Optical model description of momentum transfer in relativistic heavy ion collisions

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An optical model description of momentum transfer in relativistic heavy ion collisions, based upon composite particle multiple-scattering theory, is presented. The imaginary component of the complex momentum transfer, which comes from the absorptive part of the optical potential, is shown to be the main contributor to the momentum loss of the projectile. Within the context of the Goldhaber formalism, predictions of fragment momentum distribution observables are made and compared with experimental data. Use of the model as a tool for estimating collision impact parameters is also discussed.

I. INTRODUCTION

Since the pioneering experiments on relativistic heavy ion fragmentation using carbon and oxygen beams,[1,2] attention has been directed toward understanding the underlying mechanisms of fragmentation processes. Over the past two decades, a substantial body of literature has resulted from studies of these phenomena, and several excellent reviews have been written.[3–6] Perhaps the most significant findings of the early experiments were the observations that the fragment momentum distributions were Gaussian in the projectile rest frame, and that the isotopic production cross sections factored into a product of target and beam-fragment terms. Initial attempts to explain these phenomena utilized a statistical model to describe the reactions.[7–9] This later evolved into a two-step model called abrasion ablation[10] where the abrasion stage can be formulated using geometric[10,11] or quantum-mechanical arguments.[12,13] In the present work, we use the impulsive excitation energy ideas of Fricke,[14] within the context of composite particle multiple-scattering theory, to derive a method of predicting momentum transfers occurring in relativistic heavy ion collisions. This momentum transfer is a function of impact parameter. A novel feature of this work is that the momentum transfer is a complex quantity. The real component is the usual transverse momentum transfer resulting from elastic scattering. The imaginary component, shown from kinematics to be the main contributor to the longitudinal momentum loss or downshift, comes from the absorptive part of the complex optical potential. Using this model as input into the Goldhaber formalism,[8] projectile nucleus fragment momentum "downshifts" resulting from the dynamics of the nuclear collision can be calculated and compared with laboratory beam measurements. In addition, modifications to the widths of the momentum distributions can also be estimated.

The outline of the paper is as follows. In Sec. II the dynamical momentum-transfer expression is derived, and representative calculations of momentum transfer as a function of impact parameter are presented. In Sec. III the connections between collisional momentum transfer and fragment momentum downshifts-widths using the Goldhaber formalism are made. A method of choosing appropriate impact parameters for each fragmentation channel is then described. Next, calculations of momentum downshifts for fragments produced by oxygen nuclei colliding with various targets are made and compared with experimental data.[2] We also compute widths of momentum distributions for ^{139}La fragments and compare with recent experimental measurements.[15] In Sec. IV we propose a method for using the momentum-transfer model to estimate collision impact parameters. Finally, in Sec. V we conclude by summarizing the current status of model development and discuss future directions for research.

II. METHOD OF CALCULATION

In Ref. 16, a coupled-channels Schrödinger equation for composite particle scattering, which relates the entrance channel to all of the excited states of the target and projectile, was derived by assuming large incident projectile kinetic energies and closure of the accessible eigenstates. The equation is written as

$$(\nabla^2+k^2)\psi_{n\mu}(\mathbf{x})=2m A_p A_T(A_p+A_T)^{-1} \times \sum_{n'\mu'} V_{n\mu,n'\mu'}(\mathbf{x})\psi_{n'\mu'}(\mathbf{x}) , \qquad (1)$$

where the subscripts n and μ (primed and unprimed) label the projectile and target eigenstates, m is the nucleon mass, A_p and A_T are the mass numbers of the projectile and target, $\mathbf{k}$ is the incident projectile momentum relative to the center of mass, and $\mathbf{x}$ is the projectile position vector relative to the target. In terms of the nucleon-nucleon scattering t matrix $t_{\alpha j}$, and the internal state vectors of the projectile $g_n^P(\xi_p)$ and target $g_\mu^T(\xi_T)$, it was also demonstrated that the potential matrix is expressible as

$$V_{n\mu,n'\mu'}(\mathbf{x})=\langle g_n^P g_\mu^T | V_{\rm opt}(\xi_P,\xi_T,\mathbf{x}) | g_{n'}^P g_{\mu'}^T \rangle \ , \tag{2}$$

