Signature of cluster substructure: $\alpha+d$ spectroscopic factor of ^{6}Li

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Through the example of the nucleus ^{6}Li taken as an $\alpha+p+n$ three-particle system, we show that the description of the disintegration of a multicluster system in terms of solely intercluster variables ("macroscopic" description) cannot be maintained. The cluster internal structure enters into the spectroscopic amplitudes of macroscopic models through norm operators. The form in which these appear depends on how the macroscopic model is related to the underlying microscopic formalism. We assert that the treatment of the Pauli effects in the commonly used $\alpha+p+n$ models implies that these models are related to the six-nucleon problem via a transformation mediated by the square root of the $\alpha+p+n$ three-cluster norm operator. The spectroscopic amplitudes will then contain this operator and, in general, the norm operator of the two-cluster fragment involved. Using $\alpha+p+n$ wave functions available from the literature, we show that the inclusion of the norm operator required enhances the $\alpha+d$ spectroscopic factor by more than 30%. This reconciles the three-particle models with the microscopic models and improves the reproduction of experimental data. We point out that a smaller enhancement is expected in the spectroscopic factor of nucleon removal as well.

I. INTRODUCTION

In the description of complex physical systems, it is often desirable to treat the composite constituents as structureless. The most obvious nuclear constituent amenable to such a "macroscopic" treatment is the α particle, because of its remarkable stability and inertness. This treatment implies an obscure point, however, in that it apparently assumes the nucleons of the α particle to be distinguishable from those of the nuclear environment. A most simple system that lends itself to a test of this assumption is ^{6}Li.

The success of the $\alpha+p+n$ three-particle model[1–17] in reproducing the measured energies and ground-state (g.s.) properties of ^{6}Li puts the soundness of this assumption beyond doubt. Nevertheless, there is still a discomforting discrepancy in the $\alpha+d$ spectroscopic factor, not so much from the experimental values as from the predictions of the fully microscopic cluster model. The three-particle estimates put this spectroscopic factor at 0.54–0.75;[3,6,8,9,12,13,18] the high-energy quasifree knockout experiments yield 0.73–1.3 (Refs. 19–22) with 0.73 looking the most reliable value,[22] while the microscopic cluster models give 0.93–1.07 (Refs. 23–25) with 0.93 being the most realistic one.[25]

Considering the overall similarities between the other predictions of these two families of models, there is a challenge to understand this particular discrepancy. Assuming that both models are realistic at their own levels, we are inclined to conclude that their disagreement is to be ascribed to the inherently different ways in which they treat the α particle. In Ref. 25 we criticized the treatment of the Pauli principle in the three-particle formula for the spectroscopic amplitude. We argued intuitively in support of a different formula for the spectroscopic amplitude and pointed out that this would probably reconcile the three-particle model with the microscopic model. In Ref. 26 we reported on preliminary results that confirmed this conjecture.

In this paper we shall tackle this problem at a deeper level. We shall show that the ambiguity in the definition of the spectroscopic amplitude can be removed by carefully relating the three-particle wave function to the microscopic wave function. As we shall see, the correspondence between the macroscopic and microscopic wave functions can be firmly established once the potential in the three-particle model is taken, as usual, to be the sum of the three Hermitean interactions assumed to act in the two-body subsystems. In the derivation of such a three-particle Hamiltonian, the huge Pauli nonlocalities are removed by a transformation applied to the intercluster wave function. The wave function resulting from this transformation is the wave function proper: the one that carries probability information. Such a three-particle wave function can, in general, be used as a wave function, regardless of its microscopic foundation. Thus this reinterpretation underpins the validity of the $\alpha+p+n$ model. The microscopic origin of the intercluster wave functions comes into play only when it is used together with subsystem wave functions, such as in the spectroscopic overlap amplitudes. As we shall see, in the $\alpha+p+n$ model the correct formulas of the spectroscopic amplitudes contain the integral operator that mediates the transformation involved in the derivation of the three-particle model. It is the kernel of this operator that contains information on the cluster internal structure. These considerations bear on all applications of macroscopic ingredients in the description of the fragmentation of a system of identical fermions. (In fact, a trivial departure from the macroscopic spectroscopic amplitude formula appears in

the description of the fragmentation of a two-cluster system into the two clusters, too.[27])

We shall focus our attention on the $\alpha+d$ spectroscopic amplitude. We establish our formula for the spectroscopic amplitude by reconsidering the problem of eliminating the internal structure of the α particle from the description of ^{6}Li (Sec. II). We then present numerical comparisons between the new and conventional formulas using $\alpha+p+n$ models formulated in two different approaches. The variational approach will be represented by the wave functions of Voronchev *et al.*[8] and of Kukulin *et al.*,[14] while the three-body approach based on the Faddeev equations will be represented by the wave function of Lehman, Rai, and Ghovanlou[5] (Sec. III). In conclusion, we extend our considerations to the ^{5}He$+p$ spectroscopic amplitudes and discuss our results in a broader context (Sec. IV).

II. MACROSCOPIC VERSUS MICROSCOPIC SPECTROSCOPIC AMPLITUDE

A. Microscopic and conventional macroscopic formulas

So far as we believe that the α particle is composed of nucleons, the spectroscopic amplitude of the $\alpha+d$ fragmentation is to be defined at the microscopic level:

$$g_{\alpha d}(\mathbf{R})=\langle \mathcal{A}_{\alpha d}\{\Psi_\alpha\Psi_d\delta(\mathbf{R}-\mathbf{r}_{\alpha d})\}|\Psi_6\rangle \tag{1a}$$

$$=\begin{bmatrix}6\\2\end{bmatrix}^{1/2}\langle \Psi_\alpha\Psi_d\delta(\mathbf{R}-\mathbf{r}_{\alpha d})|\Psi_6\rangle\ , \tag{1b}$$

where $\Psi_\alpha(\xi_\alpha)$,

$$\Psi_d(\xi_p,\xi_n,\mathbf{r}_{pn})=\mathcal{A}_{pn}\{\Psi_p(\xi_p)\Psi_n(\xi_n)\Phi_d(\mathbf{r}_{pn})\}\ ,$$

and $\Psi_6(\xi_\alpha,\xi_p,\xi_n,\mathbf{r}_{pn},\mathbf{r}_{\alpha d})$ are the (antisymmetrical and normalized) wave functions of the α particle, deuteron, and ^{6}Li g.s.'s, respectively, each being a function of the respective intrinsic coordinates. We formulate the problem of ^{6}Li and of its subsystems in the respective intrinsic frames throughout so that there will be no need to carry the center-of-mass (c.m.) coordinates. The $\Psi_p(\xi_p)$ and $\Psi_n(\xi_n)$ are just nucleon spin-isospin eigenfunctions. The angular momentum and isospin couplings are suppressed. The operators $\mathcal{A}_{ab}$ are antisymmetrization operators acting between clusters a and b. We define them so that they contain a statistical factor $n^{-1/2}$, where n is the number of permutations involved. In (1b) we used

$$\mathcal{A}_{ab}^2=n^{1/2}\mathcal{A}_{ab}\ . \tag{2}$$

The overlaps in (1) involve integrations over all intrinsic variables involved in ^{6}Li, viz., $\{\xi_\alpha,\xi_p,\xi_n,\mathbf{r}_{pn},\mathbf{r}_{\alpha d}\}$.

