Variational Monte Carlo calculations for the binding energy of $^{31}_{\Lambda\Lambda}$Si

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The binding energy of the $\Lambda\Lambda$ hypernucleus $^{31}_{\Lambda\Lambda}$Si has been calculated variationally with a ^{28}Si$+n+\Lambda+\Lambda$ four-body model. The integrations have been carried out with the help of a Monte Carlo technique. Three different types of Λ-Λ and Λ-N potentials have been used. n-^{28}Si and Λ-^{28}Si potentials have been generated by folding the N-N and Λ-N potentials into the harmonic-oscillator shell-model density distribution of ^{28}Si. The calculated values of the binding energy for the three different potentials are 40.19, 46.30, and 39.90 MeV. These values are compared with the reported experimental value of 38.2 ± 6.3 MeV. The dependence of the binding energy on the depth of the Λ-Λ interaction has also been investigated.

I. INTRODUCTION

Until now knowledge of the Λ-Λ interaction has been gathered by studying two light hypernuclei, namely, $^{10}_{\Lambda\Lambda}$Be and $^{6}_{\Lambda\Lambda}$He which have been experimentally identified.[1-4] The total separation energy of both the Λ's from the above two hypernuclei is given as $B_{\Lambda\Lambda}(^{10}_{\Lambda\Lambda}\text{Be})=1.17\pm0.4$ MeV and $B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\text{He})=10.9\pm0.5$ MeV, respectively. The binding-energy calculations for these hypernuclei have been made with the aim of obtaining information about the Λ-Λ interaction.[5-9] More recently, Mondal *et al.*[10] reported an event in nuclear emulsion identified as evidence of a very heavy hypernucleus. They predicted it to be $^{31}_{\Lambda\Lambda}$Si and calculated its binding energy from range measurements to be 38.2 ± 6.3 MeV. Since it is the only heavy double hypernucleus for which there is some experimental evidence, it would be interesting to calculate the binding energy of $^{31}_{\Lambda\Lambda}$Si and compare the results with the reported value. Since no other experimental evidence on this hypernucleus is available, it is difficult to comment on the absolute reliability of the status of the supposed event.

For $^{31}_{\Lambda\Lambda}$Si we use a four-body ^{28}Si$+n+2\Lambda$ model, since a realistic $29N+2\Lambda$ calculation is virtually impossible. We calculate its binding energy by variational Monte Carlo techniques.

A three-body calculation for $^{31}_{\Lambda\Lambda}$Si taking it to be a ^{29}Si$+2\Lambda$ has been made and its binding energy is calculated to be 39.7 MeV.[11] However, a four-body model for $^{31}_{\Lambda\Lambda}$Si is much more sensible. In a four-body calculation it becomes very difficult to evaluate the multidimensional integrals of the Ritz variational method analytically and that is why a Monte Carlo technique is developed and reported here.

II. MATHEMATICAL FORMULATION

The Ritz variational principle, which is usually employed to calculate the properties of light nuclei, demands that one minimizes the expression

$$E=\frac{\int \Psi^* H \Psi \, d\tau}{\int \Psi^* \Psi \, d\tau} \tag{1}$$

with respect to the parameters in the trial wave function Ψ, H being the total Hamiltonian of the system. The analytical calculation of the integrals is extremely difficult for nuclei more massive than the triton or α particle. Therefore, a Monte Carlo method has been developed for heavier nuclei which allows one to calculate E from Eq. (1) without analytical calculations. The integrations are performed by a random process. The basic idea of the method is the following:

The integrands $\Psi^* H\Psi$ and $\Psi^*\Psi$ are written in the form $p(r_1,\ldots,r_n)w(r_1,\ldots,r_n)$, where p is called the probability density function and contains essentially Ψ^*, w is called the weight function and contains essentially $H\Psi$ or Ψ, respectively. Random coordinates are generated with the probability distribution $p(r_1,\ldots,r_n)$. For these random numbers the weight functions are calculated. The values obtained in this way are estimates for the integrals $\int \Psi^* H\Psi \, d\tau$ and $\int \Psi^*\Psi \, d\tau$ and the averages of a large number of these estimates converge towards the value of the integrals.

