Covariant Feynman rules at finite temperature: Application to nuclear matter

R. J. Furnstahl
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

Brian D. Serot
Physics Department and Nuclear Theory Center, Indiana University, Bloomington, Indiana 47405
(Received 16 August 1990)

A unified treatment of relativistic many-body systems at finite temperature and density, incorporating both real- and imaginary-time formalisms, is applied to hadronic field theories of nuclear matter (quantum hadrodynamics). Covariant Feynman rules are given, which permit direct calculations in any convenient reference frame or in manifestly covariant form. The real-time rules are illustrated by the derivation of covariant expressions for the one-loop energy-momentum tensor. Next, the partition function is evaluated at one-loop order, which yields the thermodynamic potential and pressure in covariant form and verifies the virial theorem. Finally, covariant imaginary-time rules are shown to reproduce the real-time one-loop calculations.

I. INTRODUCTION

The description of hadronic matter under extreme conditions of temperature and density is an interesting and important theoretical problem, and one that is increasingly relevant for nuclear physicists. New opportunities for exploring the phase diagram of bulk nuclear matter are emerging with the advent of ultrarelativistic heavy-ion colliders, such as the existing SPS facility at CERN and the proposed Relativistic Heavy Ion Collider. An exciting possibility is that a quark-gluon plasma can be created in the laboratory, so that the transition between hadronic and subhadronic degrees of freedom can be studied. Knowledge of the nuclear matter equation of state at temperatures and densities far from those encountered in ordinary nuclei is also fundamental to the study of astrophysical systems, such as neutron stars.

The interpretation of future data from heavy-ion experiments presents a great challenge, since the description of hadronic matter and nuclear dynamics at high energies and baryon densities is beyond the scope of the conventional Schrödinger-equation approach to nuclear physics. While the physics of the quark-gluon plasma is being studied in the framework of finite-temperature QCD, techniques to study the hadronic phase using QCD directly are very limited at present and theoretical progress has been slow. In contrast, a description based on hadronic degrees of freedom is attractive for several reasons. These variables are the most efficient at lower densities and temperatures and for describing particle emission and absorption, as hadrons and not quarks are observed experimentally. In addition, hadronic calculations can be calibrated by comparing to empirical nuclear properties; we can then extrapolate to extreme conditions. Finally, an accurate hadronic description is required to isolate and identify true signatures of the quark-gluon phase transition and other manifestations of quark-gluon degrees of freedom.

A consistent microscopic treatment of strongly interacting, relativistic, quantum-mechanical systems is needed, no matter what degrees of freedom are used. We propose applying the methods of quantum field theory at finite temperature and density to a hadronic description of nuclear matter. Although we concentrate on hadronic degrees of freedom, most of the issues and formalism discussed here have broader application (e.g., to QCD and QED).

In a series of papers, we develop a systematic framework for studying hot, dense nuclear matter described by a relativistic quantum field theory of mesons and baryons, which is known as quantum hadrodynamics (QHD).[1] This framework can be used to compute both static thermodynamic properties (like energy, pressure, and entropy) and dynamical characteristics (such as viscosity, transport coefficients, and collective modes and their damping). Ultimately, we would like to treat nonequilibrium systems, in order to describe the development of two isolated nuclei into a single system, possibly in equilibrium. Our approach is based on finite-temperature Feynman rules in both real and imaginary time.

At zero temperature, one can use Feynman rules for either Minkowski or Euclidean Green's functions, and there is little practical difference between the formalisms. At finite temperature (and density), however, the distinctions are significant. Thus there are two basic approaches to finite-temperature field theory: the imaginary-time (Euclidean) formalism and the real-time formalism (also known as the time-path formalism or thermofield dynamics). Practitioners of finite-temperature field theory (both relativistic and nonrelativistic) are often familiar with only one approach and apply it exclusively. In fact, there are advantages and disadvantages to each, and the choice between them should be based on the problem under consideration. In some cases, applying both real- and imaginary-time methods is appropriate, to check calculations or to resolve ambiguities that can arise at finite temperature and density.[2] We therefore present a unified treatment.

 ©1991 The American Physical Society

We specialize our discussion to the Walecka model[3] (also called QHD-I). This is a renormalizable relativistic quantum field theory that describes the nuclear system using a local Lagrangian density containing baryons and neutral scalar and vector mesons. It incorporates some basic elements of hadronic theories of nuclei and has been widely applied at the mean-field level with much phenomenological success.[4] By working with a renormalizable model, we have a self-contained framework that can be used to study general features of the relativistic nuclear many-body problem at finite temperature and density. These features will be relevant even if the dynamical assumption of renormalizability at the hadronic level turns out to be too restrictive.

We emphasize general aspects of relativistic many-body systems at finite temperature and density, such as covariance and thermodynamic consistency.[5] Consistency implies that the "thermodynamic" pressure calculated from the thermodynamic potential agrees with the "hydrostatic" pressure computed from the trace of the stress tensor. This is sometimes called the virial theorem.[2] We are particularly interested in the construction of nonperturbative approximations that preserve thermodynamic consistency and covariance (and other physical properties). The Walecka model mean-field theory is one such approximation (as we demonstrate), but consistency is difficult to maintain in other approximations to the relativistic many-body problem.[6]

A covariant formulation allows calculations to be carried out directly in any convenient reference frame. This may be useful for describing the collision of two heavy ions, for which there is no frame in which all the matter is at rest. A covariant description also introduces features of many-body thermodynamics not usually considered, such as the role of the fluid velocity and the momentum density as thermodynamic parameters.[5] Furthermore, when calculating Green's functions, it provides a clear identification of the correct Lorentz structure and associated invariant functions, which can be obscured at finite temperature and density by working in a fixed frame. (The importance of a covariant formalism in finite-temperature QED is stressed in Ref. 7.)

It is often stated in the literature that one cannot work covariantly at finite temperature or density because there is a preferred reference frame, which is the rest frame of the heat bath or nuclear matter. This is a misconception. In a previous paper,[5] we used a canonical formulation to analyze hot, flowing nuclear matter in the mean-field approximation to the Walecka model. The covariance of the description, although not manifest, was demonstrated explicitly. Those results will serve as benchmarks for calculations in the present work.

In this paper, we present covariant real- and imaginary-time Feynman rules for QHD-I, which provide a natural extension of the rules in Ref. 1 to systems at finite temperature and to an arbitrary reference frame. (Imaginary-time rules have been discussed previously in Refs. 8 and 9 and real-time rules in Refs. 10 and 11, but only for the rest frame of nuclear matter.) We illustrate the rules by generating manifestly covariant expressions that reproduce the mean-field nuclear matter calculations of Ref. 5. We also construct the partition function in covariant form and extend our previous results to include one-loop vacuum corrections.

In a forthcoming paper, we complete the formalism by deriving the real- and imaginary-time Feynman rules using path integrals, which provide a natural framework for a unified treatment. Each set of rules is developed in manifestly covariant form for nuclear matter. We treat the derivation of the rules separately from their application, because various technical considerations must be discussed that may not be of interest to all readers. We note, however, that formal details in thermal field theory, such as the regularization of delta functions, are often critical in practice, and it is dangerous to be cavalier about these subtleties.

The outline of this paper is as follows: In Sec. II, we compare the real- and imaginary-time approaches to thermal field theory and we argue for a unified treatment. In Sec. III, we introduce the QHD model and list the covariant real-time Feynman rules. These rules are applied to nuclear matter in Sec. IV by self-consistently summing the real-time "tadpole" diagrams to generate the one-loop energy-momentum tensor in manifestly covariant form. The partition function is presented in Sec. V and evaluated at one-loop order, which yields the thermodynamic grand potential and pressure in covariant form. From these calculations, we can reproduce results generated with canonical methods[5] and verify the virial theorem for the mean-field approximation. Finally, covariant imaginary-time rules are listed in Sec. VI and used to reproduce the real-time, one-loop results. Some final comments and a summary are given in Sec. VII.

II. FINITE-TEMPERATURE FORMALISMS

Quantum field theories at finite temperature and density were first studied using the imaginary-time or Euclidean formalism developed by Matsubara and others.[12–14] (See the review in Ref. 2 for an excellent bibliography.) In this approach, the time variable is replaced by an imaginary parameter ($t=-i\tau$), which is restricted to a finite interval determined by the inverse temperature β ($0\le\tau\le\beta$). Thermal boundary conditions dictate that boson (fermion) propagators are (anti)periodic with period β in this interval.

The Euclidean approach is natural and appealing because of the identification of β with an imaginary time. The Feynman diagrams are combinatorially and topologically identical to those at zero temperature, and the rules are similar to $T=0$ rules. For nonrelativistic field theories, the Feynman rules can be derived straightforwardly (see, for example, Ref. 15), and the extension to relativistic theories has been described by several authors.[16,17,9] By applying these rules, static thermodynamic quantities can be calculated directly in terms of the imaginary-time propagators, with no analytic continuation. High-temperature expansions are also easily derived.[16]

There are, however, several difficulties with this approach. Dynamical properties of the system (such as collective modes and transport coefficients) are the consequences of genuine real-time evolution. In practice, these require an analytic continuation of the imaginary-time functions to real time.[18–20,15] This continuation can be cumbersome and difficult, particularly when one is dealing with n-point functions that depend on several complex variables. (The primary difficulty is proving that the analytic continuation is unique.[2]) Moreover, since the imaginary-time variable runs over a finite interval, leading to discrete frequencies in a Fourier representation, it is not obvious how to combine temporal and spatial variables to create a manifestly covariant formalism (see Sec. VI). Furthermore, low-temperature expansions are awkward to compute in imaginary time. Finally, to carry out renormalization in relativistic theories, one must isolate the divergent expressions that persist in the zero-temperature and zero-density limits. This requires a conversion of discrete sums to contour integrals, together with a careful selection of contours,[21] and so introduces an extra layer of complexity beyond what is needed in zero-temperature calculations. Although one can prove that the usual (vacuum) renormalization procedures lead to well-defined (finite) results at finite temperature, technical complications arise in practice (see Refs. 21 and 16).

These difficulties can be avoided by using finite-temperature and finite-density techniques that work directly in real time (Minkowski space). Such techniques were pioneered by Schwinger,[22] Keldysh,[23] and Mills,[24] who applied them to nonequilibrium systems. An algebraic operator approach to the problem, known as thermofield dynamics (TFD), was developed recently in a comprehensive series of papers.[25–30] The essential aspect of this work was to discard the idea of describing the system with a single, fixed Hilbert space at all temperatures and densities. Instead, one begins with the algebra of the operators that define observables and constructs the Hilbert space around the desired state, for example, the equilibrium state at a given temperature.[2] This construction is carried out by introducing a fictitious system, identical to the one under consideration, and then working in the direct product space, so that the resulting formalism appears to describe twice as many degrees of freedom. Subsequently, a real-time finite-temperature formalism was developed using Feynman path integrals along contours in the complex time plane[31,2] (which is often called the "time-path" formalism). The resulting Feynman rules have a 2×2 matrix structure, corresponding to the doubling of the degrees of freedom, and are identical to the Feynman rules obtained in thermofield dynamics.[32] Although the TFD derivation is straightforward, the time-path approach contains various subtleties that have led to controversies.

There are several attractive features of the real time approach. First, since calculations are performed with real time variables, the cumbersome analytic continuations of the Euclidean formalism are avoided. The Feynman diagrams are topologically (and combinatorially) identical to those at zero temperature; however, the propagators and vertices now have a 2×2 matrix structure. The extra degrees of freedom, sometimes called thermal "ghosts," never appear on external lines. Ambiguities can arise in a naive application of real-time Feynman rules at finite temperature (that is, by using the analytically continued Euclidean propagators *without* matrix structure[35]), but are removed by correctly including the ghost degrees of freedom.[31,2]

Since the real time variables take values from $-\infty$ to $+\infty$, the corresponding frequencies are continuous. This eliminates the unwieldy computation of sums over discrete frequencies that arise in the Euclidean approach. (Various techniques have been developed, however, for performing such sums.[16,36–40]) Since both time and space variables are real, the construction of manifestly covariant Feynman rules at finite temperature and density is straightforward. Moreover, both high-temperature and low-temperature expansions can be computed systematically in the real-time formalism.[31] The zero-temperature and zero-density limits follow immediately, so renormalization can be carried out in the standard fashion, and the removal of all divergences in renormalizable theories is straightforward. (The proof is given in Ref. 31.) Techniques for studying nonequilibrium systems can also be formulated in the time-path approach.[2,41]

There are, however, some shortcomings of this formalism. For example, real-time Feynman diagrams have no direct connection to the partition function of the system;[2] thus, special techniques must be used to compute the thermodynamic potential. These techniques involve an integral over the strength of the coupling constant[15,2] or over the value of a classical field.[31] While this integration is straightforward in perturbation theory and for simple summations of diagrams, it is not useful for various *self-consistent* approximations often used in the nuclear matter problem.[1,42] This is because the self-consistency procedure is cumbersome to carry out for a series of coupling strengths, and problems may arise when the (nonlinear) self-consistent equations have either no solution or multiple solutions for various values of the scaled couplings.

In addition, the computation of real-time multiloop diagrams requires regularization procedures to define various "generalized functions" (for example, products and derivatives of Dirac δ functions).[43,31,2] Although a consistent set of rules can be obtained without this regularization procedure,[33] the resulting calculations are much more cumbersome in practice.[44,34] In all known cases, the regularized results are unique, but it is useful to have alternative (imaginary-time) methods to verify these computations.

In view of the various advantages and disadvantages enumerated above, we develop a unified treatment of finite-temperature nuclear matter that allows calculations to be performed in either the real- or imaginary-time approach and in any reference frame. The most convenient formalism can be applied to the computation of an observable, and the results verified by calculating in more than one way. In particular, as we have shown in an earlier paper,[45] the partition function can be computed directly from a path integral involving imaginary time and can be compared with results computed from real-time methods. The thermodynamic consistency of various approximations to the relativistic nuclear many-body problem can also be assessed, as we describe below.

III. COVARIANT REAL-TIME FEYNMAN RULES

In this section, we present Feynman rules for the QHD-I Lagrangian in the real-time formalism. We list the Feynman rules (and apply them) in covariant form, but we postpone some details of the covariant formulation to later sections and the formal derivation of the rules to a forthcoming paper.[34] We begin by briefly reviewing some aspects of covariant thermodynamics and statistical mechanics, which are discussed in greater detail in Ref. 5.

A. Covariant thermodynamics

In a covariant treatment of thermodynamics,[46] the primary thermodynamic functions for a system in equilibrium are the energy-momentum tensor $T^{\mu\nu}$, the entropy flux vector S^{μ}, and the baryon current density vector B^{μ}, which involve no specification of a particular reference frame. These quantities are generally functions of six variables: the baryon thermal potential α, the inverse temperature β, the fluid four-velocity u^{μ}, and the volume $\mathcal{V}$. (The volume is taken to infinity at the end to define the "thermodynamic limit" and restore invariance under translations.) The variables α and β are Lorentz scalars defined by

$$\beta \equiv \frac{1}{T'} , \quad \alpha \equiv \frac{\mu'}{T'} , \tag{3.1}$$

where T' and μ' are the temperature and baryon chemical potential in the comoving frame, where the fluid three-velocity $\mathbf{v}$ is zero. (When we refer to a quantity that may be defined by an observer in any frame, the "proper" value taken in the comoving frame will be denoted with a prime). We also define a timelike thermal four-vector

$$\beta^{\mu} \equiv \beta u^{\mu} \equiv \frac{1}{T'} u^{\mu} . \tag{3.2}$$

The secondary thermodynamic functions are defined in the comoving frame and are thus Lorentz scalars. They are the pressure p, the proper energy density $\mathcal{E}'$, the proper entropy density σ', the proper baryon density ρ'_{B}, and the scalar density ρ_{s}. In the thermodynamic limit, these secondary quantities are functions of α and β (or μ' and T') only. The secondary thermodynamic functions can be used to construct the primary functions in any frame:

$$T^{\mu\nu} = (\mathcal{E}' + p) u^{\mu} u^{\nu} - p g^{\mu\nu} , \tag{3.3}$$

$$S^{\mu} = \sigma' u^{\mu} , \tag{3.4}$$

$$B^{\mu} = \rho'_B u^{\mu} . \tag{3.5}$$

In the thermodynamic limit, the primary quantities are functions of α, β, and u^{μ}, or equivalently, α and β^{μ}.

To compute the thermodynamic functions in terms of ensemble averages of quantum-mechanical operators, a *grand partition function* Z and a four-vector thermodynamic potential $\Phi^{\mu}(\alpha,\beta^{\nu})$ are defined through

$$\begin{aligned} Z &\equiv \exp\left[-\int d\Lambda_{\mu} \Phi^{\mu}(\alpha,\beta^{\nu}) \right] \\ &\equiv \mathrm{Tr}\left[\exp\left[-\int d\Lambda_{\mu} (\beta_{\nu} \hat{T}^{\nu\mu} - \alpha \hat{B}^{\mu}) \right] \right] , \end{aligned} \tag{3.6}$$

which are manifestly Lorentz-invariant expressions. Here Λ is a spacelike hypersurface on which the theory (i.e., $\hat{T}^{\mu\nu}, \hat{B}^{\mu}$) is quantized. In the comoving frame, with $d\Lambda_{\mu} = d^3x' \delta_{\mu 0}$, Eq. (3.6) reduces to the familiar result

$$Z = \mathrm{Tr}\{ \exp[-\beta(\hat{H} - \mu' \hat{B})] \} , \tag{3.7}$$

where $\hat{H}$ is the Hamiltonian and $\hat{B}$ is the baryon number operator. Ensemble averages are computed from

$$\begin{aligned} A &\equiv \langle\langle \hat{A} \rangle\rangle \\ &= Z^{-1} \mathrm{Tr}\left[\hat{A} \exp\left[-\int d\Lambda_{\mu} (\beta_{\nu} \hat{T}^{\nu\mu} - \alpha \hat{B}^{\mu}) \right] \right] , \end{aligned} \tag{3.8}$$

where $\hat{A}$ is any operator built from the fields of the theory.

