Two-nucleon W matrices

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The free two-nucleon W matrices are studied for a number of two-nucleon channels, including coupled channels, and over a range of laboratory energies to 250 MeV. For realistic interactions these W matrices are shown to be simple, smooth, and real functions of (half-on-shell) momenta. The free two-nucleon t matrices are expanded in separable form by the W matrices and the (fully off-shell) remainder matrices left thereby should have a negligible effect in any (low-energy) few-body calculation.

I. INTRODUCTION

Microscopic theories of the structure of and reactions from nuclei are predicated upon two-nucleon interactions occuring within the nuclear medium. For bound-state studies the relevant quantities are the two nucleon G–matrices while for scattering conditions the (complex) two-nucleon t matrices are required. Ignoring nuclear medium modifications those matrices are solutions of Lippmann-Schwinger (LS hereafter) equations. Medium modifications due to Pauli blocking and an average background field in which those nucleons propagate are very important, and there the G and t matrices are defined by variations from the LS equations; for example, the Bethe-Goldstone equations. Such medium corrections are not expected to be severe for few-body systems whence the LS solutions are appropriate as input to the Faddeev equations for three- and four-body problems.[1] Therein, however, one needs those solutions for a range of energies that spans both the negative (bound) and the positive (scattering) regimes. This causes the technical difficulty of having a homogeneous equation whenever the relevant energy is less than zero and inhomogeneous ones with poles in their energy denominators whenever the relevant energy is positive.[2,3] Of course, the conventional approach makes this of no consequence since by using principal-value integration to obtain the (purely real) reaction matrices (R matrices hereafter) one needs only to use unitarity and the Heitler equation to specify the (complex) t matrices. But this technical difficulty can be removed by a recently developed formalism[4] in which the LS equation is modified so that both bound and continuum cases involve solutions of nonsingular, real but inhomogeneous integral equations, the solutions of which are the W matrices.

Of possibly greater significance however, is the fact that the t matrices required in few-body calculations are defined as a separable product of (half off shell) W matrices[4,5] plus remainder (X) matrices.[6] Those X matrices are exactly zero half off shell. Thus if conditions permit one to ignore the X matrices, the separable representation of t matrices that result by approximation is very convenient to use in few-body calculations.

In this study we have extended previous work to include the coupled two nucleon channels and consider the W and X matrices for a select set of energies and for the Reid soft core potential,[7] the more realistic parameterized Paris interaction,[8] and for the (S-wave) model interactions of Malfliet and Tjon.[9] A brief review of the defining integral equations and of the interrelationships between the various two nucleon matrix sets is given in the next section and our calculated results are presented and discussed in the ensuing sections.

II. DERIVATION OF W AND X MATRICES

In momentum space and after angular momentum projection, the LS equations reduce to a set of (coupled) channel equations in the magnitudes of momenta. With L^* designating a complete NN channel set of quantum numbers (i.e. JL^*ST) whenever necessary, the 1D LS equations are

$$t_{LL'}(p',p;E) = V_{LL'}(p',p) - \left(\frac{2}{\pi}\right)\sum_{l}\int_0^\infty \frac{q^2dq}{(q^2-E)}V_{Ll}(p',q)t_{lL'}(q,p;E), \tag{2.1}$$

for any pair of momenta p and p'. The energy E may be positive or negative with the former giving a pole via the denominator at the on-shell momentum, k, where

$$|E| = k^2. \tag{2.2}$$

A conventional approach to the scattering problem[2,3] is then to solve the reaction matrix equations

$$R_{LL'}(p',p;E) = V_{LL'}(p',p) - \left(\frac{2}{\pi}\right)\sum_l P\int_0^\infty \frac{q^2 dq}{(q^2-E)} V_{Ll}(p',q) R_{lL'}(q,p;E), \tag{2.3}$$

wherein P denotes the principal value integration. These reaction matrices (for real $V_{LL'}$) are purely real and relate to the positive-energy, t-matrices by unitarity and the Heitler equation, viz.,

$$t_{LL'}(p',p;E) = R_{LL'}(p',p;E) - ik\sum_l R_{Ll}(p',k;E) t_{lL'}(k,p;E). \tag{2.4}$$

In the W-matrix formalism,[4] one considers a momentum scaled interaction term, viz.,

$$U_{LL'}(p',p) = p^{-L'} V_{LL'}(p',p) \tag{2.5}$$

to define W-matrix equations

$$W_{LL'}(p',p;E) = U_{LL'}(p',p) - \left(\frac{2}{\pi}\right)\sum_l \int_0^\infty \frac{q^{l+2} dq}{(q^2-E)} \left\{U_{Ll}(p',q) - U_{Ll}(p',k)\right\} W_{lL'}(q,p;E). \tag{2.6}$$

Therein there is no divergence in the integral as the kernel goes to zero as q approaches k. The W matrices are real and solutions can be found by the same methods of integration irrespective of whether the energy is positive or negative. In terms of these W matrices, it can be shown that

$$t_{LL'}(p',p;E) = p^{L'} W_{LL'}(p',p;E) - \left(\frac{2}{\pi}\right)\sum_{ll'} W_{Ll}(p',k;E) k^{l-l'} F^{-1}_{ll'}(E) p^{L'} \int_0^\infty \frac{q^{l'+2} dq}{(q^2-E)} W_{l'L'}(q,p;E) \tag{2.7}$$

in which $F^{-1}_{ll'}(E)$ is the inverse of the Jost type function which satisfies

$$F_{ll'}(E) = \delta_{ll'} + \left(\frac{2}{\pi}\right) k^{l'-l} \int_0^\infty \frac{q^{l+2} dq}{(q^2-E)} W_{ll'}(q,k;E). \tag{2.8}$$