where

$$V_{\rm opt}(\xi_P,\xi_T,\mathbf{x})=\sum_{\alpha j} t_{\alpha j} \ . \tag{3}$$

This same formalism can be used to investigate relativistic heavy ion collision momentum transfers. Within the context of eikonal scattering theory, the solution to the Schrödinger equation

$$H\Psi(\mathbf{x},\xi_P,\xi_T)=E\Psi(\mathbf{x},\xi_P,\xi_T) \tag{4}$$

at relativistic energies is

$$\Psi(\mathbf{x},\xi_P,\xi_T)=(2\pi)^{-3/2}\exp\left[-\frac{i}{v}\int_{-\infty}^{z} V_{\rm opt}(\mathbf{x},\xi_P,\xi_T)dz'\right] \times g_n^P(\xi_P) g_\mu^T(\xi_T) e^{i\mathbf{k}\cdot\mathbf{x}} \ , \tag{5}$$

where v is the velocity. The total momentum of the projectile is then given by the matrix element involving the sum of the projectile single-nucleon momentum operators as

$$\mathbf{P}_{\rm tot}=\left\langle \Psi \left| -\sum_{\alpha=1}^{A_p} \nabla_{P,\alpha} \right| \Psi \right\rangle \ , \tag{6}$$

where the subscript P on the gradient operator denotes that the gradient is to be taken with respect to the projectile internal coordinates ξ_P. Equation (6) actually denotes a potential matrix $\mathbf{P}_{n\mu,n'\mu'}$ in analogy with (2). Therefore, substituting (5) into (6) yields

$$\mathbf{P}_{n\mu,n'\mu'}=\left\langle g_n^P(\xi_P) g_\mu^T(\xi_T) \left| e^{-iS} \left[\sum_{\alpha=1}^{A_p} \nabla_{P,\alpha} \right] e^{iS} \right| g_{n'}^P(\xi_P) g_{\mu'}^T(\xi_T) \right\rangle \ , \tag{7}$$

where

$$S=\frac{1}{v}\int_{-\infty}^{z} V_{\rm opt}(\mathbf{x}',\xi_P,\xi_T)dz' \ . \tag{8}$$

Using the chain rule for differentiation, Eq. (7) can be further expressed as

$$\mathbf{P}_{n\mu,n'\mu'}=\mathbf{P}_0+\left\langle g_n^P g_\mu^T \left| \left[-\sum_{\alpha}^{A_p} \nabla_{P,\alpha} S \right] \right| g_{n'}^P g_{\mu'}^T \right\rangle \ , \tag{9}$$

where the incident projectile momentum before the collision is

$$\mathbf{P}_0=\left\langle g_n^P g_\mu^T \left| \left[-i \sum_{\alpha=1}^{A_p} \nabla_{P,\alpha} \right] \right| g_{n'}^P g_{\mu'}^T \right\rangle \ . \tag{10}$$

The total momentum transfer to the projectile is then given by

$$\mathbf{Q}_{n\mu,n'\mu'}=\mathbf{P}_{n\mu,n'\mu'}-\mathbf{P}_0=\left\langle g_n^P g_\mu^T \left| \left[-\sum_{\alpha} \nabla_{P,\alpha} S \right] \right| g_{n'}^P g_{\mu'}^T \right\rangle \ . \tag{11}$$

For high-energy collisions, dominant scattering processes occur near the forward directions, since the momentum transferred is small when compared to the incident momentum of the projectile; hence, couplings between excited states are small and can be neglected.[17] The total momentum transfer to the projectile is then approximated by

$$\mathbf{Q}\approx\mathbf{Q}_{00,00}=\left\langle g_0^P g_0^T \left| \left[-\sum_{\alpha} \nabla_{P,\alpha} \right] \right| g_0^P g_0^T \right\rangle \ . \tag{12}$$