The four-vector potential Φ^{μ} is related to the thermodynamic pressure through

$$\Phi^{\mu}(\alpha,\beta^{\nu}) = -p\beta^{\mu} , \tag{3.9}$$

and the covariant form of Gibbs' relation is

$$\Phi^{\mu} = \beta_{\nu} T^{\nu\mu} - S^{\mu} - \alpha B^{\mu} . \tag{3.10}$$

B. The model

The relativistic quantum theory QHD-I, including scalar meson self-couplings, is defined by the Lagrangian density[1]

$$\begin{aligned} \mathcal{L} = & \bar{\psi}[\gamma_{\mu}(i\partial^{\mu} - g_{\mathrm{v}} V^{\mu}) - (M - g_{\mathrm{s}}\phi)]\psi \\ & + \tfrac{1}{2}(\partial_{\mu}\phi\partial^{\mu}\phi - m_{\mathrm{s}}^2\phi^2) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + \tfrac{1}{2}m_{\mathrm{v}}^2 V_{\mu}V^{\mu} - V(\phi) + \delta\mathcal{L} . \end{aligned} \tag{3.11}$$

Here $F_{\mu\nu} \equiv \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu}$, $\delta\mathcal{L}$ contains renormalization counterterms that will be suppressed until needed, and

$$V(\phi) = \frac{\kappa}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4 . \tag{3.12}$$

In Ref. 5, the (renormalized) scalar self-couplings were chosen to be zero ($\kappa = \lambda = 0$), but for now we will consider the more general case. The field equations from this Lagrangian are

$$(\partial_{\mu}\partial^{\mu} + m_{\mathrm{s}}^2)\phi + \frac{\kappa}{2}\phi^2 + \frac{\lambda}{6}\phi^3 = g_{\mathrm{s}}\bar{\psi}\psi , \tag{3.13}$$

$$(\partial_{\nu}\partial^{\nu} + m_{\mathrm{v}}^2)V^{\mu} = g_{\mathrm{v}}\bar{\psi}\gamma^{\mu}\psi , \tag{3.14}$$

$$[\gamma^{\mu}(i\partial_{\mu} - g_{\mathrm{v}}V_{\mu}) - (M - g_{\mathrm{s}}\phi)]\psi = 0 . \tag{3.15}$$

The final equation implies that the baryon current $B^{\mu} = \bar{\psi}\gamma^{\mu}\psi$ is conserved ($\partial_{\mu}B^{\mu} = 0$), which has been used to arrive at Eq. (3.14). At this stage, all these expressions involve c-number fields.

The energy-momentum tensor follows from the canonical definition as

$$T^{\mu\nu}=\tfrac{1}{2}(-\partial_\lambda\phi\partial^\lambda\phi+m_s^2\phi^2+\tfrac{1}{2}F_{\lambda\sigma}F^{\lambda\sigma}-m_v^2V_\lambda V^\lambda)g^{\mu\nu}+V(\phi)g^{\mu\nu}+i\bar{\psi}\gamma^\mu\partial^\nu\psi+\partial^\mu\phi\partial^\nu\phi+\partial^\nu V_\lambda F^{\lambda\mu}\ , \quad (3.16)$$

and the field equations (3.13)–(3.15) imply that the four-momentum

$$P^\mu\equiv(H,\mathbf{P})\equiv\int d^3x T^{0\mu} \quad (3.17)$$

is a constant of the motion. [To work in a fully covariant manner, the integral in Eq. (3.17) is taken over the spacelike hypersurface Λ, so that $P^\mu\equiv\int d\Lambda_\nu T^{\nu\mu}$. By choosing $d\Lambda_\mu=d^3x\,\delta_{\mu 0}$, one reproduces the usual expression.] States of the system can therefore be labeled by their energy, three-momentum, and baryon number: E, $\mathbf{P}$, and B.

This theory can be quantized in the canonical fashion by imposing equal-time commutation relations (or, more generally, by imposing commutation relations on an arbitrary spacelike hypersurface Λ).[47] For example, for the scalar field, we have

$$[\hat{\Pi}(t,\mathbf{x}),\hat{\phi}(t,\mathbf{y})]=-i\delta^{(3)}(\mathbf{x}-\mathbf{y})\ ,$$
$$[\hat{\phi}(t,\mathbf{x}),\hat{\phi}(t,\mathbf{y})]=[\hat{\Pi}(t,\mathbf{x}),\hat{\Pi}(t,\mathbf{y})]=0\ , \quad (3.18)$$

where $\Pi(t,\mathbf{x})=\partial\phi(t,\mathbf{x})/\partial t$.

The composite operators $\hat{B}$, $\hat{H}$, and $\hat{\mathbf{P}}$ must be defined by subtracting infinite constants to choose a particular operator ordering. Since $\hat{B}$ is conserved due to an internal symmetry, a single vacuum subtraction is sufficient, which is equivalent to normal ordering the operator. In contrast, $\hat{H}$ and $\hat{\mathbf{P}}$ are conserved because of spacetime symmetries, so a vacuum subtraction alone is insufficient,[2] and various counterterm subtractions must be included.[1]

We consider only uniform equilibrium systems, so the energy-momentum-tensor operator resulting from Eq. (3.16) can be simplified. Diagonal matrix elements of the total divergence of an operator are zero between states of good four-momentum, so we can omit such contributions from $T^{\mu\nu}$ and write

$$\hat{T}^{\mu\nu}=\hat{T}^{\mu\nu}_B+\hat{T}^{\mu\nu}_s+\hat{T}^{\mu\nu}_v\ , \quad (3.19)$$

$$\hat{T}^{\mu\nu}_B=i\hat{\bar{\psi}}\gamma^\mu\partial^\nu\hat{\psi}\ , \quad (3.20)$$

$$\hat{T}^{\mu\nu}_s=-\tfrac{1}{2}(\partial_\sigma\hat{\phi}\partial^\sigma\hat{\phi}-m_s^2\hat{\phi}^2)g^{\mu\nu}+V(\hat{\phi})g^{\mu\nu}+\partial^\mu\hat{\phi}\partial^\nu\hat{\phi}\ , \quad (3.21)$$

$$\hat{T}^{\mu\nu}_v=\tfrac{1}{2}(\partial_\sigma\hat{V}_\lambda\partial^\sigma\hat{V}^\lambda-m_v^2\hat{V}_\sigma\hat{V}^\sigma)g^{\mu\nu}-\partial^\mu\hat{V}_\lambda\partial^\nu\hat{V}^\lambda\ . \quad (3.22)$$

Here we have also used $\partial_\mu\hat{V}^\mu=0$, which follows from Eq. (3.14) and the conservation of the baryon current. In situations where the matter is not uniform, a correctly symmetrized version of $\hat{T}^{\mu\nu}$ must be used.[8,48]

C. Finite-temperature propagators

We would like to define three classes of interacting propagators:

$$i\Delta^{(c)}(x_1,\ldots,x_n)\equiv\langle\langle T_c\hat{\phi}(x_1)\cdots\hat{\phi}(x_n)\rangle\rangle\ , \quad (3.23)$$

$$iG^{(c)}(x_1,\ldots,x_n,x_1',\ldots,x_n')\equiv\langle\langle T_c\hat{\psi}(x_1)\cdots\hat{\psi}(x_n)\hat{\bar{\psi}}(x_1')\cdots\hat{\bar{\psi}}(x_n')\rangle\rangle\ , \quad (3.24)$$

$$iD^{(c)}_{\mu\cdots\nu}(x_1,\ldots,x_n)\equiv\langle\langle T_c\hat{V}_\mu(x_1)\cdots\hat{V}_\nu(x_n)\rangle\rangle\ , \quad (3.25)$$

that generalize the conventional finite-temperature propagators defined in the comoving frame to an arbitrary frame. For real time arguments, we can interpret the $(x_i)^\mu=(t_i,\mathbf{x}_i)$ as four-vectors, but we would also like to apply these definitions to times $t_1,\ldots,t_n$ and $t_1',\ldots,t_n'$ that lie on contours in the complex plane. [To simplify the notation, the "prime" convention introduced in Sec. III A will be suspended for spacetime coordinates, unless otherwise noted.] This permits Feynman rules to be derived for both real and imaginary times. (The thermal contour propagators will be denoted with superscripts "c," and T_c is a contour-ordering operator.) At this stage, however, it is unclear how to make the contour propagators (3.23)–(3.25) covariant for complex times, since the quadruplet $(t,\mathbf{x})$ with complex t *does not* define a four-vector. Furthermore, the role of the hypersurface Λ in the covariant statistical operator [see Eq. (3.8)] and in the definition of Heisenberg field operators is also unclear. Rather than resolve all of these questions at once, we will start with the more familiar definitions of a comoving observer and then consider the generalization to an arbitrary observer as we proceed.

In the comoving frame, the *grand canonical Hamiltonian* $\hat{K}$ is defined as

$$\hat{K}=\hat{H}-\mu'\hat{B}\ , \quad (3.26)$$

and the ensemble average of an operator $\hat{A}$ is given by

$$\langle\langle\hat{A}\rangle\rangle\equiv\frac{\mathrm{Tr}\{e^{-\beta\hat{K}}\hat{A}\}}{\mathrm{Tr}\{e^{-\beta\hat{K}}\}}\equiv Z^{-1}\mathrm{Tr}\{e^{-\beta\hat{K}}\hat{A}\}\ , \quad (3.27)$$

where the trace is over any complete set of states. Here $\beta=1/T'$, so that the propagators are functions of μ' and T', or equivalently, α and β defined in Sec. III A. The volume $\mathcal{V}'$ of the system will be considered fixed throughout the calculation, with the thermodynamic limit $\mathcal{V}'\to\infty$ taken at the end.

The grand Heisenberg-picture operators are

$$\hat{\phi}(x)\equiv\hat{\phi}(t,\mathbf{x})=e^{it\hat{K}}\hat{\phi}(\mathbf{x})e^{-it\hat{K}}\ , \quad (3.28)$$

$$\hat{\psi}(x)\equiv\hat{\psi}(t,\mathbf{x})=e^{it\hat{K}}\hat{\psi}(\mathbf{x})e^{-it\hat{K}}\ , \quad (3.29)$$

$$\hat{\psi}^\dagger(x)\equiv\hat{\psi}^\dagger(t,\mathbf{x})=e^{it\hat{K}}\hat{\psi}^\dagger(\mathbf{x})e^{-it\hat{K}}\ , \quad (3.30)$$

and similarly for the vector field. Since t may be complex, $\hat{\psi}$ and $\hat{\psi}^\dagger$ must be defined separately, and $[\hat{\psi}^\dagger]^\dagger\neq\hat{\psi}$ in general. The meson field operators commute with $\hat{B}$, so the baryon chemical potential μ' cancels out in Eq. (3.28) and can be set to zero when discussing the meson propagators.

The propagators of Eqs. (3.23)–(3.25) are defined using *grand* Heisenberg-picture operators and so differ from propagators defined in the usual (canonical) Heisenberg picture, which involves only the Hamiltonian $\hat{H}$. The propagators can be used to evaluate ensemble averages of various operators and thus compute observables. Different choices of time arguments are convenient in different situations. For example, by associating β with an imaginary time and $\hat{K}$ with the time-translation operator, a consistent set of imaginary-time Feynman rules can be derived. In contrast, real-time propagators can be used to compute $\langle\langle \hat{T}^{\mu\nu} \rangle\rangle$ in much the same way as at zero temperature.[1] For real-time calculations, it is convenient to extract a phase factor $\exp[i\mu'(t-t')]$ from the Fourier transform of the fermion two-point function (and analogous phases for n-point functions), so that the momentum-space Green's functions correspond to those defined in the canonical Heisenberg picture. This simplifies the real-time rules and generates Green's functions with well-defined zero-temperature limits that are equivalent to the usual $T=0$ functions.[1] Since the canonical ($\hat{H}$) and grand canonical ($\hat{K}$) Heisenberg-picture fields differ by a unitary transformation (for real times), observables are not affected.

How can we generalize from the comoving frame to an arbitrary frame? Various authors have observed that the only nonscalar present in the noninteracting real-time propagators is βk_0, which appears in thermal distribution functions. For example, the distribution function for bosons takes the form

$$N_b(k_0)=\frac{1}{e^{|\beta k_0|}-1} \ . \tag{3.31}$$

Making the replacement $\beta k_0 \to \beta\cdot k$ everywhere gives a set of rules that is apparently covariant; we list the corresponding rules for QHD-I in the next subsection. However, the precise nature of the generalized propagator (e.g., what is the generalization of the Heisenberg picture?) and the role of the spacelike hypersurface in Eq. (3.8) are not clear. In fact, the rules are correct, and we apply them in Sec. IV to reproduce mean-field results obtained through explicit calculations in the laboratory frame in Ref. 5. In Secs. V and VI, after deriving expressions for the grand partition function [Eq. (3.6)] in an arbitrary frame, we will be able to justify the covariant real-time rules and derive imaginary-time rules in covariant form.

D. Covariant real-time Feynman rules

In this section, we list real-time, momentum-space Feynman rules in covariant form for QHD-I. The Feynman rules allow one to express the interacting propagators as order-by-order expansions in the coupling constants and noninteracting propagators. In practical calculations, various diagrams must be summed to all orders, since QHD-I is a strong-coupling theory. As discussed above, the rules given here generate propagators in the canonical Heisenberg picture. Real-time rules for QHD-I have also been presented in Refs. 10 and 11, based on a thermofield-dynamics derivation, but not in covariant form.

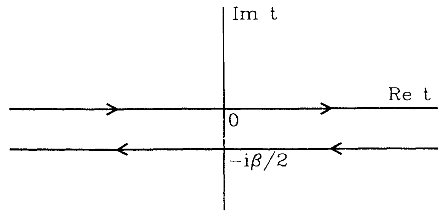

FIG. 1. Contour in the complex time plane for deriving real-time Feynman rules.

Our presentation parallels that of Ref. 1, where rules are given for QHD-I at $T=0$. The rules given below are generalizations both to $T\neq 0$ and to an arbitrary reference frame, which is characterized by the fluid four-velocity u^μ. The rules of Ref. 1 can be recovered by taking the $T\to 0$ limit and specializing to the comoving frame with $u^\mu=(1,0,0,0)$. The new features at $T\neq 0$ are that each vertex now comes in two types, which will be labeled by $r=1,2$ and which differ only by a sign, and that each propagator has a corresponding 2×2 matrix structure (e.g., $\Delta^{(rr')}\equiv\underline{\Delta}$). (Note that we interchangeably employ this matrix notation or exhibit the indices explicitly.)

The matrix structure arises by grouping together ensemble averages with various combinations of time arguments lying on the contour shown in Fig. 1. For example, in the comoving frame, the (matrix) baryon propagator is defined by

$$i\underline{G}(x-x')\equiv\begin{bmatrix} \langle\langle T\hat{\psi}(t,\mathbf{x})\hat{\bar{\psi}}(t',\mathbf{x}')\rangle\rangle & -\langle\langle \hat{\bar{\psi}}(t'-i\beta/2,\mathbf{x}')\hat{\psi}(t,\mathbf{x})\rangle\rangle \\ \langle\langle \hat{\psi}(t-i\beta/2,\mathbf{x})\hat{\bar{\psi}}(t',\mathbf{x}')\rangle\rangle & \langle\langle \bar{T}\hat{\psi}(t-i\beta/2,\mathbf{x})\hat{\bar{\psi}}(t'-i\beta/2,\mathbf{x}')\rangle\rangle \end{bmatrix} \tag{3.32}$$

$$=e^{i\mu'(t-t')}\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-x')}i\underline{G}(k) \ , \tag{3.33}$$

where $\bar{T}$ denotes anticausal time ordering. Thus the (1,1) matrix element involves only real time arguments and allows us to compute physical ensemble averages.

We begin with a variety of explicit expressions for the noninteracting propagators. These expressions include *regularized* delta functions, denoted δ_ϵ, which are defined by

$$\delta_\epsilon(\omega)\equiv\frac{i}{2\pi}\left[\frac{1}{\omega+i\epsilon}-\frac{1}{\omega-i\epsilon}\right] \quad (\omega \text{ real}) \ , \tag{3.34}$$

where $\epsilon\to 0$ eventually. In most cases, ϵ must be kept finite until the end of a calculation to ensure well-defined results; this is certainly true for diagrams involving products of propagators with the same momentum arguments.

The propagators also involve thermal factors, which are Lorentz scalars and are defined covariantly, as follows. The Bose thermal distribution function $N_b(\beta\cdot k)$ is

$$N_b(\beta\cdot k)=\frac{1}{e^{|\beta\cdot k|}-1}\ , \tag{3.35}$$

and its fermion counterpart $N_f(\beta\cdot k,\alpha)$ is

$$\begin{aligned} N_f(\beta\cdot k,\alpha)&\equiv\theta(\beta\cdot k)n_f(\beta\cdot k,\alpha)+\theta(-\beta\cdot k)\bar{n}_f(\beta\cdot k,\alpha)\\ &\equiv\theta(\beta\cdot k)\frac{1}{e^{\beta\cdot k-\alpha}+1}+\theta(-\beta\cdot k)\frac{1}{e^{-\beta\cdot k+\alpha}+1}\\ &=\frac{1}{e^{|\beta\cdot k|-\alpha\,\mathrm{sgn}(\beta\cdot k)}+1}\ . \end{aligned} \tag{3.36}$$

These distribution functions depend on the baryon thermal potential $\alpha\equiv\beta\mu'$, a scalar, and the temperature four-vector $\beta^\mu\equiv\beta u^\mu$, where β is a scalar equal to the inverse proper temperature, and μ' is the proper baryon chemical potential.[5] When evaluated in the laboratory frame [where $u^\mu=(1-\mathbf{v}^2)^{-1/2}(1,\mathbf{v})$], these expressions have the same form as those given in Ref. 5.

Note, however, that the four-momenta appearing in the distribution functions in noninteracting propagators are "off shell," that is, they involve four independent components k^0 and $\mathbf{k}$. The use of off-shell rather than on-shell momenta $[k^0=(\mathbf{k}^2+m^2)^{1/2}]$ has been a controversial topic in the literature[2,31,33] that has been resolved in a recent paper[44] in favor of the off-shell form. This is one of the subtle points arising in the time-path derivation of the Feynman rules that we will consider in a forthcoming paper.[34]

We also define a unimodular "thermal" matrix for bosons $\underline{M}_b$ as

$$\underline{M}_b\equiv\begin{bmatrix}\cosh\Theta(k) & \sinh\Theta(k)\\ \sinh\Theta(k) & \cosh\Theta(k)\end{bmatrix}, \tag{3.37}$$

where

$$\begin{aligned}\sinh\Theta(k)&\equiv\sqrt{N_b(\beta\cdot k)}=\frac{1}{(e^{|\beta\cdot k|}-1)^{1/2}}\ ,\\ \cosh\Theta(k)&=e^{|\beta\cdot k|/2}\sqrt{N_b(\beta\cdot k)}=\frac{e^{|\beta\cdot k|/2}}{(e^{|\beta\cdot k|}-1)^{1/2}}\ ,\end{aligned} \tag{3.38}$$

so that $\Theta(k)\geq 0$, and an analogous matrix $\underline{M}_f$ for fermions

$$\underline{M}_f\equiv\begin{bmatrix}\cos\Theta(k) & -\sin\Theta(k)\\ \sin\Theta(k) & \cos\Theta(k)\end{bmatrix}, \tag{3.39}$$

where

$$\begin{aligned}\sin\Theta(k)&\equiv\sqrt{N_f(\beta\cdot k,\alpha)}\\ &=\frac{1}{(e^{|\beta\cdot k|-\alpha\,\mathrm{sgn}(\beta\cdot k)}+1)^{1/2}}\ ,\\ \cos\Theta(k)&=[\theta(\beta\cdot k)-\theta(-\beta\cdot k)]\sqrt{1-N_f(\beta\cdot k,\alpha)}\\ &=\frac{\mathrm{sgn}(\beta\cdot k)e^{[|\beta\cdot k|-\alpha\,\mathrm{sgn}(\beta\cdot k)]/2}}{(e^{|\beta\cdot k|-\alpha\,\mathrm{sgn}(\beta\cdot k)}+1)^{1/2}}\ ,\end{aligned} \tag{3.40}$$

and $0\leq\Theta(k)\leq\pi$. The utility of these thermal matrices was first discovered in the context of thermofield dynamics, where they arise naturally as Bogoliubov transformation matrices.