The formula of the three-particle model identified conventionally with Eqs. (1) is

$$G_{\alpha d}(\mathbf{R})=\langle \Psi_p(\xi_p)\Psi_n(\xi_n)\Phi_d(\mathbf{r}_{pn})\delta(\mathbf{R}-\mathbf{r}_{\alpha d})| \times\sum_{p'n'6'}\Psi_{p'}(\xi_p)\Psi_{n'}(\xi_n)\Phi_{6'}(\mathbf{r}_{pn},\mathbf{r}_{\alpha d})\rangle \tag{3a}$$

$$=\int d\mathbf{r}\,\Phi_d^*(\mathbf{r})\Phi_6(\mathbf{r},\mathbf{R})\ , \tag{3b}$$

where $\Phi_{6'}$ are relative wave-function components of the $\alpha+p+n$ three-particle system, belonging to $\Psi_{p'}$ and $\Psi_{n'}$, which may differ from Ψ_p and Ψ_n in the spin projections. [If the angular momentum coupling were explicit, (3b) would be a sum over different angular momentum values.] For simplicity, in the following we shall suppress all terms of the ^{6}Li wave function but the one containing $\Psi_p\Psi_n$. Equation (3) would indeed be the amplitude that enters into the description of the $\alpha+d$ fragmentation if the α particle were really structureless. Since the intrinsic degrees of freedom of the α particle are not involved here, in this formula the nucleons that are outside the α particle seem to be distinguished from those that are inside. Indeed, the same formula [Eq. (3b)] would be obtained if we inserted $\Psi_\alpha(\xi_\alpha)$ both in the bra and in the ket of (3a) so long as *no intercluster antisymmetrization* is included. To see whether this treatment complies with the Pauli principle, one has to go back to the derivation of the three-particle model from the six-particle problem.

B. Derivation of the three-particle model

The three-particle model to be derived should be similar to the ones actually used. It should thus comprise a structureless α particle, a proton, and a neutron interacting via Hermitean, energy-independent two-body interactions that describe the behavior of the three two-particle subsystems correctly. The structureless α particle of the three-particle approach implies a nonexcitable α particle in the corresponding microscopic approach. In such a microscopic approach, the ^{6}Li wave function can be expanded as

$$\Psi_6=\int d\mathbf{r}\int d\mathbf{R}\,\varphi(\mathbf{r},\mathbf{R})\Psi_{\mathbf{r},\mathbf{R}}\ , \tag{4}$$

where, for each value of $\mathbf{r}$ and $\mathbf{R}$, the function $\Psi_{\mathbf{r},\mathbf{R}}$ is the element of a basis, labeled by the continuous indices $\mathbf{r}$ and $\mathbf{R}$,

$$\Psi_{\mathbf{r},\mathbf{R}}(\xi_\alpha,\xi_p,\xi_n,\mathbf{r}_{pn},\mathbf{r}_{\alpha d}) =\mathcal{A}_{\alpha pn}\{\Psi_\alpha(\xi_\alpha)\Psi_p(\xi_p)\Psi_n(\xi_n)\delta(\mathbf{r}-\mathbf{r}_{pn})\delta(\mathbf{R}-\mathbf{r}_{\alpha d})\}\ , \tag{5}$$

and $\varphi(\mathbf{r},\mathbf{R})$ is the corresponding expansion coefficient. Since this is the standard cluster-model ansatz, in the ensuing exposition we can follow the cluster-model formalism.[28,29] Projected onto this subspace, the six-particle Schrödinger equation $H\Psi_6=E\Psi_6$ reduces to

$$\mathcal{H}\varphi(\mathbf{r},\mathbf{R})=E\mathcal{N}\varphi(\mathbf{r},\mathbf{R})\ , \tag{6}$$

where $\mathcal{H}$ and $\mathcal{N}$ are double-integral operators whose kernels are

$$H(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')=\langle \Psi_{\mathbf{r},\mathbf{R}}|H|\Psi_{\mathbf{r}',\mathbf{R}'}\rangle\ , \tag{7a}$$

$$N(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')=\langle \Psi_{\mathbf{r},\mathbf{R}}|\Psi_{\mathbf{r}',\mathbf{R}'}\rangle\ . \tag{7b}$$

[E.g., $\mathcal{N}f(\mathbf{r},\mathbf{R})\equiv\int d\mathbf{r}'\int d\mathbf{R}'N(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')f(\mathbf{r}',\mathbf{R}')$.] In Eq. (6) we had to have recourse to operators acting on the parameter coordinates $\mathbf{r}$ and $\mathbf{R}$. In this way we managed to introduce relative variables without violating the Pauli principle. The effect of the Pauli principle is incorporat-

ed in the definitions of the kernels.

The appearance of $\mathcal{N}$ on the right-hand side requires Eq. (6) to be rewritten in some way before it can be identified with a three-particle Schrödinger equation. One possibility would be to write

$$[\mathcal{H}+(1-\mathcal{N})E]\varphi(\mathbf{r},\mathbf{R})=E\varphi(\mathbf{r},\mathbf{R}) , \tag{8}$$

with the operator in the square brackets considered a redefined Hamiltonian. This, however, can by no means be identified with the three-particle Hamiltonian either, because it is energy dependent. Alternatively, one can multiply Eq. (6) from the left by $\mathcal{N}^{-1}$ and regard $\mathcal{N}^{-1}\mathcal{H}$ as the Hamiltonian. For the inversion operation to be correct, one should exclude the subspace $\{\varphi_i^{(0)}\}$ of $\{\varphi\}$ for which $\mathcal{N}\varphi_i^{(0)}=0$. This is not an actual restriction because $\mathcal{N}\varphi_i^{(0)}=0$ implies $\mathcal{A}_{\alpha pn}\{\Psi_\alpha\Psi_p\Psi_n\varphi_i^{(0)}(\mathbf{r}_{pn},\mathbf{r}_{\alpha d})\}=0$, and thus φ and $\varphi+\sum_i c_i\varphi_i^{(0)}$ describe the same six-particle state. (The exclusion of the subspace $\{\varphi_i^{(0)}\}$ implies that $\mathcal{N}$, having no zero eigenvalue on the restricted state space, can be inverted.) The Hamiltonian $\mathcal{N}^{-1}\mathcal{H}$, however, is not Hermitean, although both $\mathcal{N}^{-1}$ and $\mathcal{H}$ are Hermitean, because they do not commute. Thus, again, this is not the proper reduction.

One can obtain a three-particle Schrödinger equation with Hermitean Hamiltonian by multiplication of (6) from the left by $\mathcal{N}^{-1/2}$ and redefining the wave function and the Hamiltonian as

$$\chi(\mathbf{r},\mathbf{R})=\mathcal{N}^{1/2}\varphi(\mathbf{r},\mathbf{R}) , \tag{9a}$$

$$h=\mathcal{N}^{-1/2}\mathcal{H}\mathcal{N}^{-1/2} , \tag{9b}$$

with the result

$$h\chi(\mathbf{r},\mathbf{R})=E\chi(\mathbf{r},\mathbf{R}) . \tag{10}$$

It is this transformation that implies, for the three-particle wave function, the same normalization as is valid for Ψ_6:

$$\langle\Psi_6|\Psi_6\rangle=(\varphi|\mathcal{N}|\varphi)=(\chi|\chi) , \tag{11}$$

where the parentheses denote matrix elements involving integrations over the parameter coordinates.

This transformation is the only one that produces a Hermitean effective Hamiltonian, apart from unitary transformations U applied to χ and h. The function χ is distinguished from the other possible functions $\bar{\chi}=U\chi$ not only by aesthetical merits. An example shows[30] that in matrix elements the many-particle wave function is represented faithfully just by χ.