The t-matrix equation [Eq. (2.7)] can be recast into the convenient form

$$t_{LL'}(p',p;E) = X_{LL'}(p',p;E) + \sum_{ll'l''} W_{Ll}(p',k;E)\left\{W^{-1}_{ll'}(k,k;E)\Delta_{l''l'}(E)\right\} W_{L'l''}(p,k;E), \tag{2.9}$$

in which

$$\Delta_{ll'}(E) = k^l F^{-1}_{ll'}(E) \tag{2.10}$$

and the remainder matrix is given by

$$X_{LL'}(p',p;E) = p^{L'}\left(W_{LL'}(p',p;E) - \sum_{ll'} W_{Ll}(p',k;E) W^{-1}_{ll'}(k,k;E) W_{l'L'}(k,p;E)\right). \tag{2.11}$$

Details are given in the Appendix. This remainder term vanishes identically half on the energy shell whence the t-matrix equation [Eq. (2.9)] is then simply

$$t_{LL'}(p,k;E) = \sum_l W_{Ll}(p,k;E)\Delta_{lL'}(E). \tag{2.12}$$

Thus since W- and X-matrix elements are both real, the complex nature of the t matrices is fixed by the Jost solutions and with half-on-shell conditions. Finally, we note that both the t matrices and the X matrices defined by Eqs. (2.9) and (2.11) involve the reciprocals of the on-shell W matrices. The singularities that occur at energies where the W matrices vanish nevertheless cancel[6] as they must since the t matrices are always finite.

All the above derivation is equally valid for both positive and negative energies, with the latter having the additional requirement that when determining the bound-state energy ($E_B < 0$), we must find the zero determinant of

$$\det|F_{ll'}(E_B)| = 0 \tag{2.13}$$

with $l, l' \equiv J \pm 1$.

One of the purposes of this study was to see whether or not the W-matrix formalism justifies a form of separable approximation to realistic t matrices. This is justified if the remainder matrices are small in comparison to the actual t matrices themselves at least in an appreciable region around the on-shell value momentum. We therefore consider the purely real "correction factor"

$$C_{LL'}(p',p;E) = \frac{|X_{LL'}(p',p;E)|}{|t_{LL'}(p',p;E)|}. \quad (2.14)$$

Note that we are concerned with a ratio to the absolute magnitude of the t matrices and with that value at the specific fully off-shell point in the momentum plane. Contour plots of these correction factors in contours of 5% of the moduli of the t matrices will be given later.

III. RESULTS AND DISCUSSION

From the structure of the integrand in Eq. (2.6) it is clear that, for large values of the integrating variable and as the second term has a scale factor of $(q/k)^L$, there may be slow convergence in numerical evaluations of the W matrices. We have used the matrix inversion procedure as specified by Haftel and Tabakin[2] to evaluate both the W and R matrices and our investigations do reveal a much slower convergence in evaluating the former compared to evaluations of the latter. Nevertheless, with a manageable number of quadrature points, stable values of the W matrices were achieved for all starting interactions. The number of quadrature points required was dependent upon the angular momentum (L) channel also, so care must be exercised in choice of quadrature points even with a given starting interaction as one makes evaluations for increasing channel quantum numbers.

The half on shell W, R, and t matrices obtained for the $^1S_0$, $^3P_1$, and $^3S_1$ channels at a laboratory energy of 50 MeV and for the Reid and Paris interactions are given for a set of off shell momenta in Table I. Those t matrix values were calculated using both the Heitler equation and W matrix relations to ensure that the W matrices, in particular, were correct. Noting that 0.7764 fm^{-1} is the on shell momentum value, this table shows clearly that the W matrices can be very large. When that is so, the Jost-like functions must also be very large to produce the known (small) values of the t matrix. Therein lies a second numerical problem when use is to be made of W matrices; namely that accurate calculations of Jost-like functions are necessary. The Reid interaction in particular gives very large values for W matrices and this is very evident in the $^3S_1$ (coupled) channel. The Reid and Paris W matrix elements are very different but their corresponding R and t matrices are quite similar as they must be since both interactions give reasonable agreement with the measured phase shifts in these channels and at 50 MeV (laboratory) energy.

The on-shell values of various W matrices and in a number of uncoupled two-nucleon channels for a range of energies are given in Table II. The results presented were calculated starting with the MT-1 and MT-3

TABLE I. The half on shell W, R, and t matrix values for the Reid and Paris interactions at 50 MeV (laboratory) energy ($k = 0.7764$ fm^{-1}). The number in parentheses is the power of ten that scales the given value.

q		Reid			Paris	
$^1S_0$	W	R	t	W	R	t
0.0098	−6.047(3)	−1.307	−0.801 −0.636i	−5.871	−1.276	−0.794 −0.619i
0.7764	−4.738(3)	−1.024	−0.627 −0.499i	−4.615	−1.003	−0.624 −0.486i
1.4488	−1.821(3)	−0.394	−0.241 −0.192i	−1.796	−0.390	−0.243 −0.189i
2.8801	2.744(3)	0.593	0.363 +0.289i	2.387	0.519	0.323 +0.251i
$^3P_1$						
0.0098	0.045	0.003	0.003 −0.000i	0.135	0.004	0.004 −0.001i
0.7764	2.482	0.184	0.180 −0.026i	6.878	0.196	0.191 −0.029i
1.4488	2.137	0.158	0.155 −0.022i	5.137	0.146	0.143 −0.022i
2.8801	0.878	0.065	0.064 −0.009i	3.058	0.087	0.085 −0.013i
$^3S_1$						
0.0098	−3.805(7)	−3.198	−0.680 −1.331i	2.037(2)	−3.126	−0.655 −1.291i
0.7764	−1.159(7)	−2.435	−0.530 −1.005i	5.708(1)	−2.466	−0.527 −1.011i
1.4488	−7.571(5)	−1.013	−0.223 −0.416i	−0.385	−1.117	−0.242 −0.456i
2.8801	3.654(4)	1.010	0.223 +0.414i	0.933	0.835	0.181 +0.341i

TABLE II. The on shell values of the *W* matrices, for various uncoupled two body channels and the Reid, Paris, and MT-1 interactions. The number in parentheses is the power of ten that scales the given result.