With these definitions, the noninteracting scalar meson propagator is given by

$$\Delta_0^{(rr')}(k)\equiv\underline{\Delta}_0(k)=\underline{M}_b\begin{bmatrix}\Delta_{0F}(k) & 0\\ 0 & -\Delta_{0F}^\dagger(k)\end{bmatrix}\underline{M}_b\ , \tag{3.41}$$

where the scalar Feynman (F) propagator is

$$\Delta_{0F}(k)=\frac{1}{k^2-m_s^2+i\epsilon},\quad \Delta_{0F}^\dagger(k)=\frac{1}{k^2-m_s^2-i\epsilon}\ . \tag{3.42}$$

Note that the central matrix in Eq. (3.41) contains only causal and anticausal Feynman propagators; all the temperature dependence is in $\underline{M}_b$. The propagator is conveniently decomposed into Feynman (F) and temperature-dependent (T) parts:

$$\begin{aligned}\underline{\Delta}_0(k)&=\begin{bmatrix}\Delta_{0F}(k) & 0\\ 0 & -\Delta_{0F}^\dagger(k)\end{bmatrix}-2\pi i\delta_\epsilon(k^2-m_s^2)\sinh\Theta(k)\begin{bmatrix}\sinh\Theta(k) & \cosh\Theta(k)\\ \cosh\Theta(k) & \sinh\Theta(k)\end{bmatrix}\\ &=\begin{bmatrix}\Delta_{0F}(k) & 0\\ 0 & -\Delta_{0F}^\dagger(k)\end{bmatrix}-2\pi i\delta_\epsilon(k^2-m_s^2)N_b(\beta\cdot k)\begin{bmatrix}1 & e^{|\beta\cdot k|/2}\\ e^{|\beta\cdot k|/2} & 1\end{bmatrix} \qquad (3.43)\\ &\equiv\underline{\Delta}_{0F}(k)+\underline{\Delta}_{0T}(k)\ . \qquad (3.44)\end{aligned}$$

These expressions are more useful than Eq. (3.41) for some applications.

Let us check the zero-temperature limit $(\beta\to\infty)$ of the scalar propagator. Since $N_b(\beta\cdot k)\to 0$, $\sinh\Theta(k)\to 0$, and $\cosh\Theta(k)\to 1$ in this limit, the second term in Eq. (3.43) disappears, and $\underline{\Delta}_0(k)$ becomes *diagonal*. This also follows from Eq. (3.41), as $\underline{M}_b$ becomes the unit matrix at zero temperature. Since $\underline{\Delta}_0(k)$ is diagonal and external points of physical Green's functions have $r=1$ only, connected diagrams with real-time propagators will reproduce the familiar zero-temperature Feynman rules[1] in terms of $\Delta_{0F}(k)$.

The noninteracting vector meson propagator is concisely expressed in terms of the scalar meson propagator,

$$D_0^{(rr')\mu\nu}(k)\equiv\underline{D}_0^{\mu\nu}(k)=\left[-g^{\mu\nu}+\frac{k^\mu k^\nu}{m_v^2}\right]\Delta_0^{(rr')}(k)\Bigg|_{m_s^2\to m_v^2}\ , \tag{3.45}$$

where the prescription is to replace the scalar meson mass m_s by the vector meson mass m_v in $\Delta_0^{(rr')}(k)$ [Eqs. (3.41)–(3.44)]. It is reasonable to expect that the $k^\mu k^\nu$ factor can be dropped when calculating observables, as at $T=0$, because the baryon current is conserved. To our knowledge, however, there is no proof that this prescription is valid for $T>0$.

Finally, the noninteracting baryon propagator is given by

$$G_0^{(rr')}(k)_{\xi\xi'} \equiv \underline{G}_0(k)_{\xi\xi'} = \underline{M}_f \begin{bmatrix} G_{0F}(k)_{\xi\xi'} & 0 \\ 0 & -G_{0F}^{\dagger}(k)_{\xi\xi'} \end{bmatrix} \underline{M}_f , \tag{3.46}$$

where the fermion Feynman (F) propagator is

$$G_{0F}(k)_{\xi\xi'} = \frac{(\not{k}+M)_{\xi\xi'}}{k^2-M^2+i\epsilon}, \quad G_{0F}^{\dagger}(k)_{\xi\xi'} = \frac{(\not{k}+M)_{\xi\xi'}}{k^2-M^2-i\epsilon} , \tag{3.47}$$

and there is no complex conjugation of the Dirac matrices in the anticausal propagator. The Dirac matrix indices ξ and ξ' should not be confused with the thermal matrix indices r and r'. We can again decompose the propagator into Feynman (F) and temperature-dependent (T) parts:

$$\begin{aligned} \underline{G}_0(k)_{\xi\xi'} &= \begin{bmatrix} G_{0F}(k)_{\xi\xi'} & 0 \\ 0 & -G_{0F}^{\dagger}(k)_{\xi\xi'} \end{bmatrix} + 2\pi i(\not{k}+M)_{\xi\xi'}\delta_\epsilon(k^2-M^2)\sin\Theta(k) \begin{bmatrix} \sin\Theta(k) & \cos\Theta(k) \\ -\cos\Theta(k) & \sin\Theta(k) \end{bmatrix} \\ &= \begin{bmatrix} G_{0F}(k)_{\xi\xi'} & 0 \\ 0 & -G_{0F}^{\dagger}(k)_{\xi\xi'} \end{bmatrix} \\ &\quad + 2\pi i(\not{k}+M)_{\xi\xi'}\delta_\epsilon(k^2-M^2)N_f(\beta\cdot k,\alpha) \begin{bmatrix} 1 & \mathrm{sgn}(\beta\cdot k)e^{[|\beta\cdot k|-\alpha\,\mathrm{sgn}(\beta\cdot k)]/2} \\ -\mathrm{sgn}(\beta\cdot k)e^{[|\beta\cdot k|-\alpha\,\mathrm{sgn}(\beta\cdot k)]/2} & 1 \end{bmatrix} \end{aligned} \tag{3.48}$$

$$\equiv \underline{G}_{0F}(k)_{\xi\xi'} + \underline{G}_{0T}(k)_{\xi\xi'} . \tag{3.49}$$

In the zero-temperature limit ($\beta \to \infty$) with positive chemical potential ($\alpha > 0$),

$$\sin^2\Theta(k) = N_f(\beta\cdot k,\alpha) \to \theta(\beta\cdot k)\theta(\alpha-\beta\cdot k) , \tag{3.50}$$

$$\sin\Theta(k)\cos\Theta(k) \to 0 . \tag{3.51}$$

Thus, from Eq. (3.48), $\underline{G}_0(k)_{\xi\xi'}$ becomes diagonal, and the (1,1) component gives a covariant generalization of the finite-density propagator in Eq. (5.5) of Ref. 1.

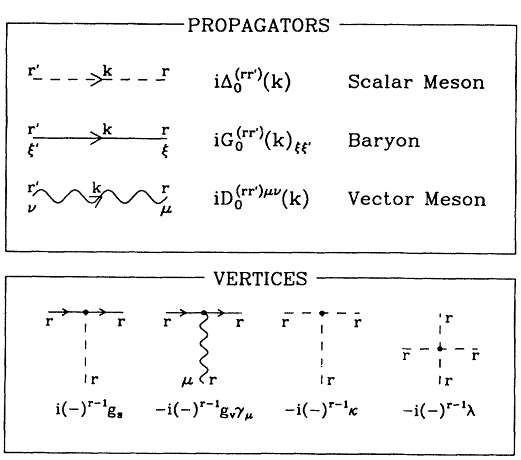

FIG. 2. Components of real time Feynman diagrams for QHD-I.

Given these definitions, we can list the rules for the two-point functions in momentum space, in the canonical Heisenberg picture. Other Green's functions (higher n-point functions) can be calculated using the same rules, by changing the number of external lines appropriately [generalizing rule (1) below] and defining the overall phase consistently.

The rules for the nth *order contribution to* $(i)\times$ *(real-time propagator) in* QHD-I *are as follows* (see Fig. 2):

(1) Draw all topologically distinct diagrams with two external propagator lines and n vertices connected by internal propagator lines. Do not include diagrams with completely disconnected pieces (that is, not connected to *any* external line).

(2) Assign a direction to each line. Associate a directed

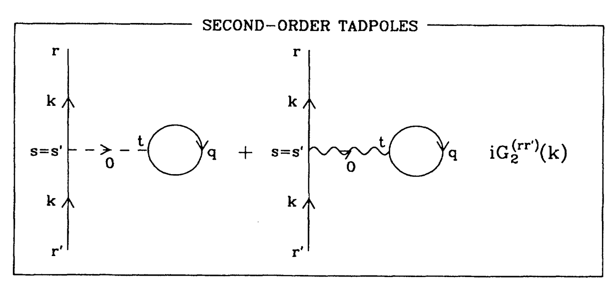

FIG. 3. Second-order tadpole contributions to the baryon propagator.

four-momentum with each line and conserve energy and momentum at each vertex. This will leave only independent four-momenta in the diagram.

(3) Each *internal* vertex is given a label r, which is summed over $r=1,2$. External points of physical real-time Green's functions have $r=1$ only. Each vertex is assigned a factor indicated in Fig. 2. [σNN vertex: $i(-1)^{r-1}g_s$; ωNN vertex: $-i(-1)^{r-1}g_v\gamma_\mu$; σ^3 vertex: $-i(-1)^{r-1}\kappa$; σ^4 vertex: $-i(-1)^{r-1}\lambda$.] These are the same factors as at $T=0$, except for the r-dependent sign. Feynman rules for counterterm contributions follow directly.

(4) Each directed line carrying momentum k gets a factor of (i) times the appropriate noninteracting propagator, $\Delta_0^{(rr')}(k)$, $D_0^{(rr')\mu\nu}(k)$, or $G_0^{(rr')}(k)_{\xi\xi'}$, as indicated in Fig. 2. The labels r and r' are those of the vertices at the ends of the propagator. Explicit expressions for these propagators are listed above.

(5) Sum over all repeated spacetime $(\mu,\nu,\ldots)$, Dirac $(\xi,\xi',\ldots)$, and thermal matrix $(r,r',\ldots)$ indices.

(6) Integrate $\int d^4k/(2\pi)^4$ over all independent four-momenta.

(7) Include a factor of (-1) for each closed fermion loop.

(8) Any single-fermion line forming a tadpole loop, as in Fig. 3, *may* be interpreted as $e^{ik_0\eta}iG_0^{(rr')}(k)_{\xi\xi'}$, where $\eta\to0^+$ at the end of the calculation. However, such factors are unnecessary if integrals are regularized dimensionally, as we will do, and they are omitted in the sequel.

(9) A symmetry factor may be needed in diagrams with σ^3 and σ^4 vertices.[48(a)] The factor is the same as at $T=0$.

It is customary to include a factor of $e^{ik_0\eta}$ (with $\eta\to0^+$ at the end of the calculation) in closed tadpole loops. However, the operator ordering implied by this factor ($\hat{\bar{\psi}}$ appears to the left of $\hat{\psi}$) does not produce normal-ordered operators, so one must still define the tadpole integral by performing a subtraction. It is therefore equally acceptable to omit this factor and define the integral by subtraction anyway, which is the preferred procedure when dimensional regularization is used. An exception to this procedure involves the vertex for a conserved current; this can be normal ordered in the Lagrangian so that divergent (vacuum) tadpoles never appear.

Similar Feynman rules (for the comoving frame) have been given elsewhere,[31,2,10] with several different phase conventions. As discussed in Sec. 3.2.2 of Ref. 2, the 2×2 matrix structure of the propagators removes pathologies that would arise if one kept only the (1,1) operator components in the Feynman rules.

IV. APPLICATION TO NUCLEAR MATTER

We now calculate the energy-momentum tensor $T^{\mu\nu}$ and other observables of QHD-I at the one-loop level, by applying the real-time Feynman rules of Sec. III. Since the momentum-space propagators we have defined are the Fourier transforms of canonical Heisenberg-picture propagators (as opposed to grand Heisenberg-picture propagators), the calculation of observables closely parallels the calculation at $T=0$, and it is convenient to follow the development in Chap. 5 of Ref. 1. As in Ref. 1, we choose the renormalized scalar self-couplings to be zero, $\kappa=\lambda=0$, but this is easily generalized.

When using real-time matrix propagators to calculate thermal averages for physical observables, only the (1,1) matrix element enters. We can therefore write the contributions to $T^{\mu\nu}$ defined in Eqs. (3.19)–(3.22) as[1]

$$\langle\langle \hat{T}_B^{\mu\nu}\rangle\rangle = -i\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu G^{(11)}(k)]k^\nu\,, \qquad (4.1)$$

$$\langle\langle \hat{T}_s^{\mu\nu}\rangle\rangle = -i\int\frac{d^4k}{(2\pi)^4}[\tfrac{1}{2}(k^2-m_s^2)g^{\mu\nu}-k^\mu k^\nu]\Delta^{(11)}(k)\,, \qquad (4.2)$$

$$\langle\langle \hat{T}_v^{\mu\nu}\rangle\rangle = i\int\frac{d^4k}{(2\pi)^4}[\tfrac{1}{2}(k^2-m_v^2)g^{\mu\nu}-k^\mu k^\nu]D_\sigma^{(11)\sigma}(k)\,, \qquad (4.3)$$

where Tr indicates a trace over both isospin and Dirac indices (which will not be indicated explicitly). Similarly, the baryon current and scalar density follow from

$$\langle\langle \hat{B}^\mu\rangle\rangle \equiv \langle\langle \hat{\bar{\psi}}\gamma^\mu\hat{\psi}\rangle\rangle = -i\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu G^{(11)}(k)]\,, \qquad (4.4)$$

$$\langle\langle \hat{\rho}_s\rangle\rangle \equiv \langle\langle \hat{\bar{\psi}}\hat{\psi}\rangle\rangle = -i\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[G^{(11)}(k)]\,. \qquad (4.5)$$

The factors of $e^{ik_0\eta}$, which appear in the $T=0$ expressions for these matrix elements,[1] are omitted here because we will use dimensional regularization to define divergent integrals over the baryon propagator. [See rule (8) in Sec. III D.]

The integrals in Eqs. (4.1)–(4.5) are, in general, divergent. We define physical observables by making subtractions to render these expressions finite. For the conserved baryon current, one need only normal order the operator, which is equivalent to a vacuum expectation value (VEV) subtraction:

$$B^\mu = \langle\langle \hat{B}^\mu\rangle\rangle - \lim_{\substack{\beta\to\infty\\ \alpha\to0}}\langle\langle \hat{B}^\mu\rangle\rangle \equiv \langle\langle \hat{B}^\mu\rangle\rangle - \mathrm{VEV}\,. \qquad (4.6)$$

For $T^{\mu\nu}$, even though this is a conserved quantity, both counterterm contributions (CTC) and a vacuum subtraction are needed:

$$T^{\mu\nu} = \langle\langle \hat{T}^{\mu\nu}+\delta\hat{T}^{\mu\nu}\rangle\rangle - \lim_{\substack{\beta\to\infty\\ \alpha\to0}}\langle\langle \hat{T}^{\mu\nu}+\delta\hat{T}^{\mu\nu}\rangle\rangle \equiv \langle\langle \hat{T}^{\mu\nu}\rangle\rangle + \mathrm{CTC} - \mathrm{VEV}\,. \qquad (4.7)$$

The scalar density ρ_s is not conserved, so the observable is defined with counterterms, which include a vacuum subtraction, as at $T=0$:[1]

$$\rho_s = \langle\langle \hat{\rho}_s\rangle\rangle + \mathrm{CTC}\,. \qquad (4.8)$$

We define the mean-field theory (MFT) and the relativistic Hartree approximation (RHA) from a diagrammatic analysis, using the real-time rules. These approximations are based on the self-consistent summation of "tadpole" contributions to the baryon self-energy Σ, using Dyson's equations. The RHA is equivalent to a full one-loop calculation, while the MFT neglects contributions to the tadpoles from the filled Dirac sea. As in Ref. 1, we first evaluate $T^{\mu\nu}$ in the MFT and compare to the results of Ref. 5, which were obtained using a canonical approach. Then we generalize to the RHA.