In terms of projectile and target number densities, and the constituent-averaged two-nucleon transition amplitude[18] $\tilde{t}$, Eq. (12) becomes

$$\mathbf{Q}(\mathbf{b})=-A_P A_T \int d^3\xi_P \rho_P(\xi_P) \int d^3\xi_T \rho_T(\xi_T) \left[\nabla_P \int_{-\infty}^{\infty} \tilde{t}(\mathbf{b}+\mathbf{z}'+\xi_P-\xi_T) \frac{dz'}{v} \right] = Q\hat{\mathbf{b}} \ , \tag{13}$$

where the integration limit in the longitudinal direction has been extended to infinity. The transverse momentum transfer in (13) is therefore only a function of the impact parameter of the collision. The projectile and target number densities (ρ_P and ρ_T) are normalized to unity as

$$\int \rho(\mathbf{x}) d^3x = 1 \ . \tag{14}$$

The constituent-averaged two-nucleon transition amplitude is obtained from the impulsive first-order t matrix used in our previous studies[13] of nucleus-nucleus collisions as

$$\tilde{t}(e,\mathbf{x})=-(e/m)^{1/2}\sigma(e)[\alpha(e)+i] \times [2\pi B(e)]^{-3/2}\exp[-x^2/2B(e)] \ , \tag{15}$$

where e is the two-nucleon kinetic energy in their center-of-mass frame, $\sigma(e)$ is the nucleon-nucleon total cross section, $\alpha(e)$ is the ratio of the real-to-imaginary part of the forward-scattering amplitude, and $B(e)$ is the nucleon-nucleon slope parameter. Values for these parameters taken from various compilations are listed in Ref. 18.

The dynamical transverse momentum transfer to the projectile, given by Eq. (13), results from interactions with the target. Note that it is a complex quantity which is consistent with the use of a complex optical potential.[19] The real part of the momentum transfer, which comes from the real part of the complex optical potential, is the contribution arising from elastic scattering. The imaginary component, which comes from the absorptive part of the complex optical potential, arises mainly from absorption and inelastic-scattering processes. At high energies the latter are mainly breakup (fragmentation) reactions, since these account for over 95% of the total reaction cross section. Physically this imaginary component represents attenuation of the incident wave front in analogy with the usual discussions for a complex index of refraction in an absorptive medium.[19] Concomitant with this attenuation of the incident wave by these absorptive processes, there is a loss of momentum from the wave front in the beam direction. This beam longitudinal momentum transfer (loss) is interpreted as arising mainly from the imaginary component of $\mathbf{Q}$. To see this kinematically, we note that the final momentum of the projectile is

$$\mathbf{P}_f = \mathbf{P}_0 + \mathbf{Q} \ , \tag{16}$$

where the incident momentum $\mathbf{P}_0$ is in the beam direction and the momentum transfer $\mathbf{Q}$ is transverse to it. The associated kinetic energy is $P_f^2/2M$, where

$$P_f^2 = P_0^2 + Q^2 \ . \tag{17}$$

For a complex momentum transfer,

$$Q^2 = Q_R^2 - Q_I^2 + 2iQ_R Q_I \ , \tag{18}$$

where the subscripts denote the real (R) and imaginary (I) components of (13). For inelastic collisions, the projectile loses kinetic energy such that $P_f^2 < P_0^2$. Therefore, the loss in kinetic energy is $(Q_T^2 - Q_R^2)/2M$. [The imaginary part of the right-hand side of (18) is related to the

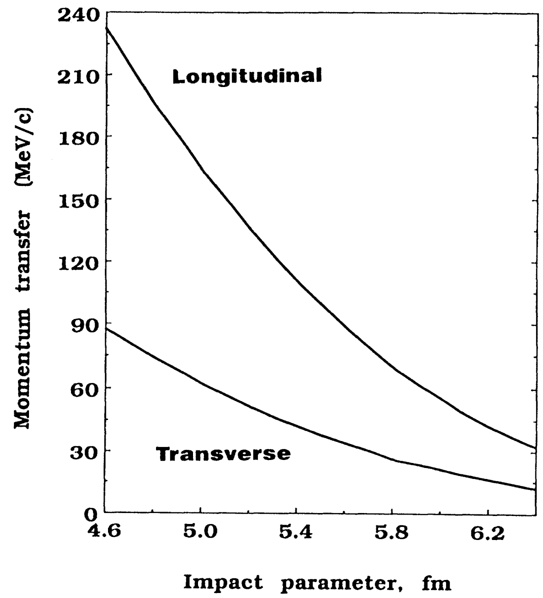

FIG. 1. Momentum transfer to the ^{16}O projectile, as a function of impact parameter, for 2.1 A GeV oxygen colliding with a beryllium target.