E(lab)	$^1S_0$	$^1P_1$	$^1D_2$	$^3P_0$	$^3P_1$	$^3D_2$
1.0 MeV						
MT-1	−21.586					
Paris	−8.370	5.561	−0.163	−2.877	4.737	−0.397
Reid	−8.599(3)	2.347(1)	−2.315	3.683(6)	1.526	−0.056
50 MeV						
MT-1	−9.936					
Paris	−4.615	6.232	−0.557	−3.656	6.878	−1.562
Reid	−4.738(3)	2.318(1)	−7.850	5.020(6)	2.482	−0.221
100 MeV						
MT-1	−4.941					
Paris	−2.729	4.445	−0.436	−1.826	5.482	−1.137
Reid	−2.766(3)	3.370(1)	−5.986	2.609(6)	2.095	−0.148
200 MeV						
MT-1	−0.664					
Paris	−0.485	3.060	−0.314	−0.060	4.436	−0.730
Reid	−5.620(2)	3.815(1)	−4.207	−1.940(4)	1.599	−0.806
400 MeV						
MT-1	1.880					
Paris	1.686	2.116	−0.174	1.132	3.717	−0.387
Reid	1.241(3)	3.018(1)	−2.342	−1.981(6)	1.042	−0.036
1000 MeV						
MT-1	2.488					
Paris	3.088	1.574	0.047	1.444	2.706	−0.025
Reid	−2.006(3)	1.275(1)	−0.270	−2.234(6)	0.447	−0.009

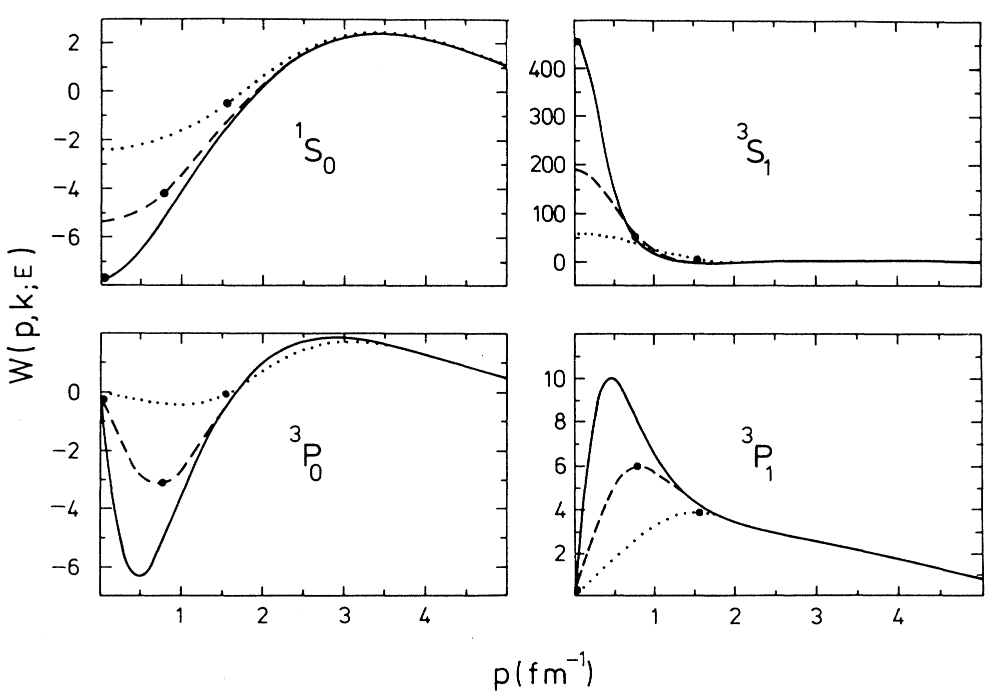

FIG. 1. The half on shell *W* matrices as functions of the off shell momentum and with (laboratory) energies of 0.5, 50, and 200 MeV given by the continuous, dashed, and dotted curves, respectively. These results were all obtained using the Paris interaction and have units of fm^{L+1}. The large dots identify the on shell momentum values.

interactions[9] for the $^1S_0$ and $^3S_1$ (uncoupled) channels, respectively, and with the Reid[7] and Paris[8] interactions as well. Again, in select channels, the W matrices determined using Reid's soft core interaction are very large. The energy variations, and with the exception of the $^3P_0$ channel, the actual signs of the W matrices, are very similar. Coupled-channel calculations have also been made and the on shell results for just the $^3S_1 + ^3D_1$ coupled equations case are displayed in Table III. The results are asymmetric in the channel coupling. Again the Reid interaction values are abnormally large, being many orders of magnitude greater than those found using the Paris interaction. The results obtained from the $^3P_2 + ^3F_2$ coupled channel show similar characteristics.