In the real-time formalism, Dyson's equation for the baryon propagator is[2]

$$G^{(rr')}(k)=G_0^{(rr')}(k)+G_0^{(rs)}(k)\Sigma^{(ss')}(k)G^{(s'r')}(k)\ , \tag{4.9}$$

where repeated thermal indices are summed (from 1 to 2) and we have suppressed the Dirac matrix indices. In matrix form,

$$\underline{G}(k)=\underline{G}_0(k)+\underline{G}_0(k)\underline{\Sigma}(k)\underline{G}(k)\ . \tag{4.10}$$

Dyson's equation and the self-energy in the real-time formalism are discussed for a general Lagrangian in Ref. 2 (and for the Walecka model in Ref. 10), and we will simply quote results obtained there. In particular, by using translational invariance, *CPT* invariance, and the antiperiodicity condition on the fermion propagator, one can prove that the exact matrix propagator has the same general form as the noninteracting propagator [Eq. (3.46)]:

$$\underline{G}(k)=\underline{M}_f\begin{bmatrix} G(k) & 0 \\ 0 & -G^{\dagger}(k)\end{bmatrix}\underline{M}_f\ . \tag{4.11}$$

Thus the self-energy

$$\underline{\Sigma}(k)=[\underline{G}_0(k)]^{-1}-[\underline{G}(k)]^{-1} \tag{4.12}$$

has the form

$$\underline{\Sigma}(k)=\underline{M}_f^{-1}\begin{bmatrix} \Sigma(k) & 0 \\ 0 & -\Sigma^{\dagger}(k)\end{bmatrix}\underline{M}_f^{-1}\ , \tag{4.13}$$

which implies that the self-energy has only one independent thermal matrix component. We can also write the inverse propagator as

$$\begin{aligned}[\underline{G}(k)]^{-1} &=[\underline{G}_0(k)]^{-1}-\underline{\Sigma}(k) \\ &=\underline{M}_f^{-1}\begin{bmatrix} \not{k}-M-\Sigma(k) & 0 \\ 0 & -[\not{k}-M-\Sigma^{\dagger}(k)]\end{bmatrix}\underline{M}_f^{-1}\ .\end{aligned} \tag{4.14}$$

The self-energy is particularly simple when only tadpole contributions are included, because $\Sigma^{(rr')}(k)$ is real, independent of momentum, and diagonal in the thermal matrix indices. To see how this works in practice, follow the development in Chap. 5 of Ref. 1. Start by applying the real-time rules of Sec. III to the second-order tadpole contributions to the baryon propagator [which we call $iG_2^{(rr')}(k)$], as illustrated in Fig. 3. If we define the second-order self-energy by

$$iG_2^{(rr')}(k)=iG_0^{(rs)}(k)\Sigma_2^{(ss')}G_0^{(s'r')}(k)\ , \tag{4.15}$$

where

$$\Sigma_2^{(ss')}=\Sigma_{2s}^{(ss')}-\gamma_\mu\Sigma_{2v}^{\mu(ss')}\ , \tag{4.16}$$

we find that $\Sigma_2^{(ss')}$ is a diagonal matrix, independent of momentum, with components

$$\Sigma_{2s}^{(11)}=-\Sigma_{2s}^{(22)}=i\frac{g_s^2}{m_s^2}\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[G_0^{(11)}(q)]\ , \tag{4.17}$$

$$\Sigma_{2v}^{\mu(11)}=-\Sigma_{2v}^{\mu(22)}=i\frac{g_v^2}{m_v^2}\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu G_0^{(11)}(q)]\ . \tag{4.18}$$

In this approximation, it is not necessary to regularize the delta functions in the meson propagators. This implies that the thermal parts of the meson propagators vanish at zero momentum, so that these propagators are diagonal,

$$\Delta_0^{(st)}(0)=-\frac{1}{m_s^2}(-1)^{s-1}\delta_{st}\ . \tag{4.19}$$

Note also that

$$\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[\Gamma G_0^{(11)}(q)]=\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[\Gamma G_0^{(22)}(q)]\ , \tag{4.20}$$

where $\Gamma=1$ or γ^μ, which has been used in Eqs. (4.17) and (4.18). [Equation (4.20) is satisfied by the Feynman parts of $\underline{G}_0$ when the integral is regularized dimensionally, since $G_{0F}(q)$ and $G_{0F}^{\dagger}(q)$ require Wick rotations in opposite directions.]

We sum the tadpole contributions to all orders using Dyson's equation. Self-consistency dictates that the Hartree self-energy $\underline{\Sigma}^H$ is defined by evaluating Eqs. (4.17) and (4.18) with the Hartree matrix propagator

$$\underline{G}^H(k)=\underline{G}_0(k)+\underline{G}_0(k)\underline{\Sigma}^H\underline{G}^H(k)\ . \tag{4.21}$$

Thus $\underline{\Sigma}^H$ is independent of momentum. Just as at $T=0$, $\underline{\Sigma}^H$ is the average interaction felt by a propagating nucleon, which means that $\underline{G}^H(k)$ should have the same analytic structure as $\underline{G}_0(k)$, but with a shifted mass and spectrum. Furthermore, it follows that $\underline{\Sigma}^H$ has the same diagonal form as $\underline{\Sigma}_2$, with $\Sigma^{H(11)}=-\Sigma^{H(22)}$.

Thus $\underline{G}^H(k)$ takes a form similar to Eq. (3.49):

$$\underline{G}^H(k)=\begin{bmatrix} G_F^H(k) & 0 \\ 0 & -G_F^{H\dagger}(k)\end{bmatrix}+2\pi i(\not{k}^*+M^*)\delta(k^{*2}-M^{*2})\sin\Theta^*(k)\begin{bmatrix}\sin\Theta^*(k) & \cos\Theta^*(k) \\ -\cos\Theta^*(k) & \sin\Theta^*(k)\end{bmatrix} \tag{4.22}$$

$$\equiv\underline{G}_F^H(k)+\underline{G}_T^H(k)\ , \tag{4.23}$$

where

$$k^{*\mu}\equiv k^{\mu}+\Sigma_{\rm v}^{H\mu},\quad M^{*}\equiv M+\Sigma_{\rm s}^{H}\ , \tag{4.24}$$

$$\Sigma^{H(11)}=\Sigma^{H}=\Sigma_{\rm s}^{H}-\gamma_{\mu}\Sigma_{\rm v}^{H\mu}\ , \tag{4.25}$$

and

$$G_F^H(k)=\frac{(\not k^{*}+M^{*})}{k^{*2}-M^{*2}+i\epsilon},\quad G_F^{H\dagger}(k)=\frac{(\not k^{*}+M^{*})}{k^{*2}-M^{*2}-i\epsilon}\ . \tag{4.26}$$

(Note that the superscript "$*$," which is historical, does *not* mean complex conjugation.) We also define new thermal angles

$$\sin^2\Theta^{*}(k)=\theta(\beta\cdot k^{*})\frac{1}{e^{\beta\cdot k-\alpha}+1}+\theta(-\beta\cdot k^{*})\frac{1}{e^{-\beta\cdot k+\alpha}+1} \tag{4.27}$$

and

$$\cos\Theta^{*}(k)=[\theta(\beta\cdot k^{*})-\theta(-\beta\cdot k^{*})]\sqrt{1-\sin^2\Theta^{*}(k)}\ , \tag{4.28}$$

with the same phase conventions for $\Theta^{*}(k)$ as for $\Theta(k)$ in the noninteracting propagators. We emphasize that the thermal distribution functions in Eqs. (4.27) and (4.28) involve the canonical four-momentum k_{μ} in the exponents and *not* the kinetic four-momentum k_{μ}^{*}. This is because the distribution functions compare true single-particle energies to the chemical potential, which means comparing $\beta\cdot k$ to α.

The self-energy integrals involving the Feynman piece $G_F^H(k)$ are divergent, but this is the *only* source of divergences in the calculation. We can include counterterms and renormalize to compute RHA results, but since the integrals have no explicit temperature or density dependence, and since

$$\int\frac{d^nk}{(2\pi)^4}\mathrm{Tr}[\Gamma G_F^H(k)]=-\int\frac{d^nk}{(2\pi)^4}\mathrm{Tr}[\Gamma G_F^{H\dagger}(k)]\ , \tag{4.29}$$

all vacuum (zero-point) contributions to the one-loop observables will have the same form as at $T=0$.

We *define* the MFT by keeping only the temperature- and density-dependent parts of the Hartree propagator $\underline{G}_T^H(k)$ in evaluating the Hartree self-energies. To calculate observables, we need only the (1,1) matrix element:

$$\begin{aligned}G_T^{H(11)}(k)&=2\pi i(\not k^{*}+M^{*})\delta(k^{*2}-M^{*2})\sin^2\Theta^{*}(k)\\&=2\pi i(\not k^{*}+M^{*})\delta(k^{*2}-M^{*2})\left[\theta(\beta\cdot k^{*})\frac{1}{e^{\beta\cdot k-\alpha}+1}+\theta(-\beta\cdot k^{*})\frac{1}{e^{-\beta\cdot k+\alpha}+1}\right]\ .\end{aligned} \tag{4.30}$$

Then the MFT self-energies (and $\rho_{\rm s}$ and B^{μ}) are

$$\begin{aligned}\Sigma_{\rm s}^H&=i\frac{g_{\rm s}^2}{m_{\rm s}^2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[G_T^{H(11)}(k)]=-\frac{g_{\rm s}^2}{m_{\rm s}^2}\rho_{\rm s}\\&=-\frac{2g_{\rm s}^2}{m_{\rm s}^2}M^{*}\frac{\gamma}{(2\pi)^3}\int d^4k\ \delta(k^{*2}-M^{*2})\sin^2\Theta^{*}(k)\\&\equiv M^{*}-M\end{aligned} \tag{4.31}$$

and

$$\begin{aligned}\Sigma_{\rm v}^{H\mu}&=i\frac{g_{\rm v}^2}{m_{\rm v}^2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^{\mu}G_T^{H(11)}(k)]=-\frac{g_{\rm v}^2}{m_{\rm v}^2}B^{\mu}\\&=-\frac{2g_{\rm v}^2}{m_{\rm v}^2}\frac{\gamma}{(2\pi)^3}\int d^4k\ k^{*\mu}\delta(k^{*2}-M^{*2})\sin^2\Theta^{*}(k)\ ,\end{aligned} \tag{4.32}$$

where γ is the spin-isospin degeneracy. Note that $\Sigma_{\rm s}^H$ is manifestly a Lorentz scalar and $\Sigma_{\rm v}^{H\mu}$ is manifestly a four-vector. We also note that Eq. (4.31) is the MFT self-consistency equation for M^{*}, written covariantly. Finally, Eqs. (4.31) and (4.32) give us manifestly covariant expressions for the scalar density and baryon current:

$$\rho_{\rm s}=2M^{*}\frac{\gamma}{(2\pi)^3}\int d^4k\,\delta(k^{*2}-M^{*2})\sin^2\Theta^{*}(k)\ , \tag{4.33}$$

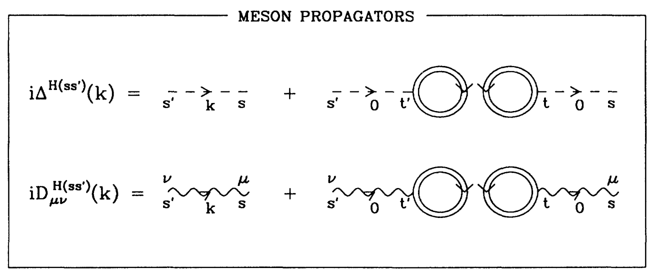

FIG. 4. The Hartree meson propagators $i\Delta^{H(ss')}(k)$ and $iD_{\mu\nu}^{H(ss')}(k)$. The double solid line represents the Hartree thermal baryon propagator $iG_T^{H(tt)}$.

$$B^{\mu}=2\frac{\gamma}{(2\pi)^3}\int d^4k\, k^{*\mu}\delta(k^{*2}-M^{*2})\sin^2\Theta^*(k)$$
$$=\left[2\frac{\gamma}{(2\pi)^3}\int d^4k(k^*\cdot u)\delta(k^{*2}-M^{*2})\sin^2\Theta^*(k)\right]u^{\mu}$$
$$=\rho'_B u^{\mu}\ . \tag{4.34}$$

It is a simple exercise to verify that these equations reduce to the expected results in the comoving frame and in the zero-temperature limit.

The second line in Eq. (4.34) follows from the general result

$$\int d^4k\, k^{*\mu}f(k^2,k\cdot u)=u^{\mu}\int d^4k(k^*\cdot u)f(k^2,k\cdot u)\ , \tag{4.35}$$

which is valid because u^{μ} is the only four-vector available after integration. We can also deduce

$$\int d^4k\, k^{*\mu}k^{*\nu}f(k^2,k\cdot u)=u^{\mu}u^{\nu}\int d^4k(k^*\cdot u)^2f(k^2,k\cdot u)+\tfrac{1}{3}(g^{\mu\nu}-u^{\mu}u^{\nu})\int d^4k[k^{*2}-(k^*\cdot u)^2]f(k^2,k\cdot u)\ , \tag{4.36}$$

which will be useful in simplifying $T^{\mu\nu}$.

To evaluate the energy-momentum tensor $T^{\mu\nu}$, we will need the tadpole contributions to the (1,1) meson propagators.[1] We apply the real-time Feynman rules to the diagrams in Fig. 4, using $G_T^{H(11)}(q)$ to compute the loops (which is consistent with our definition of the MFT), with the results

$$\Delta^{H(11)}(k)=\Delta_0^{(11)}(k)-i(2\pi)^4\delta^{(4)}(k)(\Sigma_s^H)^2/g_s^2\ , \tag{4.37}$$

$$D_{\mu\nu}^{H(11)}(k)=D_{0\mu\nu}^{(11)}(k)-i(2\pi)^4\delta^{(4)}(k)(\Sigma_{v\mu}^H\Sigma_{v\nu}^H)/g_v^2\ . \tag{4.38}$$

There are no counterterm corrections in the MFT, but vacuum subtractions are needed to define $T_s^{\mu\nu}$ and $T_v^{\mu\nu}$. After performing the subtractions, there will be terms in $T_s^{\mu\nu}$ and $T_v^{\mu\nu}$ coming from the thermal excitation of free mesons. In principle, these free-meson contributions should be included in $T^{\mu\nu}$. Nevertheless, we omit them on physical grounds because the contribution from thermal pions will dominate these contributions at any temperature, and because we expect a transition to a quark-gluon phase before they become important. Thus we keep only

$$T_s^{\mu\nu}+T_v^{\mu\nu}=\frac{m_s^2}{2g_s^2}g^{\mu\nu}(\Sigma_s^H)^2-\frac{m_v^2}{2g_v^2}g^{\mu\nu}(\Sigma_{v\sigma}^H\Sigma_v^{H\sigma}) \tag{4.39}$$
$$=g^{\mu\nu}\left[\frac{m_s^2}{2g_s^2}(M-M^*)^2-\frac{g_v^2}{2m_v^2}B_{\sigma}B^{\sigma}\right]\ . \tag{4.40}$$

For the baryon contribution, there are neither counterterm contributions nor a vacuum subtraction (in the MFT). Thus we have

$$T_B^{\mu\nu}=-i\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^{\mu}G_T^{H(11)}(k)]k^{\nu}$$
$$=2\frac{\gamma}{(2\pi)^3}\int d^4k\, k^{*\mu}k^{\nu}\delta(k^{*2}-M^{*2})\sin^2\Theta^*(k)$$
$$=\frac{g_v^2}{m_v^2}B^{\mu}B^{\nu}+2\frac{\gamma}{(2\pi)^3}\int d^4k\, k^{*\mu}k^{*\nu}\delta(k^{*2}-M^{*2})\sin^2\Theta^*(k)\ , \tag{4.41}$$

where we have used the definition of $k^{*\mu}$ and Eq. (4.32) to obtain the last line.

Combining these results, we find a manifestly covariant expression for the energy-momentum tensor in the MFT of QHD-I:

$$T_{\mathrm{MFT}}^{\mu\nu}=g^{\mu\nu}\left[\frac{m_s^2}{2g_s^2}(M-M^*)^2-\frac{g_v^2}{2m_v^2}B_{\sigma}B^{\sigma}\right]+\frac{g_v^2}{m_v^2}B^{\mu}B^{\nu}+2\frac{\gamma}{(2\pi)^3}\int d^4k\, k^{*\mu}k^{*\nu}\delta(k^{*2}-M^{*2})\sin^2\Theta^*(k) \tag{4.42}$$
$$=g^{\mu\nu}\left[\frac{m_s^2}{2g_s^2}(M-M^*)^2-\frac{g_v^2}{2m_v^2}B_{\sigma}B^{\sigma}\right]+\frac{g_v^2}{m_v^2}B^{\mu}B^{\nu}+u^{\mu}u^{\nu}\left[2\frac{\gamma}{(2\pi)^3}\int d^4k(k^*\cdot u)^2\delta(k^{*2}-M^{*2})\sin^2\Theta^*(k)\right]$$
$$+\tfrac{1}{3}(g^{\mu\nu}-u^{\mu}u^{\nu})\left[2\frac{\gamma}{(2\pi)^3}\int d^4k[k^{*2}-(k^*\cdot u)^2]\delta(k^{*2}-M^{*2})\sin^2\Theta^*(k)\right]\ , \tag{4.43}$$

where we have applied Eq. (4.36). By shifting integration variables from k^{μ} to $k^{*\mu}$ and doing the k_0^* integration, one can reproduce the results in Ref. 5, which were calculated in an arbitrary reference frame but not in manifestly covariant form.

It is straightforward to verify that $T^{\mu\nu}_{\rm MFT}$ has the form

$$T^{\mu\nu}_{\rm MFT}=(\mathcal{E}'_{\rm MFT}+p_{\rm MFT})u^{\mu}u^{\nu}-p_{\rm MFT}g^{\mu\nu}\ , \tag{4.44}$$

and to obtain covariant expressions for the proper energy density $\mathcal{E}'_{\rm MFT}$ and the hydrostatic pressure $p_{\rm MFT}$ by projecting the results for $T^{\mu\nu}_{\rm MFT}$ parallel and orthogonal to u^{μ}. In particular,

$$\mathcal{E}'_{\rm MFT}=u_{\mu}u_{\nu}T^{\mu\nu}_{\rm MFT}\ , \tag{4.45}$$

$$p_{\rm MFT}=-\tfrac{1}{3}(g_{\mu\nu}-u_{\mu}u_{\nu})T^{\mu\nu}_{\rm MFT}\ . \tag{4.46}$$

Thus we obtain for $\mathcal{E}'_{\rm MFT}$ and $p_{\rm MFT}$,

$$\mathcal{E}'_{\rm MFT}=\frac{m_{\rm s}^2}{2g_{\rm s}^2}(M-M^*)^2-\frac{g_{\rm v}^2}{2m_{\rm v}^2}B_{\mu}^2+\frac{g_{\rm v}^2}{m_{\rm v}^2}(B\cdot u)^2+2\frac{\gamma}{(2\pi)^3}\int d^4k(k^*\cdot u)^2\delta(k^{*2}-M^{*2})\sin^2\Theta^*(k)\ , \tag{4.47}$$

$$p_{\rm MFT}=-\frac{m_{\rm s}^2}{2g_{\rm s}^2}(M-M^*)^2+\frac{g_{\rm v}^2}{2m_{\rm v}^2}B_{\mu}^2-\frac{2}{3}\frac{\gamma}{(2\pi)^3}\int d^4k[k^{*2}-(k^*\cdot u)^2]\delta(k^{*2}-M^{*2})\sin^2\Theta^*(k)\ . \tag{4.48}$$

[Note that $(B\cdot u)^2=(\rho'_B)^2=B_{\mu}^2$.] These expressions are manifestly covariant and determine the equation of state for an observer in an arbitrary reference frame.

An alternative expression for the pressure is obtained after some changes of variables and a partial integration:

$$p_{\rm MFT}=-\frac{m_{\rm s}^2}{2g_{\rm s}^2}(M-M^*)^2+\frac{g_{\rm v}^2}{2m_{\rm v}^2}B_{\mu}^2+\frac{2}{\beta}\frac{\gamma}{(2\pi)^3}\int d^4k(k^*\cdot u)\delta(k^{*2}-M^{*2})\times[\theta(\beta\cdot k^*)\ln(1+e^{-\beta\cdot k+\alpha})-\theta(-\beta\cdot k^*)\ln(1+e^{\beta\cdot k-\alpha})]\ . \tag{4.49}$$

This form will provide a direct comparison with the thermodynamic pressure calculated in Sec. V.

