PHYSICAL REVIEW C VOLUME 43, NUMBER 3 MARCH 1991

Origin of phase patterns in πd elastic scattering

Leopold Mathelitsch
Institut für Theoretische Physik, Universität Graz, A-8010 Graz, Austria

Humberto Garcilazo
Institut für Theoretische Physik, Universität Hannover, D-3000 Hannover, Federal Republic of Germany
(Received 16 July 1990)

Specific patterns have been found in the relative phases of the amplitudes of various strong-interaction reactions like proton-proton, pion-nucleon, or pion-deuteron scattering. Here we investigate such patterns for the case of π-d elastic scattering. We show that these phase patterns result essentially from the dominance of the delta resonance.

The improvement of experimental facilities, in particular in performing polarization experiments, made it possible to carry out complete experiments; that means that enough experimental observables are measured in order to perform amplitude analyses (almost) unambiguously. One outcome of these analyses was the finding by Moravcsik and Goldstein that specific amplitudes show a peculiar and interesting behavior: They are predominantly pure real or imaginary relative to each other.[1] This feature appears in various reactions and at different energies: for proton-proton elastic scattering[2] from 600 MeV to 6 GeV, for pion-nucleon scattering up to an energy[3] of 45 GeV, for pion-deuteron scattering at such low energies[4] as between 100 and 300 MeV, and to a lesser extent for pion photoproduction.[5] The behavior is best seen if the amplitudes are calculated in the so-called "planar-transverse optimal" frame (the quantization direction of the particles is perpendicular to the helicity direction and also perpendicular to the transversity direction),[6] but they show up also in the helicity frame.[7]

Until now there did not exist an explanation for these patterns and Arash *et al.*[4] see three possibilities in general: (i) The patterns are fortuitous; (ii) the patterns are the outcome of some physics which is already known; (iii) the patterns give insight into some new phenomena. A definite answer to these three points is not easy [even to point (i)], since the amplitudes carry error bars stemming from experimental uncertainties or from the analysis. An approach from the theoretical side, namely, starting from a well-defined model and calculating the amplitudes straightforwardly, would address this problem. Fortunately, one of the above-mentioned findings of the phase patterns was done via a theoretical calculation: The elastic pion-deuteron amplitudes resulted from a model calculation that reproduces the experimental data rather accurately.[8]

This theoretical model for elastic π-d scattering is based on a relativistic Faddeev theory implementing the constraints of unitarity and Lorentz invariance. The latter is connected to the use of isobars for the πN and NN states. The three-particle system is reduced to a quasi-two-particle problem where the transition potentials consist either of the exchange of a pion between two πN isobars or the exchange of a nucleon between a πN isobar and a NN isobar (see Fig. 1). For the actual calculation six πn isobars (corresponding to the S_{11}, S_{31}, P_{11}, P_{31}, P_{33}, and P_{13} pion-nucleon channels) and two NN isobars (the ${}^1S_0$ and the ${}^3S_1$-${}^3D_1$ nucleon-nucleon channels) are taken into account. The experimental pion-nucleon phase shifts are used for the πN states and unitarity pole approximations are constructed for the NN states reproducing the results of the Paris potential.[9]

Figure 2 shows the phases of the helicity amplitudes A,C,D relative to the amplitude[10] B at a pionic kinetic energy of $T_\pi = 256$ MeV. The phase patterns as observed by Moravcsik *et al.* show up clearly, namely, that the relative phases are predominantly multiples of 90°. In fact, one observes plateaus at 0°, $\pm 180°$, and $\pm 360°$ with some narrow transitions from one plateau to the other. A look at the actual phases of the amplitudes A,B,C,D themselves (Fig. 3) gives more insight: All amplitudes start with the same phase at zero degrees (which, of course, gives the almost zero value of the difference) and are constant for a great part of the forward region. Φ_A, Φ_B, and Φ_D show abrupt jumps of 180° around 70°, 90°, and 50°, respectively, whereas a second jump in Φ_D and a phase change in Φ_C are less pronounced.

