β^+ decays of very proton-rich *sd*-shell nuclei

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Allowed β^+-decay branches of very proton-rich *sd*-shell nuclei up to the proton drip line are calculated by the complete *sd*-space shell model. The β^+-decay half-lives calculated with the global quenching factor $(g_A^{\text{eff}}/g_A)^2=0.60$ are in reasonable agreement with existing experimental data. However, considerable deviations of the quenching factor from the global value are obtained in a comparison of the theoretical and experimental ft values in the large-Q-value window. Probable candidates for the β-delayed two-proton emission are suggested with the predicted β^+-decay branching strengths to states above the proton-emission threshold energy.

Recent β^+-decay experiments have defined the proton drip line in the *sd*-shell region.[1-8] Beta decays of those nuclei near the drip line are expected to be subsequently followed by the emission of protons and, in some cases, proton pairs. The two-proton radioactivity has long been proposed as a radioactive decay for very proton-rich nuclei, and some potential candidates for this decay mode have been suggested.[9] So far, the β-delayed two-proton emission in the light-mass region has been observed[3,4] in the odd-odd $T_z=-2$ series for ^{22}Al and ^{26}P and for a $T_z=-\frac{5}{2}$ nuclide ^{35}Ca. The analysis of the proton spectra indicates that the two protons are emitted in a sequential process from the isobaric analog state (IAS) in the daughter nucleus. It is only in β^+ decays of light nuclei with $Z>N$ that the IAS is located in the Q-value window.

In this Brief Report, we show the β^+/electron capture-(β^+/EC-) decay properties of a systematic shell-model calculation for very proton-rich *sd*-shell nuclei, up to the proton drip line, which are (expected to be) stable or slightly unbound with regard to both one- and two-proton emissions. Beta-decay branches for nuclei with $T_z\geq-1$ and some $T_z=-\frac{3}{2}$ nuclei have been systematically studied by the shell model.[10,11]

We have performed a shell-model calculation in the full $(sd)^{A-16}$ configurations on the ^{16}O inert core, with the effective Hamiltonian of Wildenthal.[12] The Hamiltonian matrix is constructed in the basis states with definite isospin and angular momentum. The energy eigenvalues and wave functions are calculated for the parent states and for all daughter states which appear in the Q_{EC}-value window. When the ground-state angular momentum of the parent nucleus is not known experimentally, it is assumed to be identical to that of the mirror nucleus. Reduced transition probabilities of the Fermi and Gamow-Teller (GT) transitions, half-lives, and branching ratios are evaluated in the same procedure as Brown and Wildenthal.[10] We introduce the quenching factor $\gamma^2=(g_A^{\text{eff}}/g_A)^2$ multiplying $B(\text{GT})$ values which are calculated with the Gamow-Teller transition operator $t_+\sigma$, and assume the global value[12] of $\gamma^2=0.60$.

The ground-state Q_{EC} values are calculated with atomic masses of the parent and daughter nuclei. The masses are taken, if available, from the compilation by Wapstra, Audi, and Hoekstra[13] (masses predicted by systematics are not taken). Unknown masses are evaluated by applying the Kelson-Garvey mass equation based on charge symmetry,[14] which connects masses of higher-order mirror nuclei with those of ordinary mirror nuclei near the $Z=N$ line, with known experimental masses.[13] The mass equation predicts that, in the *sd*-shell region, the $A=4n+1$, $T_z=-\frac{5}{2}$ nuclei and all nuclei with $T_z\leq-3$, except ^{22}Si, are unstable for the emission of protons and/or proton pairs by more than 1 MeV.

