for this interpretation are (i) validity of the gradient expansion, (ii) long-lived quasiparticle excitations, and (iii) validity of the Kadanoff-Baym *Ansatz*. We emphasize that the three assumptions are not independent of each other. Indeed, in motivating the *Ansatz*, the gradient expansion was used and the quasiparticle pole was assumed to dominate.

Finally, we would like to mention that the Kadanoff-Baym *Ansatz* is not the only *Ansatz* on the market. Lipavski *et al.*[24] have constructed an *Ansatz* which overcomes some of the limitations of the Kadanoff-Baym *Ansatz*. Namely, the Kadanoff-Baym *Ansatz* violates causality[24] and, in addition, it is not clear how to improve it. The *Ansatz* of Lipavski *et al.* is causal and one can derive correction terms for it. Moreover, unlike the Kadanoff-Baym *Ansatz*, it is not limited to the gradient expansion. However, unlike the Kadanoff-Baym *Ansatz*, it leads to a *non-Markovian* Boltzmann equation. Since the principle goal of this work is to derive a Markovian Boltzmann equation, the non-Markovian *Ansatz* of Lipavski *et al.* will not be discussed here. For more details, the reader is referred to their paper. For a discussion of the Markovian *Ansatz* that we employ here as it pertains to heavy-ion collisions, the reader is referred to the conclusion of this paper.

This section completes the formal part of this paper. The procedure for the construction of a transport equation has been outlined. Up to this point, we have not introduced a specific model and it is to this that we now turn.

III. APPLICATION TO THE YUKAWA MODEL

In this section, we apply the preceding ideas to a Yukawa model. We derive the contour Dyson equations for this model and discuss the energy-momentum tensor. Then we go on to use the Kadanoff-Baym *Ansatz* to derive Boltzmann equations for this system.

A. The Yukawa model

The model we will concentrate on is a QFT composed of spin-half Dirac fermions interacting with a massive scalar boson field via a Yukawa coupling. The Lagrangian density for this system is

$$\begin{aligned}\mathcal{L}(x)=&\,\overline{\Psi}(x)(i\gamma_\mu\partial^\mu-m)\Psi(x)-\tfrac{1}{2}\Phi(x)(\Box+m_s^2)\Phi(x)\\&+g\overline{\Psi}(x)\Phi(x)\Psi(x)+j(x)\Phi(x)\\&+\overline{\zeta}(x)\Psi(x)+\overline{\Psi}(x)\zeta(x)\ .\end{aligned} \tag{3.1}$$

In anticipation of transforming to the sourceless representation, we have coupled the fields to the external sources $j(x)$, $\zeta(x)$, and $\overline{\zeta}(x)$. Here $j(x)$ is an ordinary c-number function while $\zeta(x)$ and $\overline{\zeta}(x)$ are Grassmann functions. The field equations follow from the Lagrangian, Eq. (3.1), in the usual way. The Euler-Lagrange equations yield

$$(\Box+m_s^2)\Phi(x)-g\overline{\Psi}(x)\Psi(x)=j(x) \tag{3.2}$$

for the boson field and

$$(i\gamma_\mu\partial^\mu-m)\Psi(x)+g\Phi(x)\Psi(x)=-\zeta(x) \tag{3.3}$$

for the fermion field. There is an analogous equation for the adjoint field $\overline{\Psi}(x)$. Quantization is achieved by *requiring* that the field equations, Eqs. (3.2) and (3.3), be consistent with the corresponding Heisenberg equations of motion—namely, Eq. (2.1) suitably generalized to include fermion fields. This consistency requirement leads to the usual *equal-time* (anti-)commutation relations,

$$\{\Psi_\alpha(t,\mathbf{x}),\overline{\Psi}_\beta(t,\mathbf{y})\}=\gamma^0_{\alpha\beta}\delta^3(\mathbf{x}-\mathbf{y}) \tag{3.4}$$

and

$$\left[\frac{\partial}{\partial t}\Phi(t,\mathbf{x}),\Phi(t,\mathbf{y})\right]=-i\delta^3(\mathbf{x}-\mathbf{y})\ , \tag{3.5}$$

with all other equal-time (anti-)commutators vanishing. We now proceed to derive a set of contour Dyson equations. After taking expectation values of the field equations (3.2) and (3.3) and extending them to the contour, we arrive at a set of functional differential equations on the closed-time path, i.e.,

$$\left[(i\gamma_\mu\partial^\mu-m)_{\mathcal{C}}\frac{1}{i}\frac{\delta}{\delta\overline{\zeta}_{\mathcal{C}}(x)}+g\frac{1}{i^2}\frac{\delta^2}{\delta j_{\mathcal{C}}(x)\delta\overline{\zeta}_{\mathcal{C}}(x)}+\zeta_{\mathcal{C}}(x)\right]Z[j_{\mathcal{C}},\zeta_{\mathcal{C}},\overline{\zeta}_{\mathcal{C}}]=0 \tag{3.6}$$

and

$$\left[(\Box+m_s^2)_{\mathcal{C}}\frac{1}{i}\frac{\delta}{\delta j_{\mathcal{C}}(x)}-g\frac{1}{i^2}\frac{\delta^2}{\delta\overline{\zeta}_{\mathcal{C}}(x)\delta\zeta_{\mathcal{C}}(x)}-j_{\mathcal{C}}(x)\right]Z[j_{\mathcal{C}},\zeta_{\mathcal{C}},\overline{\zeta}_{\mathcal{C}}]=0\ . \tag{3.7}$$

In obtaining Eqs. (3.6) and (3.7), we have extended the differential operators $\Box+m_s^2$ and $i\gamma_\mu\partial^\mu-m$ to the contour as indicated by the subscript $\mathcal{C}$. The contour-generating functional is defined as

$$Z_\mathcal{C}\equiv Z[j_\mathcal{C},\zeta_\mathcal{C},\bar{\zeta}_\mathcal{C}] \equiv\left\langle \mathcal{T}_\mathcal{C}\exp\left[i\int_\mathcal{C}(j_\mathcal{C}\Phi'+\bar{\Psi}'\zeta_\mathcal{C}+\bar{\zeta}_\mathcal{C}\Psi')\right]\right\rangle , \quad (3.8)$$

with $\bar{\Psi}'$, Ψ', and Φ' are the field operators expressed in the sourceless representation described in Sec. II A of this paper.

We define the two-point contour fermion Green's function as

$$iG_\mathcal{C}(x,y)\equiv\langle \mathcal{T}_\mathcal{C}\Psi(x)\bar{\Psi}(y)\rangle=\frac{1}{Z_\mathcal{C}}\frac{\delta^2}{\delta\bar{\zeta}(x)\delta\zeta(y)}Z_\mathcal{C} . \quad (3.9)$$

Similarly, we define the boson Green's function as

$$i\Delta'_\mathcal{C}(x,y)\equiv\langle \mathcal{T}_\mathcal{C}\Phi(x)\Phi(y)\rangle \equiv i\Delta_\mathcal{C}(x,y)+\phi(x)\phi(y) =\frac{1}{i^2Z_\mathcal{C}}\frac{\delta^2 Z_\mathcal{C}}{\delta j_\mathcal{C}(x)\delta j_\mathcal{C}(y)} , \quad (3.10)$$

explicitly indicating the presence of the *mean field*

$$\phi(x)\equiv\langle\Phi(x)\rangle=\frac{1}{iZ_\mathcal{C}}\frac{\delta}{\delta j_\mathcal{C}(x)}Z_\mathcal{C} . \quad (3.11)$$

In addition, due to complications introduced by the background field ϕ, we have introduced a *connected* contour Green's function $\Delta_\mathcal{C}$ for which the Dyson equations simplify. Also, we should emphasize that all contour Green's functions defined up to this point depend on the external sources, although this fact has not been explicitly indicated.

By rewriting Eq. (3.7) using the definitions (3.9) and (3.11) we obtain

$$(\Box+m_s^2)\phi_\mathcal{C}(x)+ig\,\mathrm{tr}G_\mathcal{C}(x,x)=0 \quad (3.12)$$

for the mean field ϕ in the absence of an external source. Upon functional differentiation of Eqs. (3.6) and (3.7) with respect to $\zeta_\mathcal{C}(y)$ and $j_\mathcal{C}(y)$, respectively, and using definitions (3.9)–(3.11), we obtain

$$(i\gamma_\mu\partial^\mu-m+g\phi)_\mathcal{C}G_\mathcal{C}(x,y)-ig\frac{\delta G_\mathcal{C}(x,y)}{\delta j_\mathcal{C}(x)}=\delta_\mathcal{C}(x-y) \quad (3.13)$$

for the fermion Green's function and

$$(\Box+m_s^2)_\mathcal{C}\Delta_\mathcal{C}(x,y)-ig\,\mathrm{tr}\frac{\delta G_\mathcal{C}(x,x)}{\delta j_\mathcal{C}(y)}=-\delta_\mathcal{C}(x-y) \quad (3.14)$$

for the connected boson Green's function. We note that, in obtaining these equations, we have set the external sources to zero *after* functional differentiation and assume that $\langle\Psi(x)\rangle=0$ in their absence. These equations may be put in a more familiar form by defining a contour self-energy

$$\Sigma_\mathcal{C}(x,y)\equiv g\int_\mathcal{C}dz\,dz'G_\mathcal{C}(x,z)\Gamma_\mathcal{C}(z,y;z')\Delta_\mathcal{C}(z',x) \quad (3.15)$$