We will show below that these patterns arise from two features, (a) the dominance of the $\Delta(3,3)$ intermediate state in the impulse approximation, which makes all the helicity amplitudes have approximately the same phase; (b) the dominance of the $J^P = 2^+$ channel in the π-d sys-

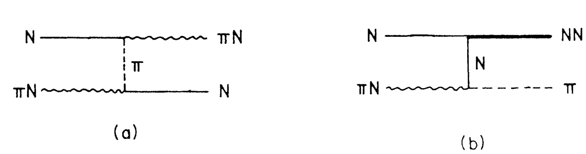

FIG. 1. Transition potentals of the πNN system: (a) transition from a πN isobar to another πn isobar; (b) transition from an πN isobar to an NN isobar.

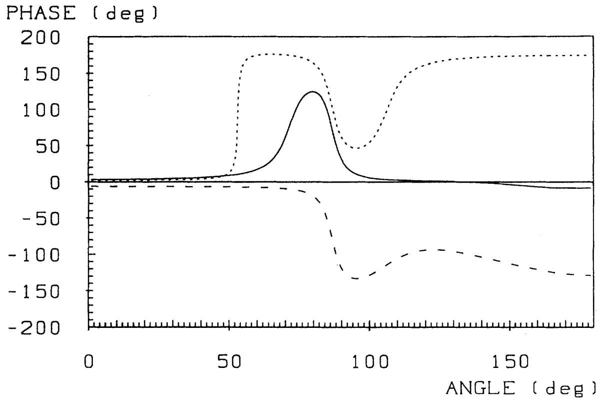

FIG. 2. Relative phases of πd helicity amplitudes at $T_\pi=256$ MeV: $\Phi_A-\Phi_B$ (solid line), $\Phi_C-\Phi_B$ (dashed line), $\Phi_D-\Phi_B$ (dotted line).

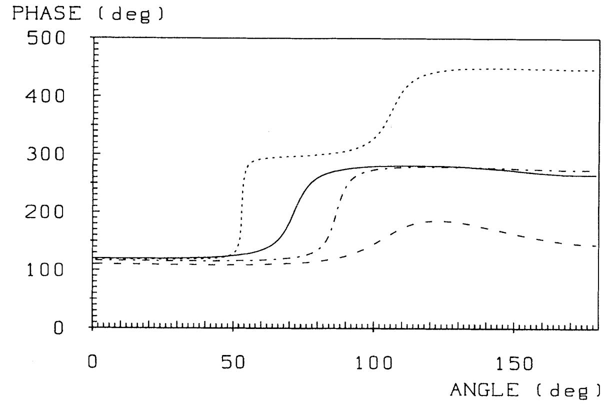

FIG. 3. Phases of the πd helicity amplitudes at $T_\pi=256$ MeV: Φ_A (solid line), Φ_B (dash-dotted line), Φ_C (dashed line), Φ_D (dotted line).

tem, which gives rise to the jumps observed in Fig. 2 originating from the zeros of the Wigner rotation matrices for $J=2$.

The amplitudes A, B, C, and D are the helicity amplitudes $F_{M_f M_i}(S,\theta)$, where M_i and M_f are the initial and final helicities of the deuteron:

$$A=F_{11}(S,\theta), \quad B=F_{10}(S,\theta) ,$$

$$C=F_{1-1}(S,\theta), \quad D=F_{00}(S,\theta) .$$

S and θ are the invariant mass squared of the system and the scattering angle in the c.m. system, respectively. The amplitudes can be expanded into partial waves