It remains to relate the Hamiltonian h of the three-particle model to those of the two-body subsystems. To this end, let us write the microscopic Hamiltonian, in self-explanatory notation, in the asymmetric form

$$H=H_\alpha(\xi_\alpha)+T_{\alpha d}(\mathbf{r}_{\alpha d})+T_{pn}(\mathbf{r}_{pn})+V_{\alpha p}(\xi_\alpha,\mathbf{r}_{\alpha p})+V_{\alpha n}(\xi_\alpha,\mathbf{r}_{\alpha n})+V_{pn}(\mathbf{r}_{pn}) . \tag{12}$$

Substituting (5) and (12) into (7a) and using (2), we arrive at

$$H(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')=[E_\alpha+T_{\alpha d}(\mathbf{R})+T_{pn}(\mathbf{r})+V_{pn}(\mathbf{r})]N(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')+V_{\alpha p}(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')+V_{\alpha n}(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}') , \tag{13}$$

where E_α is the g.s. energy of the α particle and

$$\begin{aligned}V_{\alpha N}(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')&=\left[\frac{6!}{4!}\right]^{1/2}\langle\Psi_\alpha\Psi_p\Psi_n\delta(\mathbf{r}-\mathbf{r}_{pn})\delta(\mathbf{R}-\mathbf{R}_{\alpha d})|V_{\alpha N}|\Psi_{\mathbf{r}',\mathbf{R}'}\rangle\\&=\langle\mathcal{A}_{\alpha pn}\{\Psi_\alpha\Psi_p\Psi_n\delta(\mathbf{r}-\mathbf{r}_{pn})\delta(\mathbf{R}-\mathbf{r}_{\alpha d})V_{\alpha N}\}|\Psi_{\mathbf{r}',\mathbf{R}'}\rangle\quad(N=p,n) .\end{aligned} \tag{14}$$

Denoting the integral operators belonging to the kernels by the corresponding script letters, we can express h as

$$h=E_\alpha+\mathcal{N}^{-1/2}(T_{\alpha d}+T_{pn}+V_{pn})\mathcal{N}^{1/2}+\mathcal{N}^{-1/2}(\mathcal{V}_{\alpha p}+\mathcal{V}_{\alpha n})\mathcal{N}^{-1/2} . \tag{15}$$

Except for E_α, the terms of h are all three-body operators, and there seems to be no obvious way of reducing them to sums of two-body terms unless approximations are introduced. In particular, neglecting the exchanges between n and the rest of the system in $V_{\alpha p}(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')$, we can integrate over ξ_n, and after a coordinate transformation from $\{\mathbf{r}_{pn},\mathbf{r}_{\alpha d}\}$ to $\{\mathbf{r}_{\alpha n},\mathbf{r}_{\alpha p}\}$, whose Jacobian is unity, we can also integrate over $\mathbf{r}_{\alpha n}$:

$$\begin{aligned}V_{\alpha p}(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')&\approx\langle\mathcal{A}_{\alpha p}\{\Psi_\alpha\Psi_p\Psi_n\delta(\mathbf{r}-\mathbf{r}_{pn})\delta(\mathbf{R}-\mathbf{r}_{\alpha d})V_{\alpha p}\}|\mathcal{A}_{\alpha p}\{\Psi_\alpha\Psi_p\Psi_n\delta(\mathbf{r}'-\mathbf{r}_{pn})\delta(\mathbf{R}'-\mathbf{r}_{\alpha d})\}\rangle\\&=\delta(\mathbf{r}-\mathbf{r}')\langle\mathcal{A}_{\alpha p}\{\Psi_\alpha\Psi_p\delta(\mathbf{R}-\tfrac{1}{2}\mathbf{r}-\mathbf{r}_{\alpha p})V_{\alpha p}\}|\mathcal{A}_{\alpha p}\{\Psi_\alpha\Psi_p\delta(\mathbf{R}'-\tfrac{1}{2}\mathbf{r}'-\mathbf{r}_{\alpha p})\}\rangle\\&\equiv\delta(\mathbf{r}-\mathbf{r}')V_{\alpha p}(\mathbf{R}-\tfrac{1}{2}\mathbf{r};\mathbf{R}'-\tfrac{1}{2}\mathbf{r}') .\end{aligned} \tag{16}$$

The kernel $V_{\alpha p}(\mathbf{x};\mathbf{x}')$ is the one that appears in the Hamiltonian kernel, of the type of Eq. (13), of the $\alpha+p$ two-cluster problem. By neglecting certain exchange terms similarly, one can also obtain

$$\begin{aligned}V_{\alpha n}(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')&\approx\delta(\mathbf{r}-\mathbf{r}')V_{\alpha n}(\mathbf{R}+\tfrac{1}{2}\mathbf{r};\mathbf{R}'+\tfrac{1}{2}\mathbf{r}') ,\\N(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')&\approx\delta(\mathbf{r}-\mathbf{r}')N_{\alpha p}(\mathbf{R}-\tfrac{1}{2}\mathbf{r};\mathbf{R}'-\tfrac{1}{2}\mathbf{r}')\\&\approx\delta(\mathbf{r}-\mathbf{r}')N_{\alpha n}(\mathbf{R}+\tfrac{1}{2}\mathbf{r};\mathbf{R}'+\tfrac{1}{2}\mathbf{r}') .\end{aligned} \tag{17}$$

Here the kernels $N_{\alpha N}(\mathbf{x};\mathbf{x}')$ $(N=p,n)$ are two-particle analogs of $N(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')$. Since this decomposition can only be achieved at the expense of neglecting exchange terms, the three-body nature of the microscopically deduced Hamiltonian is obviously seen to be a consequence of the Pauli principle.

Such exchange terms are always very important in microscopic cluster-model calculations, and so these approximations seem to be very rough, at least for small values of the arguments. Detailed analyses for several three-cluster systems,[31-33] including $\alpha+p+n$,[33] have revealed that the three-body potential term of $\mathcal{H}$ is indeed strong, but that of h is very weak. More precisely, it was shown that the matrix elements of the residual term $\mathcal{V}_{\rm res}$ of h, written in the form

$$h=E_\alpha+T_{\alpha d}+T_{pn}+V_{pn}+\mathcal{N}_{\alpha p}^{-1/2}\mathcal{V}_{\alpha p}\mathcal{N}_{\alpha p}^{-1/2}+\mathcal{N}_{\alpha n}^{-1/2}\mathcal{V}_{\alpha n}\mathcal{N}_{\alpha n}^{-1/2}+\mathcal{V}_{\rm res}\ , \tag{18}$$

is virtually negligible. Here $\mathcal{N}_{\alpha p}^{-1/2}\mathcal{V}_{\alpha p}\mathcal{N}_{\alpha p}^{-1/2}$ and $\mathcal{N}_{\alpha n}^{-1/2}\mathcal{V}_{\alpha n}\mathcal{N}_{\alpha n}^{-1/2}$ are the potentials appearing in the two-body analogs of Eq. (10), with $\mathcal{N}_{\alpha p}$ and $\mathcal{N}_{\alpha n}$ being the operators whose kernels are $N_{\alpha p}$ and $N_{\alpha n}$ of (17). This finding not only shows that the three-particle approach to ^{6}Li is reasonable, but also confirms that it must be considered an approximation to Eq. (10).

The reduction of the three-body term as a result of the transformation (9) can be made plausible by studying a physically important limiting case. To come to this case, let us identify Ψ_α with an intrinsic harmonic-oscillator (h.o.) state composed of 0s orbits. In cluster models this is a most usual approximation. Furthermore, let us represent the relative motion by

$$\psi(\mathbf{r},\mathbf{R})=\sum_{i,j}c_{ij}\phi_i(\mathbf{r})\phi_j(\mathbf{R})\ , \tag{19}$$

where ϕ_i and ϕ_j are h.o. single-particle states belonging to the same oscillator quantum $\hbar\omega$ as is involved in Ψ_α, and ψ carries a definite number N of $\hbar\omega$'s and definite SU_3 labels $(\lambda,\mu),\kappa,L$.[29] The wave function

$$\int d\mathbf{r}\int d\mathbf{R}\,\psi(\mathbf{r},\mathbf{R})\Psi_{\mathbf{r},\mathbf{R}}=\mathcal{A}_{\alpha pn}\{\Psi_\alpha\Psi_p\Psi_n\psi\} \tag{20}$$

may be considered to represent a state of ^{6}Li in the h.o. cluster model. Since this is a reasonable approximation for the first few states,[34] $\mathcal{A}_{\alpha pn}\{\Psi_\alpha\Psi_p\Psi_n\psi\}$ approximately satisfies the Schrödinger equation with an eigenvalue $E_{\rm h.o.}$:

$$H\mathcal{A}_{\alpha pn}\{\Psi_\alpha\Psi_p\Psi_n\psi(\mathbf{r}_{pn},\mathbf{r}_{\alpha d})\}\approx E_{\rm h.o.}\mathcal{A}_{\alpha pn}\{\Psi_\alpha\Psi_p\Psi_n\psi(\mathbf{r}_{pn},\mathbf{r}_{\alpha d})\}\ . \tag{21}$$