for the fermion field and a contour polarization insertion

$$\Pi_\mathcal{C}(x,y)\equiv -g\int_\mathcal{C}dz\,dz'\mathrm{tr}G_\mathcal{C}(x,z)\Gamma_\mathcal{C}(z,z';y)G_\mathcal{C}(z',x) \quad (3.16)$$

for the boson field. Here the three-point contour vertex function is defined as

$$\Gamma_\mathcal{C}(x,y;z)\equiv i\frac{\delta G_\mathcal{C}^{-1}(x,y)}{\delta\phi_\mathcal{C}(z)} , \quad (3.17)$$

where the *inverse* Green's function $G_\mathcal{C}^{-1}(x,y)$ is regarded as a functional of the background field ϕ. In terms of these functions, we may write the functional equations (3.13) and (3.14) as

$$(i\gamma_\mu\partial^\mu-m+g\phi)_\mathcal{C}G_\mathcal{C}(x,y)-\int_\mathcal{C}dz\,\Sigma_\mathcal{C}(x,z)G_\mathcal{C}(z,y) =\delta_\mathcal{C}(x-y) , \quad (3.18)$$

$$(\Box+m_s^2)\Delta_\mathcal{C}(x,y)+\int_\mathcal{C}dz\,\Pi_\mathcal{C}(x,z)\Delta_\mathcal{C}(z,y)=-\delta_\mathcal{C}(x-y) , \quad (3.19)$$

which are a set of coupled Dyson equations for the contour Green's functions. A schematic representation of these equations is shown in Fig. 2. Although it looks as though we have a closed set of equations for the Green's functions, we do not because the vertex function hides all of our ignorance. The vertex function can be found in perturbation theory from its definition Eq. (3.17). We shall approximate the vertex by the *bare* vertex—the lowest-order term in the perturbation expansion. We find

$$\Gamma_\mathcal{C}(x,y;z)=ig\delta_\mathcal{C}(x-y)\delta_\mathcal{C}(x-z)+O(g^2) . \quad (3.20)$$

It should be emphasized that there is no justification for such an approximation, particularly in a strongly interacting field theory. Any reasonable discussion of this point would lead us far astray.

We now turn to a discussion of the stress-energy tensor. We will use the *symmetric* stress-energy tensor as opposed to the canonical one which, in general, is not symmetric. The canonical tensor is derived[25] by assuming that under a Lorentz transformation, $x_\mu\to x_\mu+\delta x_\mu$, the fields transform as Lorentz scalars. Clearly, this is incorrect for fields with nonzero spin. For higher spin fields, the transformation properties of the fields themselves under the Lorentz transformation must be properly taken into account.[25] When this is done, one ends up with a symmetric stress-energy tensor. For the Lagrang-

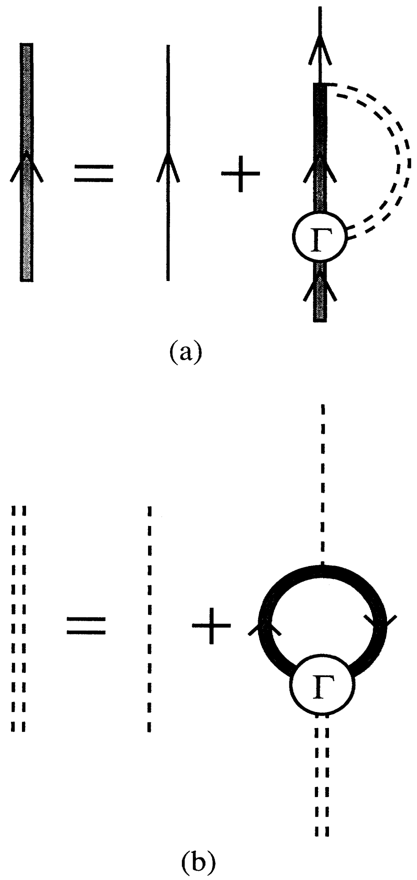

FIG. 2. The Schwinger-Dyson equation for the fermion Green's function is represented in (a). The equation for the boson propagator is shown in (b). The single full line represents the bare fermion Green's function which includes the mean-field contribution. The double full line represents the dressed fermion Green's function. Similarly, the double dashed line is the dressed boson propagator while the bare boson Green's function is given by the single dashed line. Also shown is the vertex function labeled by Γ. In this work we replace the vertex function by the bare vertex. Integrations are over the contour shown in Fig. 1.

ian, Eq. (3.1), it is straightforward, though tedious, to show that

$$T_{\mu\nu}=T^F_{\mu\nu}+T^B_{\mu\nu} \ , \tag{3.21}$$

where

$$T^F_{\mu\nu}=\frac{i}{4}\overline{\Psi}(\gamma_\mu\overleftrightarrow{\partial}_\nu+\gamma_\nu\overleftrightarrow{\partial}_\mu)\Psi \tag{3.22}$$

is the fermion contribution and

$$T^B_{\mu\nu}=\partial_\mu\Phi\partial_\nu\Phi-\tfrac{1}{2}g_{\mu\nu}(\partial_\lambda\Phi\partial^\lambda\Phi-m_s^2\Phi^2) \tag{3.23}$$

is the boson piece. Here we are using the notation

$$A\overleftrightarrow{\partial}_\nu B=A\cdot\partial_\mu B-\partial_\mu A\cdot B$$

and, in addition, we have used the field equations (3.2) and (3.3). It is easy to see why the stress-energy tensor must be symmetric. For example, physically, the kth component of momentum gives rise to an energy flux in the kth direction; this is expressed by the statement $T_{0k}=T_{k0}$. Finally, we mention that when scalar fields are involved, one should use the so-called "improved" stress tensor of Callan, Coleman, and Jackiw.[26] This tensor is obtained from the one above by the substitution

$$T^B_{\mu\nu}\to T^B_{\mu\nu}-\tfrac{1}{6}(\partial_\mu\partial_\nu-g_{\mu\nu}\Box)\Phi^2 \ . \tag{3.24}$$

It seems that, without the modification, the stress tensor is not finite even when counter-term contributions are included. With the addition of the extra term, it can be shown[26] that the counter-terms necessary to make the Green's functions finite will also guarantee finiteness of the "improved" stress tensor.

B. Distribution function for the scalar field

In this section, we construct a quasiparticle distribution function for the scalar field. The analogous construction for the fermions is more difficult due to the Dirac matrix structure. We postpone this problem until the next section.

The equation for the connected contour boson propagator, Eq. (3.19), can be put in the form

$$(\Delta^0_{\mathcal{C}})^{-1}\Delta_{\mathcal{C}}=1+\Pi_{\mathcal{C}}\Delta_{\mathcal{C}} \ , \tag{3.25}$$

where

$$(\Delta^0_{\mathcal{C}})^{-1}(x,y)=-(\Box+m_s^2)\delta_{\mathcal{C}}(x-y) \tag{3.26}$$

with integration over the contour implied. The generalized Boltzmann equation, Eq. (2.34), becomes, upon Wigner transformation and gradient expansion,

$$i\hbar[\Delta^{>,<},p^2-m_s^2-\mathrm{Re}\Pi_R\,]_{\mathrm{PB}}(R,p)+i\hbar[\Pi^{>,<},\mathrm{Re}\Delta_R\,]_{\mathrm{PB}} =[\Delta^>\Pi^<-\Delta^<\Pi^>](R,p)+O(\hbar^2) \ , \tag{3.27}$$

where we have used the properties of the Wigner transform listed in the Appendix. We also keep the $\hbar$ from the gradient expansion for bookkeeping purposes.

In accordance with the general recipe outlined in previous sections, we now implement the Kadanoff-Baym ansatz. To this end we find, from Wigner transforming and gradient expanding, Eq. (3.27) for the retarded function Δ_R,

$$[p^2-m_s^2-\Pi_R(R,p)]\Delta_R(R,p) =1-i\hbar[p^2-m_s^2-\Pi_R,\Delta_R\,]_{\mathrm{PB}}(R,p)+O(\hbar^2) \ . \tag{3.28}$$

In addition, one can show that the retarded function $\Delta_R(R,p)$ satisfies the dispersion relation

$$\Delta_R(R,p)=-\frac{1}{\pi}\int_{-\infty}^{\infty}dq_0\frac{\mathrm{Im}\Delta_R(R,\mathbf{p},q_0)}{p_0-q_0+i\varepsilon} \ . \tag{3.29}$$

In fact, such a relation holds for *any* retarded function defined analogously. Here we have assumed that $\mathrm{Im}\Delta_R(R,p)$ vanishes sufficiently rapidly as p_0 goes to ∞. In the general case of a polynomial growth for large p_0, one must use a subtracted dispersion relation and we leave it as a simple exercise for the reader to show that the argument still goes through. Using the Poisson bracket property, $[A, f(A)]_{\rm PB}=0$ for $A=A(R,p)$, we can write the solution to Eq. (3.28) as

$$\Delta_R=\frac{1}{p^2-m_s^2-{\rm Re}\Pi_R-i\,{\rm Im}\Pi_R}+O(\hbar^2)\ . \quad (3.30)$$

