Neutron radius analysis in the trinucleon system from pion scattering

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We have analyzed recent data for pion elastic scattering on the three-nucleon system to extract the relative neutron-proton radii in ^{3}He and ^{3}H. Using standard multiple-scattering analysis techniques we determine the difference between the odd radii with an uncertainty of ± 0.007 fm and that between the even radii within ± 0.010 fm from the existing data. Additional data and improvements in the scattering theory can be expected to lead to a more precise determination. To assess the significance of the extracted radius differences we compare them with radii determined from Faddeev calculations that include charge-symmetry-breaking forces.

I. INTRODUCTION

The ratios of differential cross sections for the elastic scattering of π^+ and π^- from the ^{3}H, ^{3}He isodoublet have been reported from two separate experiments.[1,2] A third measurement is being analyzed.[3] The motivation for those experiments was to search for evidence of charge symmetry breaking (CSB) in the hadronic strong force. Such CSB has apparently been observed in the binding energy, because only some 650 keV out of the 760 keV ^{3}H-^{3}He binding-energy difference is accounted for by the replusive Coulomb interaction acting between the two protons in ^{3}He. Indeed, the measured ratios differ substantially from unity, the value one would anticipate if isospin were strictly conserved.

Kim, Kim, and Landau[4] studied the effect on the measured ratios of the Coulomb force in the pion-nucleus interaction and showed it to be non-negligible. That is, the ratios must deviate from unity when the pion-nucleus Coulomb interaction is included in a model calculation. Such Coulomb effects do not completely cancel in any of the ratio measurements.

However, there is a second reason that the measured ratios must differ from unity: The proton and neutron radii (distributions) in ^{3}H and ^{3}He are not identical. It is this latter effect that we emphasize in this report. In particular, it appears on the basis of our investigation that one can extract the difference between $r_p(^3\text{H})$ and $r_n(^3\text{He})$ and that between $r_p(^3\text{He})$ and $r_n(^3\text{H})$ with a precision that exceeds the uncertainty in the difference between $r_p(^3\text{H})$ and $r_p(^3\text{He})$ as determined by existing absolute measurements in elastic electron scattering.

In the next section we review what is known about the ^{3}H and ^{3}He proton and neutron radii. In Sec. III we discuss the information content of the pion elastic-scattering ratios, including meson-exchange-current contributions. Section IV describes our method of analysis in terms of a nonlocal pion-nucleus optical model, and Sec. V addresses the main variables in the scattering theory. Our results are discussed in Sec. VI and our conclusions are summarized in Sec. VII.

II. TRINUCLEON RADII

The structure of the trinucleon bound states is determined by the properties of the underlying nuclear forces.[5] One qualitatively understands the relative sizes of ^{3}He and ^{3}H from our knowledge of the nucleon-nucleon (*NN*) interactions: neutron-proton (*n-p*), neutron-neutron (*n-n*), and proton-proton (*p-p*) interactions.[6] The *n-p* force is slightly stronger than the *n-n* (or *p-p*) force. The deuteron is bound and the *n-p* spin-singlet scattering length (-23.7 fm) is larger in magnitude than the *nn* scattering length ($\simeq -18$ fm), where $a \to -\infty$ implies a bound state with zero energy binding. Therefore, one expects the radius of the odd nucleon in the trinucleon ground states to be smaller than the radius of the like pair of (even) nucleons. We use the "odd" and "even" radii nomenclature of Schiff[7] to denote the different radii for the unlike nucleon (proton in ^{3}H or neutron in ^{3}He) and like pair of nucleons. It follows that the proton radius of ^{3}H should be smaller than the neutron radius. State-of-the-art Faddeev calculations[6] employing contemporary nucleon-nucleon force models yield a difference between these two radii of about 0.15 fm.