usual decay width.[19]] Associated with the loss of projectile kinetic energy is a momentum loss

$$P_{\text{loss}} = (Q_I^2 - Q_R^2)^{1/2} = Q_I(1 - Q_R^2/Q_I^2)^{1/2} \ . \tag{19}$$

From Eqs. (13) and (15), we note that

$$Q_R = \alpha(e) Q_I \ , \tag{20}$$

yielding

$$P_{\text{loss}} = Q_I(1-\alpha^2)^{1/2} \ . \tag{21}$$

At Bevelac energies, $\alpha(e) \approx 0.35$ giving

$$P_{\text{loss}} = 0.94 Q_I \approx Q_I \ . \tag{22}$$

Therefore, from Eq. (13), the real transverse component is

$$Q_\perp = -\left| A_P A_T \int d^3\xi_P \rho_P(\xi_P) \int d^3\xi_T \rho_T(\xi_T) \nabla_P \int_{-\infty}^{\infty} \text{Re}\tilde{t}(\mathbf{b} + \mathbf{z}' + \xi_P - \xi_T) \frac{dz'}{v} \right| , \tag{23}$$

and the momentum loss component is

$$P_{\text{loss}} \cong Q_\parallel = -\left| A_P A_T \int d^3\xi_P \rho_P(\xi_P) \int d^3\xi_T \rho_T(\xi_T) \nabla_P \int_{-\infty}^{\infty} \text{Im}\tilde{t}(\mathbf{b} + \mathbf{z}' + \xi_P - \xi_T) \frac{dz'}{v} \right| . \tag{24}$$

Calculated momentum transfers obtained using Eqs. (23) and (24) are displayed in Fig. 1 for ^{16}O at 2.1 A GeV colliding with a beryllium target. These calculations utilize the harmonic-well nuclear densities from our previous work.[13,18] From the figure, two features are readily apparent. First, the longitudinal momentum transfer (loss) is larger than the transverse indicating the primarily absorptive nature of the nuclear collision at this energy. Second, the predicted momentum transfers decrease rapidly with increasing impact parameter. This will be a subject of further discussion in subsequent sections of this paper, but its occurrence is not surprising since the nuclear optical potential decreases rapidly with increasing separation of the colliding nuclei.

III. RESULTS

The collisional momentum transfers computed using the model described in the previous section can be related to experimentally measured, heavy ion fragment momentum downshifts-widths through considerations of energy and momentum conservation. As has been formulated elsewhere,[8,20] a momentum transfer in any direction Q_j modifies the width h_j of the fragment momentum distribution in that direction by

$$(h_j')^2 = h_j^2 + \frac{F^2 Q_j^2}{A^2} , \tag{25}$$

and the mean by

$$\mathbf{P}_j' = \mathbf{P}_j + \frac{F}{A}\mathbf{Q}_j . \tag{26}$$

From the latter, the longitudinal momentum downshift is given by

$$\Delta P_{\parallel} = P_{\parallel}' - P_{\parallel} = \frac{F}{A} Q_{\parallel} , \tag{27}$$

where $Q_{\parallel}$ is the magnitude of the longitudinal momentum transfer [obtained from Eq. (24)], F is the fragment mass number, and A is the initial mass number of the fragmenting nucleus. Recalling that $Q_{\parallel}$ is a function of impact parameter, an appropriate method for choosing the impact parameter for each fragmentation channel is necessary. Recently, a semiempirical abrasion-ablation fragmentation model, NUCFRAG was proposed.[21] Although it assumes simple uniform density distributions for the colliding ions, and a zero-range (delta-function) interaction, it does include frictional-spectator interactions (FSI) and agrees with experimental cross-section data to the extent that they agree among themselves. Also, and most importantly for this work, it is easily modified to yield impact parameters for each fragmentation channel. Hence, the procedure for evaluating Eqs. (25) and (27) is to extract impact parameters from NUCFRAG for each nucleon removal corresponding exactly to $\Delta A = 1, 2, 3, \ldots$. These "most probable" impact parameters are then inserted into Eqs. (23) and (24) to obtain the corresponding momentum transfers for use in evaluating Eqs. (25) and (27). Because NUCFRAG uses uniform densities, uniform densities are also used in evaluating (23) and (24). In addition, the zero-range interaction in NUCFRAG is simulated for numerical integration purposes in (23) and (24) through the use of a very narrow Gaussian form for the t matrix given by Eq. (15). This narrow Gaussian is the same width for all collision pairs and therefore is not an arbitrarily adjusted parameter. We have checked the validity of using the "most probable" impact parameter in the calculations by actually computing the momentum transfers averaged over a range of impact parameters from NUCFRAG corresponding to $\Delta A - 0.5$ to $\Delta A + 0.5$. The differences between the estimates using averaged and "most probable" values are negligible.[22]