Since Δ_R is a retarded function, it must be analytic in the upper-half complex plane of p_0, so poles must occur in the lower-half plane. Suppose

$$z_{\bf p}=\omega_{\bf p}(R)-i\gamma_{\bf p}(R)$$

for $\gamma_{\bf p}>0$ is a pole in the lower-half plane of p_0. Then

$$z^2-{\bf p}^2-m_s^2-\Pi_R({\bf p},z)=0\ , \quad (3.31)$$

where, for clarity, we have not explicitly indicated the R dependence. Using the property

$$\Delta^>(R,p)=\Delta^<(R,-p)$$

and the identity $\Delta^>-\Delta^<=\Delta_R-\Delta_A$ together with the dispersion relation Eq. (3.29), it is easily verified that

$$\Delta_R(R,{\bf p},z)=[\Delta_R(R,-{\bf p},-z^*)]^* \quad (3.32)$$

for every complex z. Then, as the reader can readily show, if $z_{\bf p}$ is a pole, then $-z^*_{-\bf p}$ is also a pole. This method of constructing one pole from the other cannot return the same pole since, for a noninteracting system this would imply $\omega_{\bf p}=-\omega_{-\bf p}$ which is incorrect. Hence, we conclude that the poles are distinct. Assuming $\gamma_{\pm\bf p}$ are small, an approximate solution to Eq. (3.31) is given by

$$\gamma_{\pm\bf p}=-Z_B({\bf p},p_0)|_{p_0=\pm\omega_{\pm\bf p}}{\rm Im}\Pi_R(\pm\omega_{\pm\bf p})\ , \quad (3.33)$$

with $\omega_{\pm\bf p}$ satisfying the *transcendental* equation

$$\omega_{\pm\bf p}^2-{\bf p}^2-m_s^2-{\rm Re}\Pi_R({\bf p},\pm\omega_{\pm\bf p})=0\ . \quad (3.34)$$

The residue Z_B is defined by

$$Z_B^{-1}({\bf p},p_0)=\left[2p_0-\frac{\partial\,{\rm Re}\Pi_R({\bf p},p_0)}{\partial p_0}\right] \quad (3.35)$$

with the property

$$Z_B({\bf p},p_0)=-Z_B(-{\bf p},-p_0)\ . \quad (3.36)$$

Assuming there are only two poles and neglecting off-pole contributions, we can write the retarded function Eq. (3.30) as

$$\Delta_R(R,p)=\frac{Z_B({\bf p},\omega_{\bf p})}{p_0-\omega_{\bf p}+i\gamma_{\bf p}}+\frac{Z_B({\bf p},-\omega_{-\bf p})}{p_0+\omega_{-\bf p}+i\gamma_{-\bf p}}\ , \quad (3.37)$$

where, for clarity, the R dependence has been omitted on the right-hand side of the equation. The spectral function may be found by taking the imaginary part of Eq. (3.37). In the limit of vanishing width, we can write

$$\begin{aligned}{\rm Im}\Delta_R(R,p)&\\ &=-\pi\,{\rm sgn}[Z_B({\bf p},p_0)]\delta(p^2-m_s^2-{\rm Re}\Pi_R)\ , \quad (3.38)\end{aligned}$$

where for meaningful results, all distributions are considered to be regulated as in the finite-temperature QFT.[27]

Finally, we define distribution functions $N^{>,<}(R,p)$ through

$$\begin{aligned}\Delta^{>,<}(R,p)&=2iN^{>,<}(R,p){\rm Im}\Delta_R(R,p) \quad (3.39)\\ &=-2\pi i\,{\rm sgn}[Z_B({\bf p},p_0)]N^{>,<}(R,p)\\ &\quad\times\delta(p^2-m_s^2-{\rm Re}\Pi_R)\ , \quad (3.40)\end{aligned}$$

where

$$N^>(R,p)=N^<(R,p)+1\ , \quad (3.41)$$

completing the implementation of the Kadanoff-Baym *Ansatz.*

Inserting the *Ansatz*, Eq. (3.40), into the Boltzmann equation, Eq. (3.27), we can write

$$\begin{aligned}&{\rm sgn}[Z_B({\bf p},p_0)]\delta(p_0^2-\omega_p^2)[N^{>,<}(R,p),p^2-m_s^2-{\rm Re}\Pi_R]_{\rm PB}\\ &\qquad=-i\,{\rm sgn}[Z_B({\bf p},p_0)]\delta(p_0^2-\omega_p^2)[N^>(R,p)\Pi^<(R,p)-N^<(R,p)\Pi^>(R,p)]\ , \quad (3.42)\end{aligned}$$

where, by construction, the delta function commutes with the Poisson bracket. In writing this expression, we note that the correction to the drift, the term involving ReΔ in Eq. (3.27), does not contribute in the pole approximation.[8,9] Using the easily derived identity

$$\begin{aligned}&{\rm sgn}\left[\frac{\partial f}{\partial p_0}\right]^{-1}\delta(f)[N(R,p),f(R,p)]_{\rm PB}\\ &\quad=\sum_r\left[-\frac{\partial n_r}{\partial T}+\frac{\partial n_r}{\partial{\bf p}}\cdot\frac{\partial\omega_r}{\partial{\bf R}}-\frac{\partial n_r}{\partial{\bf R}}\cdot\frac{\partial\omega_r}{\partial{\bf p}}\right]\delta(p_0-\omega_r)\ ,\end{aligned}$$

where

$$f(R,{\bf p},\omega_r)=0\ ,$$

$$n_r(R,{\bf p})=N(R,{\bf p},\omega_r)\ ,$$

and all roots ω_r are summed over, we can write Eq. (3.42) in the form

$$\begin{aligned}&\left[\frac{\partial}{\partial T}-\frac{\partial\omega_{\bf p}}{\partial{\bf R}}\cdot\frac{\partial}{\partial{\bf p}}+\frac{\partial\omega_{\bf p}}{\partial{\bf p}}\cdot\frac{\partial}{\partial{\bf R}}\right]n({\bf p})\\ &\quad=iZ_B({\bf p},\omega_{\bf p})\{[1+n({\bf p})]\Pi^<({\bf p},\omega_{\bf p})-n({\bf p})\Pi^>({\bf p},\omega_{\bf p})\}\ . \quad (3.43)\end{aligned}$$

Here we have introduced a boson quasiparticle distribution function

$$n(\mathbf{p})=N^{<}(\mathbf{p},\omega_{\mathbf{p}})=N^{>}(\mathbf{p},\omega_{\mathbf{p}})-1\ , \tag{3.44}$$

and exploited the fact that $\Delta^{>}(R,p)=\Delta^{<}(R,-p)$ to write

$$N^{<}(\mathbf{p},\omega_{\mathbf{p}})=-N^{>}(-\mathbf{p},-\omega_{\mathbf{p}})\ . \tag{3.45}$$

Before construction of the distribution function for the fermions, we discuss the physical interpretation of the boson function $n(R,\mathbf{p})$ by expressing the energy-momentum stress tensor Eq. (3.23) in terms of $n(R,\mathbf{p})$. To this end, we introduce the *two-point* symmetric tensor

$$T^{B}_{\mu\nu}(x,y)\equiv\tfrac{1}{2}[\partial^x_\mu\partial^y_\nu+\partial^x_\nu\partial^y_\mu-g_{\mu\nu}(\partial^x\cdot\partial^y-m_s^2)]\langle\Phi(y)\Phi(x)\rangle\ , \tag{3.46}$$

which, upon setting $x=y$, becomes the stress tensor Eq. (3.23). By Wigner transforming this expression and integrating over all p, we can write a *local average*

$$\begin{aligned}T^{B}_{\mu\nu}(R)&\equiv\int\frac{d^4p}{(2\pi)^4}T^{B}_{\mu\nu}(R,p)\\&=\partial_\mu\phi(R)\partial_\nu\phi(R)-\tfrac{1}{2}g_{\mu\nu}[\partial_\lambda\phi(R)\partial^\lambda\phi(R)-m_s^2\phi^2(R)]+\tfrac{1}{4}(\partial^R_\mu\partial^R_\nu-\tfrac{1}{2}g_{\mu\nu}\Box_R)\int\frac{d^3p}{(2\pi)^3}2Z_B(\mathbf{p},\omega_p)(n_{\mathbf{p}}+\tfrac{1}{2})\\&\quad+\int\frac{d^3p}{(2\pi)^3}2Z_B(\mathbf{p},\omega_p)[p_\mu p_\nu-\tfrac{1}{2}g_{\mu\nu}(p^2-m_s^2)]|_{p_0=\omega_{\mathbf{p}}}(n_{\mathbf{p}}+\tfrac{1}{2})\ .\end{aligned} \tag{3.47}$$

In deriving this expression, we have used the "lesser" component $\Delta'^{<}(x,y)$ of the contour Green's function $\Delta_{\mathcal{C}}(x,y)$, Eq. (3.10), to rewrite $\langle\Phi(y)\Phi(x)\rangle$ and we have employed the ansatz for $\Delta^{<}(R,p)$, Eq. (3.40), together with Eq. (3.44) for $n(\mathbf{p})$. Finally, we have exploited the symmetry of $Z_B(p)$, Eq. (3.36).

Each of these terms has a straightforward interpretation. The first term provides the familiar mean-field energy and momentum. The remaining terms are easily understood by realizing that $[d^3p/(2\pi)^3]2Z_B(\mathbf{p},\omega_p)$ is the Lorentz-invariant momentum-space volume element at R, while $n_{\mathbf{p}}$ is the boson number density at space-time point R. The second term gives the energy momentum associated with space-time inhomogeneities of the boson number density, while the third term corresponds to energy momentum of the quasiparticles at R.