Projecting this equation onto $\Psi_{\mathbf{r},\mathbf{R}}$, we can cast it in a form analogous to (6):

$$\mathcal{H}\psi(\mathbf{r},\mathbf{R})\approx E_{\rm h.o.}\mathcal{N}\psi(\mathbf{r},\mathbf{R})\ . \tag{22}$$

The eigenvalue $E_{\rm h.o.}$ in these approximate equations may be identified with the exact eigenvalue, belonging to the exact eigenfunctions ψ, of the model problem

$$H_{\rm h.o.}(\mathbf{r},\mathbf{R})\psi(\mathbf{r},\mathbf{R})=E_{\rm h.o.}\psi(\mathbf{r},\mathbf{R}) \tag{23}$$

that involves h.o. interactions $V_{pn}^{\rm h.o.}(\mathbf{r})$ and $V_{\alpha d}^{\rm h.o.}(\mathbf{R})$ between p and n and between α and d, respectively:

$$H_{\rm h.o.}(\mathbf{r},\mathbf{R})=E_\alpha^{\rm h.o.}+T_{pn}(\mathbf{r})+T_{\alpha d}(\mathbf{R})+V_{pn}^{\rm h.o.}(\mathbf{r})+V_{\alpha d}^{\rm h.o.}(\mathbf{R})\ , \tag{24}$$

with $E_\alpha^{\rm h.o.}$ being the model α energy. With (23) multiplied from the left by $\mathcal{N}$, its right-hand side will be identical with that of (22). Equating the left-hand sides, we obtain

$$\mathcal{H}\psi(\mathbf{r},\mathbf{R})\approx\mathcal{N}H_{\rm h.o.}(\mathbf{r},\mathbf{R})\psi(\mathbf{r},\mathbf{R})=\mathcal{N}^{1/2}H_{\rm h.o.}(\mathbf{r},\mathbf{R})\mathcal{N}^{1/2}\psi(\mathbf{r},\mathbf{R})\ . \tag{25}$$

In the second step we exploited the fact that $\psi(\mathbf{r},\mathbf{R})$ is an exact eigenfunction of not only $H_{\rm h.o.}(\mathbf{r},\mathbf{R})$, but also of $\mathcal{N}$.[29] With (25) used in (22), one obtains

$$\mathcal{N}^{1/2}H_{\rm h.o.}(\mathbf{r},\mathbf{R})\mathcal{N}^{1/2}\psi(\mathbf{r},\mathbf{R})\approx E_{\rm h.o.}\mathcal{N}\psi(\mathbf{r},\mathbf{R})\ . \tag{26}$$

The Hamiltonian $\mathcal{N}^{1/2}H_{\rm h.o.}(\mathbf{r},\mathbf{R})\mathcal{N}^{1/2}$ contains the large three-body Pauli terms implied by $\mathcal{N}$, but the transformation (9) applied to it results in $h=H_{\rm h.o.}(\mathbf{r},\mathbf{R})$.

Thus, within the validity of the h.o. cluster model, the six-particle Schrödinger equation exactly reduces to a three-particle equation, of the type of (10), which contains no three-body Pauli term.[35] At the same time, the equation of the type of (6) is contaminated with a large three-body Pauli term. In these considerations we had to resort to the h.o. cluster model because the norm operator and the Hamiltonian have a complete set of common eigenfunctions only in that model. But, as is shown by the numerical tests mentioned earlier,[31-33] the realistic models tend to mimic this behavior.

C. Conflicting interpretation

We have now established that the three-particle model can be derived from the microscopic approach with a good approximation, and that the three-particle Schrödinger equation is to be regarded as an approximation to Eq. (10). There is a consensus on the first part of this statement, but there is none on the second part. On the contrary, the three-particle wave function Φ_6 is usually believed to be an approximation to $\varphi(\mathbf{r},\mathbf{R})$ of Eq. (6) or (8) (see, e.g., Refs. 1, 6, and 36). Since the correct microscopic interpretation of Φ_6 is crucial from the point of view of the spectroscopic amplitude, we should revise the arguments, for this alternative identification, which were expounded by Wackman and Austern.[1]

This identification hinges on the hypothesis that the Hamiltonian in (8), $\mathcal{H}+(\mathbf{1}-\mathcal{N})E$, can be reduced, with a good approximation, to an energy-independent Hermitean Hamiltonian that contains no three-body term. Let us cast this Hamiltonian into a form resembling (18):

$$\mathcal{H}+(\mathbf{1}-\mathcal{N})E=E_\alpha+T_{\alpha d}+T_{pn}+V_{pn}+\tilde{\mathcal{V}}_{\alpha p}+\tilde{\mathcal{V}}_{\alpha n}+\tilde{\mathcal{V}}_{\rm res}\ , \tag{27}$$

where $\tilde{\mathcal{V}}_{\alpha p}$ and $\tilde{\mathcal{V}}_{\alpha n}$ are the potential terms that appear in the two-particle analogs of (8),

$$[\mathcal{H}_{\alpha N}+(1-\mathcal{N}_{\alpha N})E_{\alpha N}]\varphi_{\alpha N}(\mathbf{r})\equiv[T_{\alpha N}+\tilde{V}_{\alpha N}]\varphi_{\alpha N}(\mathbf{r}) =E_{\alpha N}\varphi_{\alpha N}(\mathbf{r}) \quad (N=p,n)\ , \tag{28}$$

and $\tilde{V}_{\rm res}$ is a residual three-body term. The last three terms on the right-hand side of (27) are obviously nonlocal and energy dependent.

The conventional arguments are as follows.[1] The α-nucleon phase shifts are well reproduced by local and energy-independent phenomenological potentials. From this it can be inferred that the nonlocal and energy-dependent terms of $\tilde{V}_{\alpha N}$ have little effect, at least on the phase shift. It has also been recognized, however, that these terms must influence the wave functions in the interaction region so as to remove the 0s state, which is Pauli forbidden. By eliminating this state separately, e.g., via a projection operator, the effect of these neglected terms seems duly allowed for. The residual three-body term, whether large or not, looks negligible because it is of short range, and in ^{6}Li all three constituents are not likely to stay too long within its range simultaneously.

While some of these statements hold for equations of both types, some are only valid if they are dissociated from Eqs. (8) and (28) and are associated with (10). Thus reinterpreted, these arguments will still serve as a justification of the $\alpha+p+n$ model, but come to be in full accord with our derivation proposed here.

To be more specific, one should note the following. Since the equations of the two types differ only in an inner region, the phase shift fits only imply the existence of well-behaved potentials without telling which equation they belong to. The reasoning ceases to be valid for (8) and (28) when the inner region is referred to. Indeed, there is no general reason to prevent the solution $\varphi_{\alpha N}(\mathbf{r})$ of Eq. (28) from being of 0s character. E.g., in the h.o. limit of Ψ_α, the function $\varphi_{\alpha N}(\mathbf{r})$ may have an arbitrarily large 0s component $\varphi^{(0)}$ since $\mathcal{H}_{\alpha N}\varphi^{(0)}=\mathcal{N}_{\alpha N}\varphi^{(0)}=0$. [Cf. the discussion following Eq. (8).] This component is, however, suppressed by $\mathcal{N}_{\alpha N}^{1/2}$ in the two-particle analog of Eq. (9a), and that is why the equation with Pauli projection should be likened to Eq. (10), rather than to Eq. (8) or (28).