Half on shell W matrices obtained using the Paris interactions and for laboratory energies of 0.5, 50, and 200 MeV are shown in Fig. 1 by the continuous, dashed, and dotted lines, respectively. The large dots indicate the on-shell momentum values. A selection is shown therein for a range of off shell momenta to 5 fm^{-1}. The channel quantum numbers are as indicated in the figure. It is evident that the overall magnitudes of these W matrices decrease in the low momentum range with increasing laboratory energy but show little change with energy at the higher off shell momentum values. Indeed above 2–3 fm^{-1} the W matrices, to a good approximation, can be taken to be energy independent. The magnitudes in these channels are quite large, especially in the $^3S_1$ (deuteron) channel, stressing again the significant role of the Jost-like solution in the definition of the t matrices per Eq. (2.7). Interestingly, these half off shell variations are of a structural form that leads to a possibility of their representation by simple function combinations, e.g., two or more Gaussians, with which the use of phenomenological, separable t matrices in few-body calculations[1] may be justified. But the remainder matrices must be small, at least in the regions of off shell momenta that are important in any application. That region we shall assume to be a circle of radius 0.5 fm^{-1} in the off shell momentum plane and centered about the on shell point at each and every energy.

The correction factors, as defined by Eq. (2.14) and for the four two-nucleon channels whose half on shell W matrices were presented before, are given in Figs. 2 through 5. Those correction factors are displayed as contour plots in the fully off shell momentum plane and for six laboratory energies between 1.0 and 250 MeV. In each plot the energy is identified as is the on shell momentum value (by a large dot). Contour lines of 5, 10, 15, and 25% for the correction factors are displayed and recall that the

TABLE III. The on shell values of the W matrices, for the $^3S_1 + ^3D_1$ coupled channels and the Reid and Paris interactions. The MT-3 interaction represents the uncoupled $^3S_1$ channel. The number in parentheses is the power of ten that scales the given result.

E(lab)	$^3S_1$	$^3D_1$	$^3S_1$-$^3D_1$	$^3D_1$-$^3S_1$
1.0 MeV				
MT-3	−1.566(1)			
Paris	4.577(2)	0.303	−1.768(1)	−8.077
Reid	−3.924(4)	−0.000	−0.003	2.460(2)
50 MeV				
MT-3	−8.814			
Paris	5.708(2)	1.416	−2.409	−3.820(1)
Reid	−1.159(7)	−3.473	5.771	6.954(6)
100 MeV				
MT-3	−5.603			
Paris	1.960(1)	0.970	−0.914	−2.654(1)
Reid	−7.966(6)	−3.982	3.967	7.973(6)
200 MeV				
MT-3	−2.538			
Paris	4.708	0.520	−0.251	−1.478(1)
Reid	−8.371(6)	−6.669	4.175	1.336(7)
400 MeV				
MT-3	−0.294			
Paris	1.675	0.213	−0.292	−6.822
Reid	−8.848(6)	−1.465(1)	4.418	2.935(7)
1000 MeV				
MT-3	0.962			
Paris	2.564	0.064	0.035	−2.797
Reid	1.077(7)	−3.075(1)	−5.369	6.125(7)

remainder matrices, and therefore the correction factors are exactly zero half on shell.

The correction factors for the $^1S_0$ channel are shown in Fig. 2. For laboratory energies of 100 MeV and less there exist minimal zones of about 0.5 fm^{-1} in radius about the on-shell momentum point, whence a separable (W matrix) approximation to the t matrix in this channel is reasonable in that energy regime. At energies of 200–250 MeV this is clearly not the case but in that energy region the $^1S_0$ t matrix itself is very small (the free NN phase shift changes sign therein). A similar anomalous effect is observed with Kowalski–Noyes f ratios whenever they are used to assess the significance of off-shell properties of t matrices.[3] Therefore the $^1S_0$ channel contribution to calculations involving that 200-250 MeV energy region should be small whence use of a convenient separable