In the absence of the Coulomb interaction between the two protons in ^{3}He, charge symmetry would require that the ^{3}H and ^{3}He systems be identical. (At this point we are neglecting in this discussion any small charge symmetry breaking in the strong interaction; i.e., the *n-n* force is assumed to be identical to the strong *p-p* force.) Including the repulsive Coulomb interaction in the Faddeev calculations for ^{3}He leads to an increase in the proton radius (over the neutron radius in ^{3}H) estimated to be 0.03–0.04 fm, based upon separate least-squares fits to a number of ^{3}H and ^{3}He model calculations.[8] This Coulomb interaction also affects the ^{3}He neutron radius, since the increased separation of the two protons means that the neutron is less bound; that is, the neutron distribution is also expanded. The neutron radius is increased (over the proton radius in ^{3}H) by 0.02–0.03 fm in the model Faddeev calculations. Thus, to investigate the Coulomb effect it is natural to compare the two even or two odd radii.

The sizes of ^{3}H and ^{3}He have been studied experimentally[9-19] in elastic-electron-scattering experiments. A recent analysis of the charge-scattering data by the Saclay group[20] yielded point proton radii values of $r_p(^3\mathrm{He})=1.75\pm0.04$ fm and $r_p(^3\mathrm{H})=1.61\pm0.04$ fm. Point proton charge radii have also been calculated using ^{3}He and ^{3}H one-body densities generated from wave-function solutions of the Faddeev equations for various contemporary nucleon-nucleon potential models both with and without including two-pion-exchange three-body-force interactions.[5,6,8] Using an interpolating fit to estimate the point proton radii yields[8]

$$r_p(^3\mathrm{He})=1.77 \text{ fm} , \tag{1a}$$

$$r_p(^3\mathrm{H})=1.58 \text{ fm} . \tag{1b}$$

The 1.77-fm value for ^{3}He includes an 0.04-fm increase in its size due to point-Coulomb repulsion between the two protons. The model difference of 0.19 fm is consistent with the results of the Saclay analysis. A ^{3}He/^{3}H charge-scattering ratio measurement would better define this difference experimentally.

What is known about the neutron radii of ^{3}He and ^{3}H? The spin of the trinucleon is given essentially by that of the odd nucleon. However, it is difficult to extract a neutron radius for ^{3}He from magnetic elastic electron scattering, because meson-exchange-current corrections are sizable.[21-24] In fact, the magnetic properties of ^{3}He provide one of the few examples where meson-exchange currents manifest themselves in an unmistakable way. Using only nucleon one-body currents, one is unable to account for the experimental ^{3}He magnetic moment: irrespective of the *NN* and three-body-force models employed, the theoretical value for the ^{3}He magnetic moment in impulse approximation (no meson-exchange-current correction included) is 20% smaller in magnitude than the experimental value $(-2.1275\,\mu_N)$.[25] Even if one had full confidence in the available models for calculating the meson-exchange-current corrections to ^{3}He magnetic scattering, the uncertainty in the ^{3}He magnetic radius is a factor of 2 larger than that attributed to the charge radius measurement,[26] which would imply that the neutron-proton radius difference in ^{3}He is essentially unknown. The uncertainty in our knowledge of the triton magnetic radius is even larger. However, that is irrelevant for our purpose, because it is the proton (the odd nucleon) in ^{3}H that carries most of the spin. That is, for ^{3}H it is impossible to extract a neutron radius from electron-scattering data.

III. PION-SCATTERING RATIOS

For the reasons discussed in the previous section, one is led to pursue pion scattering to determine the relative neutron-proton radii in the $A=3$ system, where meson-exchange-current contamination is expected to be minimal. Near resonance, the π^+-p interaction dominates π^+ scattering and the π^--n interaction dominates π^- scattering. That is, the π-nucleon coupling is much stronger than the π-π coupling. Hence, the situation is very different from that in electron scattering where the coupling of the electron to the pion is the same as that of the electron to the proton. In fact, we can place experimental limits on the strong-interaction contributions of pion-exchange currents in pion scattering. If we look at the double-charge-exchange channel, calculations[27] and experiments[28] limit the contribution of this effect to less than 1 μb/sr at forward angles (and the cross section decreases with angle). Since the smallest cross section to be considered here is of the order of 1 mb/sr, it is clear that this contamination is no more than 0.1% in cross section (3% in amplitude). Assuming that pion multiple-scattering effects can be properly accounted for, ratio measurements should be very sensitive to differences in the odd-nucleon and even-nucleon matter distributions. Similar techniques have been used in electron-scattering ratio measurements to explore the charge-distribution differences of isotope and isotone sequences.