Note that, in the vacuum limit, the energy-momentum tensor does not vanish due to the additional sums of $\frac{1}{2}$ in the integrands of Eq. (3.47) which are the familiar zero-point vacuum fluctuation contributions. In ordinary vacuum quantum field theory these terms are made to vanish by *normal ordering* with respect to the vacuum. However, in more general situations, as the ones we consider here, such terms must persist. The best one can do is to subtract the vacuum expectation value of $T_{\mu\nu}$ and then, with the appropriate counter terms, $T_{\mu\nu}$ can be made finite.[27] We emphasize that this is *not* the same as simply ignoring the extra terms of $\frac{1}{2}$ in the integrands of Eq. (3.47); only by properly dealing with the vacuum and through proper renormalization of the theory can the energy-momentum tensor be made finite.

Finally, we make the remark that the *local averaging* procedure employed above for a composite operator is equivalent to the *point-splitting* technique of Schwinger.[28] To see this, it is sufficient to consider the composite operator $\Phi^2(x)$ in one dimension with an obvious generalization to higher dimensions. The point-splitting technique consists of *defining*

$$\langle\Phi^2(x)\rangle\equiv\lim_{\varepsilon\to0^+}\langle\Phi(x-\varepsilon)\Phi(x+\varepsilon)\rangle\ . \tag{3.48}$$

We will prove that the right-hand side of this equation is equivalent to

$$\int\frac{dp}{2\pi}\Delta'(R,p)\ , \tag{3.49}$$

where $\Delta'(R,p)$ is the Wigner transform of

$$\Delta'(x,y)\equiv\langle\Phi(y)\Phi(x)\rangle\ . \tag{3.50}$$

Indeed, from the inverse Wigner transform we find

$$\begin{aligned}&\int\frac{dp}{2\pi}\int dr\,e^{ipr}\left\langle\Phi\left[R-\frac{r}{2}\right]\Phi\left[R+\frac{r}{2}\right]\right\rangle\\&\quad=\lim_{\varepsilon\to0^+}\int dr\,\delta_\varepsilon(r)\left\langle\Phi\left[R-\frac{r}{2}\right]\Phi\left[R+\frac{r}{2}\right]\right\rangle\ ,\end{aligned} \tag{3.51}$$

where we have regularized the delta function according to the prescription

$$\delta_\varepsilon(r)=\begin{cases}(2\varepsilon)^{-1} & \text{for } |r|\le\varepsilon\ ,\\ 0 & \text{otherwise}\ .\end{cases} \tag{3.52}$$

Using the mean-value theorem of integral calculus, we can write the right-hand side of Eq. (3.51) as

$$\lim_{\varepsilon\to0^+}(2\varepsilon)^{-1}\cdot2\varepsilon\left\langle\Phi\left[R-\frac{\eta_\varepsilon}{2}\right]\Phi\left[R+\frac{\eta_\varepsilon}{2}\right]\right\rangle\ , \tag{3.53}$$

where the mean value theorem guarantees that this expression holds for at least one $\eta_\varepsilon\in[-\varepsilon,\varepsilon]$. From this, the result follows trivially.

C. Fermion distribution function

In this section we extend the Kadanoff-Baym *Ansatz* to the Dirac field. Because of the matrix structure of the Green's functions, this is not as straightforward as in the boson case. In the latter case, upon Wigner transformation and gradient expansion, commutators were replaced by Poisson brackets with ordinary products replacing anticommutators. In this case, due to lack of commutivity, this prescription fails. For simplicity, in the following we adopt the *spin-symmetric ansatz* of Bezzerides and Dubois.[7] This ansatz consists of decomposing fermion Green's functions as

$$G(R,p)=\mathcal{G}(R,p)+\mathcal{G}_\mu(R,p)\gamma^\mu \ , \tag{3.54}$$

neglecting tensor, pseudoscalar, and pseudovector contributions. It is obvious from Eq. (2.33) that the retarded Green's function G_R satisfies the anticommutation relation

$$\tfrac{1}{2}\{G_0^{-1}-\Sigma_R,G_R\}(x,y)=\delta(x-y) \ . \tag{3.55}$$

Upon Wigner transformation and gradient expansion, this equation decomposes as

$$1=[p^\mu-\sigma_R^\mu(R,p)]\mathcal{G}_{R\mu}(R,p) -[m-g\phi(R)+\sigma_R(R,p)]\mathcal{G}_R(R,p) \ , \tag{3.56}$$

and

$$0=[p^\mu-\sigma_R^\mu(R,p)]\mathcal{G}_R(R,p) -[m-g\phi(R)+\sigma_R(R,p)]\mathcal{G}_R^\mu(R,p) \ , \tag{3.57}$$

which may be solved immediately to yield

$$\mathcal{G}_R(R,p)=\frac{m-g\phi(R)+\sigma_R(R,p)}{[p^\mu-\sigma_R^\mu(R,p)][p_\mu-\sigma_{R\mu}(R,p)]-[m-g\phi(R)+\sigma_R(R,p)]^2} \ , \tag{3.58}$$

$$\mathcal{G}_R^\mu(R,p)=\frac{p^\mu-\sigma_R^\mu(R,p)}{[p^\nu-\sigma_R^\nu(R,p)][p_\nu-\sigma_{R\nu}(R,p)]-[m-g\phi(R)+\sigma_R(R,p)]^2} \ . \tag{3.59}$$

Here, Σ_R has the form

$$\Sigma_R(R,p)=\sigma_R(R,p)+\gamma_\mu\sigma_R^\mu(R,p) \ . \tag{3.60}$$

To find the dispersion relation for the quasiparticles, we must look for poles in the retarded propagator. We assume there are only two poles at $p_0=\epsilon_{\mathbf{p}}^+-i\Gamma_{\mathbf{p}}^+$ and $p_0=-\epsilon_{-\mathbf{p}}^-$, or, more concisely, at $p_0=\pm\epsilon_{\pm\mathbf{p}}^\pm-i\Gamma_{\pm\mathbf{p}}^\pm$, where $\Gamma_{\mathbf{p}}^\pm>0$. Note that, in the boson case, the two poles are related. This is due to the charge-conjugation symmetry of the scalar field. Here, this is not the case and we must introduce two distribution functions—one for particles and one for antiparticles.

For small $\Gamma_{\pm\mathbf{p}}^\pm$ we find

$$\Gamma_{\pm\mathbf{p}}^\pm=-2Z_F(\mathbf{p},\pm\epsilon_{\pm\mathbf{p}}^\pm) \times[p_\mu^*(\mathbf{p},\pm\epsilon_{\pm\mathbf{p}}^\pm)\mathrm{Im}\sigma^\mu(\mathbf{p},\pm\epsilon_{\pm\mathbf{p}}^\pm) +m^*(\mathbf{p},\pm\epsilon_{\pm\mathbf{p}}^\pm)\mathrm{Im}\sigma(\mathbf{p},\pm\epsilon_{\pm\mathbf{p}}^\pm)] \ , \tag{3.61}$$

where $\pm\epsilon_{\pm\mathbf{p}}^\pm$ satisfies the *mass-shell* condition

$$p^{*2}(\mathbf{p},\pm\epsilon_{\pm\mathbf{p}}^\pm)-m^{*2}(\mathbf{p},\pm\epsilon_{\pm\mathbf{p}}^\pm)=0 \ . \tag{3.62}$$

Here we have introduced the momentum variable

$$p_\mu^*(R,\mathbf{p},p_0)=p_\mu-\mathrm{Re}\sigma_\mu(R,\mathbf{p},p_0) \ , \tag{3.63}$$

and the *effective mass*

$$m^*(R,\mathbf{p},p_0)=m-g\phi(R)+\mathrm{Re}\sigma(R,\mathbf{p},p_0) \ . \tag{3.64}$$

The value of the residue at the pole Z_F is given by

$$Z_F^{-1}(\mathbf{p},p_0)=\frac{\partial}{\partial p_0}(p^{*2}-m^{*2}) \ . \tag{3.65}$$

Hence, neglecting off-pole contributions, we can write the imaginary part of the retarded function as

$$\mathrm{Im}G_R(R,p) =-\pi\,\mathrm{sgn}Z_F(R,p)(p_\mu^*\gamma^\mu+m^*)\delta(p^{*2}-m^{*2}) \ , \tag{3.66}$$

where we have taken the width $\Gamma_{\pm\mathbf{p}}^\pm$ to be vanishingly small. We point out that, when taking complex conjugates, it is understood that Dirac gamma matrices are not conjugated, e.g.,

$$\mathrm{Im}G_R=\mathrm{Im}\mathcal{G}_R+\gamma_\mu\mathrm{Im}\mathcal{G}_R^\mu \ .$$

In accordance with the Kadanoff-Baym *Ansatz*, Eq. (2.44), we define quasiparticle distribution functions $\pm F^{>,<}(R,p)$ through

$$G^{>,<}(R,p)=\pm 2iF^{>,<}(R,p)\mathrm{Im}G_R(R,p) =\mp 2\pi iF^{>,<}(R,p) \times\mathrm{sgn}Z_F(R,p)[p_\mu^*\gamma^\mu+m^*]\delta(p^{*2}-m^{*2}). \tag{3.67}$$