We can generalize our results to the RHA by including the zero-point (ZP) corrections. As explained earlier, these corrections have the same form as the $T=0$ expressions in Refs. 1 and 49:

$$\{\Delta\rho_{\rm s}\}_{\rm ZP}=-\frac{1}{\pi^2}[M^{*3}\ln(M^*/M)-M^2(M^*-M)-\tfrac{5}{2}M(M^*-M)^2-\tfrac{11}{6}(M^*-M)^3] \tag{4.50}$$

and

$$\{\Delta T^{\mu\nu}\}_{\rm ZP}=-\frac{1}{4\pi^2}[M^{*4}\ln(M^*/M)+M^3(M-M^*)-\tfrac{7}{2}M^2(M-M^*)^2+\tfrac{13}{3}M(M-M^*)^3-\tfrac{25}{12}(M-M^*)^4]g^{\mu\nu} \tag{4.51}$$

$$\equiv\Delta\mathcal{E}'_{\rm ZP}g^{\mu\nu}\ . \tag{4.52}$$

Note that the "Casimir energy" $\Delta\mathcal{E}'_{\rm ZP}$ is a Lorentz scalar. (Note also that a specific renormalization prescription has been applied, as described in Ref. 1, and an isospin degeneracy of 2 has been assumed for the vacuum.) The baryon current is unchanged, since $\hat{B}^{\mu}$ is assumed normal ordered through Eq. (4.6).

Thus we can write the RHA (one-loop) results as

$$T^{\mu\nu}_{\rm RHA}=T^{\mu\nu}_{\rm MFT}+\{\Delta T^{\mu\nu}\}_{\rm ZP}\ , \tag{4.53}$$

$$\mathcal{E}'_{\rm RHA}=\mathcal{E}'_{\rm MFT}+\Delta\mathcal{E}'_{\rm ZP}\ , \tag{4.54}$$

$$p_{\rm RHA}=p_{\rm MFT}-\Delta\mathcal{E}'_{\rm ZP}\ , \tag{4.55}$$

which agree with the results in Ref. 8.

V. COVARIANT CALCULATION OF THE GRAND POTENTIAL

A. Partition function and grand potential

To verify that a calculation satisfies the virial theorem and is thermodynamically consistent, one must compare the pressure computed from the energy-momentum tensor (the hydrostatic pressure) with that computed directly from the partition function (the thermodynamic pressure). If a calculation is also Lorentz covariant, this comparison can be made by performing the calculation in any reference frame. In Sec. IV, we derived frame-independent expressions for the RHA hydrostatic pressure starting from $T^{\mu\nu}$. [See Eqs. (4.48), (4.49), and (4.55).] In this section, we derive a covariant expression for the thermodynamic pressure starting with the partition function of Eq. (3.6). This calculation will help justify the covariant real time Feynman rules given earlier and also help determine the covariant imaginary-time rules.

Recall that in our discussion of the real-time Feynman rules, it was straightforward to generalize the momentum-space propagators from the comoving frame to an arbitrary reference frame. The subtleties arise in the interpretation of the results. In particular, how does one define the Heisenberg picture (both canonical and grand canonical) in an arbitrary Lorentz frame? How does one allow for quantization on hypersurfaces that are not moving with the relativistic fluid? The answers to these questions will determine the form of the covariant propagators, which contain information on the time evolution of the system and the properties of the surrounding fluid. To obtain the answers, it is easiest to begin with the partition function Z given in Eq. (3.6) and generalize later to the generating functional and the resulting Feynman rules.

We will simplify the discussion by considering spacelike hyperplanes σ rather than more general hypersurfaces Λ. Four-vectors x^μ that determine the hyperplane are defined by a unit normal vector n^μ and a time parameter t through[50]

$$\sigma_t : t - n^\mu x_\mu = 0, \quad n^\mu n_\mu = 1, \quad n^0 > 0 . \tag{5.1}$$

Note that the time (or "instant") parameter t is not necessarily equal to the coordinate x^0, since the spacetime coordinate axes can be chosen in many different ways; however, t is a Lorentz scalar quantity that measures the time evolution for observers living on hyperplanes normal to n^μ. Although only real time variables enter in the definition of σ, we will generalize the discussion to allow for complex time variables in order to study the partition function and the grand potential. The imaginary time is related to the scalar quantity $n\cdot\beta = 1/T$, where β^μ is defined in Eq. (3.2), and T is the temperature measured by observers living on the hyperplanes σ. This generalizes the result of Ref. 5 that a laboratory frame observer with $n^\mu = (1,0,0,0)$ measures a temperature $T = 1/\beta^0$.

The first step in the construction of a covariant grand potential is to rewrite Eq. (3.6) in terms of an imaginary-time variable τ appropriate for the hyperplanes σ. From the definitions of β^μ and the thermal potential α, we have

$$-\int d\sigma_\mu(\beta_\nu \hat{T}^{\mu\nu} - \alpha \hat{B}^\mu) = -\int_0^\beta d\tau' \int d\sigma_\mu (u_\nu \hat{T}^{\mu\nu} - \mu' \hat{B}^\mu) , \tag{5.2}$$

where $d\sigma_\mu = n_\mu d\sigma$. This result follows because $\hat{T}^{\mu\nu}$ and $\hat{B}^\mu$ are conserved. The parameter τ' is the imaginary counterpart of the *proper* time t' that determines the evolution for observers living on hyperplanes normal to u^μ (comoving observers). The relationship between τ and τ' can be deduced by observing that an infinitesimal timelike translation between two hyperplanes σ_t and σ_{t+dt} can be written as $n^\mu dt = u^\mu dt'$, so that $dt = n\cdot u\ dt'$ (see Fig. 5) and $d\tau = n\cdot u\ d\tau'$. Thus we can write the partition function as

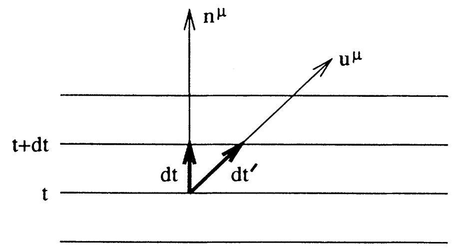

FIG. 5. Two-dimensional illustration of hyperplanes at successive times. Infinitesimal translations along the normal n^μ and fluid four-velocity u^μ are indicated by dt and dt', respectively.

$$Z = \mathrm{Tr}\exp\left\{-\int_0^{n\cdot\beta} d\tau \int d\sigma_\mu [(n\cdot u)^{-1} u_\nu \hat{T}^{\mu\nu} - \mu \hat{B}^\mu]\right\} , \tag{5.3}$$

where we have used $\mu' = \mu(n\cdot u)$. The chemical potential μ is that measured by an observer living on hyperplanes normal to n^μ.

Equation (5.3) expresses the partition function as the trace of an (imaginary) time evolution operator with a grand canonical Hamiltonian, where all quantities are defined for an observer living on hyperplanes normal to n^μ. This trace can be recast as a path integral over fields using well-known techniques.[51,45] The only subtlety arises because the quantization is carried out on an arbitrary hyperplane, which may not be expressed in terms of purely spatial coordinates. We will illustrate the procedure with free fermions and then generalize to model QHD-I.

Since the time evolution is along n^μ, we must make the replacements

$$\dot{\psi} \rightarrow n_\mu \partial^\mu \psi, \quad \Pi \rightarrow n_\mu \Pi^\mu = n_\mu i \bar{\psi}\gamma^\mu \tag{5.4}$$

when computing the time derivatives of fields and the canonical momenta. Thus, for free fermions, the path-integral representation of the partition function becomes

$$\begin{aligned}(Z_0)_B &= \int_{-\Psi}^{\Psi} D[\bar{\psi},\psi] \exp\left[\int_0^{n\cdot\beta} d\tau \int d\sigma \left[i n_\mu \bar{\psi}\gamma^\mu n_\nu \partial^\nu \psi - \frac{1}{n\cdot u} n_\mu u_\nu T^{\mu\nu} + \mu n_\mu B^\mu\right]\right] \\ &= \int_{-\Psi}^{\Psi} D[\bar{\psi},\psi] \exp\left[\int_0^{n\cdot\beta} d\tau \int d\sigma \{\mathcal{L}_0 + n_\mu [n_\nu - u_\nu/(n\cdot u)] i\bar{\psi}\gamma^\mu \partial^\nu \psi + \mu n_\mu B^\mu\}\right] .\end{aligned} \tag{5.5}$$

The limits on the path integral reflect the usual antiperiodic boundary condition for fermions, and we have used[50] [compare Eq. (3.16)]

$$T^{\mu\nu} = -g^{\mu\nu}\mathcal{L}_0 + \Pi^\mu \partial^\nu \psi , \tag{5.6}$$

where $\mathcal{L}_0$ is the free fermion Lagrangian.

Equation (5.5) can be cast into a more intuitive form by defining the derivatives

$$\tilde{\partial}^\mu \equiv \partial^\mu - n^\mu (n\cdot\partial) , \tag{5.7}$$

$$\mathcal{D} \equiv n\cdot\partial - \frac{u\cdot\tilde{\partial}}{n\cdot u} , \tag{5.8}$$

$$d^\mu \equiv n^\mu \mathcal{D} + \tilde{\partial}^\mu , \tag{5.9}$$

so that

$$(Z_0)_B=\int_{-\Psi}^{\Psi}D[\bar{\psi},\psi]\exp\left[\int_0^{n\cdot\beta}d\tau\int d\sigma([\mathcal{L}_0]_{\partial\to d}+\mu n_\mu B^\mu)\right] . \tag{5.10}$$

Here the Lagrangian is to be evaluated by replacing the usual partial derivative ∂^μ with the new derivative d^μ, which contains a gradient $\tilde{\partial}^\mu$ in the hyperplane and a "convective" derivative $\mathcal{D}$ normal to the hyperplane. This is a natural result because the fluid is flowing past the observer when $n^\mu\neq u^\mu$, and the convective derivative automatically incorporates the Lagrange multiplier involving the fluid velocity.[5] (Note that the convective derivative collapses to $u\cdot\partial$ if $n^\mu=u^\mu$.) Moreover, the final term involves the familiar $\mu\rho_B$ constraint, in which both of these quantities are defined naturally for this observer.

The extension of the preceding derivation to the scalar and vector bosons is straightforward, but somewhat tedious, and we will simply sketch the development and quote the results. For the scalar field, one must first write a path integral over the field ϕ and its conjugate momentum density Π, defined by

$$\Pi\equiv n_\mu\Pi^\mu,\quad \Pi^\mu\equiv\frac{\partial\mathcal{L}_0}{\partial(\partial_\mu\phi)} . \tag{5.11}$$

Note that the commutation relations imposed in the hyperplane constrain only the component of Π^μ normal to the hyperplane. The path integral can be cast into a form involving the Lagrangian by using a relation analogous to (5.6) and then by integrating over Π, with proper care regarding momentum and coordinate variables. In particular, since only the component of Π^μ along n^μ is a momentum, one must take

$$\Pi^\mu=\Pi n^\mu+(\Pi^\mu-n^\mu\Pi)=\Pi n^\mu+\tilde{\partial}^\mu\phi \tag{5.12}$$

when integrating over Π. The result is an expression analogous to (5.10), with $\mu=0$.

For the vector field, things become even more cumbersome, as the only dynamical components of the field are those that lie in the hyperplane. The corresponding conjugate momenta are given by

$$\Pi^\lambda\equiv n_\mu\frac{\partial\mathcal{L}_0}{\partial(\partial_\mu V_\lambda)}=n_\mu F^{\lambda\mu} , \tag{5.13}$$

which indeed illustrates that the field variable $n\cdot V$ has no conjugate momentum, since $F^{\mu\nu}$ is antisymmetric. One therefore begins with a path integral over the spacelike vector fields $V^\mu-(n\cdot V)n^\mu$ and the corresponding electric fields Π^λ, and expresses the energy-momentum tensor $T^{\mu\nu}$ in terms of these variables alone. To arrive at an expression analogous to (5.10), one introduces an auxiliary field as a Lagrange multiplier[34] to remove terms involving $(\partial_\lambda\Pi^\lambda)^2$. The (Gaussian) path integral over the electric fields can then be easily done, and in the end, one renames the auxiliary field $n\cdot V$ to arrive at a result corresponding to Eq. (5.10), with a path integral over all four field components V^μ. [Note that most of the complication here is independent of the covariant formulation. Even if one chooses coordinates such that $n^\mu=(1,0,0,0)$, one must start with a path integral over **V** and **E**, and introduce V^0 as an auxiliary field.]

Interactions can be included easily, since the QHD-I Lagrangian is renormalizable. Thus the interaction terms contain no derivatives, and they are carried along trivially through the manipulations discussed above. In the end, the QHD-I result for the partition function has the form of Eq. (5.10), with $\mathcal{L}_0$ replaced by the full Lagrangian (3.11). [Conversely, more care is needed if one considers a Lagrangian with derivative interactions; one should not assume that the interacting partition function will have the form of Eq. (5.10).] This result is manifestly covariant, is valid for observers living on hyperplanes normal to any timelike n^μ, and requires no particular definition of coordinates.

If we now add source terms for the scalars and vectors, we can write the corresponding result for the Euclidean generating functional in QHD-I as

$$Z[j,J^\mu]=\int D[\bar{\psi},\psi]D[\phi]D[V^\mu]\exp\left[\frac{1}{\hbar}\int_0^{n\cdot\beta}d\tau\int d\sigma([\mathcal{L}]_{\partial\to d}+\mu n\cdot B+j\phi+J\cdot V)\right] . \tag{5.14}$$

Here we have exhibited a factor of $\hbar$ to enable us to isolate the one-loop contribution. The Lagrangian density is as written in Eq. (3.11) (with all terms bilinear in the fermion fields proportional to $\hbar$), and we are instructed to shift the derivative operators from ∂^μ to d^μ defined in Eq. (5.9). We can rewrite the integral in the exponential using

$$\int_0^{n\cdot\beta}d\tau\int d\sigma=i\int_0^{-in\cdot\beta}dt\int d\sigma=i\int_c d^4x , \tag{5.15}$$

where c denotes the familiar Matsubara contour. We will consistently suppress the (anti)periodic boundary conditions on the (fermion) boson path integrals, as well as some normalization constants arising from the integrals over boson momenta. These normalization factors are easy to determine (see Ref. 45), but they will not be needed in our discussion.

As expected, the generating functional depends on both the external sources j and J^μ and the thermodynamic parameters $T=(n\cdot\beta)^{-1}$, μ, u^μ, and $\mathcal{V}\equiv\int d\sigma$. The connected generating functional $W[j,J^\mu]$ is related to $Z[j,J^\mu]$ by

$$Z[j,J^\mu]\equiv\exp\left[\frac{i}{\hbar}W[j,J^\mu]\right] , \tag{5.16}$$

and the partition function Z follows by turning off the sources:

$$Z[0,0]\equiv Z\equiv\exp\left[-\int d\sigma_\mu\Phi^\mu(\alpha,\beta^\nu)\right], \tag{5.17}$$

with α and β^μ defined in Eqs. (3.1) and (3.2).

It is convenient to make a Legendre transformation from the "Lagrange multipliers" j and J^μ to the corresponding fields ϕ and V^μ by defining the effective action

$$\Gamma[\phi,V^\mu]\equiv W[j,J^\mu]-\int_c d^4x[j(x)\phi(x)+J_\nu(x)V^\nu(x)] . \tag{5.18}$$

As usual,[52] the sources on the right-hand side of (5.18) must be eliminated in terms of the fields, and the observed fields in the absence of sources are determined by extremizing the resulting effective action

$$\left[\frac{\delta\Gamma[\phi,V^\mu]}{\delta\phi(x)}\right]=0, \quad \left[\frac{\delta\Gamma[\phi,V^\mu]}{\delta V^\nu(x)}\right]=0 . \tag{5.19}$$

If we consider only constant and uniform external fields and sources (as appropriate for nuclear matter), $\Gamma[\phi,V^\mu]$ becomes proportional to the effective potential, and Eqs. (5.16) to (5.18) allow us to relate this to a four-vector grand potential through

$$\Gamma[\phi,V^\lambda]\equiv i\hbar\int d\sigma_\mu\Phi^\mu(\alpha,\beta^\nu;\phi,V^\lambda) . \tag{5.20}$$

Here $\Phi^\mu(\alpha,\beta^\nu;\phi,V^\lambda)$ depends parametrically on the field variables; the physical grand potential is to be determined by applying the extremization conditions (5.19).

B. One-loop approximation

An exact determination of the generating functional $Z[j,J^\mu]$ is not feasible. Various schemes have been devised for making systematic approximations to the grand potential (see Ref. 2 for a bibliography), but for the present discussion, we require only the one-loop term. (Feynman rules for imaginary time, which can be adapted to the calculation of the grand potential, are discussed in Sec. VI.) We will therefore carry out the conventional expansion of $W[j,J^\mu]$ around the classical fields and retain only the first quantum correction. Although our procedure follows that of the usual loop expansion, the higher-order quantum terms that we neglect here need not be treated order-by-order in $\hbar$ and can be summed in any desired fashion.

We begin by defining the Euclidean action (including the chemical potential) from Eq. (5.14) as

$$S[\phi,V^\mu]=i\int_c d^4x\{\hbar\bar{\psi}[\gamma_\mu(id^\mu-g_vV^\mu)-(M-g_s\phi)+\mu n_\mu\gamma^\mu]\psi+\tfrac{1}{2}(d_\mu\phi d^\mu\phi-m_s^2\phi^2)$$
$$-\tfrac{1}{4}(d_\mu V_\nu-d_\nu V_\mu)(d^\mu V^\nu-d^\nu V^\mu)+\tfrac{1}{2}m_v^2V_\mu V^\mu\} , \tag{5.21}$$

where we have set the meson self-interactions to zero (as in the original model of Walecka) and suppressed the counterterms $\delta\mathcal{L}$. The contour c is understood to be the Matsubara contour. The time derivatives in d^μ are defined as $\partial/\partial t$.

The equations defining the classical fields ϕ_0 and V_0^μ follow from extremizing the Euclidean action in the presence of the sources. Henceforth, we will assume that the external sources and classical fields are constants, so that the classical action becomes

$$S_{\rm cl}[\phi_0,V_0^\mu]=i\int_c d^4x(-\tfrac{1}{2}m_s^2\phi_0^2+\tfrac{1}{2}m_v^2V_0^\mu V_{0\mu}) . \tag{5.22}$$

We then expand the generating functional around the classical fields by defining

$$\phi(x)\equiv\phi_0+\hbar^{1/2}\sigma(x) \quad\text{and}\quad V^\mu(x)\equiv V_0^\mu+\hbar^{1/2}\eta^\mu(x) , \tag{5.23}$$

with the result

$$\exp\left[\frac{i}{\hbar}W[j,J^\mu]\right]=\exp\left[\frac{1}{\hbar}\left[S_{\rm cl}[\phi_0,V_0^\mu]+i\int_c d^4x[j\phi_0+J_\mu V_0^\mu]\right]\right]$$
$$\times\int D[\bar{\psi},\psi]D[\sigma]D[\eta^\mu]\exp\Big[i\int_c d^4x\{\bar{\psi}[\gamma_\mu(id^\mu-g_vV_0^\mu)-(M-g_s\phi_0)+\mu n\cdot\gamma]\psi$$
$$+\tfrac{1}{2}\sigma(-d_\mu d^\mu-m_s^2)\sigma-\tfrac{1}{4}(d_\mu\eta_\nu-d_\nu\eta_\mu)^2$$
$$+\tfrac{1}{2}m_v^2\eta^\mu\eta_\mu-\hbar^{1/2}g_v\bar{\psi}\gamma_\mu\psi\eta^\mu+\hbar^{1/2}g_s\bar{\psi}\psi\sigma\}\Big] , \tag{5.24}$$

where the classical action is given by Eq. (5.22). This expression is still exact (for uniform matter), although we have suppressed the counterterms and vacuum subtractions needed to render $W[j,J^\mu]$ finite; these will be included as needed.