The calculated half-lives and the errors due to the Q-value uncertainty are listed in Table I, together with available experimental values.[1-6,15-20] Since the half-lives and Q values are well determined experimentally for nuclei with $T_z=-1$ and $-\frac{3}{2}$ near the stability line, a comparison of the calculated and experimental half-lives of those nuclei shows the predictive power of the present shell-model calculation for the half-life. The agreement is excellent and no systematic deviations are observed. Values of the deviation, $t_{1/2}^{\text{calc}}/t_{1/2}^{\text{expt}}-1$, are distributed in a small range from -20% to $+27\%$ with a standard deviation of 13%. For more proton-rich nuclei, the large uncertainties of the existing experimental half-lives make it difficult to compare with the theoretical values. The reliable shell-model prediction provides a useful guide to experimental half-life measurements.

For the nuclei in the vicinity of the proton drip line, the calculated half-lives could have systematic uncertainties originating from the Q_{EC} value. While the Kelson-Garvey mass equation gives good agreement with experiment for 27 known pairs with $|T_z|\geq1$ in the *sd*-shell region, with a standard deviation of 0.11 MeV and the largest deviation of 0.32 MeV for the $|T_z|=\frac{5}{2}$ pair ^{35}Ca-^{35}P, it systematically overestimates masses of proton-unstable nuclei by up to about 1 MeV.[21,22] This energy shift is due to a Coulomb perturbation in the nuclear wave function (Thomas-Ehrman shift). If the parent mass is smaller than the prediction, the calculation gives accordingly a longer half-life. The errors of half-life shown in Table I are calculated with a Q-value uncertainty of 300 keV for those nuclei whose masses are not known experimentally.

The two parent nuclides in the odd-odd $T_z=-2$ series, ^{30}Cl and ^{34}K, are predicted by the mass equation to be unstable with regard to direct one-proton emission by 0.716 and 0.628 MeV, respectively (both are stable for two-proton emission). However, these nuclei could be less unbound owing to the Thomas-Ehrman shift. Then the β^+ decay would be observable, being the dominant decay mode or competitive with the direct proton emission. The shell-model prediction of the half-life would be helpful in an experimental determination of the main decay mode of these nuclei.

An advantage of a study of β^+ decays of very proton-rich nuclei is that it enables a direct comparison of theoretical and experimental Gamow-Teller strengths not only near the ground state, but also in higher-energy regions. Therefore, the β^+ decays with large Q values provide a sensitive test of the shell-model calculation. We have calculated the ratio of the sum of experimental B(GT) values to the theoretical sum in the same energy range $\gamma^2=\sum B(\mathrm{GT})^{\mathrm{expt}}/\sum B(\mathrm{GT})^{\mathrm{calc}}$. For *sd*-shell nuclei with $-1\leq T_z\leq\frac{5}{2}$,[10] the weighted average is $\gamma^2=0.64\pm0.09$, where the error is due to uncertainties in the experimental sum. This is consistent with the global value, which was obtained[12] by an extensive analysis of low-lying β transitions in a number of *sd*-shell nuclei, using wave functions from the same Hamiltonian as in the present shell-model calculation. ^{32}Ar is the most proton-rich nucleus whose β-decay branches have precisely been determined experimentally.[20] For the ^{32}Ar decay, $\gamma^2=0.74\pm0.08$ is obtained, corresponding to $\sum B(\mathrm{GT})^{\mathrm{expt}}=3.8\pm0.4$ below the limit of detection ($E_x=8.75$ MeV).[20] A shell-model calculation[23] with another effective Hamiltonian gave[20] a smaller value of $\gamma^2=0.49\pm0.05$. For β^+ decays of $T_z=-\frac{3}{2}$ nuclei,[16,24–27] we obtain $\gamma^2(^{21}\mathrm{Mg})=0.38\pm0.07$, $\gamma^2(^{29}\mathrm{S})=1.21\pm0.30$, $\gamma^2(^{33}\mathrm{Ar})=0.86\pm0.28$, $\gamma^2(^{35}\mathrm{K})=0.34\pm0.10$, and $\gamma^2(^{37}\mathrm{Ca})=0.67\pm0.11$. The significant deviations of the deduced quenching factors indicate uncertainties involved in the shell-model calculation. A guideline of a refinement of the effective interaction is suggested in a sum-rule analysis of Gamow-Teller excitations[28] and nuclear structure studies of the double-beta decay,[29] which is described as successive Gamow-Teller transitions through virtual intermediate nuclear states with $J^\pi=1^+$.