One might ask how three-nucleon forces affect this analysis. Contemporary two-pion-exchange three-nucleon-force models were included in a number of the above-mentioned Faddeev calculations.[5] These three-body-force models are isoscalar in nature. Thus, they tend to decrease the difference between the proton and neutron radii. One can see in Fig. 1 from Ref. 29 that, while the introduction of a three-body force can improve the model binding energy (and therefore low-energy properties such as radii), three-body forces do not resolve the discrepancy between theory and experiment for the higher-momentum-transfer region of the charge form factors. The pion-scattering data we analyze does not encompass momentum transfers approaching those in the region of the charge-form-factor discrepancy. We shall compare results based on trinucleon densities suggested by Faddeev calculations which both include and omit model three-nucleon forces. However, one anticipates that these shape differences will have only a small effect on radii differences.

Let us consider three ratios of pion-trinucleon elastic-scattering cross sections. First, the ratio

$$r_1=\frac{\sigma(\pi^{+}\,^3\mathrm{H})}{\sigma(\pi^{-}\,^3\mathrm{He})} \tag{2}$$

primarily involves the pion strong interaction with the *odd nucleon* in each nucleus. That is, in the region of the (3,3) resonance, π^+p and π^-n scattering dominate over π^-p and π^+n. Clearly, the coherent Coulomb scattering does not cancel from the ratio, but away from forward angles the strong interaction should be much more important. Thus, r_1 should be sensitive to the ratio of the odd-nucleon form factors—in the single scattering (impulse) approximation, this is exactly what one would measure if only the dominant π^+p and π^-n interactions were retained. Both spin-flip and non-spin-flip scattering from the odd nucleon are important, so that the individual cross sections and r_1 are sensitive to both processes.

Second, the ratio

$$r_2=\frac{\sigma(\pi^{-}\,^3\mathrm{H})}{\sigma(\pi^{+}\,^3\mathrm{He})} \tag{3}$$

primarily involves the pion strong interaction with the *even nucleons* in each nucleus. Again the Coulomb effects do not cancel in the ratio. However, because the like nucleons are essentially paired in spin (to spin 0), spin-flip scattering is minimal. Thus, r_2 is sensitive to the ratio of the even-nucleon form factors and is dominated by non-spin-flip scattering.

Finally, the "super ratio"

$$R = r_1 r_2 \tag{4a}$$

$$= \frac{\sigma(\pi^+ \, ^3\text{H})\sigma(\pi^- \, ^3\text{H})}{\sigma(\pi^- \, ^3\text{He})\sigma(\pi^+ \, ^3\text{He})} \tag{4b}$$

should be least sensitive to model uncertainties in the pion-nucleus-scattering theory (as well as experimental normalizations). While the Coulomb interaction does *not* cancel, the calculation of R is less sensitive to any model dependence in those effects than are the individual ratios r_1 and r_2.

Because the ^{3}He nucleus is expected to be larger than that of ^{3}H, so that its form factor falls faster, we anticipate (in general) that $R > 1$. Similar conclusions can be reached for r_1 and r_2, although they are subject to greater uncertainty due to Coulomb interference effects.

Also useful from the point of view of the scattering theory, although they have no direct relationship to the neutron radii, are the ratios

$$\rho^+ = \sigma(\pi^+ \, ^3\text{H})/\sigma(\pi^+ \, ^3\text{He})$$

and

$$\rho^- = \sigma(\pi^- \, ^3\text{H})/\sigma(\pi^- \, ^3\text{He}) \, .$$

IV. ANALYSIS OF THE SCATTERING DATA

In this investigation we use an approach similar to that of Kim, Kim, and Landau[4] in that a potential is constructed to represent the strong-interaction scattering. We use this method because the Coulomb interaction modifies the ratios from their isospin-conserving values of one and it can be included in a potential model. The ratios also deviate from one because the ^{3}H/^{3}He odd radii and even radii are not expected to be equal, as discussed in Sec. II. (In Ref. 4 these radii were assumed to be equal in order to concentrate on the Coulomb effect alone.) This second reason for a deviation of the ratios from one is perhaps more interesting, since it is directly related to the structure of the three-nucleon system.