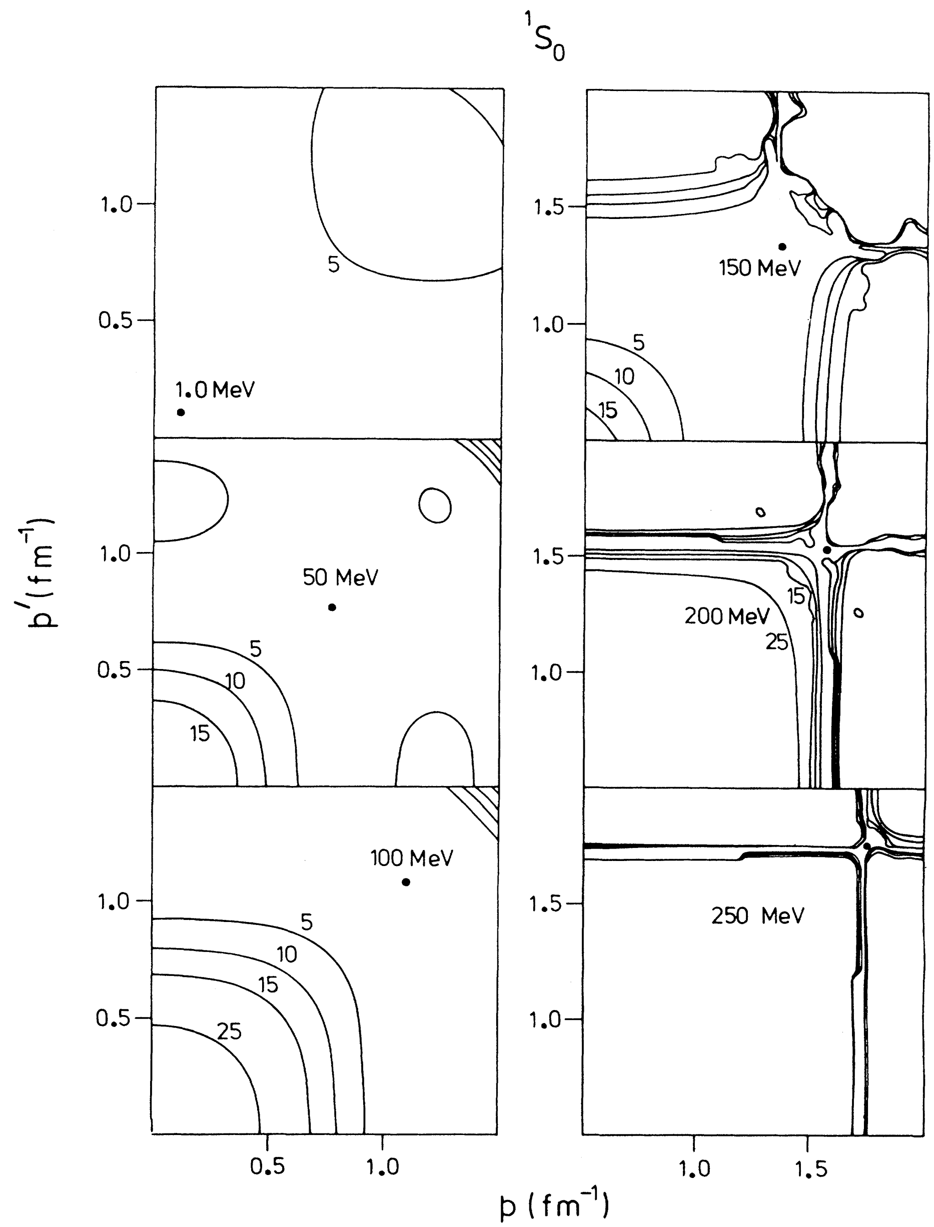

FIG. 2. Contour plots of the correction factor to the separable approximation for the (Paris interaction) $^1S_0$ t matrices at a variety of laboratory energies. The contours shown are 5%, 10%, 15%, and 25%.

approximation suitably scaled to give the strength of the exact t matrices, though formally problematic, is only representing small effects.

The correction factor for the $^3P_0$ channel is shown in Fig. 3. The results are very similar to those obtained with the $^1S_0$ channel; the major difference being a smaller minimal zone around the on shell momenta for energies up to 100 MeV. But in this channel the free NN phase shifts are relatively small and, as with the $^1S_0$ channel, cross the energy axis in the vicinity of 200 MeV. Overall then, in few body calculations, this channel may not be too significant whence a separable approximation (neglect of the remainder matrices) would be reasonable.

In contrast, the $^3P_1$ channel results that are shown in Fig. 4 are all well behaved. At all energies the correction factors are smooth and small within a circle of 0.5 fm^{-1} radius about the on-shell momenta. In this channel the free NN phase shifts are negative and increase in magnitude smoothly. The $^1P_1$ channel gives almost identical results.

Finally, we show in Fig. 5 the correction factor for the $^3S_1$ channel. At all energies to 200 MeV, the correction factor from this (coupled) channel is small within the 0.5 fm^{-1} selected circles. Only at 250 MeV does this vary substantially. But again, at 250 MeV, the $^3S_1$ t matrix (and free NN phase shift) is quite small on the energy