A similar statement can be said about the last argument. As is shown by the normalization equation (11), probability interpretation[31] can only be attributed to χ. So $|\chi(\mathbf{r},\mathbf{R})|^2$ is bound to be small in the region where both $\mathbf{r}$ and $\mathbf{R}$ are small. For $|\varphi(\mathbf{r},\mathbf{R})|^2$, however, there being no probability argument, there is nothing to forbid it to be large in regions where all relative coordinates are small, and so the three-body term $\tilde{V}_{\rm res}$ in the operator may have a significant effect.

In short, the reason why the three-particle Schrödinger equation is related to Eq. (10) rather than to (6) or (8) is that (6) and (8) incorporate the Pauli principle in the operators only, whereas (10) contains the Pauli effects in the solution, too. The function $\varphi(\mathbf{r},\mathbf{R})$ need not obey the Pauli principle because it is to be written *behind the antisymmetrizer,* which will project out its Pauli-forbidden component. The function $\chi(\mathbf{r},\mathbf{R})$, on the other hand, is like a wave function in its own right, amenable to probability interpretation. On these grounds the identification of Φ_6 with φ is to be rejected.

D. Well-founded macroscopic formula

Having established the correspondence of Φ_6 to χ, we can express the spectroscopic amplitude in (1a) in terms of Φ_6. First, $\Psi_d=\mathcal{A}_{pn}\{\Psi_p\Psi_n\Phi_d\}$ and Ψ_6 of Eqs. (4) and (5) are to be substituted in (1a):

$$g_{\alpha d}(\mathbf{R})=\int d\mathbf{r}\int d\mathbf{r}'\int d\mathbf{R}'\Phi_d^*(\mathbf{r})\langle\mathcal{A}_{\alpha d}\mathcal{A}_{pn}\{\Psi_\alpha\Psi_p\Psi_n\delta(\mathbf{r}-\mathbf{r}_{pn})\delta(\mathbf{R}-\mathbf{r}_{\alpha d})\} \times|\mathcal{A}_{apn}\{\Psi_\alpha\Psi_p\Psi_n\delta(\mathbf{r}'-\mathbf{r}_{pn})\delta(\mathbf{R}'-\mathbf{r}_{\alpha d})\}\rangle\varphi(\mathbf{r}',\mathbf{R}') \tag{29a}$$

$$=\int d\mathbf{r}\int d\mathbf{r}'\int d\mathbf{R}'\Phi_d^*(\mathbf{r})N(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')\varphi(\mathbf{r}',\mathbf{R}') \tag{29b}$$

$$=\int d\mathbf{r}\Phi_d^*(\mathbf{r})\mathcal{N}\varphi(\mathbf{r},\mathbf{R})\ . \tag{29c}$$

In the second step we used $\mathcal{A}_{\alpha d}\mathcal{A}_{pn}=\mathcal{A}_{apn}$ as well as Eq. (7b). Now, inverting (9a), we obtain $\varphi=\mathcal{N}^{-1/2}\chi$. Inserting this in (29c), we arrive at

$$g_{\alpha d}(\mathbf{R})=\int d\mathbf{r}\,\Phi_d^*(\mathbf{r})\mathcal{N}^{1/2}\chi(\mathbf{r},\mathbf{R}) \tag{30a}$$

$$\approx\int d\mathbf{r}\,\Phi_d^*(\mathbf{r})\mathcal{N}^{1/2}\Phi_6(\mathbf{r},\mathbf{R})\ . \tag{30b}$$

This differs from (3b) just in $\mathcal{N}^{1/2}$ and agrees with the result of the intuitive derivation.[25]

Since $\mathcal{N}$ differs from the unity operator due to antisymmetrization, the appearance of $\mathcal{N}^{1/2}$ in the formula for $g_{\alpha d}(\mathbf{R})$ is a consequence of the Pauli principle in the effective three-particle Schrödinger equation (or Faddeev equations). This $\mathcal{N}^{1/2}$ is an extra requirement of the Pauli principle, which appears even though one uses the solution $\chi(\mathbf{r},\mathbf{R})$ of the exact microscopic approach [see Eq. (30a)]. Thus the appearance of $\mathcal{N}^{1/2}$ is not a compensation for any violation of the Pauli principle in the three-particle dynamics, nor does it imply a double counting of the Pauli effects. It will always appear provided the $\alpha+p+n$ wave function is derived from an energy-independent Hermitean Hamiltonian with no (or little) three-body potential.

From (29c) it is immediately seen that the alternative interpretation of the three-particle wave function, $\Phi_6\sim\varphi$, would lead to $g_{\alpha d}(\mathbf{R})=\int d\mathbf{r}\,\Phi_d^*(\mathbf{r})\mathcal{N}\Phi_6(\mathbf{r},\mathbf{R})$, which departs from the conventional formula (3) even more. Equation (3) would only be consistent with this if $\mathcal{N}$ were

a mere projector, and the Pauli principle were taken into account just by imposing $\mathcal{N}\Phi_6=\Phi_6$. Then Eq. (3) would also coincide with Eq. (30b). In the next section we shall show that the effect of $\mathcal{N}$ differs from that of a projector appreciably.

III. RESULTS

We have seen that the intrinsic structure of the α particle, eliminated though from the dynamics, reappears in the spectroscopic amplitude, embodied in the operator $\mathcal{N}^{1/2}$. We had shown,[37] however, that the effect of the norm operator depends only slightly on the actual description of the α particle. It thus suffices to use, for Ψ_α, the simplest h.o. $0s$ shell model with a realistic size parameter. We used $m\omega/\hbar=0.528$ fm^{-2}. The operator $\mathcal{N}$ is tractable through its spectral representation:

$$\mathcal{N}=\sum_{N\lambda\mu\kappa Li}|\psi^i_{N(\lambda,\mu)\kappa L}\rangle\nu^i_{N(\lambda,\mu)}\langle\psi^i_{N(\lambda,\mu)\kappa L}| \ . \tag{31}$$

With this choice of Ψ_α, the eigenfunctions of $\mathcal{N}$ are of the form of Eq. (19), where ϕ are h.o. single-particle functions carrying the same oscillator quanta $\hbar\omega$. The eigenvalues can be calculated by diagonalization of $\mathcal{N}$ in each subspace characterized by the set of quantum numbers $N(\lambda,\mu)\kappa L$.

The matrix elements of $\mathcal{N}$ between products of h.o. functions were calculated with a technique[38] that is based on the generating function of the h.o. eigenfunctions. This technique requires the analytical calculation of the overlap of multicenter Slater determinants of normalized h.o. single-particle functions as functions of the positions of the potential centers. With all h.o. wells chosen to have the same parameter, the c.m. wave functions can be factored out, so that the overlap will also be a c.m. factor times a factor coming from the internal motion. This latter factor depends solely on the relative positions of the h.o. wells. Its Taylor expansion in terms of these relative positions can also be performed analytically. It has been shown that the expansion coefficients, for which explicit formulas can be obtained, are nothing but the required matrix elements of the norm operator of the corresponding multicluster problem formulated in the intrinsic frame.[29,38]

We have calculated the radial spectroscopic amplitudes $g(r),G(r)$, belonging to αd relative orbital momentum 0, defined as

$$g_{\alpha d}(\mathbf{r})=r^{-1}g(r)Y_{00}, \quad G_{\alpha d}(\mathbf{r})=r^{-1}G(r)Y_{00} \ , \tag{32}$$

with $g_{\alpha d}(\mathbf{r})$ and $G_{\alpha d}(\mathbf{r})$ given by (30b) and (3b), respectively, and the corresponding momentum-space intensity functions, the so-called "momentum distributions," $f(q)$ and $F(q)$ defined as

$$\begin{aligned} f(q)(Y_{00})^2&=\frac{1}{4\pi}\left|\frac{1}{(2\pi)^{3/2}}\int d\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}g_{\alpha d}(\mathbf{r})\right|^2 , \\ F(q)(Y_{00})^2&=\frac{1}{4\pi}\left|\frac{1}{(2\pi)^{3/2}}\int d\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}G_{\alpha d}(\mathbf{r})\right|^2 . \end{aligned} \tag{33}$$