In our investigation we have treated the even and odd radii (actually their difference as discussed below) as variables by rescaling the individual proton and neutron densities obtained from Faddeev equation calculations. Thus, we implicitly assume a shape for the densities given by theory. We have used two different densities as starting points:[29] one from the solution including both two- and three-body forces (the "standard" case) and one using two-body forces alone (denoted by a subscript "2" on the letter identifying the case). Since the momentum transfer probed is much smaller than that at which the first zero in the form factor occurs, the basic shape is primarily determined by the radius. The sensitivity to this assumption is tested by comparing results with the two forms just mentioned.

While there is, unavoidably, a degree of uncertainty due to the manner in which the strong scattering process is treated (and its effect on the Coulomb interference), we have found that the sensitivity to changes in the three-nucleon form factors, at a certain scale of variation of the radii, is considerably greater than this uncertainty. In order to determine the uncertainties due to the approximations in our scattering theory, we have varied the parameters controlling its structure within what we believe to be reasonable bounds, keeping in mind that it should provide an acceptable representation of the differential cross sections. We note that the errors quoted for these (latter) measured cross sections are substantially larger than those of the measured ratios.

While we find that there is relatively little sensitivity of "R" to the scattering theory, for the ratios r_1 and r_2 (and for ρ^+ and ρ^-), this is not true. We discuss this point in more detail below. In the future we may anticipate that the scattering calculation can be made with greater certainty using multiple-scattering techniques,[30] but for the present investigation we have simply varied the relevant controlling parameters. The different cases are defined in Table I with a letter denoting each.

For each case a χ^2 comparison with the recent LAMPF data of Nefkens *et al.*[1] and of Pillai *et al.*[2] for the observable "R" was constructed as a function of the even (r_e) and odd (r_o) neutron radii. The corresponding even (^{3}He) and odd (^{3}H) proton radii were held fixed at values within the errors quoted from the electron-scattering determination. This χ^2 function was found to be well represented by a parabolic form in two dimensions:

$$\chi^2 = [(r_e - r_e^c)\cos(\phi) + (r_0 - r_0^c)\sin(\phi)]^2/a^2 + [-(r_e - r_e^c)\sin(\phi) + (r_0 - r_0^c)\cos(\phi)]^2/b^2 + \chi^2_{\min} \, . \tag{5}$$

The quantities r_e^c, r_o^c, ϕ, a, b, and $\chi^2_{\min}$ which characterize each fit are quoted in the second half of Table I. They were obtained by fitting this function to the χ^2 calculated with the scattering theory for selected values of the even- and odd-neutron radii. It is estimated that the uncertainty in the values of r_e^c and r_o^c extracted in this second fitting procedure is ± 0.001 fm.

We observe that it is only the differences between the two odd and two even radii which are relevant for the fits. While Table I lists results for several different radii, a comparison of cases J and K demonstrates the effect of this change alone. This character of the fits is essential to the analysis, since the uncertainties in the proton radii determined from the electron scattering are of the same order as the effect that we are investigating.

The underlying scattering theory is based on a nonlocal optical potential that has frequently been used in investigations of pion scattering from heavier nuclei.[30–32] The optical potential for either neutrons or protons in momentum space can be written as

$$V(\mathbf{q},\mathbf{q}')=[k^2 b_0 v_0(q) v_0(q') + b_1 \mathbf{q}\cdot\mathbf{q}' v_1(q) v_1(q')]\rho(|\mathbf{q}-\mathbf{q}'|) ,$$

where b_0 and b_1 are written terms of the π-nucleon phase shifts as

$$b_0=(e^{2i\delta_0}-1)/k^3 ,$$

$$b_1=(e^{2i\delta_1}-1)/k^3 ,$$

the pion-nucleon form factors are

$$v_{0,1}=\frac{\alpha_{0,1}^2+k^2}{\alpha_{0,1}^2+q^2} ,$$

and ρ is the Fourier transform of the density [$\rho(r)$ is normalized to 4π]. In the present case the potential is the sum of the potential from the neutron density (with neutron strengths) and the potential from the proton density (with proton strengths). In practice, this potential is transformed into a nonlocal form in coordinate space, partial wave by partial wave, where it is added to a Coulomb potential and a local absorptive potential proportional to the square of the density. The scattering problem is then solved in coordinate space using standard matrix solution techniques.