Since $G^> - G^< = G_R - G_A = 2i\,\mathrm{Im}G_R$, it is easy to see that

$$F^>(R,p)+F^<(R,p)=1\ . \tag{3.68}$$

After Wigner transformation, gradient expansion, and standard manipulations with the Dirac algebra, we can write the scalar piece of the generalized Boltzmann equation, Eq. (2.34), in the form

$$i[\mathcal{G}^{>,<},-m^*]_{\mathrm{PB}}(R,p)+i[\mathcal{G}_\mu^{>,<},p^{*\mu}]_{\mathrm{PB}}(R,p) = (\sigma^<\mathcal{G}^> + \sigma_\mu^<\mathcal{G}^{>\mu})(R,p)-(\sigma^>\mathcal{G}^<+\sigma_\mu^>\mathcal{G}^{<\mu})(R,p)\ . \tag{3.69}$$

Upon substituting the *Ansatz*, Eq. (3.67), into the previous equation and after making the observation that $\mathrm{Im}G_R$ commutes with the Poisson bracket, we find

$$\mathrm{sgn}Z_F(R,p)\delta(p^{*2}-m^{*2})[\pm F^{>,<}(R,p),p^{*2}-m^{*2}]_{\mathrm{PB}} = \mathrm{sgn}Z_F(R,p)\delta(p^{*2}-m^{*2}) \times[(\sigma^<m^*+\sigma_\mu^<p^{*\mu})F^>(R,p) +(\sigma^>m^*+\sigma_\mu^>p^{*\mu})F^<(R,p)]\ . \tag{3.70}$$

Using the identity, Eq. (3.43), we see that the *particle* distribution function $f(\mathbf{p})$ defined by

$$f(R,\mathbf{p})=F^<(R,\mathbf{p},\epsilon_\mathbf{p}^+) \tag{3.71}$$

satisfies

$$\left[\frac{\partial}{\partial T}-\frac{\partial\epsilon_\mathbf{p}^+}{\partial\mathbf{R}}\cdot\frac{\partial}{\partial\mathbf{p}}+\frac{\partial\epsilon_\mathbf{p}^+}{\partial\mathbf{p}}\cdot\frac{\partial}{\partial\mathbf{R}}\right]f(\mathbf{p})=-2iZ_F(\mathbf{p},\epsilon_\mathbf{p}^+)\{[1-f(\mathbf{p})][\sigma^<(\mathbf{p},\epsilon_\mathbf{p}^+)m^*(\mathbf{p},\epsilon_\mathbf{p}^+)+\sigma_\mu^<(\mathbf{p},\epsilon_\mathbf{p}^+)p^{*\mu}(\mathbf{p},\epsilon_\mathbf{p}^+)] +f(\mathbf{p})[\sigma^>(\mathbf{p},\epsilon_\mathbf{p}^+)m^*(\mathbf{p},\epsilon_\mathbf{p}^+)+\sigma_\mu^>(\mathbf{p},\epsilon_\mathbf{p}^+)p^{*\mu}(\mathbf{p},\epsilon_\mathbf{p}^+)]\}\ . \tag{3.72}$$

Similarly, the *antiparticle* distribution function $\bar f(\mathbf{p})$ given by

$$\bar f(R,-\mathbf{p})=F^>(R,\mathbf{p},-\epsilon_{-p}^-) \tag{3.73}$$

satisfies the equation

$$\left[\frac{\partial}{\partial T}-\frac{\partial\epsilon_\mathbf{p}^-}{\partial\mathbf{R}}\cdot\frac{\partial}{\partial\mathbf{p}}+\frac{\partial\epsilon_\mathbf{p}^-}{\partial\mathbf{p}}\cdot\frac{\partial}{\partial\mathbf{R}}\right]\bar f(\mathbf{p})=-2iZ_F(-\mathbf{p},-\epsilon_\mathbf{p}^-) \times\{[1-\bar f(\mathbf{p})][\sigma^>(-\mathbf{p},-\epsilon_\mathbf{p}^-)m^*(-\mathbf{p},-\epsilon_\mathbf{p}^-)+\sigma_\mu^>(-\mathbf{p},-\epsilon_\mathbf{p}^-)p^{*\mu}(-\mathbf{p},-\epsilon_\mathbf{p}^-)] +\bar f(\mathbf{p})[\sigma^<(-\mathbf{p},-\epsilon_\mathbf{p}^-)m^*(-\mathbf{p},-\epsilon_\mathbf{p}^-)+\sigma_\mu^<(-\mathbf{p},-\epsilon_\mathbf{p}^-)p^{*\mu}(-\mathbf{p},-\epsilon_\mathbf{p}^-)]\}\ . \tag{3.74}$$

We can get some insight into the meaning of the distribution functions $f(\mathbf{p})$ and $\bar f(\mathbf{p})$ by writing the fermion contribution to the energy-momentum stress tensor, Eq. (3.22). We proceed as in the boson case by defining the two-point function

$$T_{\mu\nu}^F(x,y)\equiv\frac{i}{4}(\partial_\mu^x-\partial_\mu^y)\bar\Psi(y)\gamma_\nu\Psi(x)+\frac{i}{4}(\partial_\nu^x-\partial_\nu^y)\bar\Psi(y)\gamma_\mu\Psi(x) =\tfrac14(\partial_\mu^x-\partial_\mu^y)\mathrm{tr}\gamma_\nu G^<(x,y)+\tfrac14(\partial_\nu^x-\partial_\nu^y)\mathrm{tr}\gamma_\mu G^<(x,y)\ , \tag{3.75}$$

which reduces to the ordinary stress tensor upon setting $x=y$. Upon Wigner transforming with the help of Eq. (3.67) for $G^<(R,p)$ along with the distribution functions, Eqs. (3.71) and (3.73), we find

$$T_{\mu\nu}^F(R)\equiv\int\frac{d^4p}{(2\pi)^4}T_{\mu\nu}^F(R,p) =\gamma\int\frac{d^3p}{(2\pi)^3}2Z_F(\mathbf{p},\epsilon_p^+)[p_\mu p_\nu^*(\mathbf{p},\epsilon_p^+)+p_\nu p_\mu^*(\mathbf{p},\epsilon_p^+)]|_{p_0=\epsilon_p^+}f_\mathbf{p} +\gamma\int\frac{d^3p}{(2\pi)^3}2Z_F(-\mathbf{p},-\epsilon_p^-)[p_\mu p_\nu^*(-\mathbf{p},-\epsilon_p^-)+p_\nu p_\mu^*(-\mathbf{p},-\epsilon_p^-)]|_{p_0=\epsilon_p^-}\bar f_\mathbf{p}+\text{vacuum contribution}\ , \tag{3.76}$$

where we have not explicitly written down the divergent zero-point vacuum fluctuation contribution. As in the boson case, some caution has to be exercised here because the vacuum contribution cannot be eliminated by normal ordering the field operators. Also, γ, the isospin degeneracy parameter arising from the trace operation, takes on a value of n for a system with n flavors of fermions. So, for nuclear matter with only neutrons and protons, $\gamma=2$.

To provide a more transparent interpretation of the distribution functions we consider the energy-momentum tensor in the mean-field or *collisionless* approximation. This approximation is realized by the substitutions $p_\mu^* \rightarrow p_\mu$ and $m^* \rightarrow m-g\phi$. We find

$$T_{\rm MF}^{\mu\nu}=2\gamma \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{E^*(\mathbf{p})}(f_{\rm p}+\bar{f}_{\rm p}) , \tag{3.77}$$

where

$$E^*(\mathbf{p})=+[\mathbf{p}^2+(m-g\phi)^2]^{1/2} . \tag{3.78}$$

We see that, at least in the mean-field approximation, $f_{\rm p}$ and $\bar{f}_{\rm p}$ represent the number of fermions and antifermions, respectively, with momentum p. We emphasize that f and $\bar{f}$ also depend upon the space-time point R but this dependence has been suppressed for clarity.

D. Boltzmann equations

Now that we have constructed distribution functions for the particles, we can complete the task of finding Boltzmann equations for the interacting fermion-boson system. As previously mentioned, we approximate the self-energies by using the bare vertex, Eq. (3.20). In this lowest-order approximation, the contour self-energies, Eqs. (3.15) and (3.16), are given by

$$\Sigma_{\mathcal{C}}(x,y)=ig^2G_{\mathcal{C}}(x,y)\Delta_{\mathcal{C}}(y,x) \tag{3.79}$$

and

$$\Pi_{\mathcal{C}}(x,y)=-ig^2{\rm tr}G_{\mathcal{C}}(x,y)G_{\mathcal{C}}(y,x) . \tag{3.80}$$