Our trial wave function is a product of six two-body correlation functions

$$\Psi(^{31}_{\Lambda\Lambda}\text{Si})=\prod_{i=0}^{2}\prod_{j=t+1}^{3} f_{ij}(r_{ij}) , \tag{2}$$

where 0 represents the ^{28}Si core, 1 stands for neutron, and

the two Λ's are denoted by 2 and 3. The shape of the correlation functions is taken from Ref. 11:

$$f(r)=e^{-\alpha r^2}+\beta e^{-\gamma r^2} . \tag{3}$$

In these functions, assuming that $\gamma \gg \alpha$, the second term represents the form of the wave function for the two-body systems in the region of close approach while the first term is related to the long-range part of the wave function. A product-type wave function with a correlation function of the form (3) has already been useful in earlier investigation of three-body hypernuclear systems.[12] This is, however, only a good starting point, a detailed knowledge of Λ-N repulsive core would perhaps involve a more detailed correlation function.

The total Hamiltonian of the system is given by

$$H=-\sum_{i=0}^{3}\frac{\hbar^2}{2\mu_i}\nabla_i^2+\sum_{i=0}^{2}\sum_{j=i+1}^{3}V_{ij} , \tag{4}$$

where μ_i are the reduced masses of the four pairs of particles and have the appropriate numerical values.

There are 12 position coordinates for the four particles. The center of mass takes care of three, and there are three Euler angles defining the orientation of the hypernucleus in space. This leaves six coordinates describing the tetrahedron formed by the four particles. These can be taken as the six interparticle distances. When the wave function is independent of the Euler angles, the kinetic-energy operator simplifies considerably. If ∇_i^2 is the Laplacian operator corresponding to the ith particle, and $\mathbf{r}_{ij}$ is the vector displacement from the ith to the jth particle, we have

$$\nabla^2=2\sum_{i=0}^{2}\sum_{j=i+1}^{3}\left[\frac{\partial^2}{\partial r_{ij}^2}+\frac{2}{r_{ij}}\frac{\partial}{\partial r_{ij}}\right]+\sum_{i=0}^{2}\sum_{j=i+1}^{3}\sum_{i_1=ij_1=i_1+1}^{2}\sum_{3}\delta_{iji_1j_1}\frac{\partial^2}{\partial r_{ij}\partial r_{i_1j_1}}\cos(\hat{r_{ij}r_{i_1j_1}}) , \tag{5}$$

where $\delta_{iji_1j_1}=1$ if $i=i_1$ or $j=j_1$, and $\delta_{iji_1j_1}=0$ otherwise. Using Eq. (3) for the correlation functions the expression for $\nabla^2\Psi$ becomes

$$\begin{aligned}\nabla^2\Psi=&2\sum_{i=0}^{2}\sum_{j=i+1}^{3}[2a_{ij}(2a_{ij}r_{ij}^2-1)e^{-a_{ij}r_{ij}^2}+2b_{ij}c_{ij}(2c_{ij}r_{ij}^2-1)e^{-c_{ij}r_{ij}^2}]\prod_{k\neq j}f_{ik}\\&+\sum_{i=0}^{2}\sum_{j=i+1}^{3}-4(a_{ij}e^{-a_{ij}r_{ij}^2}+b_{ij}c_{ij}e^{-c_{ij}r_{ij}^2})\prod_{k\neq j}f_{ik}\\&+2\sum_{t=0}^{2}\sum_{j=i+1}^{3}\sum_{i_1=ij_1=i_1+1}^{2}\sum_{3}(\mathbf{r}_{ij}\cdot\mathbf{r}_{i_1j_1})(a_{ij}e^{-a_{ij}r_{ij}^2}+b_{ij}c_{ij}e^{-c_{ij}r_{ij}^2})\\&\times(a_{i_1j_1}e^{-a_{i_1j_1}r_{i_1j_1}^2}+b_{i_1j_1}c_{i_1j_1}e^{-c_{i_1j_1}r_{i_1j_1}^2})\prod_{k\neq i_1,k\neq j_1}f_{ik},\quad i\leq j ,\end{aligned} \tag{6}$$