The above potentials directly describe the usual non-spin-flip pion-nucleus scattering. The spin-flip cross section (incoherent with the non-spin-flip cross section) is calculated in the distorted-wave impulse approximation by using the wave function resulting from the scattering from the two like nucleons. The logic of Kerman, McManus, and Thaler (KMT) (discussed in more detail below in the next section and in the Appendix) would im-

TABLE I. Data from the 14 cases considered. Note that cases B, F, and F_2 have extreme proton radii. All calculations use the form of the density from Faddeev calculations with a three-body potential except F_2, I_2, and J_2. It was found that the spin-flip contribution was not very sensitive to off-shell range and this quantity was held fixed at 400 MeV/c. The experimental value of ρ^+ is 0.676±0.009 and that of ρ^- is 1.604±0.021. The notation AT refers to the factor multiplying the angle transform discussed in Sec. V, feature (3) and SF refers to the factor multiplying the potential used to calculate the distorted waves discussed in Sec. IV.

Set	r_p^o (fm)	r_p^e (fm)	α (MeV/c)	ΔE (MeV)	AT	ρ^2	KMT	Coulomb shifts	SF dist.
A	1.560	1.750	600	0	1.0	7.5	2/3	on	1
B	1.610	1.790	600	10	0.6	5.0	2/3	on	$\frac{1}{2}$
C	1.560	1.750	600	0	1.0	7.5	2/3	on	$\frac{1}{2}$
D	1.560	1.750	600	0	1.0	7.5	0.673,0.595	on	$\frac{1}{2}$
E	1.560	1.750	600	0	1.0	7.5	0.673,0.595	off	$\frac{1}{2}$
F	1.610	1.790	900	10	0.6	5.0	0.673,0.595	on	$\frac{1}{2}$
F_2	1.610	1.790	900	10	0.6	5.0	0.673,0.595	on	$\frac{1}{2}$
G	1.560	1.750	600	0	1.0	7.5	0.568,0.615	on	$\frac{1}{2}$
H	1.560	1.750	600	0	1.0	7.5	1	on	$\frac{1}{2}$
I	1.560	1.750	600	0	1.0	7.5	0.673,0.555	on	$\frac{1}{2}$
I_2	1.560	1.750	600	0	1.0	7.5	0.673,0.555	on	$\frac{1}{2}$
J	1.560	1.750	800	0	1.0	6.3	0.673,0.555	on	$\frac{1}{2}$
J_2	1.560	1.750	800	0	1.0	6.3	0.673,0.555	on	$\frac{1}{2}$
K	1.580	1.750	800	0	1.0	6.3	0.673,0.555	on	$\frac{1}{2}$

Set	r_n^o (fm)	r_n^e (fm)	δ_0 (fm)	δ_e (fm)	a (fm)	b (fm)	$\chi^2_{\rm min}$	cosϕ	ρ (45°) +	ρ (45°) −
A	1.602	1.712	0.042	−0.038	0.009	0.005	2.8	0.850	0.845	1.270
B	1.654	1.761	0.044	−0.029	0.018	0.005	8.4	0.810	0.827	1.281
C	1.601	1.714	0.041	−0.036	0.009	0.005	3.5	0.860	0.859	1.251
D	1.596	1.720	0.036	−0.030	0.012	0.005	6.3	0.890	0.738	1.457
E	1.596	1.724	0.036	−0.026	0.010	0.005	13.3	0.895	0.741	1.443
F	1.646	1.763	0.037	−0.027	0.012	0.005	12.6	0.805	0.746	1.426
F_2	1.654	1.769	0.044	−0.021	0.027	0.005	11.9	0.860	0.746	1.416
G	1.599	1.712	0.039	−0.038	0.011	0.005	3.5	0.870	0.932	1.157
H	1.628	1.720	0.068	−0.030	0.013	0.005	4.9	0.710	0.905	1.194
I	1.592	1.721	0.032	−0.029	0.009	0.004	7.7	0.900	0.679	1.569
I_2	1.595	1.720	0.035	−0.030	0.010	0.005	8.4	0.870	0.682	1.577
J	1.595	1.721	0.035	−0.029	0.011	0.005	7.0	0.830	0.681	1.560
J_2	1.596	1.719	0.036	−0.031	0.009	0.005	7.0	0.840	0.684	1.567
K	1.615	1.721	0.035	−0.029	0.010	0.005	7.0	0.810	0.681	1.576