The β^+-decay branching ratios are calculated for all daughter states falling in the Q-value window. The window is divided into three regions at the one- and two-proton threshold energies of the daughter nucleus, $E_x<S_{1p}+\Delta$, $S_{1p}+\Delta<E_x<S_{2p}+\Delta$, and $S_{2p}+\Delta<E_x$, where Δ represents the effective Coulomb barrier which prevents a proton from being promptly emitted, and we assume $\Delta=0.3$ MeV. The branching strengths are summed in each energy interval, and the result is shown

TABLE I. Calculated half-lives of proton-rich *sd*-shell nuclei which are (expected to be) stable or slightly unbound with regard to both one- and two-proton emissions. The error corresponds to the Q-value uncertainty. The global quenching factor of $\gamma^2=0.60$ is used for the Gamow-Teller transitions. The available experimental values are also presented.

	$t_{1/2}^{\mathrm{calc}}$ (ms)	$t_{1/2}^{\mathrm{expt}}$ (ms)	Ref.		$t_{1/2}^{\mathrm{calc}}$ (ms)	$t_{1/2}^{\mathrm{expt}}$ (ms)	Ref.
$T_z=-1$				$T_z=-2$			
^{18}Ne	1758±12	1672±5	15	^{20}Mg	116±2	90^{+80}_{-50}	2
^{22}Mg	3085±4	3857±9	16	^{24}Si	158±2	100^{+90}_{-40}	2
^{26}Si	2033±8	2235±9	17	^{28}S	128±14	125±10	19
^{30}S	1202±4	1179±5	17	^{32}Ar	88±3	98±2	20
^{34}Ar	859±2	845±3	16	^{36}Ca	100±3	~100	2
^{38}Ca	470±2	435±9	17	^{22}Al	99±4	70^{+50}_{-35}	3
^{20}Na	525±2	446±3	15	^{26}P	37±5[a]	~20	3
^{24}Al	1830±8	2066±10	16	^{30}Cl	27±4[a]		
^{28}P	276±1	270±1	16	^{34}K	32±5[a]		
^{32}Cl	319±2	298±2	16				
^{36}K	327±2	342±2	16	$T_z=-\frac{5}{2}$			
				^{23}Si	47±7[a]		
$T_z=-\frac{3}{2}$				^{27}S	10±1[a]	16±5	6
^{21}Mg	155±1	122±3	16	^{31}Ar	10±1[a]	15±3	5
^{25}Si	233±2	220±3	16	^{35}Ca	23±1	50±30	4
^{29}S	163±5	187±4	16				
^{33}Ar	164±3	174±1	16	$T_z=-3$			
^{37}Ca	160±3	175±3	16	^{22}Si	31±5[a]		
^{23}Al	578±11	470±30	16				
^{27}P	275±7	260±80	18				
^{31}Cl	189±6	150±25	18				
^{35}K	186±3	190±30	1				

[a]The mass of the parent nucleus is evaluated by the Kelson-Garvey mass equation, and a Q-value uncertainty of 300 keV is assumed. Masses of the other parent nuclei and all daughter nuclei are known experimentally (Ref. 13).

TABLE II. Calculated β^+/EC-decay branching strengths, normalized to unity, in the three energy regions of the $Q_{\rm EC}$-value window divided at the one- (S_{1p}) and two-proton (S_{2p}) separation energies of the daughter nucleus. Region 1: $E_x < S_{1p} + \Delta$; region 2: $S_{1p} + \Delta < E_x < S_{2p} + \Delta$; region 3: $S_{2p} + \Delta < E_x$, with $\Delta = 0.3$ MeV. A bar denotes that the parent state is higher in energy than $S_{2p} + \Delta$. The quenching factor $\gamma^2 = 0.60$ is assumed for the Gamow-Teller transitions. The branching ratio of the IAS is separately shown. Experimental values (Ref. 30) of the β-delayed one-proton emission probability are given in the last column.