where $\mathbf{r}_{ij}\cdot\mathbf{r}_{i_1j_1}=\cos(\hat{r_{ij}r_{i_1j_1}})\times|r_{ij}||r_{i_1j_1}|$. The core-neutron interaction V_{01} and the core-Λ interactions V_{02} and V_{03} are generated by folding the N-N and Λ-N interactions over the shell-model density distribution of ^{28}Si core. N-N and Λ-N potentials are taken from Malfliet and Tjon[13] and Dzhibuti, Mikhelashvili, and Shitikova,[14] respectively. The volume element $d\tau$ in Eq. (1) is given by

$$d\tau=\frac{16\pi^2\prod_{i=0,j=i+1}^{i=2,j=3}r_{ij}dr_{ij}}{r_{02}(r_{13}^2-r_{13(\min)}^2)^{1/2}(r_{13(\max)}^2-r_{13}^2)^{1/2}} , \tag{7}$$

where

$$r_{13(\min)}^2=r_{03}^2+r_{01}^2-2r_{03}r_{01}\cos(\theta-\Psi') ,$$

$$r_{13(\max)}^2=r_{03}^2+r_{01}^2-2r_{03}r_{01}\cos(\theta+\Psi')$$

(see Fig. 1). Limits on θ and Ψ' are

$$0\leq\theta\leq\pi,\ 0\leq\Psi'\leq\pi .$$

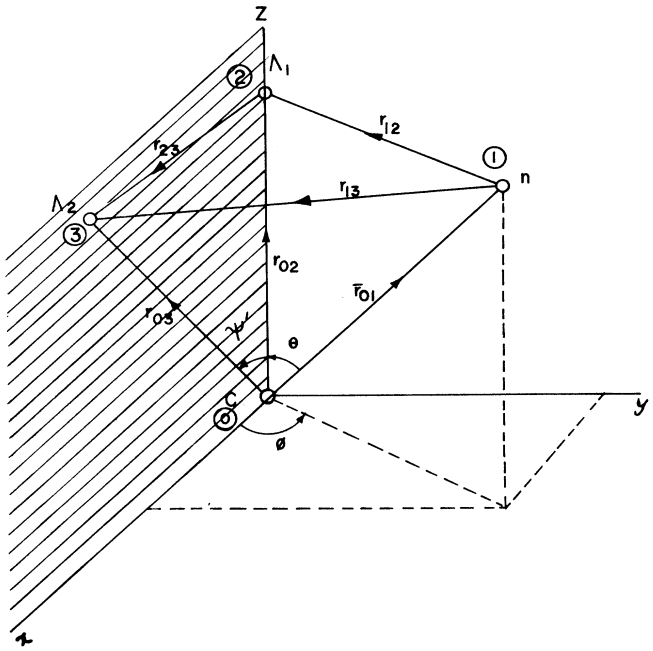

FIG. 1. Coordinate system for the four-body calculations. r_{01} is the n-core separation, r_{02} and r_{03} are the Λ-core separations, r_{12} and r_{13} are the n-Λ separations while r_{23} is the Λ-Λ separation.

III. THE Λ- AND *n*-CORE INTERACTIONS

The lambda-core interactions V_{02} and V_{03} and the *n*-core interaction V_{01} are obtained by folding Gaussian Λ-*N* and *N*-*N* interactions[13–15] into the ^{28}Si core density distribution. For this purpose, a harmonic-oscillator density distribution for the configuration $1s^2 1p^4_{3/2} 1p^2_{1/2} 1d^6_{5/2}$ is calculated to be

$$\rho(r)=4\left[\frac{1}{\pi a^2}\right]^{3/2}\left[\frac{9}{8}+\frac{3r^2}{2a^2}+\frac{29r^4}{30a^4}\right]e^{-r^2/a^2}, \quad (8)$$

where $a=1.873$ fm is the oscillator size parameter. The distribution of the form (8) is found to reproduce well the electron-scattering data from the nucleus under consideration (see Ref. 16).