We have, furthermore, calculated the spectroscopic factors belonging, respectively, to the two amplitudes,

$$s_{\alpha d}=\int d\mathbf{r}|g_{\alpha d}(\mathbf{r})|^2, \quad S_{\alpha d}=\int d\mathbf{r}|G_{\alpha d}(\mathbf{r})|^2 \ . \tag{34}$$

We have made calculations with the $\alpha+p+n$ wave functions of Voronchev *et al.*,[8] Kukulin *et al.*,[14] and Lehman, Rai, and Ghovanlou.[5] These are composed of terms that carry various total orbital momenta L and summed nucleon spins S. Since the relationship between $g_{\alpha d}(\mathbf{r})$ and $G_{\alpha d}(\mathbf{r})$ cannot depend on the small components of Φ_d and Φ_6, we treated these terms in a somewhat simplified manner. In the models of Voronchev *et al.*[8] and Kukulin *et al.*[14] the weights of the $L\neq0$, $S\neq1$ terms of Φ_6 add up to 0.5% and 4.5%, respectively, and we simply omitted them, renormalized Φ_6 and combined it with a deuteron wave function of no d-wave admixture.[39] As for the model of Lehman, Rai, and Ghovanlou,[5] we used the version called "full model with $P_d=0$," which includes all $\alpha+N$ partial waves, but excludes the tensor term of the nucleon-nucleon interaction both in Φ_6 and in Φ_d. We furthermore disregarded the 0.28% d-wave component of the $\alpha+d$ spectroscopic amplitude.

The $\alpha+p+n$ radial wave functions of Voronchev *et al.*[8] and Kukulin *et al.*[14] are given in terms of combinations of $\Gamma_n(r_{pn})\Gamma'_{n'}(r_{\alpha d})$, where Γ_n and $\Gamma'_{n'}$ are Gaussians of diverse sizes. The deuteron wave function[39] used in combination with them is also a sum of such Gaussians. The overlaps of the Gaussians with the h.o. functions involved in $\mathcal{N}^{1/2}$ were calculated analytically.[29]

The ^{6}Li state of Lehman, Rai, and Ghovanlou[5] is given in momentum representation as a sum over the three partitions $\alpha(pn)$, $(\alpha p)n$, and $(\alpha n)p$, each to be denoted by Φ_6 bearing a partition superscript. The deuteron state[5] is given like the $p+n$ wave function in the $\alpha(pn)$ component. In calculating the overlaps of these with $\psi^i_{N(\lambda,\mu)\kappa L}$, the functions $\psi^i_{N(\lambda,\mu)\kappa L}$ were transformed to momentum representation. The integrations over the angular part $\hat{\mathbf{k}}_{pn}$ of the relative number $\mathbf{k}_{pn}$ in $\langle\Phi_d|\psi^i_{N(\lambda,\mu)\kappa L}\rangle$ and over $\hat{\mathbf{k}}_{pn}$ and $\hat{\mathbf{k}}_{\alpha d}$ in $\langle\psi^i_{N(\lambda,\mu)\kappa L}|\Phi_6^{\alpha(pn)}\rangle$ reduce to Kronecker deltas. The integrations involved in $\langle\psi^i_{N(\lambda,\mu)\kappa L}|\Phi_6^{(ap)n}\rangle$ and in $\langle\psi^i_{N(\lambda,\mu)\kappa L}|\Phi_6^{(an)p}\rangle$ were performed via transformation of the integration variable $\mathbf{k}_{pn}$ to $\mathbf{k}_{^5\mathrm{He}p}$ and $\mathbf{k}_{^5\mathrm{Li}n}$, respectively. The spherical harmonic $Y_{lm}(\hat{\mathbf{k}}_{pn})$ is then expressible in terms of the new angles with the use of rotation matrices. The angular integrations were thus performed analytically. The integrations over the radial wave numbers and the Fourier transformation to coordinate space were carried through numerically.

It is clear from Eq. (31) that $\mathcal{N}$ differs from the unity operator through its eigenvalues $\nu^i_{N(\lambda,\mu)}\neq1$ and differs from a projection operator through its eigenvalues $\nu^i_{N(\lambda,\mu)}\neq1,0$. The three-cluster norm operator is a noncompact operator with an infinite number of eigenfunctions belonging to eigenvalues substantially different from unity, among which there are an infinite number of eigenfunctions belonging to $\nu^i_{N(\lambda,\mu)}=0$ as well as to $\nu^i_{N(\lambda,\mu)}\neq0$. The $\alpha+p+n$ wave functions, however, have appreciable overlaps with only a finite number of these eigenfunctions. The overlap of Φ_6 with the $\leq N\hbar\omega$ subspace of

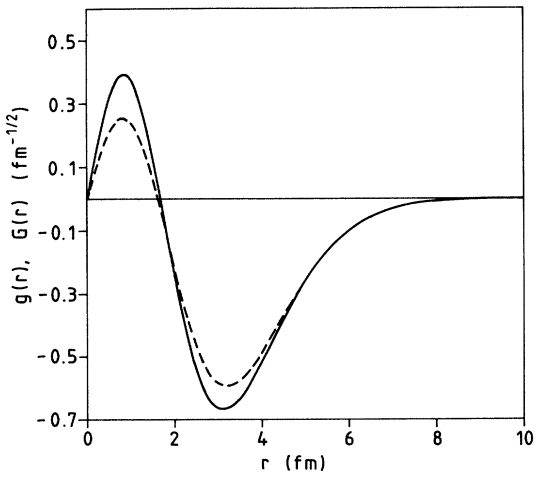

FIG. 1. Alpha + deuteron spectroscopic amplitudes $g(r)$ (solid line) and $G(r)$ (dashed line) with the $\alpha+p+n$ wave function of Voronchev *et al.* (Ref. 8). The deuteron wave function is from Ref. 39.

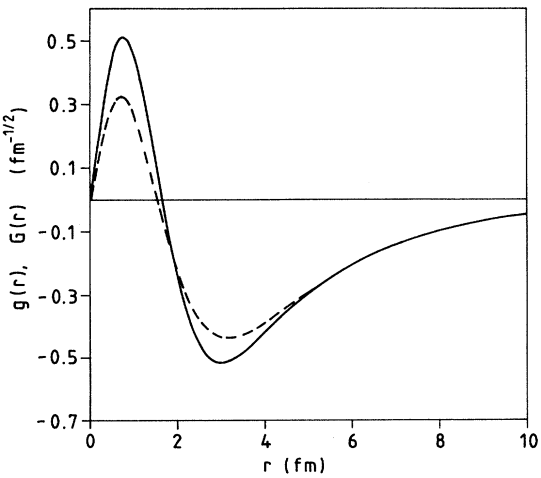

FIG. 3. Alpha + deuteron spectroscopic amplitudes $g(r)$ (solid line) and $G(r)$ (dashed line) with the $\alpha+p+n$ and deuteron wave functions of Lehman, Rai, and Ghovanlou (Ref. 5).

eigenfunctions can be characterized by $\langle\Phi_6|P_N|\Phi_6\rangle$, where P_N is the projector onto this subspace. We have found that $\langle\Phi_6|P_N|\Phi_6\rangle\approx\langle\Phi_6|\Phi_6\rangle$ within 0.01% for $N=100$ and within 2–3 % for $N=14$. We have solved the eigenvalue problem for $N\leq 14$ and approximated $\mathcal{N}^{1/2}$ by $P_{14}\mathcal{N}^{1/2}P_{14}+1-P_{14}$. We estimated the error of this approximation for the spectroscopic factor $s_{\alpha d}$ to be within 1%.