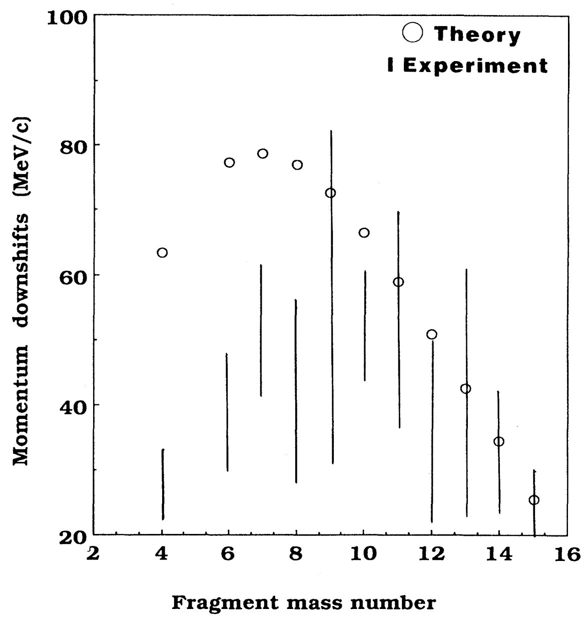

FIG. 2. Target-averaged longitudinal momentum downshifts as a function of projectile fragment mass number for $2.1A$ GeV ^{16}O colliding with Be, C, Al, Cu, Ag, and Pb targets. The experimental data, taken from Ref. 2, are averaged over isotopes for each fragment mass.

Representative calculations for momentum downshifts as a function of fragment mass number are displayed in Fig. 2 for ^{16}O projectiles at 2.1A GeV colliding with targets of Be, C, Al, Cu, Ag, and Pb. These momentum downshifts are target averaged using simple arithmetic averaging. For comparison, the target-averaged experimental data from Ref. 2 are also displayed. For display and comparison purposes, the latter are also averaged over all isotopes contributing to each fragment mass number using

$$(\Delta P_{\parallel})_{\mathrm{avg}} = \frac{\sum_i \sigma_i (\Delta P_{\parallel}^i)}{\sum_i \sigma_i} , \tag{28}$$

where σ_i is the experimental production cross section for the ith fragment isotope. Comparing the theoretical estimates to the experimental data, reasonable agreement is obtained considering the simplified form of the nuclear fragmentation model used in the calculations and the overall sensitivity of the calculated momentum transfer to the choice of impact parameter. Improved agreement is expected if impact parameters from a fragmentation model using realistic nuclear densities and interactions were available. This is especially true for collisions involving lighter ions, such as carbon, oxygen, and beryllium, which are poorly represented by simple uniform nuclear distributions.