A schematic diagram of the folding model calculation is shown in Fig. 2. The Λ-^{28}Si potential can be written as

$$V_{0i}(R)=\int V_{\Lambda N}(\mathbf{r}-\mathbf{R})r\rho(r)d^3r \quad \text{for } i=2,3\ , \quad (9)$$

where **r** and **R** are related by $\mathbf{r}+\mathbf{r}'=\mathbf{R}$. $v_{\Lambda N}$ is taken of the form[14]

$$v_{\Lambda N}(r')=v^0_{\Lambda N}e^{-r'^2/b^2}\ , \quad (10)$$

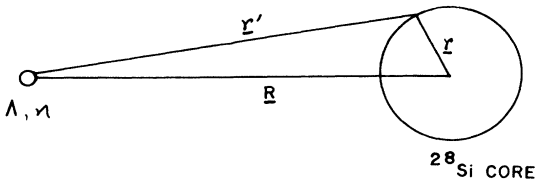

FIG. 2. Coordinate system for the folding model calculations for *n*-^{28}Si and Λ-^{28}Si. **r** is the position of a nucleus relative to the center of mass of the ^{28}Si core, **r**′ is the Λ-*N* or *n*-*N* separation while **R** is the position of *n* or Λ relative to the center of mass of the core.

where $b=1.034$ fm and $v^0_{\Lambda N}=-38.19$ MeV.

Using this potential the binding energy and rms radius of $^{6}_{\Lambda\Lambda}$He have been reproduced excellently.[12] The $^{6}_{\Lambda\Lambda}$He radius has not been determined experimentally. Actually the theoretical value of the rms radius (1.66 fm) was compared with the ^{4}He radius. This radius being somewhat greater (but not too much) than the ^{4}He radius (1.61 fm) which is understandable. After using (8) and (10) in (9) and evaluating the integral, the potential is found to be

$$V_{0i}(R)=-V^0_{0i}\left[\frac{1}{2a^3\delta^3}\left[9+\frac{18}{a^2\delta^2}+\frac{27}{a^4\delta^4}\right]+\frac{1}{3a^5b^4\delta^7}\left[18+\frac{58}{a^2b^2}\right]R^2+\frac{58}{15a^7b^8\delta^{11}}R^4\right]e^{-R^2/a^2b^2\delta^2}\ , \quad (11)$$

where $\delta^2=1/a^2+1/b^2$ and V^0_{0i} is the potential depth which is calculated from the two-body variational calculations. For the *n*-^{28}Si potential an almost similar form of equation as Eq. (11) is calculated. We use the *N*-*N* potential from Afzal and Ali[17] which is of the form

$$V_{NN}=v^0_{NN}e^{-r^2/b^2}\ , \quad (12)$$

where $b=1.474$ fm and $V^0=72.98$ MeV. It is a spin-averaged potential and singlet and triplet *N*-*N* parameters are excellently reproduced with this potential.

IV. TWO-BODY SYSTEMS

For the Λ-^{28}Si and *n*-^{28}Si systems variational calculations are carried out with the potentials in Eq. (11) and trial wave functions as in Eq. (3). The variational principle takes the form

$$V^0_{0i}\le(B_i\langle f|f\rangle+\langle f|T|f\rangle)\langle f|V_{0i}|f\rangle\ , \quad (13)$$

where B_i is the Λ-^{28}Si or *n*-^{28}Si separation energy.

Nevertheless it may be mentioned here that the *n*-^{28}Si potential thus generated is, however, not all that realistic. A rigorous and more realistic approach would be to generate the *n*-^{28}Si potential from the first principles taking into account proper antisymmetrization effects, the latter would, however, be extremely complicated and would bring in nonlocal forces. In the present work, however, the simple-minded folded potential may be looked upon only as an effective and local one adjusted to the separation energy of ^{29}Si.

From the curve of *B* against $A^{-2/3}$ of Bhaduri *et al.*,[18] the Λ-^{28}Si separation energy is about 19.2 MeV and from Wapstra and Audi[19] the separation energy of the neutron in ^{29}Si is 8.47 MeV. In the absence of unique and more reliable data on the binding energies of heavy hypernuclei, the Λ-^{28}Si separation energy was taken from an extrapolation of *B* against the $A^{-2/3}$ curve of Bhaduri *et al.*[18] The individual terms in (13) take the form

$$\langle f|T|f\rangle=\frac{-\hbar^2\pi^{3/2}}{\mu_{0i}}\left[\frac{\alpha^2}{2\alpha^{5/2}}+\frac{2\beta(\alpha^2-\alpha\gamma+\gamma^2)}{(\alpha+\gamma)^{5/2}}+\frac{\beta^2\gamma^2}{(2\gamma)^{5/2}}\right],\quad i=1,2,3 \quad (14)$$