The three pairs of spectroscopic amplitudes are shown in Figs. 1–3, and the spectroscopic factors are collected in Table I. To illustrate the difference implied for the momentum structure, in Fig. 4 we depict $f(q)$ and $F(q)$ for the case of the wave function of Lehman, Rai, and Ghovanlou. These two functions compare with each other in much the same way in the other cases as well. Our results for the conventional spectroscopic amplitudes and factors should be very close to those published originally.[8,9,13] This is indeed so except for a minor disagreement in $S_{\alpha d}$ with Voronchev *et al.*,[8] the origin of which is obscure.

We see that in the inner region the microscopically well-founded amplitudes $g(r)$ are substantially larger than the corresponding conventional amplitudes $G(r)$. This is understood by noting that in the $N=2$ (i.e., $2\hbar\omega$) subspace, which dominates the g.s. of ^{6}Li according to the shell model, $\mathcal{N}$ has just one nonzero eigenvalue, $\nu=\frac{13}{8}$.[33] This shows that its effect on Φ_6 is indeed substantially different from that of a projector. In the $N>2$ subspaces there still are eigenvalues 0, eigenvalues smaller as well as greater than 1, and they influence $g(r)$ mainly for large r. The net effect is $s_{\alpha d}/S_{\alpha d}\approx 1.31-1.34$.

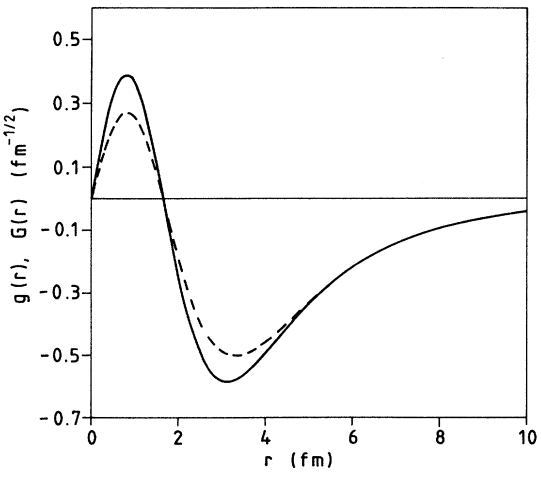

FIG. 2. Alpha + deuteron spectroscopic amplitudes $g(r)$ (solid line) and $G(r)$ (dashed line) with the $\alpha+p+n$ wave function of Kukulin *et al.* (Ref. 14). The deuteron wave function is from Ref. 39.

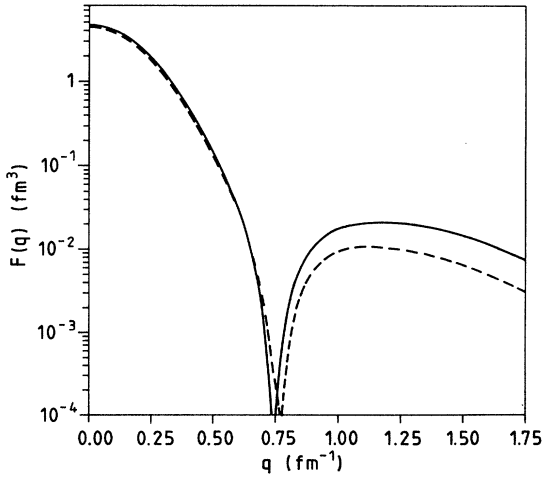

FIG. 4. Alpha-deuteron relative "momentum distributions" $f(q)$ (solid line) and $F(q)$ (dashed line) with the $\alpha+p+n$ and deuteron wave functions of Lehman, Rai, and Ghovanlou (Ref. 5).

TABLE I. Alpha + deuteron spectroscopic factors.

Source of Φ_6	$S_{\alpha d}$ (original)	$S_{\alpha d}$ (present work)	$s_{\alpha d}$ (present work)
Voronchev *et al.*[a]	0.66[a]	0.757	0.991
Kukulin *et al.*[b]	0.7487[c]	0.738	0.970
Lehman, Rai, and Ghovanlou[d]	0.632[e]	0.632	0.847

[a]Reference 8.
[b]Reference 14.
[c]Reference 13.
[d]Reference 5.
[e]Reference 9.

IV. DISCUSSION AND CONCLUSION

First of all, it is appropriate to note that the idea of incorporating the Pauli principle in spectroscopic amplitudes through norm operators, as was first proposed by Fliessbach,[40] was subsequently widely disputed.[39,41,42] That issue, however, does not bear on the present one. The norm operators involved in that controversy were those which should represent the Pauli effects in a *distorted wave* describing the motion of a fragment of ^{6}Li with respect to a partner, which contains the other fragment, in a direct reaction, whereas the norm operator invoked here is an inherent ingredient of the description of the structure of ^{6}Li itself. The subject of the dispute was whether it is correct or at least acceptable as an approximation to shift those distorted-wave norm operators onto the spectroscopic amplitudes. If it is, the definition of the spectroscopic amplitude should be modified. In accord with our view expounded elsewhere,[41] in this paper we stick to the conventional definition of the spectroscopic amplitude. [Otherwise, the modification would involve $\mathcal{N}_{\alpha d}^{-1/2}$, where $\mathcal{N}_{\alpha d}$ is the $\alpha+d$ norm operator. This would have a much smaller effect than $\mathcal{N}^{1/2}$, missing from the definition of $G_{\alpha d}(\mathbf{r})$, for the dominant eigenvalues of $\mathcal{N}_{\alpha d}$ and $\mathcal{N}$ are $\frac{9}{8}$ (Ref. 43) and $\frac{13}{8}$, respectively.]

Our finding for the $\alpha+d$ spectroscopic amplitude poses a similar question concerning the description of the other possible disintegration of the $\alpha+p+n$ system, the one that involves nucleon removal. To see this point explicitly, let us consider the amplitude of proton removal. In the three-particle model the conventional spectroscopic amplitude is defined, like in (3b), as

$$G_{5p}(\mathbf{R})=\int d\mathbf{r}\,\Phi_5^*(\mathbf{r})\Phi_6(\mathbf{r},\mathbf{R})\ , \tag{35}$$

where Φ_5 is the $\alpha+n$ analog of Φ_6. The wave functions involved in the microscopic approach are

$$\Psi_5=\mathcal{A}_{5p}\{\Psi_\alpha(\xi_\alpha)\Psi_n(\xi_n)\varphi_{\alpha n}(\mathbf{r}_{\alpha n})\}\ , \tag{36a}$$

$$\Psi_6=\mathcal{A}_{\alpha pn}\{\Psi_\alpha(\xi_\alpha)\Psi_p(\xi_p)\Psi_n(\xi_n)\varphi'(\mathbf{r}_{\alpha n},\mathbf{r}_{5p})\}\ . \tag{36b}$$

In (36b), $\varphi'(\mathbf{r}_{\alpha n},\mathbf{r}_{5p})\equiv\varphi(\mathbf{r}_{pn},\mathbf{r}_{\alpha d})$, so that this Ψ_6 is identical to that given in (4) and (5). It is convenient to introduce the kernel

$$N'(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')=\langle\mathcal{A}_{\alpha pn}\{\Psi_\alpha\Psi_p\Psi_n\delta(\mathbf{r}-\mathbf{r}_{\alpha n})\delta(\mathbf{R}-\mathbf{r}_{5p})\}|\mathcal{A}_{\alpha pn}\{\Psi_\alpha\Psi_p\Psi_n\delta(\mathbf{r}'-\mathbf{r}_{\alpha n})\delta(\mathbf{R}'-\mathbf{r}_{5p})\}\rangle\ , \tag{37}$$

which defines an operator $\mathcal{N}'$. The spectroscopic amplitude is then calculated after the pattern of Eqs. (29) and (30):

$$g_{5p}(\mathbf{R})=\int d\mathbf{r}\int d\mathbf{r}'\int d\mathbf{R}'\varphi_{\alpha n}^*(\mathbf{r})N'(\mathbf{r},\mathbf{R};\mathbf{r}',\mathbf{R}')\varphi'(\mathbf{r}',\mathbf{R}') \tag{38a}$$