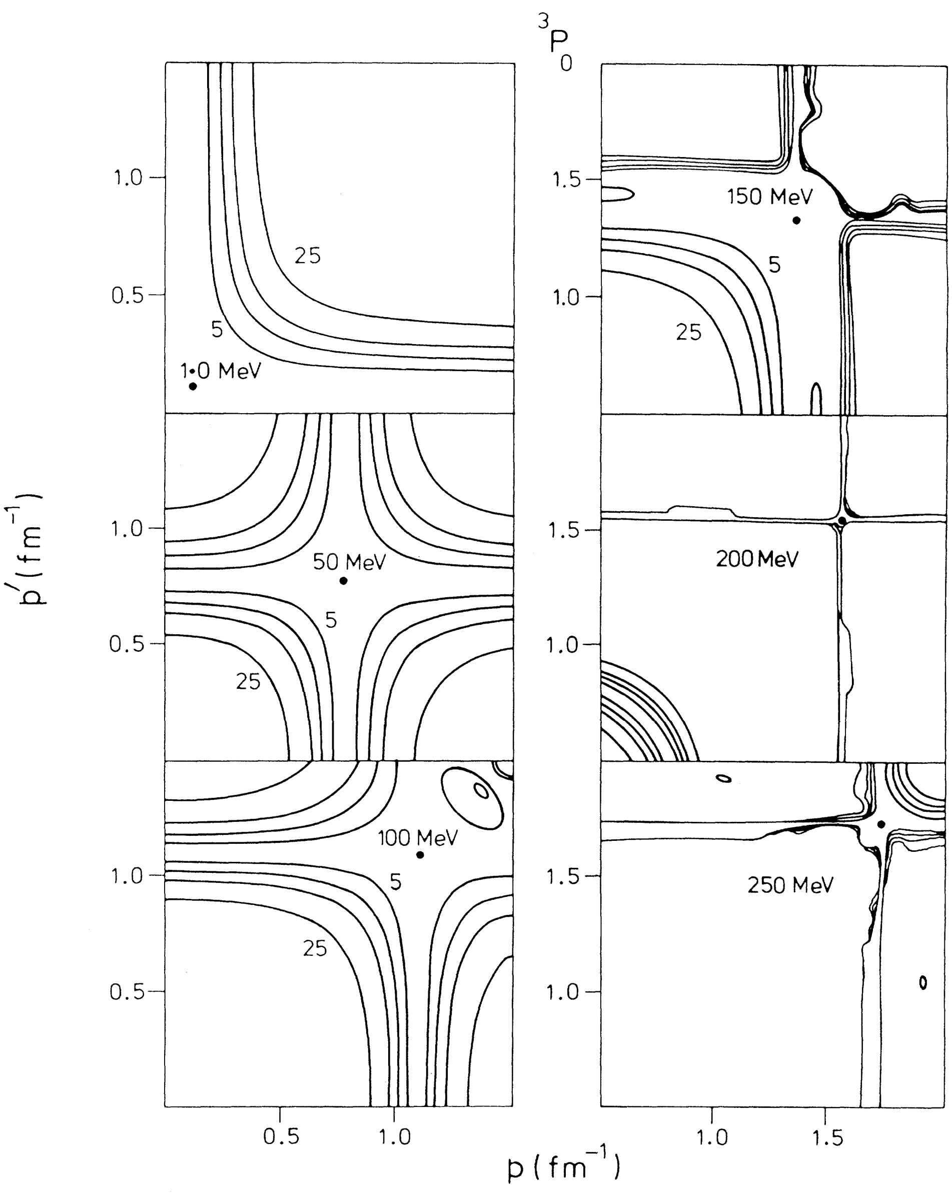

FIG. 3. As for Fig. 2 but for the $^3P_0$ channel.

shell. Contributions to few-body calculations from the $^3S_1$ t matrix for energies around 250 MeV will therefore be of minor significance whence a separable approximation will not be problematic.

We have also considered the S- and P-channel W matrices for negative energies of -2.27, -25, and -50 MeV. For negative energies there is no direct relationship between E and k^2 so we are free to choose values for the momentum variable. Bartnik *et al.* [5] in this study of the triton and n-d scattering found minimum binding energy for $k = 0.655$ fm^{-1}. We have used this value and $k =$ 0.0 fm^{-1} in our calculations. In all cases, the negative energy results were very much like the equivalent positive energy results. The contributions of the remainder terms were negligible around the chosen momentum, k. Thus the W-matrix separable approximation to the NN interaction is also valid at negative energies.

IV. CONCLUSION

The W matrices of realistic free two-nucleon interactions have been calculated and are found to be smooth

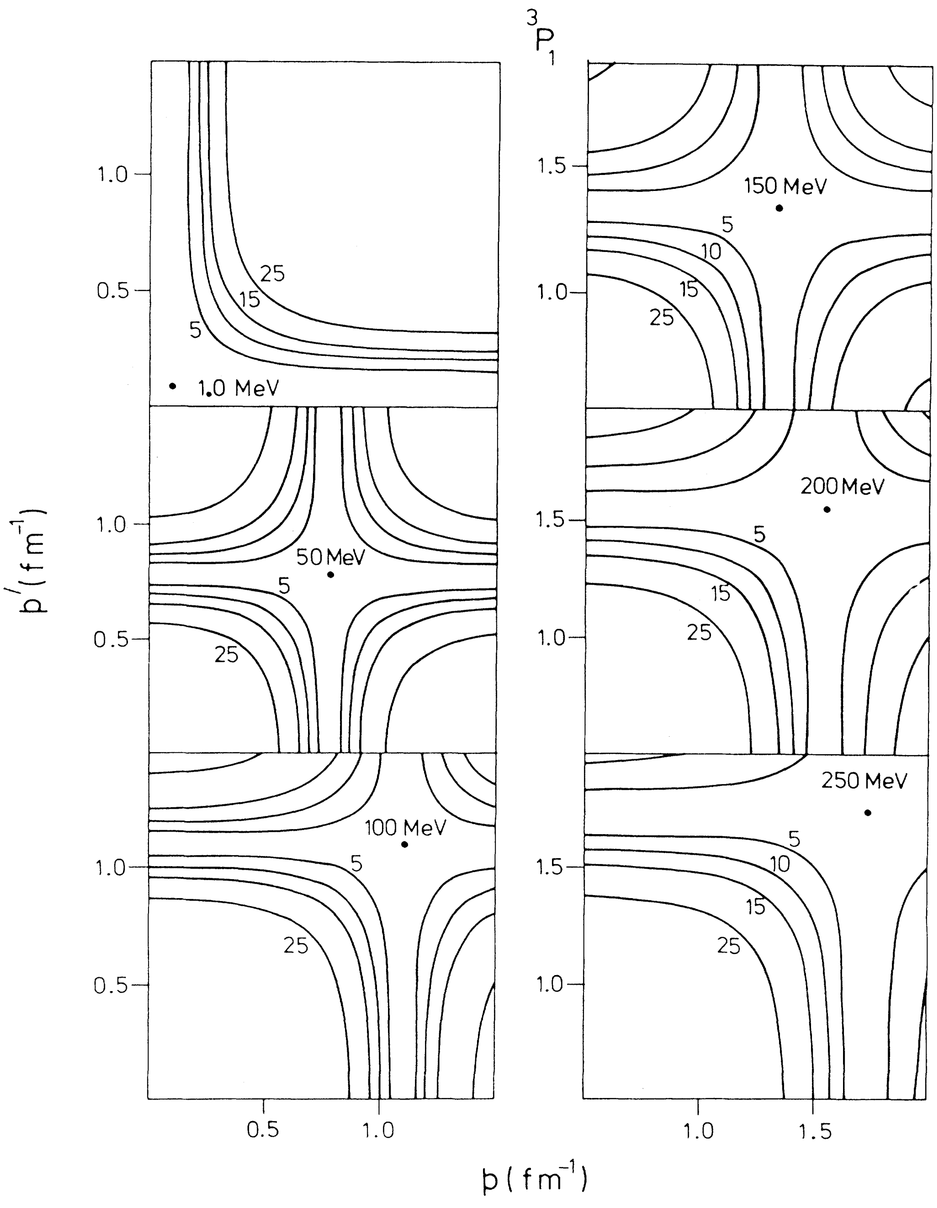

FIG. 4. As for Fig. 2 but for the $^3P_1$ channel.