Using the Langreth-Wilkins rule for antiparallel products, Eq. (2.30), we find

$$\Sigma^{>,<}(x,y)=ig^2G^{>,<}(x,y)\Delta^{<,>}(y,x) \tag{3.81}$$

and

$$\Pi^{>,<}(x,y)=-ig^2{\rm tr}G^{>,<}(x,y)G^{<,>}(y,x) . \tag{3.82}$$

Upon Wigner transforming with the help of Eq. (A5) in the Appendix and inserting Eqs. (3.40) and (3.67), we obtain, after tedious algebra,

$$\begin{aligned}\Sigma^>(p)=&-(2\pi)^4ig^2\int\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}[\gamma^\mu p_\mu^*(\mathbf{q},\epsilon_{\rm q}^+)+m^*(\mathbf{q},\epsilon_{\rm q}^+)]Z_F(\mathbf{q},\epsilon_{\rm q}^+)Z_B(\mathbf{k},\omega_{\rm k})\\&\times[n_{\rm k}(1-f_{\rm q})\delta(p_0-\epsilon_{\rm q}^++\omega_{\rm k})\delta(\mathbf{p}-\mathbf{q}+\mathbf{k})+(1+n_{\rm k})(1-f_{\rm q})\delta(p_0-\epsilon_{\rm q}^+-\omega_{\rm k})\delta(\mathbf{p}-\mathbf{q}-\mathbf{k})]\\&-(2\pi)^4ig^2\int\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}[\gamma^\mu p_\mu^*(-\mathbf{q},-\epsilon_{\rm q}^-)+m^*(-\mathbf{q},-\epsilon_{\rm q}^-)]Z_F(-\mathbf{q},-\epsilon_{\rm q}^-)Z_B(\mathbf{k},\omega_{\rm k})\\&\times[n_{\rm k}\bar{f}_{\rm q}\delta(p_0+\epsilon_{\rm q}^-+\omega_{\rm k})\delta(\mathbf{p}+\mathbf{q}+\mathbf{k})+(1+n_{\rm k})\bar{f}_{\rm q}\delta(p_0+\epsilon_{\rm q}^--\omega_{\rm k})\delta(\mathbf{p}+\mathbf{q}-\mathbf{k})] ,\end{aligned} \tag{3.83}$$

$$\begin{aligned}\Sigma^<(p)=&(2\pi)^4ig^2\int\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}[\gamma^\mu p_\mu^*(\mathbf{q},\epsilon_{\rm q}^+)+m^*(\mathbf{q},\epsilon_{\rm q}^+)]Z_F(\mathbf{q},\epsilon_{\rm q}^+)Z_B(\mathbf{k},\omega_{\rm k})\\&\times[n_{\rm k}f_{\rm q}\delta(p_0-\epsilon_{\rm q}^+-\omega_{\rm k})\delta(\mathbf{p}-\mathbf{q}-\mathbf{k})+(1+n_{\rm k})f_{\rm q}\delta(p_0-\epsilon_{\rm q}^++\omega_{\rm k})\delta(\mathbf{p}-\mathbf{q}+\mathbf{k})]\\&+(2\pi)^4ig^2\int\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}[\gamma^\mu p_\mu^*(-\mathbf{q},-\epsilon_{\rm q}^-)+m^*(-\mathbf{q},-\epsilon_{\rm q}^-)]Z_F(-\mathbf{q},-\epsilon_{\rm q}^-)Z_B(\mathbf{k},\omega_{\rm k})\\&\times[n_{\rm k}(1-\bar{f}_{\rm q})\delta(p_0+\epsilon_{\rm q}^--\omega_{\rm k})\delta(\mathbf{p}+\mathbf{q}-\mathbf{k})+(1+n_{\rm k})(1-\bar{f}_{\rm q})\delta(p_0+\epsilon_{\rm q}^-+\omega_{\rm k})\delta(\mathbf{p}+\mathbf{q}+\mathbf{k})] ,\end{aligned} \tag{3.84}$$

and

$$\begin{aligned}\Pi^<(p)=&\Pi^>(-p)\\=&-4ig^2\gamma(2\pi)^4\int\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}[\Theta(-\mathbf{q},-\epsilon_{\rm q}^-,-\mathbf{k},-\epsilon_{\rm k}^-)Z_F(-\mathbf{k},-\epsilon_{\rm k}^-)Z_F(-\mathbf{q},-\epsilon_{\rm q}^-)\\&\times\bar{f}_{\rm q}(1-\bar{f}_{\rm k})\delta(\mathbf{p}+\mathbf{q}-\mathbf{k})\delta(p_0+\epsilon_{\rm q}^--\epsilon_{\rm k}^-)\\&+\Theta(-\mathbf{q},-\epsilon_{\rm q}^-,\mathbf{k},\epsilon_{\rm k}^+)Z_F(\mathbf{k},\epsilon_{\rm k}^+)Z_F(-\mathbf{q},-\epsilon_{\rm q}^-)(1-\bar{f}_{\rm q})\\&\times(1-f_{\rm k})\delta(\mathbf{p}+\mathbf{q}+\mathbf{k})\delta(p_0+\epsilon_{\rm q}^-+\epsilon_{\rm k}^+)\\&+\Theta(\mathbf{q},\epsilon_{\rm q}^+,-\mathbf{k},-\epsilon_{\rm k}^-)Z_F(-\mathbf{k},-\epsilon_{\rm k}^-)Z_F(\mathbf{q},\epsilon_{\rm q}^+)\\&\times f_{\rm q}\bar{f}_{\rm k}\delta(\mathbf{p}-\mathbf{q}-\mathbf{k})\delta(p_0-\epsilon_{\rm q}^+-\epsilon_{\rm k}^-)\\&+\Theta(\mathbf{q},\epsilon_{\rm q}^+,\mathbf{k},\epsilon_{\rm k}^+)Z_F(\mathbf{k},\epsilon_{\rm k}^+)Z_F(\mathbf{q},\epsilon_{\rm q}^+)\\&\times f_{\rm q}(1-f_{\rm k})\delta(\mathbf{p}-\mathbf{q}+\mathbf{k})\delta(p_0-\epsilon_{\rm q}^++\epsilon_{\rm k}^+)] ,\end{aligned} \tag{3.85}$$

where, for convenience, we have defined the function

$$\Theta(\mathbf{p},p_0,\mathbf{q},q_0)=p^*_\mu(\mathbf{p},p_0)p^{*\mu}(\mathbf{q},q_0)+m^*(\mathbf{p},p_0)m^*(\mathbf{q},q_0) \; . \tag{3.86}$$

Inserting expressions (3.83) and (3.84) into Eq. (3.72) for the particle distribution function yields the Boltzmann equation

$$\left[\frac{\partial}{\partial T}-\frac{\partial\epsilon^+_{\mathbf{p}}}{\partial\mathbf{R}}\cdot\frac{\partial}{\partial\mathbf{p}}+\frac{\partial\epsilon^+_{\mathbf{p}}}{\partial\mathbf{p}}\cdot\frac{\partial}{\partial\mathbf{R}}\right]f(\mathbf{p})=2g^2\int\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\Theta(\mathbf{q},\epsilon^+_{\mathbf{q}},\mathbf{p},\epsilon^+_{\mathbf{p}})Z_F(\mathbf{p},\epsilon^+_{\mathbf{p}})Z_F(\mathbf{q},\epsilon^+_{\mathbf{q}})Z_B(\mathbf{k},\omega_{\mathbf{k}})$$
$$\times\{[n_{\mathbf{k}}f_{\mathbf{q}}(1-f_{\mathbf{p}})(2\pi)^4\delta(\mathbf{p}-\mathbf{q}-\mathbf{k})\delta(\epsilon^+_{\mathbf{p}}-\epsilon^+_{\mathbf{q}}-\omega_{\mathbf{k}})$$
$$+(1+n_{\mathbf{k}})(1-f_{\mathbf{p}})f_{\mathbf{q}}(2\pi)^4\delta(\mathbf{p}-\mathbf{q}+\mathbf{k})\delta(\epsilon^+_{\mathbf{p}}-\epsilon^+_{\mathbf{q}}-\omega_{\mathbf{k}})]$$
$$-[n_{\mathbf{k}}f_{\mathbf{p}}(1-f_{\mathbf{q}})(2\pi)^4\delta(\mathbf{p}-\mathbf{q}+\mathbf{k})\delta(\epsilon^+_{\mathbf{p}}-\epsilon^+_{\mathbf{q}}+\omega_{\mathbf{k}})$$
$$+(1+n_{\mathbf{k}})f_{\mathbf{p}}(1-f_{\mathbf{q}})(2\pi)^4\delta(\mathbf{p}-\mathbf{q}-\mathbf{k})\delta(\epsilon^+_{\mathbf{p}}-\epsilon^+_{\mathbf{q}}-\omega_{\mathbf{k}})]\}$$
$$+\text{antiparticle terms} \; . \tag{3.87}$$

Similarly, upon substitution of Eq. (3.85) for the polarization insertion into Eq. (3.43) for the boson distribution function n, we find

$$\left[\frac{\partial}{\partial T}-\frac{\partial\omega_{\mathbf{p}}}{\partial\mathbf{R}}\cdot\frac{\partial}{\partial\mathbf{p}}+\frac{\partial\omega_{\mathbf{p}}}{\partial\mathbf{p}}\cdot\frac{\partial}{\partial\mathbf{R}}\right]n(\mathbf{p})=4g^2\gamma\int\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\Theta(\mathbf{q},\epsilon^+_{\mathbf{q}},\mathbf{k},\epsilon^+_{\mathbf{k}})Z_F(\mathbf{k},\epsilon^+_{\mathbf{k}})Z_F(\mathbf{q},\epsilon^+_{\mathbf{q}})Z_B(\mathbf{p},\omega_{\mathbf{p}})$$
$$\times\{[(1+n_{\mathbf{p}})f_{\mathbf{q}}(1-f_{\mathbf{k}})-n_{\mathbf{p}}f_{\mathbf{k}}(1-f_{\mathbf{q}})]$$
$$\times(2\pi)^4\delta(\mathbf{p}-\mathbf{q}+\mathbf{k})\delta(\omega_{\mathbf{p}}-\epsilon^+_{\mathbf{q}}+\epsilon^+_{\mathbf{k}})+\text{antiparticle terms}\} \; . \tag{3.88}$$

For simplicity, in deriving Eqs. (3.87) and (3.88) we neglect any coupling to antiparticle states. It is easy to include them *formally*, but it is not clear what to do with them due to the *nonperturbative* nature of the equations. Such an inclusion would imply renormalization, but, at this point, an adequate nonperturbative renormalization scheme does not exist. Moreover, since the theory is not asymptotically free, it is questionable whether their inclusion makes any sense.[29–31] This omission is also consistent with our view of the *phenomenological* nature of hadronic field theories.