Parent nucleus	Branching strength Region 1	Region 2	Region 3	IAS	P_{1p}(expt)
$T_z = -\frac{3}{2}$					
^{21}Mg	0.675	0.325	–	0.032	0.33±0.11
^{25}Si	0.566	0.434	–	0.118	
^{29}S	0.596	0.404	0.000	0.150	0.47±0.05
^{33}Ar	0.648	0.352	0.000	0.281	0.34±0.06
^{37}Ca	0.238	0.762	0.000	0.414	0.76±0.03
^{23}Al	0.984	0.016	–	0.184	
^{27}P	0.991	0.009	–	0.166	
^{31}Cl	0.985	0.015	0.000	0.243	0.003
^{35}K	0.992	0.008	0.000	0.394	0.0037±0.0015
$T_z = -2$					
^{20}Mg	0.521	0.479	0.000	0.034	~0.03
^{24}Si	0.568	0.432	0.000	0.109	~0.07
^{28}S	0.721	0.279	0.000	0.180	
^{32}Ar	0.694	0.306	0.000	0.208	~0.17
^{36}Ca	0.484	0.516	0.000	0.387	~0.20
^{22}Al	0.414	0.282	0.303	0.041	~0.029
^{26}P	0.680	0.194	0.126	0.039	~0.019
^{30}Cl	0.440	0.332	0.228	0.050	
^{34}K	0.533	0.301	0.166	0.096	
$T_z = -\frac{5}{2}$					
^{23}Si	0.067	0.577	0.356	0.049	
^{27}S	0.569	0.292	0.139	0.020	
^{31}Ar	0.275	0.377	0.348	0.033	
^{35}Ca	0.000	0.784	0.216	0.095	
$T_z = -3$					
^{22}Si	0.000	0.741	0.259	0.044	

in Table II. For the $T_z = -\frac{3}{2}$ nuclei, the theoretical sum in the second region agrees well with the observed probability of β-delayed one-proton emission, P_{1p}(expt). In the decay of $T_z = -2$ nuclei, the calculated branching ratios to the IAS coincide fairly well with the experimental values of P_{1p}, and larger probabilities of the one-proton emission are expected through other daughter states. The present calculation predicts sizable branching strengths of the β-delayed two-proton emission for nine nuclei, the $T_z = -2$, $A = 4n + 2$ series and the $T_z = -\frac{5}{2}$ and -3 nuclides listed in Table II, and these nuclei near the proton drip line would have a rather weak β-decay branch (~5%) to the IAS which is located above S_{2p}. The two-proton emission has been observed[3,4] in the decay of ^{22}Al, ^{26}P, and ^{35}Ca. Observation of β^+ decays of ^{30}Cl and ^{34}K depends, as mentioned above, on the mass of the parent nuclei.

Summarizing, we have calculated β^+-decay branches of all daughter states falling in the $Q_{\rm EC}$-value window for very proton-rich *sd*-shell nuclei up to the proton drip line in the full $(sd)^{A-16}$ configurations with the effective interaction designed by Wildenthal. The half-lives calculated with the quenching factor $\gamma^2 = 0.60$ for the Gamow-Teller transitions agree well with existing experimental data. However, a direct comparison of the strength distributions up to the giant resonance region has revealed significant deviations of the quenching factor from the global value, implying possible uncertainties in the effective interaction. The present microscopic calculation predicts β^+-decay branching strengths to states above the threshold energy of proton emission and suggests probable candidates of the β-delayed two-proton radioactivity.

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