$$=\int d\mathbf{r}\,\varphi_{\alpha n}^*(\mathbf{r})\mathcal{N}'\varphi'(\mathbf{r},\mathbf{R}) \tag{38b}$$

$$=\int d\mathbf{r}\,\chi_{\alpha n}^*(\mathbf{r})\mathcal{N}_{\alpha n}^{-1/2}\mathcal{N}'^{1/2}\chi'(\mathbf{r},\mathbf{R}) \tag{38c}$$

$$\approx\int d\mathbf{r}\,\Phi_5^*(\mathbf{r})\mathcal{N}_{\alpha n}^{-1/2}\mathcal{N}'^{1/2}\Phi_6'(\mathbf{r},\mathbf{R})\ , \tag{38d}$$

where $\chi_{\alpha n}\equiv\mathcal{N}_{\alpha n}^{1/2}\varphi_{\alpha n}$ and $\chi'\equiv\mathcal{N}'^{1/2}\varphi'$. The function $\Phi_6'(\mathbf{r},\mathbf{R})$ is defined so as to have the same values as $\Phi_6(\mathbf{r},\mathbf{R})$ with the argument transformed like the physical coordinates in the transformation $\{\mathbf{r}_{pn},\mathbf{r}_{\alpha d}\}\rightarrow\{\mathbf{r}_{\alpha n},\mathbf{r}_{5p}\}$:

$$\Phi_6'(\mathbf{r},\mathbf{R})\delta(\mathbf{r}-\mathbf{r}_{\alpha n})\delta(\mathbf{R}-\mathbf{r}_{5p})=\Phi_6(\mathbf{r},\mathbf{R})\delta(\mathbf{r}-\mathbf{r}_{pn})\delta(\mathbf{R}-\mathbf{r}_{\alpha d})\ . \tag{39}$$

Thus the parameter coordinates involved in all primed functions N', φ', Φ', and χ' correspond to $\{\mathbf{r}_{\alpha n},\mathbf{r}_{5p}\}$. (Note that the Jacobian of this transformation is unimodular.) Such a transformation applied to the eigenvalue equation of $\mathcal{N}$ shows that the eigenvalues of $\mathcal{N}'$ are identical to those of $\mathcal{N}$ and the eigenfunctions of $\mathcal{N}'$ are related to those of $\mathcal{N}$ via a Moshinsky transformation. Thus $\mathcal{N}'^{1/2}$ in (38c) and (38d) must have an effect similar to

$\mathcal{N}^{1/2}$ in (30). This effect, however, is partly compensated for by $\mathcal{N}_{\alpha n}^{-1/2}$. [In Eq. (30) no operator corresponding to $\mathcal{N}_{\alpha n}^{-1/2}$ appears because p and n are treated as elementary particles; at this level $\mathcal{N}_{pn}=1$.] The extent of this compensation may be estimated, again, from the leading nonzero eigenvalue[43] of $\mathcal{N}_{\alpha n}$, which is $\frac{5}{4}$, in contrast with $\frac{13}{8}$ of $\mathcal{N}'$.

A more precise estimate of the joint effect of $\mathcal{N}_{\alpha n}^{-1/2}$ and $\mathcal{N}'^{1/2}$ can be obtained from the fact that the microscopic sum-rule limit of the removal of a proton from outside the α core of ^{6}Li is $\frac{6}{5}$,[44] while the three-particle picture interpreted according to Eq. (35) gives unity.[45] The departure from unity in the microscopic picture arises just from the normalization rule[46] formulated in (38). Thus the appearance of $\mathcal{N}_{\alpha n}^{-1/2}\mathcal{N}'^{1/2}$ in the amplitude causes a 20% enhancement, on an average, in the proton spectroscopic factor.

Knowing the tendency of the behavior of the microscopically well-founded spectroscopic amplitudes with respect to the conventional amplitudes, we are in a position to put their relationship into a broader perspective. We should state, first of all, that the modifications implied by the microscopic considerations change the predictions of the $\alpha+p+n$ model so as to approach those of the microscopic models. As regards the $\alpha+d$ fragmentation, the two models seem fully reconciled. Not only do the $\alpha+d$ spectroscopic factors (Table I) come close to the microscopic estimate (0.93, Ref. 25), but also the corresponding amplitudes. The remaining differences could be presumably accounted for by differences in details. In particular, the models of Voronchev *et al.*[8] and of Kukulin *et al.*[14] use local interactions with Pauli projection, while that of Lehman, Rai, and Ghovanlou[5] employs separable interactions with Pauli repulsion. Using the Faddeev approach, the model of Lehman, Rai, and Ghovanlou describes the three-body dynamics more perfectly than the others, but it is the only one that neglects the Coulomb potential. Where there still is some disagreement is in the asymptotic normalization constant.[25] From Figs. 1–3 it is clear that the inclusion of $\mathcal{N}^{1/2}$ does not change the value of this constant.

With the $\alpha+d$ spectroscopic amplitude increased, the agreement with experiment has also been improved in general. The only experiment requiring extra discussion is that of Ent *et al.*,[22] which produced $s_{\alpha d}=0.73\pm0.09$. Comparing the curves of Fig. 4 with the experimental "momentum distribution" of Ent *et al.*,[22] we would find that the inclusion of $\mathcal{N}^{1/2}$ improves the agreement for momentum values below the diffraction dip, while it may worsen beyond. Since, however, the latter region is infected by wave-distortion effects, whose description involves an amount of uncertainty, we do not assign too much significance to this region.

In the ^{5}He$+p$ spectroscopic factor, the 20% increase would also help to bring the predictions of the three-particle model[44] closer to those of the microscopic approach.[47] In the ^{6}Li$(e,e'p)^5$He reaction, the final state is the ^{5}He continuum, and the three-particle model may only be appropriate for ^{5}He energies below the $t+d$ threshold. In the region of the peak belonging to the $\frac{3}{2}^-$ g.s. of ^{5}He, the correction $\mathcal{N}_{\alpha n}^{-1/2}\mathcal{N}'^{1/2}$ could bring the two models in full harmony, with the experiment slightly overshot. But in the smooth continuum beyond this peak, the microscopic prediction overshoots experiment with a factor of 1.2–2, due apparently to deficiencies in the particular version of the microscopic model,[47] whereas the predictions of the three-particle model agree with experiment reasonably.[44] Nevertheless, $\mathcal{N}_{\alpha n}^{-1/2}\mathcal{N}'^{1/2}$ may still improve the performance of the three-particle model here, while slightly reducing the disagreement with the microscopic model.

From the example of the $\alpha+p+n$ system, one can conjecture that deviations should be observable from macroscopic descriptions of cluster removal from (or addition to) other multicluster systems as well. The most obvious candidates for showing such effects are ^{9}Be described as $\alpha+\alpha+p$ and ^{12}C described as $\alpha+\alpha+\alpha$. Similarly to the case of proton removal from ^{6}Li formulated in (38c) and (38d), these effects must be governed by the interplay of the norm operator of the full system with that of a subsystem. For that matter, such effects should also appear one level lower as a signature of nucleon structure. For instance, the spectroscopic factor of proton removal from triton should also differ from unity, the value obtained with the nucleons treated as structureless.

In conclusion, we have demonstrated that the cluster substructure does show up in nuclear processes that involve the splitting of the nucleus into clusters or the inverse process. The $\alpha+d$ spectroscopic factor of ^{6}Li has been shown to be enhanced by more than 30% owing to the nonelementary nature of the α particle. This enhancement improves the agreement of the predictions of the $\alpha+p+n$ models with experiment and restores the accord with the results of the microscopic models. The effect is caused by the Pauli principle. This particular Pauli effect is bound to appear as an extra deviation from the macroscopic models, however perfectly the Pauli principle is simulated in the formulation of the macroscopic dynamics.

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