Equations (3.87) and (3.88) are coupled Boltzmann equations for the fermion distribution function $f(R,\mathbf{p})$ and the boson distribution function $n(R,\mathbf{p})$, respectively. The left-hand sides have the familiar form of a drift term while the right-hand sides represent collision terms. The collision terms, illustrated in Fig. 3, include Pauli blocking as is evident from the presence of the $1-f$ factors; in addition, they contain the familiar terms proportional to $1+n$ indicating stimulated emission of bosons.

While the coupled Boltzmann equations, (3.87) and (3.88), present a formidable numerical challenge, it is possible, under certain conditions,[7] to "integrate" out the boson distribution function $n(R,\mathbf{p})$, reducing the task to that of solving a single Boltzmann equation. To see this, note that Eq. (2.32) for the boson function $\Delta^{><}(x,y)$ may be put in the form

$$\Delta_R^{-1}\Delta^{><}=\Pi^{><}\Delta_A \tag{3.89}$$

or, upon Wigner transformation and gradient expansion,

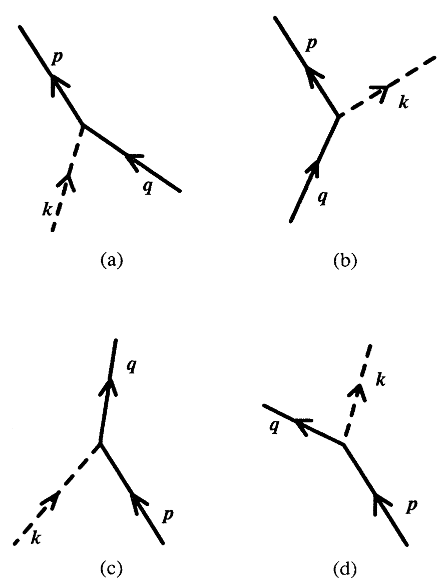

FIG. 3. Collision terms. In (a), a fermion with four-momentum q absorbs a boson of momentum k and scatters into a state of momentum p. In (b), a fermion scatters into a state p through the emission of a boson with four-momentum k. (c) and (d) represent processes in which a fermion scatters out of state p into a state q. All these processes take place "on shell" and the radiated mesons are produced by a process analogous to Cherenkov radiation. These diagrams should not be confused with Feynman diagrams.

$$\Delta^{><}(R,p)=\Delta_R(R,p)\Pi^{><}(R,p)\Delta_A(r,p)+O(\hbar) \ . \tag{3.90}$$

The $O(\hbar)$ term may be neglected if the meson distribution functions vary slowly enough that local equilibrium is maintained. It is easy to see that the approximation, Eq. (3.90), guarantees that the collision terms of the boson Boltzmann equation, Eq. (3.27), vanish provided one neglects terms of $O(\hbar)$. This defines *local equilibrium.* Indeed, inserting Eq. (3.90) into the right-hand side (rhs) of Eq. (3.27) and neglecting terms of $O(\hbar)$ yields

$$0=\Pi^{<}(R,p)\Delta_R(R,p)\Pi^{>}(R,p)\Delta_A(R,p) - \Pi^{>}(R,p)\Delta_R(R,p)\Pi^{<}(R,p)\Delta_A(R,p) \ . \tag{3.91}$$

The right-hand side of this equation vanishes and, hence, the Boltzmann equation for the boson field is trivially satisfied in this limit. With the approximation, Eq. (3.90), we can write

$$\Sigma^{>}(R,p)=-4i\gamma g^2\int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}|\Delta_R(q-p)|^2\Theta(q,q')Z_F(p')Z_F(q')Z_F(q) \times[\gamma^\mu p^*_\mu(p')+m^*(p')]f_{\mathbf{q}}(1-f_{\mathbf{p}'})(1-f_{\mathbf{q}'})(2\pi)^4\delta^4(p+q-p'-q') \tag{3.92}$$

and

$$\Sigma^{<}(R,p)=4i\gamma g^2\int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}|\Delta_R(p'-p)|^2\Theta(q,q')Z_F(p')Z_F(q')Z_F(q) \times[\gamma^\mu p^*_\mu(p')+m^*(p')]f_{\mathbf{p}'}f_{\mathbf{q}'}(1-f_{\mathbf{q}})(2\pi)^4(p+q-p'-q') \ , \tag{3.93}$$

where we have neglected antiparticle contributions. Furthermore, it is understood that all four-momenta are on shell, i.e., $p_0=\epsilon^+(\mathbf{p})$. Substitution into Eq. (3.72) yields the single Boltzmann equation

$$\left[\frac{\partial}{\partial T}-\frac{\partial\epsilon^+_{\mathbf{p}}}{\partial\mathbf{R}}\cdot\frac{\partial}{\partial\mathbf{p}}+\frac{\partial\epsilon^+_{\mathbf{p}}}{\partial\mathbf{p}}\cdot\frac{\partial}{\partial\mathbf{R}}\right]f(\mathbf{p})=8\gamma g^2\int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}|\Delta_R(p'-p)|^2 \times\Theta(q,q')\Theta(p,p')Z_F(p)Z_F(p')Z_F(q)Z_F(q') \times[f_{\mathbf{p}'}f_{\mathbf{q}'}(1-f_{\mathbf{p}})(1-f_{\mathbf{q}})-f_{\mathbf{p}}f_{\mathbf{q}}(1-f_{\mathbf{p}'})(1-f_{\mathbf{q}'})] \times(2\pi)^4\delta^4(p+q-p'-q') \ . \tag{3.94}$$

This equation has an easy interpretation. The left-hand side is the drift or streaming term. This term describes the measure-preserving evolution of a particle with momentum $\mathbf{p}$ and velocity $\partial\varepsilon^+_{\mathbf{p}}/\partial\mathbf{p}$ "drifting" through phase space under the influence of a "force" $-\partial\varepsilon^+_{\mathbf{p}}/\partial\mathbf{R}$. This "force" is due to the direct and exchange contributions of the self-consistent Hartree-Fock potential generated by the other particles in the medium. In the absence of collisions, the right-hand side is zero and we recover the classical Liouville equation. The terms in square brackets on the right-hand side describe collisions in and out of a phase-space element with Pauli blocking; the delta function ensures that energy-momentum conservation is maintained. The $Z_F(p)$ factors or, more precisely $2p_0Z_F(p)$, are renormalizations for the outgoing fermion lines due to the momentum dependence of the self-energy; this feature was absent in previous investigations.[6,11] The remaining factors represent the effective cross section for the collisions which include dynamical screening effects due to medium modifications. We emphasize that no matter how physically intuitive and appealing Eq. (3.94) is, it is only valid when the boson fields are in local equilibrium—an unlikely scenario in an energetic heavy-ion collision.

IV. DISCUSSION AND CONCLUSION

In the previous section, we present two sets of Boltzmann equations. The first set consists of two coupled equations, (3.87) and (3.88), describing the self-consistent transport of mutually interacting fermions and bosons. These two equations are derived under the assumption that the pole value of the propagators dominate the dynamics (Kadanoff-Baym *Ansatz*), thus leading to the on-shell propagation of particles. Hence, the collision terms reflect this on-shell constraint through the presence of energy-momentum conserving delta functions and, as a result, the bosons are created on shell. Such production processes cannot take place in the vacuum because energy and momentum cannot be simultaneously conserved. Creation via this process can only take place in the presence of a medium through the modification of effective masses, etc. For this reason, the production process that we describe here is analogous to Cherenkov radiation. Although it is easy to produce photons via Cherenkov radiation, it may be very difficult to produce mesons in this manner since the mesons are quite massive. Because the mesons are so massive, the only way for them to be produced by this process is for the medium to *significantly* modify the masses of the particles in such a way as to maintain energy-momentum conservation. Clearly, this could be a problem for the heavy mesons, and the utility of these equations is problematic.

The second consists of a single equation, Eq. (3.94), for the fermion distribution function. It was derived under the assumption that the mesons remain in equilibrium during transport. Stated in another way, the mesons interacting with the nucleons are virtual particles. This picture is expected to be valid at low energies below or just above pion production threshold.

Clearly, the most physical scenario lies somewhere between the two extreme pictures described above. In a reasonable picture, it should be possible to describe the creation and destruction of real as well as virtual particles and not just one or the other. In this picture, the production mechanism for mesons is bremsstrahlung, where two nucleons exchange virtual mesons with spacelike momenta producing an off-shell nucleon which then decays to an on-shell nucleon and an on-shell meson (Fig. 4). This mechanism is more physically appealing and much more likely for meson production than the Cherenkov process described above. It must be emphasized that any transport theory which employs a mass-shell constraint will inevitably lead to meson production via a Cherenkov-like process. For this reason, it is necessary to go beyond the quasiparticle approximation if a consistent transport theory is to be constructed.