$$\langle f|V_{0i}|f\rangle=-\pi^{3/2}V_{0i}\left[\frac{l}{(2\alpha+p)^{3/2}}+\frac{2\beta l}{(\alpha+\gamma+p)^{3/2}}+\frac{\beta^2 l}{(2\gamma+p)^{3/2}}+\frac{3m}{2(2\alpha+p)^{5/2}}+\frac{6\beta m}{2(\alpha+\gamma+p)^{5/2}}\right.$$
$$\left.+\frac{3\beta^2 m}{2(2\gamma+p)^{5/2}}+\frac{15n}{4(2\alpha+p)^{7/2}}+\frac{30\beta n}{4(\alpha+\gamma+p)^{7/2}}+\frac{15\beta^2 n}{4(2\gamma+p)^{7/2}}\right], \quad (15)$$

$$\langle f|f\rangle=\pi^{3/2}\left[\frac{\beta^2}{2\alpha^{3/2}}+\frac{2\beta}{(\alpha+\beta)^{3/2}}+\frac{1}{2\beta^{3/2}}\right], \tag{16}$$

where

$$l=\frac{1}{2a^3\delta^3}\left[9+\frac{18}{a^2\delta^2}+\frac{29}{a^4\delta^4}\right],$$

$$m=\frac{1}{3a^5b^4\delta^7}\left[18+\frac{58}{a^2\delta^2}\right],$$

$$n=\frac{58}{15a^7b^2\delta^2},$$

and $$p=\frac{1}{a^2b^2\delta^2}.$$

In Eq. (14) μ_{01} is the n-^{28}Si reduced mass and μ_{02} and μ_{03} are the reduced mass of Λ-^{28}Si. The optimum parameters α,β,γ and the corresponding values of V_{0i} are given in Table I.

V. Λ-Λ AND Λ-N POTENTIALS

For the ground state of Λ-Λ hypernuclei it is the $1S_0$ Λ-Λ interaction which is relevant and to which $V_{\Lambda\Lambda}$ refers. We use three different forms of $V_{\Lambda\Lambda}$. The first one is the Urbana-type 2π-exchange potential[20] which is of the form

$$V_{2\pi}=V_c-VT_\pi^2. \tag{17}$$

V_c is a Woods-Saxon repulsive core

$$V_c=W_c\{1+\exp[(r-R)/d]\}^{-1}, \tag{18}$$

with $W_c=2137$ MeV, $R=0.5$ fm, $d=0.2$ fm. A similar potential is also used for $V_{\Lambda N}$ with the same W_c, R, and d values, and these values are very close to those for the spin, isospin independent core of the N-N potential of Ref. 20. Only the strength V is different for $V_{\Lambda\Lambda}$ and $V_{\Lambda N}$. The values are $V(\Lambda\Lambda)=6.24$ MeV and $V(\Lambda N)=6.20$ MeV. T_π is the 1π-exchange tensor potential shape modified with a cutoff:

$$T_\pi(r)=\left[1+\frac{3}{x}+\frac{3}{x^2}\right]\frac{e^{-x}}{x}(1-e^{-cr^2})^2, \tag{19}$$

with $x=0.7r$, $c=2$ fm^{-2}.

TABLE I. Two-body parameters for the n-^{28}Si and Λ-^{28}Si systems. Separation energies are (for n-^{28}Si) 8.47 MeV and (for Λ-^{28}Si) 19.2 MeV, respectively.

Parameter	n-^{28}Si	Λ-^{28}Si
α (fm^{-2})	0.09	0.12
β	5.50	6.00
γ (fm^{-2})	0.60	0.90
V_{0i}^0	−24.15	−10.70

The second one we use is from Dzhibuti, Mikhelashvili, and Shitikova[14] which has a Gaussian form corresponding to the two-pion exchange

$$v=v_0e^{-r^2/b^2}. \tag{20}$$

This is used for both Λ-Λ and Λ-N cases. The parameters are $v_{\Lambda N}^0=-38.19$ MeV, $b_{\Lambda N}=1.034$ fm, $v_{\Lambda\Lambda}^0=-52.25$ MeV, $b_{\Lambda\Lambda}=1.566$ fm. They reproduced the binding energy and rms radius of $^5_{\Lambda\Lambda}$He excellently.