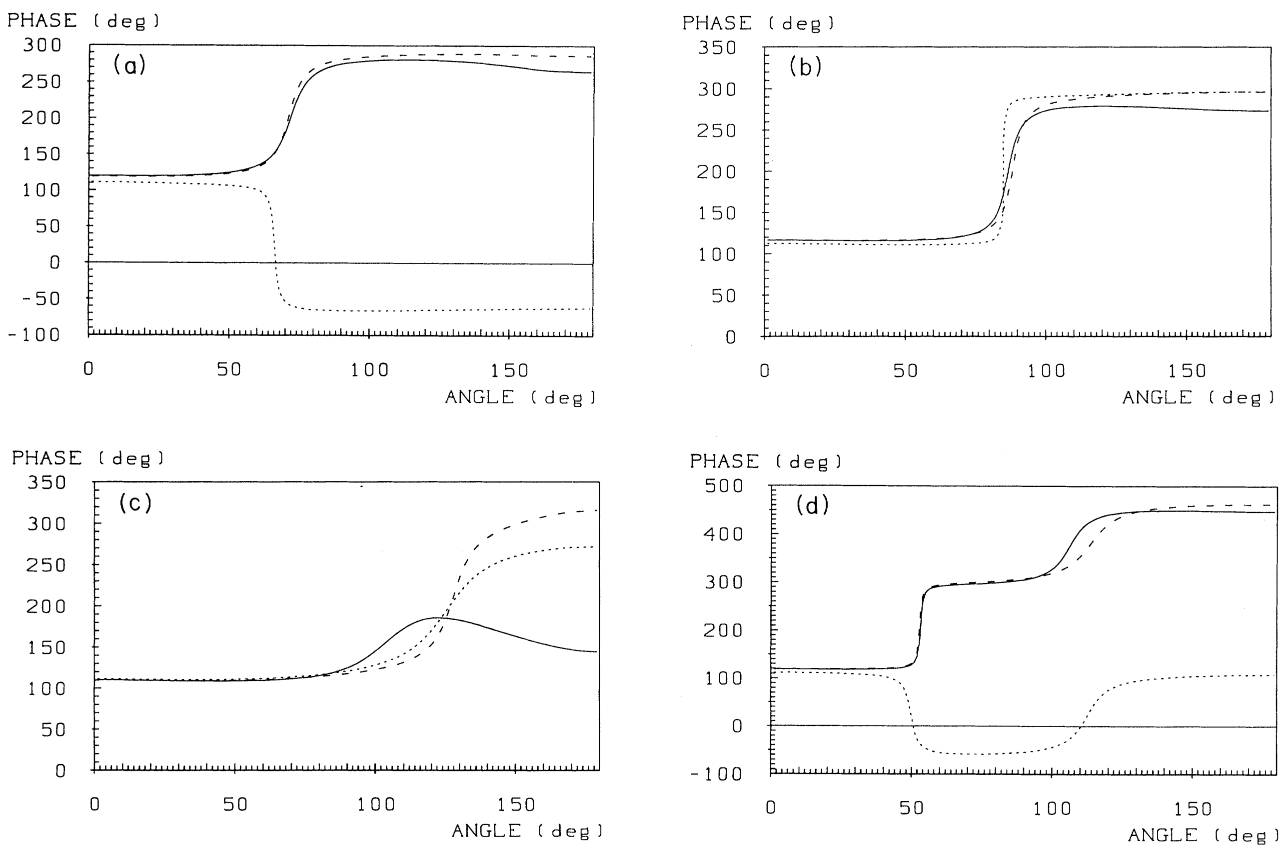

FIG. 4. Phases (a) Φ_A, (b) Φ_B, (c) Φ_C, (d) Φ_D for three models of πd scattering at $T_\pi=256$ MeV: full calculation (solid line), impulse approximation (dashed line), impulse approximation with only the P_{33} channel (dotted line).

$$F_{M_f M_i}(S,\theta)=\frac{1}{q_0}\sum_J (2J+1)F^J_{M_f M_i}(S)d^J_{M_i M_f}(\theta) ;$$

q_0 is the pion-deuteron on-shell momentum and $d^J_{M_i M_f}(\theta)$ are the Wigner rotation matrices.

Since the partial-wave amplitudes F^J are complex functions and since the rotation matrices are relatively complicated expressions of the scattering angle, it is, in general, not easy to trace back patterns of the complete amplitude to specific features of a partial wave. An exception would be if one partial wave is much larger than the other ones. Due to the dominance of the Δ resonance in pion-nuclear physics, the pion-deuteron amplitude is dominated by the state with total angular momentum and parity $J^P=2^+$. In this state the $\Delta(3,3)$ and the nucleon are in a relative S state with the spins parallel to each other. Thus, it is easy to see that this state will couple very strongly to the three-body configuration where the two nucleons are in the $^3S_1$ channel (spins parallel), while the pion is in a P wave with respect to the two nucleons, since in order to get the quantum numbers of the $J^P=2^+$ state, this configuration requires also that all the spins and orbital angular momenta be parallel to each other.