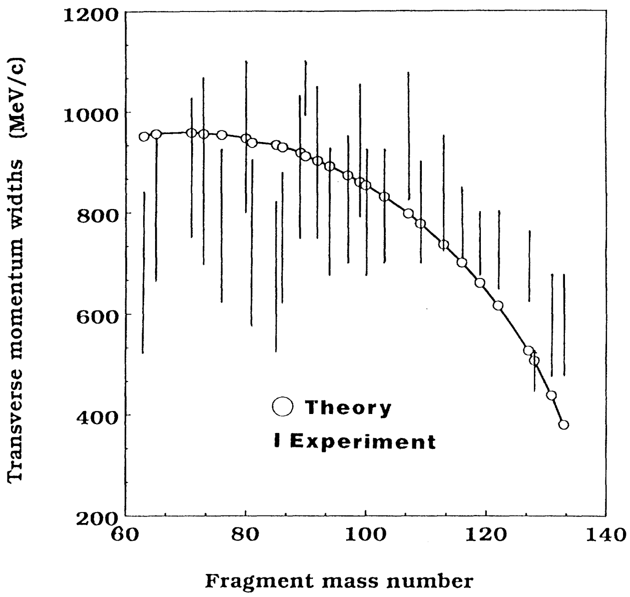

FIG. 3. Transverse momentum widths as a function of fragment mass number for 1.2 A GeV ^{139}La colliding with a carbon target. The experimental data are taken from Ref. 15.

Figure 3 displays transverse momentum widths as a function of fragment mass number for 1.2A GeV ^{139}La fragmenting in carbon targets. The experimental data are taken from Ref. 15. Again, impact parameters from NUCFRAG are used as inputs into the momentum-transfer expressions [Eqs. (23) and (24)]. For consistency with the use of these impact parameters, a narrow Gaussian t-matrix and uniform nuclear densities were again utilized in the momentum-transfer calculations. From Fig. 3, it is clear that the agreement is much better than in Fig. 2 and probably reflects the fact that a uniform nuclear density distribution is a more reasonable approximation for a heavy nucleus like lanthanum than for a light nucleus, such as oxygen.

IV. ESTIMATING COLLISION IMPACT PARAMETERS

Thus far in this work, we have used collision impact parameters as inputs into a momentum-transfer computational model which, in turn, has yielded estimates of heavy ion fragment momentum downshifts-widths for comparison with experimental data. However, this procedure can be reversed and the model used to estimate collision impact parameters from measured momentum downshifts for relativistic collisions. Let F be the fragment mass number with measured longitudinal momentum downshift $\Delta P_{\parallel}$ produced in a relativistic collision between a projectile nucleus (mass number A) and some target. Then, from Eq. (27), the longitudinal momentum transfer to the projectile from the target is

$$Q_{\parallel}=\frac{A}{F}\Delta P_{\parallel} \ . \tag{29}$$

The collision impact parameter can then be estimated from Eq. (24) by computing $Q_{\parallel}$ as a function of impact parameter (e.g., in Fig. 1) and using $Q_{\parallel}$ from Eq. (29) as the entry. To illustrate, consider a collision involving 2.1A GeV oxygen colliding with a beryllium target. The calculated momentum transfer using realistic nuclear densities is displayed in Fig. 1. If the measured (hypothetical) momentum downshift for the ^{14}N fragment is 35 ± 7 MeV/c, then Eq. (29) yields a longitudinal momentum transfer of 40 ± 8 MeV/c. From Fig. 1, the corresponding range of impact parameters is $6.1-6.4$ fm. A similar procedure incorporating measured momentum distribution widths and Eqs. (25) and (23) or (24) could also be used to estimate collision impact parameters. Note that these proposed methods for estimating collision impact parameters are similar in concept to the use of heavy fragment yields in the quantum molecular-dynamics approach of Aichelin and collaborators.[24]

V. CONCLUDING REMARKS

Beginning with composite particle multiple-scattering theory, an optical model description of collision momentum transfer in relativistic heavy ion collisions was derived. General expressions for momentum transfer, which utilize a finite-range two-nucleon interaction and realistic nuclear densities, were presented. The theory was used as input into the Goldhaber formalism to estimate heavy ion fragment momentum downshifts for relativistic oxygen and transverse momentum widths for relativistic lanthanum projectiles. The novel feature of this work was the relating of the imaginary component of the momentum transfer to the longitudinal collision momentum loss. Finally, the use of the model as a mechanism for estimating collision impact parameters was described.

The present theory is mainly applicable at intermediate or high energies because of the use of eikonal wave functions and the impulse approximation. At lower energies (below several hundred MeV/nucleon), the validity of straight-line trajectories and the assumption of a constant projectile velocity is questionable. Therefore, to compare theory with experiment at lower energies[23] revisions to the model are necessary. In particular, deceleration corrections to the constant velocity assumption are being developed. For incident energies greater than 1 A GeV, first-order deceleration corrections are small ($<1\%$). As the incident energy decreases, however, the first-order corrections increase significantly (over 50% at 100 A MeV), indicating that higher-order terms must be included.[22] Work on this is in progress and will be reported when completed.