The third one we use is the potential of Mian[21] which is a three-parameter density-dependent force

$$v_{\Lambda N}(r,\rho)=v_0\frac{1}{\pi d^{3/2}}e^{-r^2/d^2}(1-\beta'\rho^{3/2}r), \tag{21}$$

where $v_0=297.86$ MeV fm^3, $\beta'=1.92$ fm^2, $d=0.729$ fm. With this potential Mian reproduced the binding energy of $^5_\Lambda$He, $^{11}_\Lambda$B, $^{13}_\Lambda$C, and $^{15}_\Lambda$N satisfactorily.

VI. MONTE CARLO METHOD FOR THE FOUR-BODY SYSTEM

An integral

$$\int p(r_1,\ldots,r_n)w(r_1,\ldots,r_n)dr_1\ldots dr_n \tag{22}$$

can be evaluated in the following way: One generates random numbers U according to the normalized probability distribution $p(r_1,\ldots,r_n)$. The possibilities of choosing $p(r_1,\ldots,r_n)$ are restricted by the difficulty of

TABLE II. Four-body parameters. Statistical error on $B_{\Lambda\Lambda}$ is ±0.06. V_{01}^0, V_{02}^0, and V_{03}^0 are taken from two-body results.

Parameters	Urbana $V_{\Lambda N}$	Dzhibuti $V_{\Lambda N}$	Mian $V_{\Lambda N}$
α_{01} (fm^{-2})	0.022	0.070	0.065
β_{01}	6.500	6.500	4.500
γ_{01} (fm^{-2})	0.070	0.100	0.100
α_{02} (fm^{-2})	0.022	0.070	0.065
β_{02}	6.500	6.500	4.500
γ_{02} (fm^{-2})	0.070	0.100	0.100
α_{03} (fm^{-2})	0.022	0.070	0.065
β_{03}	6.500	6.500	4.500
γ_{03} (fm^{-2})	0.070	0.100	0.100
α_{12} (fm^{-2})	0.022	0.070	0.065
β_{12}	6.500	6.500	4.500
γ_{12} (fm^{-2})	0.070	0.100	0.100
α_{13} (fm^{-2})	0.022	0.070	0.065
β_{13}	6.500	6.500	4.500
γ_{13} (fm^{-2})	0.070	0.100	0.100
α_{23} (fm^{-2})	0.022	0.070	0.065
β_{23}	6.500	6.500	4.500
γ_{23} (fm^{-2})	0.070	0.100	0.100
$V_{\Lambda\Lambda}^0$ (MeV)	−2137.0	−52.25	−297.86
$B_{\Lambda\Lambda}$ (MeV)	40.19	46.30	39.90

TABLE III. Calculated values of the binding energy $B_{\Lambda\Lambda}$ for the different potentials along with potential parameters.

Potential		Depth and other parameters	Binding energy $B_{\Lambda\Lambda}$ with Monte Carlo errors (MeV)
Urbana	$V_{\Lambda N}$	$W_c=2137$ MeV, $R=0.5$ fm, $d=0.2$ fm, $V=6.20$ MeV	40.19±0.06
	$V_{\Lambda\Lambda}$	$W_c=2137$ MeV, $R=0.5$ fm, $d=0.2$ fm, $V=6.24$ MeV	
Dzhibuti	$V_{\Lambda N}$	$v^0_{\Lambda N}=-38.19$ MeV, $b_{\Lambda N}=1.034$ fm	46.30±0.06
	$V_{\Lambda\Lambda}$	$v^0_{\Lambda\Lambda}=-52.25$ MeV, $b_{\Lambda N}=1.566$ fm	
Mian	$V_{\Lambda N}$	$v_0=297.86$ MeV fm^3, $\beta'=1.92$ fm^2, $d=0.729$ fm	39.90±0.06
	$V_{\Lambda\Lambda}$	$v_0=297.86$ MeV fm^3, $\beta'=1.92$ fm^2, $d=0.729$ fm	