In this work, through a series of approximations and *Ansätze*, we derive *semiclassical* transport equations from the underlying quantum field theory. Now one must ask whether or not these approximations or *Ansätze* are valid and, if so, under what conditions.

In an attempt to describe the dynamics classically, we make the assumption that the gradient expansion is a useful expansion. The validity of this expansion hinges upon the assumption that the space-time inhomogeneities of the system vary much slower than the average space-time distance between successive collisions. Equivalently, the

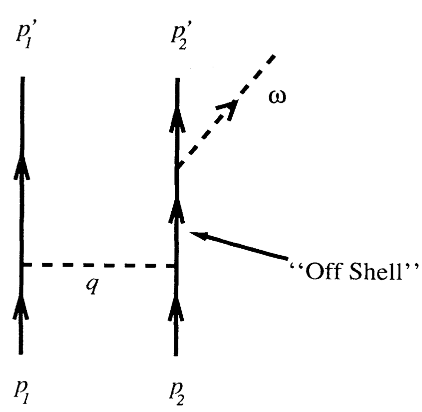

FIG. 4. A possible meson production process which is outside the domain of the Kadanoff-Baym *Ansatz* (see main text). A virtual (off-shell) meson is exchanged between two nucleons producing and off-shell nucleon which subsequently decays into a nucleon and a meson.

average distance between collisions is assumed to be much larger than the distance scale set by the collision time. In a typical *NN* collision, the mean free path between collisions, λ, is on the order of $\lambda \sim (\rho\sigma)^{-1} \sim 1\text{–}2$ fm^{-1} for energies around 1 GeV/nucleon. However, the nuclear force is of approximately the same range, hence, quantum effects are expected to be important. The argument presented here is for an individual nucleon-nucleon collision. However, in a heavy-ion collision, many such collisions take place and it is hoped that the quantum interference effects will tend to cancel out. Perhaps this is the reason for the success of semiclassical kinetic equations in the description of heavy-ion collisions; however, it has yet to be demonstrated.

The most crucial *Ansatz* which dictates a Markovian behavior of the resulting transport equations is the Kadanoff-Baym *Ansatz.* This *Ansatz* is equivalent to putting the particles on mass shell between collisions. However, from the energy-time uncertainty principle, nucleons are expected to be off shell on the order of $\hbar/\tau \sim 100$ MeV or more where $\tau = (\rho\sigma v)^{-1} \sim 2$ fm/c is the time between collisions. Since this is a substantial fraction of the nucleon mass, it is not clear whether the assumption of putting the nucleons on their mass shell between collisions is valid. This point was first emphasized by Danielewicz.[8] Moreover, as eluded to earlier, putting the mesons on mass shell results in the production of mesons via Cherenkov radiation. Any other production mechanism is outside the domain of the pole approximation.

In this paper an attempt is made to present a systematic program for constructing relativistic Boltzmann equations from first principles by generalizing the same techniques that have been successful in nonrelativistic transport theory. We present the closed-time-path method of Schwinger using a contour-functional approach which parallels the approach used in the ordinary vacuum theory. Using the Langreth-Wilkins rules to continue from the contour to the real-time axis, we derive the generalized Boltzmann equation from the contour Dyson equation, showing that the generalized Boltzmann equation is nothing more than a restatement of the Dyson equation. At this point it is necessary to introduce an *Ansatz* to relate the "off-time-diagonal components" of the Green's function $G^<(t,t')$ to its "time-diagonal" components $G^<(t,t)$ since the generalized Boltzmann equation has a two-time structure while a Boltzmann equation only depends upon a single time. The *Ansatz* fulfilling these needs used here is the Kadanoff-Baym *Ansatz.* This *Ansatz* consists of describing the system in terms of weakly interacting quasiparticles with a "delta-function-like" width for the energy spread. This is found to be equivalent to the so-called "mass-shell" constraint used by other authors.[2,6,11]

To illustrate these ideas we work with a relativistic system of spin-half particles interacting with spin-zero quanta via a Yukawa coupling. Although the program outlined above is straightforward to implement for the bosons, we find it convenient to use the spin-symmetric *Ansatz*, Eq. (3.54), of Bezzerides and Dubois to deal with the matrix structure of the fermion Green's functions. Having dealt with the discrete variables, we are then able to derive a set of coupled transport equations for the quasiparticle distribution functions. Finally, assuming local equilibrium for the boson quasiparticles, we eliminate the boson function in favor of one Boltzmann equation for the fermion distribution function. We have demonstrated that above meson production threshold, one cannot describe the dynamics of a system in terms of the fermion distribution function alone, ignoring the dynamics of the meson fields, regardless of whether or not medium-dependent cross sections are used, if the boson fields are not in equilibrium. This condition should be violated in heavy-ion collisions of sufficiently high energy, invalidating any attempt which ignores dynamical mesons, relativistic or nonrelativistic. This claim should be intuitively obvious. At high energies, mesons must carry a significant fraction of the energy and momentum after a collision. This means any description of the equilibration process should refer to the self-consistent transport in this sector. In other words, one should be forced to consider coupled Boltzmann equations for the particle distribution functions. It is possible that an energy regime exists within which only pions need to be explicitly treated.

In this work, although the Lagrangian used is renormalizable, we neglect vacuum effects such as creation and annihilation of antiparticles. Even if one were dealing with a fundamental rather than an effective theory, the nonperturbative approximations we employ would require nonperturbative renormalizations. Thus, the consideration of vacuum effects is premature at this time. Moreover, this work is aimed at phenomenological hadronic field theories in which one usually ignores the vacuum anyway. We feel that a more immediate problem is the choice of the parameters, e.g., meson masses and coupling constants. Unlike quantum electrodynamics (QED), in which the renormalized parameters are measured experimentally, hadronic field theories such as the Walecka model[10] have the peculiarity that parameters are adjusted to fit the bulk properties of nuclei; hence, the masses and coupling constants are chosen differently at each level of approximation. The Boltzmann equations derived in this paper come from a "self-consistent Hartree-Fock plus RPA" (random phase) approximation to the Dyson equation—an approximation that goes far beyond what has been done for the ground state in the Walecka model. So, before attempting a numerical solution of the Boltzmann equations presented here, one needs to study the ground state at this same level of sophistication (neglecting Dirac sea effects).

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APPENDIX

In this appendix we gather together a number of useful formulas involving the Wigner transform. The proof of these formulas is left to the reader. The Wigner transform of a two-point function $A(x,y)$ (this should not be confused with a contour two-point function) is defined

$$A(R,p) \equiv \int d^4r\, e^{ipr/\hbar} A\left[R+\frac{r}{2}, R-\frac{r}{2}\right], \tag{A1}$$

with the inverse Wigner transform

$$A(x,y) = \int \frac{d^4p}{(2\pi\hbar)^4} e^{-ipr/\hbar} A(R,p) . \tag{A2}$$

Here $R=\frac{1}{2}(x+y)$ and $r=x-y$.

From the definition of the Wigner transform it follows that

$$A(x,y) \rightarrow A(R,p) , \tag{A3}$$

$$\int d^4z\, A(x,z)B(z,y) \rightarrow \exp\left[\frac{i\hbar}{2}(\partial_p^A \cdot \partial_R^B - \partial_R^A \cdot \partial_p^B)\right] A(R,p)B(R,p) , \tag{A4}$$

$$A(x,y)B(y,x) \rightarrow \int \frac{d^4q}{(2\pi\hbar)^4} A(R,q)B(R,q-p) , \tag{A5}$$

where we use a right arrow to indicate a Wigner transformation has been performed. For the special case where the two-point functions have *no* discrete indices (i.e., no matrix structure), we find, from Eq. (A4), the useful result

$$\int d^4z [A(x,z)B(z,y) - B(x,z)A(z,y)] \rightarrow i\hbar[\partial_p A \cdot \partial_R B - \partial_R A \cdot \partial_p B](R,p) + O(\hbar^2) , \tag{A6}$$

or, in a more compact form,

$$[A,B](x,y) \rightarrow i\hbar[A,B]_{\mathrm{PB}}(R,p) + O(\hbar^2) , \tag{A7}$$

which defines the generalized Poisson bracket. Similarly, when discrete indices are absent we have

$$\{A,B\}(x,y) \rightarrow 2A(R,p)B(R,p) + O(\hbar^2) . \tag{A8}$$

In addition, one can show that

$$\int d^4x\, d^4y\, A(x,y)B(y,x) = \int \frac{d^4R d^4p}{(2\pi\hbar)^4} A(R,p)B(R,p) , \tag{A9}$$

which is useful for calculating ensemble averages.

Finally, we list a few useful transforms that are frequently encountered:

$$\delta(x-y) \rightarrow 1 , \tag{A10}$$

$$\phi(x)\delta(x-y) \rightarrow \phi(R) , \tag{A11}$$

$$(i\hbar\partial_\mu)^n \delta(x-y) \rightarrow (p_\mu)^n , \tag{A12}$$

$$\theta(x_0-y_0)\delta(\mathbf{x}-\mathbf{y}) \rightarrow \frac{-i\hbar}{p_0+i\varepsilon} , \tag{A13}$$

$$\theta(x_0-y_0) \rightarrow \frac{i\hbar}{p_0+i\varepsilon}(2\pi\hbar)^3\delta(\mathbf{p}) . \tag{A14}$$

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