We now make the relativistic Hartree (one-loop) approximation by retaining only the terms through $O(\hbar^0)$ in the exponential, which produces

$$\exp\left[\frac{i}{\hbar}W_{\rm RHA}[j,J^{\mu}]\right]=\exp\left[\frac{1}{\hbar}\left[S_{\rm cl}[\phi_0,V_0^{\mu}]+i\int_c d^4x[j\phi_0+J_{\mu}V_0^{\mu}]\right]\right](Z_0)_{\rm s}(Z_0)_{\rm v}(Z_B)_{\rm RHA}\ , \tag{5.25}$$

$$(Z_0)_{\rm s}=\int D[\sigma]\exp\left[i\int_c d^4x\tfrac{1}{2}\sigma(-d_{\mu}d^{\mu}-m_{\rm s}^2)\sigma\right]\ , \tag{5.26}$$

$$(Z_0)_{\rm v}=\int D[\eta^{\mu}]\exp\left\{i\int_c d^4x\left[-\tfrac{1}{4}(d_{\mu}\eta_{\nu}-d_{\nu}\eta_{\mu})^2+\tfrac{1}{2}m_{\rm v}^2\eta^{\mu}\eta_{\mu}\right]\right\}\ , \tag{5.27}$$

$$(Z_B)_{\rm RHA}=\int D[\bar{\psi},\psi]\exp\left[i\int_c d^4x\{\bar{\psi}[\gamma_{\mu}(id^{\mu}-g_{\rm v}V_0^{\mu})-(M-g_{\rm s}\phi_0)+\mu n_{\mu}\gamma^{\mu}]\psi\}\right]\ . \tag{5.28}$$

We compute the baryon partition function by evaluating Eq. (5.28) on the Matsubara contour. With the relations (5.15) and (5.7)–(5.9), we can rewrite this function as

$$\begin{aligned}(Z_B)_{\rm RHA}&=\int D[\bar{\psi},\psi]\exp\left[\int_0^{n\cdot\beta}d\tau\int d\sigma\{\bar{\psi}[\gamma_{\mu}(id^{\mu}-g_{\rm v}V_0^{\mu})-(M-g_{\rm s}\phi_0)+\mu n_{\mu}\gamma^{\mu}]\psi\}\right]\\&=\int D[\bar{\psi},\psi]\exp\left\{\int_0^{n\cdot\beta}d\tau\int d\sigma\left[\bar{\psi}\left[-n\cdot\gamma\frac{\partial}{\partial\tau}+i[\gamma_{\mu}-u_{\mu}(n\cdot\gamma)/(n\cdot u)]\tilde{\partial}^{\mu}\right.\right.\right.\\&\qquad\left.\left.\left.-g_{\rm v}\gamma_{\mu}\tilde{V}_0^{\mu}-M^*+(\mu-g_{\rm v}n\cdot V_0)(n\cdot\gamma)\right]\psi\right]\right\}\ .\end{aligned} \tag{5.29}$$

Here we have isolated the directional derivative $\partial/\partial\tau$ normal to the hyperplane and used the shorthands $M^*=M-g_{\rm s}\phi_0$ and $V_0^{\mu}=(n\cdot V_0)n^{\mu}+\tilde{V}_0^{\mu}$, with $\tilde{V}_0^{\mu}$ in the hyperplane σ.

We can now follow the analysis in Ref. 45 by dividing the integral over τ into N segments of length ϵ, such that $N\epsilon=n\cdot\beta$. As discussed in that reference, it is permissible to treat the spacelike hypersurface as a continuum for the algebraic manipulations (but see the discussion of normalization factors below).

Proceeding exactly as in Ref. 45, the partition function is given by the determinant of the imaginary-time baryon propagator, which can be written in a Fourier representation as

$$i\mathcal{G}_{mm'}(\tilde{x}-\tilde{y})=\frac{1}{n\cdot\beta}\sum_{j=-N/2}^{N/2-1}\frac{1}{\mathcal{V}}\sum_{\tilde{k}}e^{-i\tilde{k}\cdot(\tilde{x}-\tilde{y})}e^{-i\epsilon\omega_j(m-m'+1)}\left[\frac{in\cdot\gamma\Omega_{jk}(u)+\gamma\cdot\tilde{k}^*+M^*}{\Omega_{jk}^2(u)+E^{*2}(\tilde{k}^*)}\right]\ , \tag{5.30}$$

where

$$\omega_j\equiv\frac{(2j+1)\pi}{n\cdot\beta},\quad \tilde{k}^{*\mu}\equiv\tilde{k}^{\mu}-g_{\rm v}\tilde{V}_0^{\mu},\quad E^*(\tilde{k}^*)\equiv(-\tilde{k}^{*2}+M^{*2})^{1/2}\ , \tag{5.31}$$

and

$$\Omega_{jk}(u)\equiv\frac{i}{\epsilon}(e^{-i\epsilon\omega_j}-1)-i\nu+i\frac{u\cdot\tilde{k}^*}{n\cdot u}\ , \tag{5.32}$$

with $\nu\equiv\mu-g_{\rm v}(u\cdot V_0)/(n\cdot u)$. Here $\tilde{x}$ and $\tilde{y}$ are spacelike coordinates confined to a hyperplane σ of volume $\mathcal{V}$ normal to n^{μ}, $\tilde{k}^{\mu}$ and $\tilde{k}^{*\mu}$ are spacelike four-momenta confined to the reciprocal hyperplane, and $\sum_{\tilde{k}}$ runs only over the reciprocal hyperplane. The indices m and m' label nodes along the Matsubara contour.

Note two important points. First, the Fourier series involving the Matsubara frequencies ω_j has the same form for any n^{μ}, but the frequencies are defined in terms of the temperature $T=1/n\cdot\beta$ for observers that live normal to n^{μ}. Second, all normalization factors required to evaluate the determinant of $\mathcal{G}$ involve either the volume of the hyperplane and the length of the Matsubara contour or the corresponding infinitesimal quantities that arise when we divide spacetime into a lattice.[45] Since these objects are all "coordinate free," the proof that the path integral is normalized correctly follows here just as in Ref. 45, and we will not consider this point further.

The evaluation of the determinant of $\mathcal{G}$ proceeds straightforwardly, and various products over Matsubara frequencies can be computed using the formulas in the reference noted above. One finds

$$\begin{aligned}(Z_B)_{\rm RHA}&=\prod_{\tilde{k}}\lim_{\epsilon\to 0}\prod_{j=-N/2}^{N/2-1}\epsilon^4e^{4i\epsilon\omega_j}[\Omega_{jk}^2(u)+E^{*2}(\tilde{k}^*)]^2\\&=\exp\left\{n\cdot\beta\sum_{\tilde{k}s}[E^*(k^*)+\mu-g_{\rm v}n\cdot V_0]\right\}\prod_{\tilde{k}s}(1+\exp\{-n\cdot\beta[E^*(\tilde{k}^*)-\nu+u\cdot\tilde{k}^*/n\cdot u]\})\\&\qquad\times(1+\exp\{-n\cdot\beta[E^*(\tilde{k}^*)+\nu-u\cdot\tilde{k}^*/n\cdot u]\})\end{aligned} \tag{5.33}$$

for the baryon partition function in the RHA. (The sum and product over s incorporate the spin-isospin degeneracy.)

To arrive at this result, one must recall that gamma matrices defined for a particular set of coordinates can be transformed to another set of coordinates through a similarity transformation, which has no effect on a determinant; thus, one can pick a simple set of coordinates (such as those in Ref. 45) to evaluate the determinant.

Equation (5.33) is a manifestly covariant expression for the RHA partition function on a set of hyperplanes normal to any n^{μ}. It is also "coordinate free," since it involves only scalar products and scalar objects defined for these hyperplanes. Note, however, that our expression is not finite, since the initial exponential contains a contribution from the zero-point energy, which must be renormalized, as we discuss shortly.

Similar manipulations can be carried out for the (free) scalar and vector partition functions using the techniques discussed in Ref. 45. For example, the general form of the scalar partition function is given by

$$(Z_0)_s=\exp\left[-n\cdot\beta\sum_{\tilde{k}}\tfrac{1}{2}\omega_k\right]\prod_{\tilde{k}}(1-e^{-n\cdot\beta(\omega_k+u\cdot\tilde{k}/n\cdot u)})^{-1}\ , \tag{5.34}$$

where $\omega_k^2\equiv-\tilde{k}^2+m_s^2$ and $n\cdot\tilde{k}=0$.

We can now combine the results in Eqs. (5.25) and (5.33) to write the connected Hartree (one-loop) generating functional as

$$\begin{aligned}W_{\mathrm{RHA}}[j,J^{\mu}]=-i\left[S_{\mathrm{cl}}[\phi_0,V_0^{\mu}]+i\int_c d^4x\,[j\phi_0+J_{\mu}V_0^{\mu}]\right]-i\hbar n\cdot\beta\sum_{\tilde{k}s}[E^*(\tilde{k}^*)+\mu-g_{\mathrm{v}}n\cdot V_0]\\ -i\hbar\sum_{\tilde{k}s}\Big\{\ln(1+\exp\{-n\cdot\beta[E^*(\tilde{k}^*)-\nu+u\cdot\tilde{k}^*/n\cdot u]\})\\ +\ln(1+\exp\{-n\cdot\beta[E^*(\tilde{k}^*)+\nu-u\cdot\tilde{k}^*/n\cdot u]\})\Big\}\ .\end{aligned} \tag{5.35}$$

Here both the sources and classical fields are assumed constant and uniform. We have neglected the meson partition functions as they will be relevant only at exceedingly high temperatures ($T'\gtrsim m_s,m_v$).

The Hartree effective potential is now determined by the Legendre transform

$$\Gamma_{\mathrm{RHA}}[\phi,V^{\mu}]=W_{\mathrm{RHA}}[j,J^{\mu}]-\int_c d^4x\,(j\phi+J_{\mu}V^{\mu})\ , \tag{5.36}$$

where the new fields ϕ and V^{μ} differ from the classical ones by terms of $O(\hbar)$. Since the classical action (in the presence of sources) is stationary about ϕ_0 and V_0^{μ}, the Legendre transformation is trivial to this order, and since the fields are constants, Eq. (5.20) allows us to write

$$\begin{aligned}\int d\sigma_{\mu}\Phi^{\mu}(\alpha,\beta^{\nu};\phi,V^{\lambda})=\mathcal{V}n\cdot\beta(\tfrac{1}{2}m_s^2\phi^2-\tfrac{1}{2}m_v^2V_{\mu}V^{\mu})-n\cdot\beta\sum_{\tilde{k}s}[E^*(\tilde{k}^*)+\mu-g_{\mathrm{v}}n\cdot V]\\ -\sum_{\tilde{k}s}\Big\{\ln(1+\exp\{-n\cdot\beta[E^*(\tilde{k}^*)-\nu+u\cdot\tilde{k}^*/n\cdot u]\})\\ +\ln(1+\exp\{-n\cdot\beta[E^*(\tilde{k}^*)+\nu-u\cdot\tilde{k}^*/n\cdot u]\})\Big\}\ ,\end{aligned} \tag{5.37}$$

where we have restored the units $\hbar=1$.

The first sum in Eq. (5.37) represents the zero-point energy and can be handled in the usual fashion. The $\mu-g_v(n\cdot V)$ term arises because we failed to normal order the baryon current in the Lagrangian; the correct normal ordering can be restored by omitting this term. The remaining divergent sum is a Lorentz scalar that can be evaluated for $n^{\mu}=u^{\mu}$ by making the familiar vacuum subtraction and including Lorentz scalar counterterms.[1] At the end, the sum can be replaced by $-\mathcal{V}\Delta\mathcal{E}'_{\mathrm{ZP}}(M-g_s\phi)$, with $\Delta\mathcal{E}'_{\mathrm{ZP}}(m^*)$ from Eq. (4.52).

By converting the remaining momentum sum to an integral over the reciprocal spacelike hyperplane σ_k and cancelling an overall volume factor, we find

$$n\cdot\Phi(\alpha,\beta^{\nu};\phi,V^{\lambda})=n\cdot\beta[\tfrac{1}{2}m_s^2\phi^2-\tfrac{1}{2}m_v^2V\cdot V+\Delta\mathcal{E}'_{\mathrm{ZP}}(M-g_s\phi)]-\sum_s\int\frac{d\sigma_k}{(2\pi)^3}[\ln(1+e^{-\beta\cdot K+\alpha})+\ln(1+e^{\beta\cdot\bar{K}-\alpha})]\ . \tag{5.38}$$

Here we have introduced on-shell *canonical* four-momenta for particles and antiparticles:

$$\begin{aligned}K^{\mu}&\equiv E^*(\tilde{k}^*)n^{\mu}+g_vV^{\mu}+\tilde{k}^{*\mu}=[E^*(\tilde{k}^*)+g_vn\cdot V]n^{\mu}+\tilde{k}^{\mu}\ ,\\ \bar{K}^{\mu}&\equiv-E^*(\tilde{k}^*)n^{\mu}+g_vV^{\mu}+\tilde{k}^{*\mu}=[-E^*(\tilde{k}^*)+g_vn\cdot V]n^{\mu}+\tilde{k}^{\mu}\ ,\end{aligned} \tag{5.39}$$

and used the relation $\mu(n\cdot\beta)=\alpha$. Note that the timelike components of the canonical four-momenta contain the true single-particle energies $\epsilon^{(\pm)}(k)=g_v(n\cdot V)\pm E^*(\tilde{k}^*)$, which yield the correct values to compare with the thermal potential α in the arguments of the exponentials. Equation (5.38) gives a coordinate-free, manifestly covariant expression for the four-vector grand potential.

We are now in a position to discuss thermodynamics and the relation between Eq. (5.38) and the results of Sec. IV. We will demonstrate the following:

(1) The extremization conditions that determined the physical values of ϕ and V^μ agree with those deduced from Eqs. (4.31), (4.32), and (4.50).

(2) Equation (5.38) reproduces the covariant expression for the pressure in Eq. (4.49), which follows from $T^{\mu\nu}$. This will verify the virial theorem, since the present calculation determines p from the partition function.

(3) Results for physical observables are independent of the quantization hyperplane defined by n^μ.

As a first step, recall that thermodynamic arguments imply that Φ^μ is proportional to the fluid four-velocity u^μ. While this is apparent for the first term on the right-hand side of (5.38), it is not yet obvious for the second. We can, however, introduce on-shell *kinetic* momenta

$$K^{*\mu} \equiv E^*(\tilde{k}^*) n^\mu + \tilde{k}^{*\mu} \quad \text{and} \quad \bar{K}^{*\mu} \equiv -E^*(\tilde{k}^*) n^\mu + \tilde{k}^{*\mu} \tag{5.40}$$

that obey

$$K^{*\mu} K^*_\mu = \bar{K}^{*\mu} \bar{K}^*_\mu = E^{*2}(\tilde{k}^*) + \tilde{k}^{*2} = M^{*2} \tag{5.41}$$

for both particles and antiparticles. The on-shell kinetic momenta lie on a hyperboloid of mass M^* in four-dimensional momentum space, and rather than integrate over the spacelike hyperplane σ_k, we can integrate over *all* momentum space, as long as we enforce the condition (5.41). In this way, the integral in Eq. (5.38) can be rewritten as

$$-\sum_s \int \frac{d^4k}{(2\pi)^3} [\delta(n\cdot k^* - E^*(\tilde{k}^*)) \ln(1+e^{-\beta\cdot k+\alpha}) + \delta(n\cdot k^* + E^*(\tilde{k}^*)) \ln(1+e^{\beta\cdot k-\alpha})]$$

$$= -2n_\mu \frac{\gamma}{(2\pi)^3} \int d^4k\, k^{*\mu} \delta(k^{*2} - M^{*2}) [\theta(\beta\cdot k^*) \ln(1+e^{-\beta\cdot k+\alpha}) - \theta(-\beta\cdot k^*) \ln(1+e^{\beta\cdot k-\alpha})] , \tag{5.42}$$

where we have replaced the sum over spin and isospin by the degeneracy factor γ. As the integration runs over four-dimensional momentum space, the dependence on n^μ disappears from the integration measure. Note that k^μ and $k^{*\mu} \equiv k^\mu - g_v V^\mu$ have four independent components and are not restricted to the reciprocal hyperplane σ_k. We have also let $n\cdot k^* \to \beta\cdot k^*$ inside the θ functions; these scalars have the same sign because all vectors involved are timelike.

By combining Eqs. (5.38) and (5.42), we can express the four-vector grand potential as

$$\Phi^\mu(\alpha,\beta^\nu;\phi,V^\lambda) = \beta^\mu[\tfrac{1}{2} m_s^2 \phi^2 - \tfrac{1}{2} m_v^2 V_\sigma V^\sigma + \Delta\mathcal{E}'_{\rm ZP}(M^*)]$$

$$-2\frac{\gamma}{(2\pi)^3} \int d^4k\, k^{*\mu} \delta(k^{*2} - M^{*2}) [\theta(\beta\cdot k^*) \ln(1+e^{-\beta\cdot k+\alpha}) - \theta(-\beta\cdot k^*) \ln(1+e^{\beta\cdot k-\alpha})] . \tag{5.43}$$

Here M^* is still simply a shorthand for $M - g_s\phi$. Notice that all dependence on the original hyperplane (defined by n^μ) has disappeared.

Now the extremization conditions that determine the physical values of ϕ and V^μ [see Eq. (5.19)] can be derived by first differentiating Eq. (5.38) with respect to these variables, and then switching to on-shell momenta. The results can be written as

$$M^* = M - \frac{g_s^2}{m_s^2} \rho_s(M^*) - \frac{g_s^2}{m_s^2} \frac{\partial \Delta\mathcal{E}'_{\rm ZP}(M^*)}{\partial M^*} , \tag{5.44}$$

$$V^\mu = \frac{g_v}{m_v^2} B^\mu , \tag{5.45}$$

where $\rho_s(M^*)$ and B^μ are exactly the same as in Eqs. (4.33) and (4.34). Thus the extremization conditions on the effective potential (and hence on Φ^μ) reproduce the field equations determined from the self-consistent diagrammatic analysis. The additional zero-point contribution in Eq. (5.44) agrees with the correction to the scalar density in Eq. (4.50). This correction must be added to the MFT self-consistency condition of Eq. (4.31) when working in the RHA.