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[1]P. J. Lindstrom, D. E. Greiner, H. H. Heckman, Bruce Cork, and F. S. Beiser, Lawrence Berkeley Laboratory Report No. LBL-3650, 1975.
[2]D. E. Greiner, P. J. Lindstrom, H. H. Heckman, Bruce Cork, and F. S. Bieser, Phys. Rev. Lett. **35**, 152 (1975).
[3]A. S. Goldhaber and H. H. Heckman, Annu. Rev. Nucl. Part. Sci. **28**, 161 (1978).
[4]S. Nagamiya, J. Randrup, and T. J. M. Symons, Annu. Rev. Nucl. Part. Sci. **34**, 155 (1984).
[5]J. Hufner, Phys. Rep. **125**, 129 (1985).
[6]G. Baur, F. Rösel, D. Trautmann, and R. Shyam, Phys. Rep. **111**, 333 (1984).
[7]H. Feshbach and K. Huang, Phys. Lett. **47B**, 300 (1973).
[8]A. S. Goldhaber, Phys. Lett. **53B**, 306 (1974).
[9]R. K. Bhaduri, Phys. Lett. **50B**, 211 (1974).
[10]J. D. Bowman, W. J. Swiatecki, and C. F. Tsang, Lawrence Berkely Laboratory Report No. LBL-2908, 1973.
[11]J. Gosset, H. H. Gutbrod, W. G. Meyer, A. M. Poskanzer, A. Sandoval, R. Stock, and G. D. Westfall, Phys. Rev. C **16**, 629 (1977); L. F. Oliveira, R. Donangelo, and J. O. Rasmussen, *ibid.* **19**, 826 (1979).
[12]J. Hufner, K. Schafer, and B. Schurmann, Phys. Rev. C **12**, 1888 (1975); M. Bleszynski and C. Sander, Nucl. Phys. **A326**, 525 (1979).
[13]L. W. Townsend, Can. J. Phys. **61**, 93 (1983); L. W. Townsend, J. W. Wilson, and J. W. Norbury, *ibid.* **63**, 135 (1985); L. W. Townsend, J. W. Wilson, F. A. Cucinotta, and J. W. Norbury, Phys. Rev. C **34**, 1491 (1986).
[14]S. H. Fricke, Ph.D. dissertation, University of Minnesota, 1985 (unpublished); B. F. Bayman, P. J. Ellis, S. Fricke, and Y. C. Tang, Phys. Rev. Lett. **53**, 1322 (1984).
[15]F. P. Brady *et al.*, Phys. Rev. Lett. **60**, 1699 (1988).
[16]J. W. Wilson and L. W. Townsend, Can. J. Phys. **59**, 1569 (1981).
[17]F. A. Cucinotta, G. S. Khandelwal, K. M. Maung, L. W. Townsend, and J. W. Wilson, NASA Technical Paper No. TP-2830, 1988; J. W. Wilson, Ph.D. dissertation, College of William and Mary, 1975.
[18]L. W. Townsend and J. W. Wilson, NASA Reference Publication No. RP-1134, 1985.
[19]L. S. Rodberg and R. M. Thaler, *Introduction to the Quantum Theory of Scattering* (Academic, New York, 1967), pp. 369–389.
[20]C. Y. Wong, in Proceedings of the 5th High Energy Heavy Ion Study, Lawrence Berkeley Laboratory, Report No. LBL-12652, 1981.
[21]J. W. Wilson, L. W. Townsend, and F. F. Badavi, Nucl. Instrum. Methods B **18**, 225 (1987).
[22]F. Khan, Ph.D. dissertation, Old Dominion University, 1989.
[23]J. P. Wefel, T. G. Guzik, J. W. Mitchell, H. J. Crawford, J. Engelage, P. J. Lindstrom, and W. Schimmerling, Bull. Am. Phys. Soc. **34**, 1137 (1989).
[24]J. Aichelin, G. Peilent, A. Bohnet, A. Rosenhauer, H. Stöcker, and W. Greiner, Phys. Rev. C **37**, 2451 (1988).