and well behaved in all important (for low energy problems, i.e., $E < 250$ MeV) two-nucleon channels. The W matrices are real and defined half on the energy shell and in terms of which the free two-nucleon t matrices fully off of the energy shell and in each channel are separable; each to within a remainder matrix.

If the zone of importance in momenta for any use to be made of these (low-energy) t matrices is a circle of radius 0.5 fm^{-1} about the on-shell momentum value, then we have found that the remainder matrix for the Paris interaction at least, can be ignored. There are circumstances where the correction factors that reflect the relative size of the remainder matrices are not small in the selected momentum zone but there the free NN t matrices themselves were very small on shell (the free NN phase shifts changed sign in the energy region). Thus we believe that a separable approximation to the t matrices, made channel by channel and therefore retaining the inherent nonlocality of the fully off shell t matrices, is vindicated. Further the W matrices are in fact the separable interactions and one should construct convenient combinations to match them rather than just fitting the

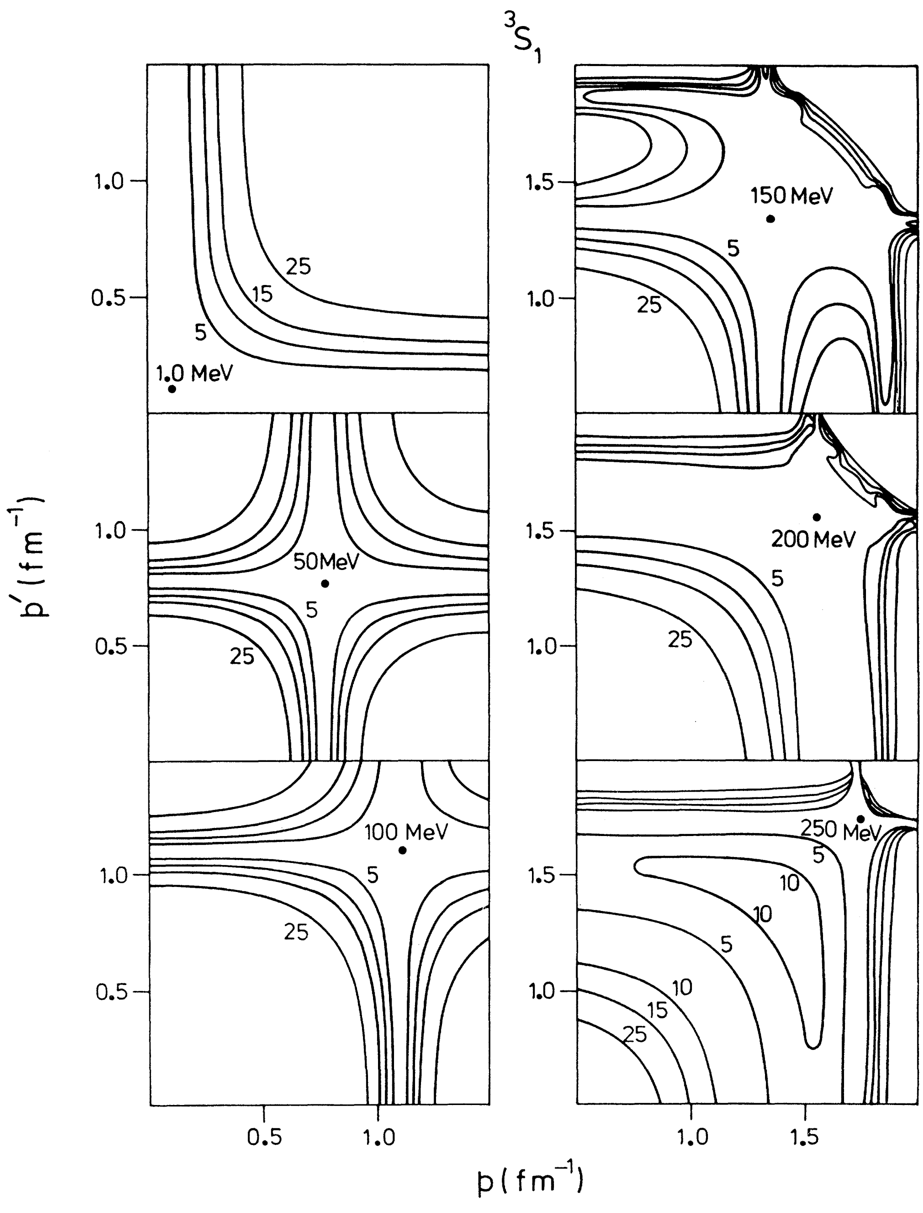

FIG. 5. As for Fig. 2 but for the $^3S_1$ channel.

two-nucleon phase shifts. This leads one to believe that the separation of the t matrices via W matrices is useful for three-body calculations for which energies above 100 MeV are often unimportant.