Equation (5.45) implies that $V^\mu \propto u^\mu$, so that Eqs. (4.35) and (5.43) imply that $\Phi^\mu \propto u^\mu$ as well. Moreover, the relation $\Phi^\mu = -p\beta^\mu$ allows us to arrive at the following covariant expression for the thermodynamic pressure,

$$p = \frac{-m_s^2}{2g_s^2}(M - M^*)^2 + \frac{g_v^2}{2m_v^2} B^\mu B_\mu - \Delta\mathcal{E}'_{\rm ZP}(M^*)$$

$$+ \frac{2}{\beta} \frac{\gamma}{(2\pi)^3} \int d^4k (k^*\cdot u) \delta(k^{*2} - M^{*2}) [\theta(\beta\cdot k^*) \ln(1+e^{-\beta\cdot k+\alpha}) - \theta(-\beta\cdot k^*) \ln(1+e^{\beta\cdot k-\alpha})] , \tag{5.46}$$

which agrees with Eqs. (4.55) and (4.49). Here Eq. (5.45) has been used to eliminate V^μ, and M^* is to be determined by solving Eq. (5.44). Moreover, by working directly with Eq. (5.43), it is easy to verify that the thermodynamic identities[5]

$$\left[\frac{\partial(p\beta^\mu)}{\partial\beta_\nu}\right]_\alpha=-T^{\mu\nu},\quad \left[\frac{\partial(p\beta^\mu)}{\partial\alpha}\right]_{\beta_\nu}=B^\mu\ , \tag{5.47}$$

are satisfied, where $T^{\mu\nu}$ and B^μ are given by the covariant expressions (4.43) and (4.34), with the additional zero-point contribution to $T^{\mu\nu}$ from Eq. (4.51).

Finally, since we have verified that the one-loop approximation is thermodynamically consistent, we can apply the covariant form of Gibbs' relation [Eq. (3.10)], together with Eqs. (5.46) and (5.47), to find the entropy flux:

$$\begin{aligned}S^\mu&=p\beta^\mu+\beta_\nu T^{\mu\nu}-\alpha B^\mu\\&=p\beta^\mu-\beta_\nu\left[\frac{\partial(p\beta^\mu)}{\partial\beta_\nu}\right]_\alpha-\alpha\left[\frac{\partial(p\beta^\mu)}{\partial\alpha}\right]_{\beta_\nu}\\&=-2\frac{\gamma}{(2\pi)^3}\int d^4k\, k^{*\mu}\delta(k^{*2}-M^{*2})\\&\qquad\times(\theta(\beta\cdot k^*)\{n_f(\beta\cdot k,\alpha)\ln n_f(\beta\cdot k,\alpha)+[1-n_f(\beta\cdot k,\alpha)]\ln[1-n_f(\beta\cdot k,\alpha)]\}\\&\qquad\quad-\theta(-\beta\cdot k^*)\{\bar n_f(\beta\cdot k,\alpha)\ln\bar n_f(\beta\cdot k,\alpha)+[1-\bar n_f(\beta\cdot k,\alpha)]\ln[1-\bar n_f(\beta\cdot k,\alpha)]\})\ ,\end{aligned} \tag{5.48}$$

where $n_f(\beta\cdot k,\alpha)$ and $\bar n_f(\beta\cdot k,\alpha)$ are defined in Eq. (3.36). As expected, S^μ is proportional to u^μ.

VI. COVARIANT FEYNMAN RULES IN IMAGINARY TIME

A. Noninteracting contour propagators

Feynman rules for the Euclidean Green's functions of QHD-I can be derived from the Euclidean generating functional $Z[j,J^\mu]$, which is given in Eq. (5.14). The procedure is standard: Remove nonquadratic terms from the action in favor of functional derivatives with respect to the sources and then evaluate the remaining Gaussian integrals to determine the noninteracting propagators. To perform the latter step we must invert the differential operators that appear in Eqs. (5.26)–(5.28), with V_0^μ and ϕ_0 set to zero in the last equation, subject to the appropriate thermal boundary conditions.

In fact, we can invert these operators for a more general contour than the Matsubara contour and in doing so relate the noninteracting Euclidean propagators to the noninteracting real-time propagators given in the Feynman rules of Sec. III. In this paper we will simply state the results without a detailed derivation, which we reserve for Ref. 34. The noninteracting contour propagators given below in spectral form are covariant generalizations of expressions given in Ref. 2.

The noninteracting scalar contour propagator is defined to satisfy [see Eq. (5.26)]

$$(d_\mu d^\mu+m_s^2)\Delta_0^{(c)}(t,\tilde x)=-\delta_c(t,\tilde x)\ , \tag{6.1}$$

where $\tilde x^\mu$ lies in $\sigma_{t=0}$, t can be complex, d^μ is defined in Eq. (5.9), and the appropriate boundary condition is

$$C_0^>(t,\tilde x)=C_0^<(t+in\cdot\beta,\tilde x)\ , \tag{6.2}$$

where

$$i\Delta_0^{(c)}(t,\tilde x)\equiv\theta_c(t)C_0^>(t,\tilde x)+\theta_c(-t)C_0^<(t,\tilde{\mathbf{x}})\ . \tag{6.3}$$

Here θ_c and δ_c are straightforward extensions of the familiar θ and δ functions to times lying on a contour c and spatial variables lying in the hyperplane $\sigma_{t=0}$. At this point the contour c has not been specified. The solution to Eq. (6.1) can be written as[34]

$$i\Delta_0^{(c)}(t,\tilde x)=\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot\tilde x}e^{-it(k\cdot u)/(n\cdot u)}\rho_0(k)\times\left[\frac{1}{1-e^{-\beta\cdot k}}\theta_c(t)+\frac{1}{e^{\beta\cdot k}-1}\theta_c(-t)\right]\ , \tag{6.4}$$

with the regularized spectral function $\rho_0(k)$ defined by

$$\rho_0(k)\equiv2\pi[\theta(n\cdot k)-\theta(-n\cdot k)]\delta_\epsilon(k^2-m_s^2)\ . \tag{6.5}$$

One can also derive Eq. (6.4) directly from grand interaction-picture fields $\hat\phi(t,\tilde x)$ quantized on the hyperplane $\sigma_{t=0}$.

The noninteracting Euclidean propagator follows by letting c be the Matsubara contour. With $t\to-i\tau$, we define

$$\Delta_0^{(E)}(\tau,\tilde x)\equiv\Delta_0^{(c)}(-i\tau,\tilde x) \tag{6.6}$$

and the Fourier transforms

$$\Delta_0^{(E)}(\nu_j,\tilde p)=\int_0^{n\cdot\beta}d\tau\int d\sigma\, e^{i\tilde p\cdot\tilde x}e^{i\nu_j\tau}\Delta_0^{(E)}(\tau,\tilde x)\ , \tag{6.7}$$

$$\Delta_0^{(E)}(\tau,\tilde x)=\frac{1}{n\cdot\beta}\sum_j\int\frac{d\sigma_p}{(2\pi)^3}e^{-i\tilde p\cdot\tilde x}e^{-i\nu_j\tau}\Delta_0^{(E)}(\nu_j,\tilde p)\ , \tag{6.8}$$

which involve the discrete frequencies $\nu_j=2j\pi/n\cdot\beta$ (see below). If one inserts (6.4) into (6.7), the distribution functions drop out, leaving

$$\begin{aligned}i\Delta_0^{(E)}(\nu_j,\tilde p)&=\int\frac{d(n\cdot p)}{2\pi}\frac{i\rho_0(p)}{\nu_j+i(p\cdot u)/(n\cdot u)}\\&=\frac{1}{-p_j^2+m_s^2}\ ,\end{aligned} \tag{6.9}$$

where p_j^μ is defined in Eq. (6.16) below. Similar expressions can be written for the noninteracting baryon and vector meson propagators. [For example, the noninteracting baryon propagator is given by the continuum limit of Eq. (5.30) with $\phi_0 = V_0^\mu = 0$.] The imaginary-time Feynman rules listed in the next subsection allow one to express interacting Euclidean propagators as order-by-order expansions in the coupling constants and these noninteracting propagators. Note that the propagators generated by $Z[j,J^\mu]$ and the imaginary-time Feynman rules involve operators in the covariant grand Heisenberg picture, in contrast to the canonical Heisenberg-picture propagators generated by the real-time rules of Sec. III.

Let us clarify the role of the quantization hyperplane and the different Heisenberg pictures. A freely falling observer in a laboratory frame will trace out a world line n_1^μ and thereby define a set of hyperplanes normal to n_1^μ, on which he makes observations. A second freely falling observer in a different laboratory frame will define a set of hyperplanes normal to n_2^μ and will appear to be moving with a finite velocity relative to the first observer. Each observer can choose to quantize a field theory on any hyperplane, and if the theory or approximation is covariant, physical observables must be independent of the choice of hyperplane. Nevertheless, the finite-temperature propagators will depend, in general, not only on the fluid four-velocity u^μ but also on the timelike four-vector n^μ that defines the quantization hyperplane.

After one quantizes on a particular hyperplane $\sigma_{t=0}$, the covariant Heisenberg picture evolves the system on successive hyperplanes σ in the direction of n^μ. The time evolution operator contains $\int d\sigma\, n_\mu n_\nu \hat{T}^{\mu\nu} = n_\nu \hat{P}^\nu$, and no explicit reference is made to the moving fluid. In contrast, time evolution in the covariant *grand* Heisenberg picture involves $\int d\sigma\, n_\mu u_\nu \hat{T}^{\mu\nu}/(n\cdot u)$, plus a term involving the chemical potential, which is the same as the grand canonical weighting operator from the partition function. Neglecting the chemical potential, we see that the grand Heisenberg picture follows the evolution of *fluid elements* along the hyperplanes σ. The physical reason for this is that the grand Heisenberg picture must incorporate the constraint that the fluid has a nonzero velocity in an arbitrary reference frame. Therefore, time evolution in the grand Heisenberg picture depends explicitly on the fluid four-velocity u^μ and on $n\cdot u$ in particular, while time evolution in the canonical Heisenberg picture is performed without reference to $n\cdot u$. The consequence is that real-time propagators in the former picture have explicit $n\cdot u$ dependence while those in the latter picture are independent of n^μ.

For real time arguments, the difference between operators in the two pictures is simply a phase, because the pictures are related by a unitary transformation. For a real time argument $t = x\cdot n$, we can rewrite the scalar contour propagator in terms of the four-vector x^μ:

$$i\Delta_0^{(c)}(x^\mu) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} e^{i(x\cdot n)[k\cdot n - (k\cdot u)/(n\cdot u)]} \times \rho_0(k) \left[\frac{1}{1-e^{-\beta\cdot k}} \theta_c(x\cdot n) + \frac{1}{e^{\beta\cdot k}-1} \theta_c(-x\cdot n) \right]. \quad (6.10)$$

The evolution operators commute with each other, so the phase factor between pictures is precisely the second exponential in Eq. (6.10). If this is removed, we are left with an expression for the canonical Heisenberg-picture contour propagator. Thus the (1,1) component of the real-time scalar propagator is given by

$$\begin{aligned} i\Delta_0^{(11)}(x^\mu) &= \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \rho_0(k) \left[\frac{1}{1-e^{-\beta\cdot k}} \theta(x\cdot n) + \frac{1}{e^{\beta\cdot k}-1} \theta(-x\cdot n) \right] \\ &= \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \left[i\theta(\beta\cdot k)\Delta_{0F}(k) + i\theta(-\beta\cdot k)\Delta_{0F}^\dagger(k) + \rho_0(k)\frac{1}{e^{\beta\cdot k}-1} \right], \end{aligned} \quad (6.11)$$

which agrees with Eq. (3.41). Note that the dependence on n^μ has disappeared in the final expression. Similar results can be obtained for the other thermal matrix elements.

B. Imaginary-time Feynman rules

In this section, we list imaginary-time momentum-space Feynman rules in covariant form for QHD-I. These rules generate propagators in the covariant *grand* Heisenberg picture, and so the rules depend explicitly on the timelike unit four-vector n^μ, as discussed above. In general, the n^μ dependence does not drop out until observables are calculated (see below). The Feynman rules can also be applied to the calculation of the thermodynamic potential, but the combinatorial factors are rather intricate (see Refs. 2 and 17 for a discussion).

The rules for the mth order contribution to $(i)\times($Euclidean propagator) in QHD-I are as follows (see Fig. 6):

(1) Draw all topologically distinct diagrams with two external propagator lines and m vertices connected by internal propagator lines. Do not include diagrams with completely disconnected pieces (that is, not connected to *any* external line).

(2) Assign a direction to each line. Associate a directed four-momentum on σ_k (i.e., $\tilde{k}^\mu$ such that $n\cdot\tilde{k}=0$) and a discrete (Matsubara) frequency with each line. The frequencies obey

$$\nu_j = \frac{2j\pi}{n\cdot\beta} \quad \text{(bosons)}, \quad (6.12)$$

$$\omega_j = \frac{(2j+1)\pi}{n\cdot\beta} \quad \text{(fermions)}, \quad (6.13)$$

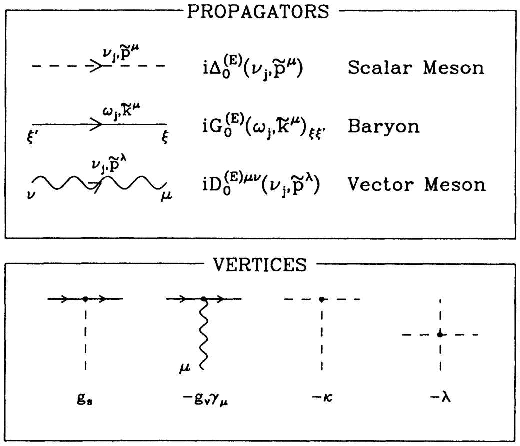

FIG. 6. Components of imaginary-time Feynman diagrams for QHD-I.

with integral j. Conserve four-momentum on σ_k and Matsubara frequency at each vertex.

(3) Each vertex is assigned a factor indicated in Fig. 6. [σNN vertex: g_s; ωNN vertex: $-g_v\gamma_\mu$; σ^3 vertex: $-\kappa$; σ^4 vertex: $-\lambda$.]

(4) Each directed line gets a factor of (i) times the appropriate noninteracting propagator, $\Delta_0^{(E)}(\nu_j,\tilde{p}^\mu)$, $D_0^{(E)\mu\nu}(\nu_j,\tilde{p}^\lambda)$, or $G_0^{(E)}(\omega_j,\tilde{k}^\mu)_{\xi\xi'}$, as indicated in Fig. 6. The propagators are given by

$$i\Delta_0^{(E)}(\nu_j,\tilde{p}^\mu)=\frac{1}{-p_j^2+m_s^2}\ , \tag{6.14}$$

$$iD_0^{(E)\mu\nu}(\nu_j,\tilde{p}^\lambda)=\frac{-g^{\mu\nu}+p_j^\mu p_j^\nu/m_v^2}{-p_j^2+m_v^2}\ , \tag{6.15}$$

where p_j^μ is defined as

$$p_j^\mu \equiv \left[i\nu_j - \frac{\tilde{p}\cdot u}{n\cdot u} \right] n^\mu + \tilde{p}^\mu\ , \tag{6.16}$$

and

$$iG_0^{(E)}(\omega_j,\tilde{k}^\mu)_{\xi\xi'} = \left[\frac{1}{-\not{k}_j+M} \right]_{\xi\xi'} = \frac{(\not{k}_j+M)_{\xi\xi'}}{-k_j^2+M^2}$$
$$= \frac{\left[(n\cdot\gamma)\left[i\omega_j - \frac{\tilde{k}\cdot u}{n\cdot u}+\mu \right] + \gamma\cdot\tilde{k}+M \right]_{\xi\xi'}}{\left[\omega_j + i\frac{\tilde{k}\cdot u}{n\cdot u} - i\mu \right]^2 + E^2(\tilde{k})} \tag{6.17}$$

with

$$k_j^\mu \equiv \left[i\omega_j - \frac{\tilde{k}\cdot u}{n\cdot u} + \mu \right] n^\mu + \tilde{k}^\mu \tag{6.18}$$

and

$$E^2(\tilde{k}) \equiv -\tilde{k}^2 + M^2\ . \tag{6.19}$$

(5) Sum over all repeated spacetime $(\mu,\nu,\ldots)$ and Dirac $(\xi,\xi',\ldots)$ indices.

(6) Integrate $\int d\sigma_k/(2\pi)^3$ over all independent four-momenta on σ_k and sum $(1/n\cdot\beta)\sum_j$ over all independent Matsubara frequencies.

(7) Include a factor of (-1) for each closed fermion loop.

(8) Any single-fermion line forming a tadpole loop must be multiplied by a convergence factor such as $e^{i\omega_j\eta}$, where $\eta\to 0^+$ at the end of the calculation. Since the convergence factor defines an ordering that is not normal ordering, a subtraction is still needed to define the tadpole loop.

(9) A symmetry factor may be needed in diagrams with σ^3 and σ^4 vertices. The factor is the same as at $T=0$.

If desired, one can replace $\int d\sigma_k$ by $\int d^4k\,\delta(n\cdot k)$ in loop integrals and then let $\tilde{k}^\mu \to k^\mu$, but this is really just an alternative notation. However, as we will see below, it is possible in some circumstances to introduce an integral over $n\cdot k$ with an appropriate delta function that eliminates the dependence on n^μ.

We observe that the covariant imaginary-time rules are quite similar to those written in the comoving frame.[8,9] Recall that there are basically three problems to be solved to write imaginary-time propagators covariantly: How does one define four-vectors when spatial parts are real and temporal parts are imaginary? How does one incorporate the Lagrange multiplier for the fluid velocity? How do general observers define the Matsubara frequencies? The first problem is solved by using two types of four-vectors: a real one in the quantization hyperplane and a complex one normal to the hyperplane. The complicated form of the complex part arises because we work in the grand canonical Heisenberg picture, and the Lagrange multipliers for the fluid velocity and chemical potential enter here as imaginary shifts in the Matsubara frequency. Finally, the Matsubara frequencies are defined using $T=1/n\cdot\beta$; thus, each observer defines the frequencies with the observed temperature, rather than the proper temperature.