generating random numbers. For a distribution function depending only on one coordinate the generation of random numbers is easy. A transformation function can be obtained by a one-dimensional numerical integration which transforms random numbers of a uniform distribution into random numbers of the desired distribution. Here in our case we take p a product of functions each of which depends only on one coordinate and every factor for itself is used to generate a random coordinate. For example, we take p as

$$\prod_{i=0}^{2}\prod_{j=i+1}^{3} e^{-\alpha_{ij}r_{ij}^2} . \tag{23}$$

A random number U in the domain (0,1) is generated so that

$$U=e^{-\alpha_{01}r_{01}^2}$$

or

$$r_{01}=-\frac{1}{\alpha_{01}}\ln U^{-1/2} . \tag{24}$$

All radius vectors are generated in this way. Triangular inequalities are maintained strictly, e.g., $r_{01}+r_{03}>r_{13}$, etc. The weight functions for the numerator and denominator of Eq. (1) are

$$w_1=\frac{1}{p}\Psi^*H\Psi, \quad w_2=\frac{1}{p}\Psi^*\Psi , \tag{25}$$

respectively. The quantities w_1 and w_2 are calculated numerically. These are calculated a large number of times n and then the average values are calculated which give the estimates of the integral in Eq. (1). The exact form of the probability function p is not very important, since it has only an influence on the variance. In this method the main contribution to the variance comes from the Hamiltonian in the region where two or more particles are close together. A somewhat different probability function would therefore change the variance slightly while the computing time will be smaller for the simpler probability density function used in this paper. The variance is calculated approximately from

$$\nu=\frac{1}{n}\sum_{j=1}^{n} w_j-\left[\frac{1}{n}\sum_{i=1}^{n} w_i\right]^2 . \tag{26}$$

The statistical error is obtained by

$$\Delta I=(\nu/n)^{1/2} . \tag{27}$$

VII. RESULTS AND DISCUSSIONS

First as a check of our Monte Carlo calculations we used the same three-body parameters and potentials as used in our previous paper[11] wherein the calculations were done analytically, and got approximately the same value for the binding energy of ${}^{31}_{\Lambda\Lambda}$Si. The result is 39.95 MeV. Four-body results for three $\Lambda-N$ potentials considered here are shown in Table II. For ${}^{31}_{\Lambda\Lambda}$Si the wave function Ψ depends on the total of 18 variational parameters, α_{01}, β_{01}, γ_{01}, α_{02}, β_{02}, γ_{02}, α_{03}, β_{03}, γ_{03}, α_{12}, β_{12}, γ_{12}, α_{13}, β_{13}, γ_{13}, α_{23}, β_{23}, γ_{23}. For simplicity, we consider f_{01}, f_{02}, and f_{03} equivalent and similarly f_{12}, f_{13}, and f_{23} equivalent which leaves six parameters for variation. The expectation value $\langle\Psi|H|\Psi\rangle$ and normalization $\langle\Psi|\Psi\rangle$ are obtained by a Monte Carlo procedure with statistical errors of about ±0.06 MeV. The optimum variational parameters are shown in Table II. These were obtained for $\nu^0_{01}=-10.7$ MeV and $\nu^0_{02}=\nu^0_{03}=-24.15$ MeV.

For the three different potentials the calculated values

TABLE IV. The experimental $B_{\Lambda\Lambda}$ and the additional binding energy $\Delta B_{\Lambda\Lambda}$.

$\Lambda\Lambda$ hypernuclear system	Experimental value of $B_{\Lambda\Lambda}$ (MeV)	Experimental value of B_Λ (MeV)	$\Delta B_{\Lambda\Lambda}$ (MeV)
${}^{6}_{\Lambda\Lambda}$He	10.9±0.5	$B_\Lambda({}^{5}_{\Lambda}\text{He})=3.12\pm0.02$	4.66±0.54
${}^{10}_{\Lambda\Lambda}$be	17.7±0.4	$B_\Lambda({}^{9}_{\Lambda}\text{Be})=6.71\pm0.04$	4.28±0.48
${}^{14}_{\Lambda\Lambda}$C (Ref. 23)	27.21±0.7	$B_\Lambda({}^{13}_{\Lambda}\text{C})=11.69\pm0.12$	3.83±0.94
${}^{31}_{\Lambda\Lambda}$Si	38.2±6.3	$B_\Lambda({}^{30}_{\Lambda}\text{Si})=19.5\pm0.02$	−0.8 ±6.7