Let us assume that the partial wave with $J=2$ dominates such that one can neglect the other terms of the expansion. This assumption yields for the amplitudes

$$A\approx\frac{5}{q_0}F^2_{11}(S)d^2_{11}(\theta)+,\quad B\approx\frac{5}{q_0}F^2_{10}(S)d^2_{01}(\theta) ,$$

$$C\approx\frac{5}{q_0}F^2_{1-1}(S)d^2_{-11}(\theta),\quad D\approx\frac{5}{q_0}F^2_{00}(S)d^2_{00}(\theta) .$$

The dominance of the Δ resonance should be largest at small angles, i.e., at peripheral collisions. Here, the partial amplitudes $F_{M_f M_i}$ should be essentially given by the contribution of the single scattering term proceeding through the (3,3) state; in particular, the phases of the amplitudes $F_{M_f M_i}$ should be equal to each other and it should be the phase of the P_{33} pion-nucleon channel. That this is true can be seen in Fig. 3. The fact that this feature also holds up to higher angles is an indication of the importance of the resonance.

But the jumps in the amplitudes can also be explained in this model: The rotation matrices $d^2_{M_i M_f}$ are real numbers; therefore a variation of the angle θ will *not* change the phase of the amplitude. There is one exception to this fact, namely, when $d^2_{M_i M}$ changes its sign. In this case the phase of the amplitude will jump by 180°. Sign changes for the rotation matrices occur for $d^2_{00}(\theta)\approx P_2(\theta)$ (Legendre polynomial) at $\theta\approx 55°$, for $d^2_{10}(\theta)\approx P^1_2(\theta)$ (associated Legendre polynomial) at $\theta\approx 90°$, and for $d^2_{11}(\theta)$ at $\theta\approx 74°$. But these are just the angles where the drastic jumps show up in the amplitudes in Figs. 2 and 3.

Our considerations were based on the assumption of the dominance of the Δ resonance. In order to check this we calculated (in addition to the full model as explained above, model 1) also the amplitudes for the impulse approximation (all πN channels included, model 2) and for the impulse approximation with only the P_{33} channel (model 3). The results are given in Figs. 4. They show that the amplitudes A and B are given entirely by the Δ contribution (that the phase change in amplitude A is counterclockwise for models 1 and 2 and clockwise for model 3 has no physical meaning). The Δ resonance dominates roughly up to $\theta\approx 90°$ for the amplitude C and D.

The dominance of the P_{33} resonance should depend on the chosen energy of the incident pion. The reason for