ply that there should be a factor of $\frac{1}{2}$ multiplying the potential used in this scattering to obtain the t matrix. For most of our distorted-wave calculations, we have used this factor: It is, however, not very important for the analysis pertaining to R, as can be seen by comparing cases A and C in Table I where this variation is the only difference.

V. PRINCIPAL FACTORS IN THE SCATTERING THEORY

Our scattering theory has been described in detail in Refs. 30–32. The primary uncertainty in our analysis is related to the treatment of the following principal features.

(1) Off-shell ranges. The π-N scattering amplitudes contain s-wave and p-wave ranges which can be varied independently. While the off-shell dependence of the t matrix can be determined from a model of the strong interaction in principle, in practice, the value obtained depends on the model used and/or on the means employed to determine it from the extant experimental data.[33] There is also the uncertainty arising because the value within the nucleus may be modified by the lifetime of the delta resonance, adding further nonlocality to the interaction.[30]

We found that the calculated ratios depended only weakly on these variables (on a scale set by changes in the radius differences), although the absolute cross section approximately scales with these off-shell ranges. Because of the lack of sensitivity in the ratios to these quantities, we set the s- and p-wave off-shell ranges equal to one another.

Since the spin-dependent contribution to the scattering (all of the spin was assumed to be carried by the odd nucleon) was treated by a distorted-wave impulse approximation in which the distorting wave was assumed to be due to scattering from the even nucleons, it was necessary to construct a transition operator based on the measured phase shifts. Since this transition operator was treated as nonlocal, there is also an off-shell range associated with it. The dependence of the ratios on this spin-flip off-shell range was even smaller than that due to the non-spin-flip range, so we fixed its value (at 400 MeV/c).

(2) Effective energy. Because of the restriction on the allowed intermediate states in the nuclear medium, the t matrix used in the scattering from a bound nucleon differs from that found in free space scattering.[34,35] The effective energy shifts, as explained in Ref. 30, can be treated as a function of the pion-nucleus angular momentum. Since such a calculation is beyond the scope of the present work, we used a single number for the energy shift[36] (in practice, usually zero).

(3) "Angle transform." Due to recoil of the nucleon in the nuclear medium, the effective s- and p-wave amplitudes are modified.[37,30] We used a rather standard prescription[38] but considered the possibility of an additional multiplying factor in this correction. A calculation of the type reported in Ref. 30 would provide a more complete description of the recoil effects, but a relativistic version would be needed for these energies.

(4) KMT factor. It is clear that a given nucleon cannot be struck successively by the projectile in a multiple-scattering series based on a t matrix. Kerman, McManus, and Thaler pointed out[39] that the geometric multiple-scattering series can be summed under this restriction by using for the potential $(A-1)/A$ times the impulse approximation for scattering and then multiplying the scattering amplitude resulting from the solution of the Schrödinger equation by the inverse of this $(A-1)/A$ factor. For large A this is a small correction, but for the three-body system it is very important. The calculation "H" in Table I shows the effect of ignoring this correction entirely. The values of ρ^+ and ρ^- at 45° (where the predictions of the theory should be the most reliable and where the dependence on the radii is the least) are in poor agreement with the data when the KMT factor is omitted.

In order to understand the physics of this factor, consider it to be a variable "g." That is, start with the (single-scattering) impluse approximation for the elastic scattering

$$[f(\mathbf{k},\mathbf{k}')S(|\mathbf{k}-\mathbf{k}'|)] ,$$

multiplied by g, as the potential in the Schrödinger equation and then divide the resulting amplitude by g. If g is very small, then the single-scattering impulse approximation is recovered. If the interaction is very strong (and $g\approx 1$), then the nucleus appears to be nearly "black," and the cross section will be approximately independent of the strength of the potential resulting in values for ρ^+ and ρ^- nearly equal to one. If the interaction is weak (or if g is small), then the cross sections will scale as the square of the strength of the potential. Thus, the factor g can be thought of as a variable that interpolates between these two extremes.