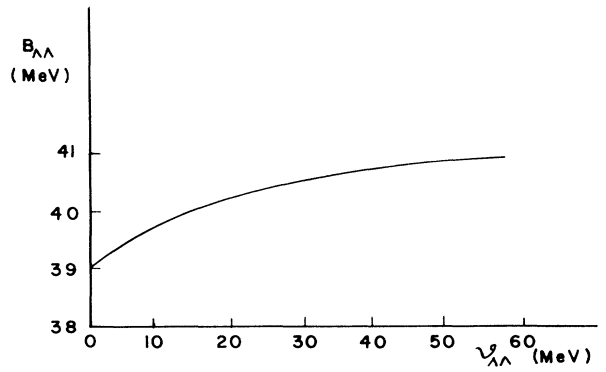

FIG. 3. Plot of $B_{\Lambda\Lambda}$ vs $U_{\Lambda\Lambda}$ for Urbana-type potential. Values of the parameters are taken from Table II.

of the binding energy $B_{\Lambda\Lambda}$ are given in Table III. These calculated values of the binding energy are not inconsistent with the reported experimental value of $B_{\Lambda\Lambda}({}^{31}_{\Lambda\Lambda}\mathrm{Si})=38.2\pm6.3$ MeV, although for the potential of Dzhibuti, Mikhelashvili, and Shitikova,[14] the values are somewhat larger. The Urbana potential and the potential of Mian seem to be equivalent for the purposes of the present calculation. The $\Lambda\Lambda$ correlation function used in our calculation does not have the flexibility to give the needed short-range correlation. This might be the reason why $B_{\Lambda\Lambda}$ for the Urbana $V_{\Lambda\Lambda}$ is significantly lower than $B_{\Lambda\Lambda}$ for the potential of Dzhibuti, Mikhelashvili, and Shitikova, which does not have any short-range repulsion. The use of a more adequate correlation function could make the result for the Urbana $V_{\Lambda\Lambda}$ come considerably closer to that for the Soviet potential. The result for the latter should therefore be considered the more reliable one. The effect of different $V_{\Lambda N}$ on $\Delta B_{\Lambda\Lambda}$ $(=B_{\Lambda\Lambda}-2B_{\Lambda})$ has been explored. For the three potentials used here the calculated values of $\Delta B_{\Lambda\Lambda}$ are 1.79, 7.9, and 1.5 MeV, respectively. For comparison with other hypernuclei, we tabulate the values of $\Delta B_{\Lambda\Lambda}$ for different hypernuclei in Table IV. From Table IV it is observed that the value of $\Delta B_{\Lambda\Lambda}$ gradually decreases with an increase of mass number. It has been shown in Ref. 22 that as the mass of the core nucleus becomes infinitely heavy, $B_{\Lambda\Lambda}$ approaches the value $2B_{\Lambda}$. Since ${}^{31}_{\Lambda\Lambda}$Si is very heavy in comparison with the other hypernucleus shown, the value of $\Delta B_{\Lambda\Lambda}$ is obviously too small.

Figure 3 is a plot of $B_{\Lambda\Lambda}$ vs $V^0_{\Lambda\Lambda}$. For $V^0_{\Lambda\Lambda}=0$ we have $B_{\Lambda\Lambda}=38.16$ MeV, which is approximately $2B_{\Lambda}$. If the core is infinitely massive then one has as expected the relation $B_{\Lambda\Lambda}=2B_{\Lambda}$ for $V^0_{\Lambda\Lambda}=0$

We conclude that the four-body model for ${}^{31}_{\Lambda\Lambda}$Si which is more realistic than the three-body model, gives a value of $B_{\Lambda\Lambda}({}^{31}_{\Lambda\Lambda}\mathrm{Si})$ for reasonable $\Lambda-N$ potentials quite consistent with the reported value. In view of the large uncertainties in the experimental value, an exploration of the finer details of the $\Lambda-N$ interaction would not be of much significance here. Nevertheless, the present calculation for ${}^{31}_{\Lambda\Lambda}$Si can be used for indicative purposes. Further experimental observations of similar events could allow one to have more definitive conclusions.

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