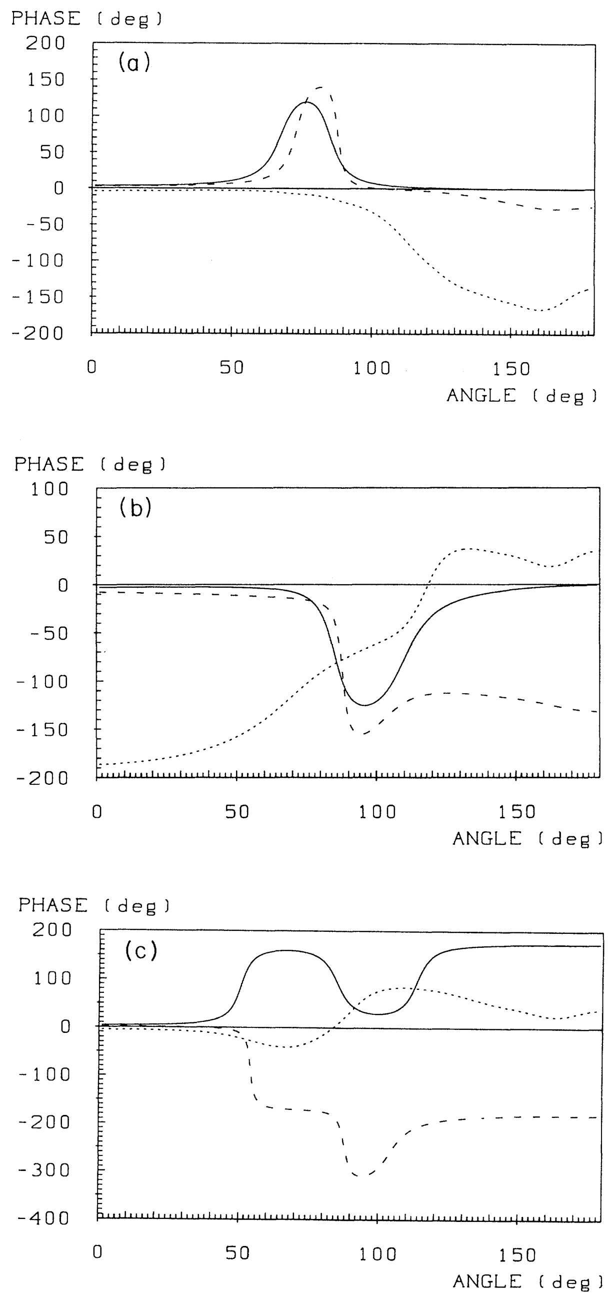

FIG. 5. Relative phases (a) $\Phi_A-\Phi_B$, (b) $\Phi_C-\Phi_B$, (c) $\Phi_D-\Phi_B$ for πd scattering: $T_\pi=180$ MeV (solid line), $T_\pi=294$ MeV (dashed line), $T_\pi=600$ MeV (dotted line).

our choice of the calculation at $T_\pi = 256$ MeV was that new experimental data were available at this energy (as well as at $T_\pi = 294$ MeV).[11] According to our explanation above, the pattern structure should be more dramatic at the Δ-resonance region and should vanish if one is far from the resonance. To prove this statement we calculated the amplitudes at $T_\pi = 180$ MeV (resonance region) and far above at $T_\pi = 600$ MeV. The results (including the amplitudes at $T_\pi = 294$ MeV) strengthen our finding (Figs. 5): The patterns are most pronounced at $T_\pi = 180$ MeV and at 600 MeV only a zero-phase difference remains for $\Phi_A - \Phi_B$ and $\Phi_D - \Phi_B$ and this just at the very forward region.

Goldstein *et al.* point out[12] that the pattern visualized in phase histograms could be understood as the outcome of some kind of a two-component model. One component is related to the dominance of a single process (coherent component) accompanied by an incoherent residual background. In our case, the coherent part is the delta dominance through the impulse approximation, and the incoherent part comes from the other pion-nucleon channels and from the effect of unitarization achieved by the use of exact three-body equations.

We have shown that the patterns of relative phases in the elastic pion-deuteron scattering amplitudes can be interpreted as the visible result of the dominance of a single process, namely, the Δ resonance in impulse approximation. Our finding suggests that the patterns seen in other reactions are of similar origin. This idea, already put forward by Moravcsik and colleagues, namely, that nucleon-nucleon and pion-nucleon scattering are dominated by single processes at energies from hundreds of MeV to tens of GeV, could lead to new insight in those reactions.

ACKNOWLEDGMENTS

The authors would like to thank Prof. G. Goldstein for valuable comments and discussions. This work was supported in part (H.G.) by the German Federal Ministry for Research and Technology (BMFT) under Contract No. 06 OH 754.

[1] F. Arash, M. J. Moravcsik, and G. R. Goldstein, Mod. Phys. Lett. A **4**, 529 (1989).
[2] N. Ghahramany, G. R. Goldstein, and M. J. Moravcsik, Phys. Rev. D **28**, 1086 (1983); M. J. Moravcsik, F. Arash, and G. R. Goldstein, *ibid.* **31**, 2360 (1985).
[3] F. Arash, M. J. Moravcsik, and G. R. Goldstein, Int. J. Mod. Phys. A **2**, 739 (1987); G. R. Goldstein and M. J. Moravcsik, Phys. Lett. B **199**, 563 (1987).
[4] F. Arash, H. Garcilazo, G. Goldstein, and M. J. Moravcsik, Mod. Phys. Lett. A **5**, 83 (1990).
[5] G. R. Goldstein and M. J. Moravcsik, Nuovo Cimento **73A**, 196 (1983).
[6] G. R. Goldstein and M. J. Moravcsik, Ann. Phys. (N.Y.) **98**, 128 (1976); **142**, 219 (1982).
[7] H. Garcilazo, Phys. Rev. Lett. **61**, 1457 (1988).
[8] H. Garcilazo, Phys. Rev. C **35**, 1804 (1987).
[9] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).
[10] H. Garcilazo, E. T. Boschitz, W. Gyles, W. List, C. R. Ottermann, R. Tacik, and M. Wessler, Phys. Rev. C **39**, 942 (1989).
[11] C. R. Ottermann *et al.*, Phys. Rev. C **38**, 2296 (1988); **38**, 2310 (1988).
[12] G. R. Goldstein, F. Arash, and M. J. Moravcsik, Tufts University Report No. TUFTS TH-G90-3, 1990; G. R. Goldstein and F. Arash, Few-Body Syst. **9**, 57 (1990).