C. Application to nuclear matter

We now apply the imaginary-time rules to the calculation of observables in the one-loop approximation to check that the real-time results are produced. Thermal matrix elements of Heisenberg-picture operators can be expressed in terms of the imaginary-time propagators using standard manipulations. For example,

$$\langle\langle \hat{\bar{\psi}}_H(-i\tau,\tilde{x}^\mu)\Gamma\hat{\psi}_H(-i\tau,\tilde{x}^\mu)\rangle\rangle = \langle\langle \hat{\bar{\psi}}_K(-i\tau,\tilde{x}^\mu)\Gamma\hat{\psi}_K(-i\tau,\tilde{x}^\mu)\rangle\rangle$$
$$= -\frac{i}{n\cdot\beta}\sum_j e^{i\omega_j\eta}\int\frac{d\sigma_k}{(2\pi)^3}\mathrm{Tr}[\Gamma G^{(E)}(\omega_j,\tilde{k}^\mu)]\ , \tag{6.20}$$

where the H and K labels refer to the covariant Heisenberg and grand Heisenberg pictures, and Γ does not contain derivatives. Thus the baryon current and scalar density follow from

$$\langle\langle \hat{B}^{\mu} \rangle\rangle = -\frac{i}{n\cdot\beta}\sum_j e^{i\omega_j\eta}\int\frac{d\sigma_k}{(2\pi)^3}\mathrm{Tr}[\gamma^{\mu}G^{(E)}(\omega_j,\tilde{k}^{\mu})] , \tag{6.21}$$

$$\langle\langle \hat{\rho}_s \rangle\rangle = -\frac{i}{n\cdot\beta}\sum_j e^{i\omega_j\eta}\int\frac{d\sigma_k}{(2\pi)^3}\mathrm{Tr}[G^{(E)}(\omega_j,\tilde{k}^{\mu})] . \tag{6.22}$$

To formulate expressions for $T^{\mu\nu}$, use

$$\langle\langle \hat{\bar{\psi}}_H(-i\tau,\tilde{x}^{\mu})\partial^{\nu}\hat{\psi}_H(-i\tau,\tilde{x}^{\mu}) \rangle\rangle = \langle\langle \hat{\bar{\psi}}_K(-i\tau,\tilde{x}^{\mu})(d^{\nu}-i\mu n^{\nu})\hat{\psi}_K(-i\tau,\tilde{x}^{\mu}) \rangle\rangle , \tag{6.23}$$

where d_{μ} is defined in Eq. (5.9). Similar results hold for scalar and vector fields, so that all field gradients produce factors of momenta p_j^{μ} and k_j^{μ} defined in Eqs. (6.16) and (6.18). Thus the contributions to $T^{\mu\nu}$ defined in Eqs. (3.19)–(3.22) follow immediately as

$$\langle\langle \hat{T}_B^{\mu\nu} \rangle\rangle = -\frac{i}{n\cdot\beta}\sum_j e^{i\omega_j\eta}\int\frac{d\sigma_k}{(2\pi)^3}\mathrm{Tr}[\gamma^{\mu}G^{(E)}(\omega_j,\tilde{k}^{\lambda})]k_j^{\nu} , \tag{6.24}$$

$$\langle\langle \hat{T}_s^{\mu\nu} \rangle\rangle = -\frac{i}{n\cdot\beta}\sum_j \int\frac{d\sigma_p}{(2\pi)^3}[\tfrac{1}{2}(p_j^2-m_s^2)g^{\mu\nu}-p_j^{\mu}p_j^{\nu}]\Delta^{(E)}(\nu_j,\tilde{p}^{\lambda}) , \tag{6.25}$$

$$\langle\langle \hat{T}_v^{\mu\nu} \rangle\rangle = \frac{i}{n\cdot\beta}\sum_j \int\frac{d\sigma_p}{(2\pi)^3}[\tfrac{1}{2}(p_j^2-m_v^2)g^{\mu\nu}-p_j^{\mu}p_j^{\nu}]D_{\sigma}^{(E)\sigma}(\nu_j,\tilde{p}^{\lambda}) , \tag{6.26}$$

where Tr indicates a trace over isospin and Dirac indices (which will not be indicated explicitly). Ensemble averages of composite operators are divergent and must be rendered finite with appropriate normalization conditions, as in Eqs. (4.6)–(4.8).

The summation of tadpole diagrams goes through just as in real time or as in the imaginary-time calculation in the comoving frame (see Ref. 8), so we can write down expressions for the Hartree propagators and self-energies immediately. The baryon propagator also follows as the continuum limit of Eq. (5.30). Thus

$$\begin{aligned} G_H^{(E)}(\omega_j,\tilde{k}^{\mu}) &= G_0^{(E)}(\omega_j,\tilde{k}^{\mu})+G_0^{(E)}(\omega_j,\tilde{k}^{\mu})(-i\Sigma^H)G_H^{(E)}(\omega_j,\tilde{k}^{\mu}) \\ &= \frac{i}{\not{k}_j^*-M^*} = \frac{i(\not{k}_j^*+M^*)}{k_j^{*2}-M^{*2}} , \end{aligned} \tag{6.27}$$

where

$$k_j^{*\mu} \equiv \left[i\omega_j - \frac{\tilde{k}^*\cdot u}{n\cdot u} + \nu \right] n^{\mu} + \tilde{k}^{*\mu} , \tag{6.28}$$

$$\tilde{k}^{*\mu} \equiv \tilde{k}^{\mu} - g_v\tilde{V}_0^{\mu}, \quad E^*(\tilde{k}^*) \equiv (-\tilde{k}^{*2}+M^{*2})^{1/2} , \tag{6.29}$$

and $\nu \equiv \mu - g_v(u\cdot V_0)/(n\cdot u)$, as in Eqs. (5.31) and (5.32).

We will calculate the scalar self-energy as an example. From the Feynman rules and Eq. (6.27),

$$\begin{aligned} \Sigma_s^H &= -g_s^2\Delta_0^{(E)}(0,0)\frac{1}{n\cdot\beta}\sum_j e^{i\omega_j\eta}\int\frac{d\sigma_k}{(2\pi)^3}\mathrm{Tr}[G_H^{(E)}(\omega_j,\tilde{k}^{\mu})] \\ &= -\frac{2g_s^2}{m_s^2}M^*\gamma\int\frac{d\sigma_k}{(2\pi)^3}\frac{1}{n\cdot\beta}\sum_j\frac{e^{i\omega_j\eta}}{k_j^{*2}-M^{*2}} . \end{aligned} \tag{6.30}$$

To evaluate the frequency sum, apply

$$\lim_{\eta\to 0}\sum_j \frac{e^{i\omega_j\eta}}{i\omega_j - x} = \frac{n\cdot\beta}{e^{(n\cdot\beta)x}+1} \tag{6.31}$$

to obtain

$$\begin{aligned} \sum_j \frac{e^{i\omega_j\eta}}{k_j^{*2}-M^{*2}} &= \frac{1}{2E^*(\tilde{k}^*)}\left[\sum_j \frac{e^{i\omega_j\eta}}{i\omega_j-(u\cdot K/n\cdot u)+\mu} - \sum_j \frac{e^{i\omega_j\eta}}{i\omega_j-(u\cdot\bar{K}/n\cdot u)+\mu} \right] \\ &= \frac{n\cdot\beta}{2E^*(\tilde{k}^*)}\left[\frac{1}{e^{\beta\cdot K-\alpha}+1} + \frac{1}{e^{-\beta\cdot\bar{K}+\alpha}+1} - 1 \right] . \end{aligned} \tag{6.32}$$

Here we have used the definitions of on-shell canonical four-momenta from Sec. V [see Eq. (5.39)] and $(n\cdot\beta)\mu=\alpha$.

Just as in Sec. V, we can introduce an integral over $n\cdot k$ with δ functions that ensure on-shell kinetic momenta. Equation (6.32) implies that there will be a contribution to Σ_s^H that has no explicit dependence on the thermodynamic variables β and α; this represents the vacuum (zero-point) contribution, and as usual, it is omitted to define the MFT and retained (with renormalization) in the RHA. The result for the MFT scalar self-energy is

$$\begin{aligned}\Sigma_s^H&=-\frac{2g_s^2}{m_s^2}M^*\frac{\gamma}{(2\pi)^3}\int d^4k\frac{1}{2E^*(\tilde{k}^*)}\left[\delta(n\cdot k^*-E^*(\tilde{k}^*))\frac{1}{e^{\beta\cdot k-\alpha}+1}+\delta(n\cdot k^*+E^*(\tilde{k}^*))\frac{1}{e^{-\beta\cdot k+\alpha}+1}\right]\\&=-\frac{2g_s^2}{m_s^2}M^*\frac{\gamma}{(2\pi)^3}\int d^4k\,\delta(k^{*2}-M^{*2})\left[\theta(\beta\cdot k^*)\frac{1}{e^{\beta\cdot k-\alpha}+1}+\theta(-\beta\cdot k^*)\frac{1}{e^{-\beta\cdot k+\alpha}+1}\right],\end{aligned}\tag{6.33}$$

which agrees with that obtained using the real-time rules and is independent of n^μ. The vector self-energy $\Sigma_v^{H\mu}$ can be calculated analogously. Notice that the zero-point contribution from Eqs. (6.30) and (6.32) can be written as

$$\begin{aligned}\frac{g_s^2}{m_s^2}2M^*\gamma\int\frac{d\sigma_k}{(2\pi)^3}\frac{1}{2E^*(\tilde{k}^*)}&=\frac{g_s^2}{m_s^2}2M^*\frac{\gamma}{(2\pi)^3}\int d^4k\,\delta(k^{*2}-M^{*2})\theta(-\beta\cdot k^*)\\&=\frac{g_s^2}{m_s^2}\frac{1}{\mathcal{V}}\frac{\partial}{\partial M^*}\left[\sum_{\tilde{k}s}E^*(\tilde{k}^*)\right];\end{aligned}\tag{6.34}$$

when the appropriate counterterms are included, this produces the vacuum correction to the scalar self-energy that appears as the final term in Eq. (5.44).

The Hartree meson propagators contain tadpoles analogous to those shown in Fig. 4, and the mesonic contributions to $T^{\mu\nu}$ can be calculated from Eqs. (6.25) and (6.26). Convergence factors of the form $e^{i\nu_j\eta}$ $(\eta\to0^+)$ must be included to properly define the vacuum contributions, which are then removed by subtraction.

The baryon contribution follows by substituting $G_H^{(E)}$ of Eq. (6.27) into (6.24). The manipulations are similar to those discussed for Σ_s^H, and after some algebra, the final result agrees with Eq. (4.53). The zero-point correction looks exactly like that in Eq. (5.37) and can be renormalized in the same fashion.

An alternative representation of the Euclidean Hartree propagator, based on the contour representation discussed earlier, is

$$G_H^{(E)}(\omega_j,\tilde{k}^\mu)=\int\frac{d(n\cdot k)}{2\pi}\frac{\rho_H(k^*)(\not{k}^*+M^*)}{\omega_j+i(k\cdot u)/(n\cdot u)-i\mu},\tag{6.35}$$

with

$$\rho_H(k^*)\equiv2\pi[\theta(n\cdot k^*)-\theta(-n\cdot k^*)]\delta_\epsilon(k^{*2}-M^{*2}).\tag{6.36}$$

(Note that $k^{*\mu}$ and not $k_j^{*\mu}$ appears in the numerator and k^μ appears in the denominator.) Substituting Eq. (6.35) into Eq. (6.30) and performing the frequency sum produces the final expression for Σ_s^H [Eq. (6.33)] immediately [plus the zero-point contribution (6.34)].

VII. SUMMARY

We have developed a unified treatment of relativistic many-body systems at finite temperature and density, incorporating both real- and imaginary-time formalisms, for a hadronic field theory of nuclear matter. Covariant Feynman rules in both real and imaginary time were given and were applied to derive covariant expressions for the one-loop energy-momentum tensor of QHD-I, including the zero-point energy contributions. These expressions allow any observer to determine the equation of state from an arbitrary reference frame. Moreover, when evaluated in the comoving frame or in the laboratory frame, they duplicate results obtained in Ref. 5 by working canonically.

A key element in our discussion was the difference between the time evolution of the system in the canonical and grand canonical Heisenberg pictures. The former evolves the system along the world line of the observer, and thus corresponds to the Eulerian formulation of hydrodynamics, whereas the latter follows fluid elements and corresponds to the Lagrangian formulation. In this way, the grand canonical picture incorporates the Lagrange multipliers for the fluid velocity and chemical potential. Although the generalization of the real-time Feynman rules from the comoving frame to an arbitrary frame is algebraically simple, it is only by recognizing the distinction between the two pictures and by utilizing canonically defined propagators that this simplicity can be understood. These canonical propagators also have the advantage that they are independent of the hyperplane used to quantize the system.

The noninteracting imaginary-time propagators are more complicated for two reasons: First, they are defined in the grand Heisenberg picture, and second, since the spatial variables are real while the temporal variables are complex, it is necessary to introduce *two* four-vectors to incorporate the variables covariantly. Thus imaginary-time propagators have an explicit dependence on n^μ, the timelike unit vector orthogonal to the quantization hyperplane. Nevertheless, contrary to what is often stated in the literature, it is possible to define these propagators covariantly, and we have shown how the dependence on n^μ drops out in the calculation of observables. One advantage of these covariant imaginary-time rules is that they allow one to correctly identify the Lorentz structure of thermodynamic quantities, which can be important in practical calculations.

We also derived a manifestly covariant expression for the QHD-I partition function and evaluated it in the one-loop approximation. The direct calculation of the grand potential yields an expression for the pressure that agrees with the result computed from the energy-momentum tensor, verifying that this approximation is consistent with the virial theorem. The self-consistency conditions and thermodynamic relations agree with those obtained from the covariant summation of tadpole diagrams. Although the one-loop calculation performed here for illustration is a simple one, the techniques we have developed can be applied straightforwardly to more sophisticated covariant calculations in QHD, as well as to other dynamical models. The general development of nonperturbative approximations in QHD and other strong-coupling theories remains an open problem.

ACKNOWLEDGMENTS

We acknowledge useful comments from D. Griegel and K. Wehrberger. This work was supported in part by the U. S. Department of Energy under Grants DE-FG02-87ER-40365 and DE-FG05-87ER-40322.

[1]B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
[2]N. P. Landsman and Ch. G. van Weert, Phys. Rep. **145**, 141 (1987).
[3]J. D. Walecka, Ann. Phys. (N.Y.) **83**, 491 (1974).
[4]*Relativistic Nuclear Many-Body Physics,* edited by B. C. Clark, R. J. Perry, and J. P. Vary (World Scientific, Singapore, 1989).
[5]R. J. Furnstahl and B. D. Serot, Phys. Rev. C **41**, 262 (1990).
[6]H. Uechi, Ph.D. thesis, Indiana University, 1988; Nucl. Phys. **A501**, 813 (1989).
[7]F. Ruiz Ruiz and R. F. Alvarez-Estrada, Z. Phys. C **34**, 131 (1987).
[8]R. A. Freedman, Ph.D. thesis, Stanford University, 1978.
[9]J. D. Walecka, in *New Vistas in Nuclear Dynamics,* edited by P. J. Brussaard and J. H. Koch (Plenum, New York, 1986), p. 229.
[10]K. Saito, T. Maruyama, and K. Soutome, Phys. Rev. C **40**, 407 (1989).
[11]K. Soutome, T. Maruyama, and K. Saito, Nucl. Phys. **A507**, 731 (1990).
[12]T. Matsubara, Prog. Theor. Phys. **14**, 351 (1955).
[13]R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957).
[14]D. J. Thouless, Phys. Rev. **107**, 1162 (1957).
[15]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).
[16]P. D. Morley and M. B. Kislinger, Phys. Rep. **51**, 63 (1979).
[17]R. E. Norton and J. M. Cornwall, Ann. Phys. (N.Y.) **91**, 106 (1975).
[18]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. **36**, 900 (1959) [Sov. Phys.—JETP **9**, 636 (1959)].
[19]E. S. Fradkin, Zh. Eksp. Teor. Fiz. **36**, 1286 (1959) [Sov. Phys.—JETP **9**, 912 (1959)].
[20]G. Baym and N. D. Mermin, J. Math. Phys. **2**, 232 (1961).
[21]M. B. Kislinger and P. D. Morley, Phys. Rev. D **13**, 2765 (1976); **13**, 2771 (1976).
[22]J. Schwinger, J. Math. Phys. **2**, 407 (1961).
[23]L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys.—JETP **20**, 1018 (1965)].
[24]R. Mills, *Propagators for Many-Particle Systems* (Gordon and Breach, New York, 1969).
[25]Y. Takahashi and H. Umezawa, Collect. Phenom. **2**, 55 (1975).
[26]I. Ojima, Ann. Phys. (N.Y.) **137**, 1 (1981).
[27]H. Umezawa, H. Matsumoto, and M. Tachiki, *Thermo Field Dynamics and Condensed States* (North-Holland, Amsterdam, 1982).
[28]H. Matsumoto, Y. Nakano, H. Umezawa, F. Mancini, and M. Marinaro, Prog. Theor. Phys. **70**, 599 (1983).
[29]H. Matsumoto, Y. Nakano, and H. Umezawa, J. Math. Phys. **25**, 3076 (1984).
[30]H. Matsumoto, I. Ojima, and H. Umezawa, Ann. Phys. (N.Y.) **152**, 348 (1984).
[31]A. J. Niemi and G. W. Semenoff, Nucl. Phys. **B230**, 181 (1984); Ann. Phys. (N.Y.) **152**, 105 (1984).
[32]G. W. Semenoff and H. Umezawa, Nucl. Phys. **B220**, 196 (1983).
[33]C. B. Iredale, Ph.D. thesis, Harvard University, 1986.
[34]R. J. Furnstahl and B. D. Serot (unpublished).
[35]L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).
[36]I. E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. **42**, 1126 (1962) [Sov. Phys.—JETP **15**, 778 (1962)].
[37]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1963).
[38]G. Baym and A. M. Sessler, Phys. Rev. **131**, 2345 (1963).
[39]E. V. Shuryak, Phys. Rep. **61**, 71 (1980).
[40]R. D. Pisarski, Nucl. Phys. **B309**, 476 (1988).
[41]P. A. Henning, Nucl. Phys. **B337**, 547 (1990).
[42]C. J. Horowitz and B. D. Serot, Nucl. Phys. **A464**, 613 (1987); **A473**, 760 (1987) (E).
[43]M. J. Lighthill, *Introduction to Fourier Analysis and Generalized Functions* (Cambridge University Press, Cambridge, 1958).
[44]A. Niégawa, Phys. Rev. D **40**, 1199 (1989).
[45]R. J. Furnstahl and B. D. Serot, Ann. Phys. (N.Y.) **185**, 138 (1988).
[46]W. Israel, Ann. Phys. (N.Y.) **100**, 310 (1976); Physica **106A**, 204 (1981).
[47]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), Chap. 11.
[48]A. F. Bielajew, Ph.D. thesis, Stanford University, 1982. (a) C. Nash, *Relativistic Quantum Fields* (Academic, New York, 1978), pp. 63–65.
[49]S. A. Chin, Ann. Phys. (N.Y.) **108**, 301 (1977).
[50]J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Springer, New York, 1976), second ed., Chap. 1.
[51]E. S. Abers and B. W. Lee, Phys. Rep. **9C**, 1 (1973).
[52]J. Iliopoulos, C. Itzykson, and A. Martin, Rev. Mod. Phys. **47**, 165 (1975).