APPENDIX

With the state-dependent modified interaction, $U_{LL'}$, the Lippmann-Schwinger equation, Eq. (2.1), can be reexpressed as

$$t_{LL'}(p',p;E) = p^{L'}U_{LL'}(p',p) - \left(\frac{2}{\pi}\right)\sum_l \int_0^\infty \frac{q^{l+2}dq}{(q^2-E)}[U_{Ll}(p',q) - U_{Ll}(p',k)]\,t_{lL'}(q,p;E) - \left(\frac{2}{\pi}\right)\sum_l \int_0^\infty \frac{q^{l+2}dq}{(q^2-E)}U_{Ll}(p',k;E)t_{lL'}(q,p;E). \quad \text{(A1)}$$

Then by using the prescription of W given in Eq. (2.6), one obtains

$$t_{LL'}(p',p;E) - p^{L'}W_{LL'}(p',p;E) = -\left(\frac{2}{\pi}\right)\sum_l \int_0^\infty \frac{q^{l+2}dq}{(q^2-E)}U_{Ll}(p',q)\left[t_{lL'}(q,p;E) - p^{L'}W_{lL'}(q,p;E)\right] - \left(\frac{2}{\pi}\right)\sum_l k^l U_{Ll}(p',k;E)\int_0^\infty \frac{q^{l+2}dq}{(q^2-E)}k^{-l}W_{lL'}(q,p;E)p^{L'} \quad \text{(A2)}$$

and we define the integral as

$$I_{ll'}(p;E) \equiv \left(\frac{2}{\pi}\right)\int_0^\infty \frac{q^{l+2}dq}{(q^2-E)}k^{-l}W_{ll'}(q,p;E) \quad \text{(A3)}$$

so that by using the half on shell conditions with Eq. (2.6) we have

$$k^{L'}U_{LL'}(p',k) \equiv k^{L'}W_{LL'}(p',k;E) + k^{L'}J_{LL'}(p',k;E) - \sum_l k^{l-L'}U_{Ll}(p',k)I_{lL'}(k;E)\ , \quad \text{(A4)}$$

wherein

$$J_{LL'}(p',k;E) \equiv \left(\frac{2}{\pi}\right)\sum_l \int_0^\infty \frac{q^{l+2}dq}{(q^2-E)}U_{Ll}(p',q)W_{lL'}(q,k;E). \quad \text{(A5)}$$

Using the definition of the Jost functions, Eq. (2.6), Eq. (A4) can be reordered to the form

$$\sum_l U_{Ll}(p',k)k^l F_{lL'}(E) \equiv k^{L'}W_{LL'}(p',k;E) + k^{L'}J_{LL'}(p',k;E). \quad \text{(A6)}$$

Matrix inversion enables this to be recast as

$$U_{LL'}(p',k)k^{L'} \equiv \sum_l W_{Ll}(p',k;E)k^l F^{-1}_{lL'}(E) + \sum_l k^l F^{-1}_{lL'}(E)J_{Ll}(p',k;E). \quad \text{(A7)}$$

This is convenient for use in recasting Eq. (A2) to a form

$$B_{LL'}(p',p;E) \equiv -\left(\frac{2}{\pi}\right)\sum_l \int_0^\infty \frac{q^{l+2}dq}{(q^2-E)}U_{Ll}(p',q)B_{lL'}(q,p;E), \quad \text{(A8)}$$

wherein

$$B_{LL'}(p',p;E) = t_{LL'}(p',p;E) - W_{LL'}(p',p;E)p^{L'} + \sum_{ll'} W_{Ll}(p',k;E)k^l F^{-1}_{ll'}(E) \times p^{L'}I_{l'L'}(p;E) \equiv 0 \quad \text{(A9)}$$

for Eq. (A8) to be true for any momentum. Then, as the integrals $I_{l'L'}$ are independent of the variable p' matrix inversion again gives

$$-p^{L'}I_{LL'}(p;E) \equiv \sum_{ll'} F_{Ll}(E)k^{-l}W^{-1}_{ll'}(k,k;E) \times \left[t_{L'l'}(p,k;E) - p^{L'}W_{l'L'}(k,p;E)\right] \quad \text{(A10)}$$

since

$$t_{LL'}(p,k;E) = t_{L'L}(k,p;E). \quad \text{(A11)}$$

Eqs. (A10) and (A11) can then be used in Eq. (2.7) to deduce Eq. (2.9) given in the text.

[1] W. Plessas, in *Few Body Methods*, edited by T. K. Lim *et al.* (World Scientific, Singapore, 1986), p. 43; I. Afnan and A. W. Thomas, in *Modern Three Hadron Physics, Topics in Current Physics 2*, edited by A. W. Thomas (Springer, Berlin, 1977), and references therein.
[2] M. I. Haftel and F. Tabakin, Nucl. Phys. **A158**, 1 (1970).
[3] K. Amos, L. Berge, F. A. Brieva, A. Katsogiannis, L. Petris, and L. Rikus, Phys. Rev. C **37**, 934 (1988).

[4]E. A. Bartnik, H. Haberzettl, and W. Sandhas, Phys. Rev. C **34**, 1520 (1986).
[5]E. A. Bartnik, H. Haberzettl, Th. Januschke, U. Kerwath, and W. Sandhas, Phys. Rev. C **36**, 1678 (1987); B. F. Gibson, B. C. Pearce, and G. L. Payne, *ibid.* **40**, 2877 (1989); H. Haberzettl, *ibid.* **40**, 1147 (1989).
[6]L. Canton, G. Pisent, and G. Rawitscher, Phys. Rev. C **41**, 427 (1990).
[7]R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).
[8]M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).
[9]R. A. Malfliet and J. A. Tjon, Nucl. Phys. **A127**, 161 (1969).