Consider the cross sections at 45° where the spin-flip contribution is small. If we postulate that we have two "strong" interactions of magnitude 3 and one "weak" interaction of magnitude 1 (which is the approximate ratio of the π^+p to π^-p p-wave amplitudes), then the cross-section ratio for the single-scattering impulse approximation is

$$(3+1+1)^2/(3+3+1)^2=0.51$$

for ρ^+ and the inverse 1.96 for ρ^-. The experimentally observed ratios are 0.676 for ρ^+ and 1.604 for ρ^-. A potential with $g=1$ gives 0.90 for ρ^+ and 1.18 for ρ^- (approaching the "black" limit) while using $g=\frac{2}{3}$ gives 0.85 and 1.27 (somewhat closer to the experimental result). As g is reduced, the phase of the strong amplitude is rotated, affecting the interference with the Rutherford amplitude as well. In fact, $g<1$ reduces this Coulomb correction for both r_1 and r_2. While the ratio R is much less affected, this dependence on g is one of the primary sources of uncertainty in the present analysis.

The situation becomes even more complex when the interactions of the projectile with the target protons and nucleons are allowed to differ. An heuristic discussion is given in the Appendix. In Table I we include calculations for both the second and third term prescriptions, (1) and (2) in the Appendix. (Compare G and D with E in Table I.) The value of the ratio t_2/t_1 was taken to be the ratio of p-wave strengths for the optical model. While this ratio is complex, the imaginary part is very small. The second prescription [Eq. (A4)] is preferred since it gives a better representation of ρ^+ and ρ^-. In fact, by reducing the "strong" scattering KMT factor to $g=0.555$, one obtains the most satisfactory agreement, and therefore several cases ($I-K$) are illustrated with this value.

(5) True absorption. A purely imaginary constant multiplying the square of the density was used to provide a potential representing the effect of pion absorption.[40] The coefficients used are in rough agreement with values determined from other nuclei.[41] [The units used in Table I for this factor are such as to yield fm^{-2} when it multiplies the square of the nucleon density (in nucleons/fm^3).] Scaling from experience with calcium and carbon (assuming a proportionality to the number of n-p pairs, NZ) would lead one to expect a value of around 12 fm^4. Variation from this value was allowed in order to obtain a satisfactory representation of the differential cross sections.

(6) Coulomb energy shifts. Since the pion will gain or lose energy approaching the nucleus, depending on the sign of its charge, the effective energy for the strong interaction will differ for π^+ and π^- (Ref. 42). The effect is not very large, but it is of opposite sign for the numerator and denominator in r_1 and r_2, so that it is important for those ratios. It is much less important for the ratio R (compare cases D and E in Table I) since the increase in one factor of the numerator or denominator is compensated by a decrease in the other. The values used for the Coulomb energy shifts are -0.9 (π^+ T), -1.8 (π^+ ^{3}He), $+0.9$ (π^- T), and $+1.8$ MeV (π^- ^{3}He).

VI. DISCUSSION

A. Super ratio R

We have discussed in the previous section the various assumptions that went into the calculations summarized in Table I. In this section we deal with the results shown in the second part of Table I. The values of $\delta_e=r_e^n-r_e^p$ and $\delta_o=r_o^n-r_o^p$ are seen to closely cluster (with the exception of "H" which is not a serious model variation). Before discussing this uncertainty, we note that the angular distributions are well represented as shown in Fig. 1. The minimum in each curve is due to the p-wave nature of pion-nucleon scattering and is unrelated to the minimum in the three-body form factor. In Figs. 2 and 3 we illustrate the effect on the predictions for R (compared with the data) due to variations in the radii of ± 0.01 fm about the best fit. Note that the odd radius difference is determined by the data around the minimum (where the spin flip, coming from the odd nucleon, dominates) whereas the even radius difference is determined by the data at many angles.

The results of our investigation are shown in graphical form in Fig. 4. The curves were computed using the average of all of the χ^2 surfaces and represent the locii of all points $\chi^2-\chi^2_{\min}=1$ and 4. The point corresponding to

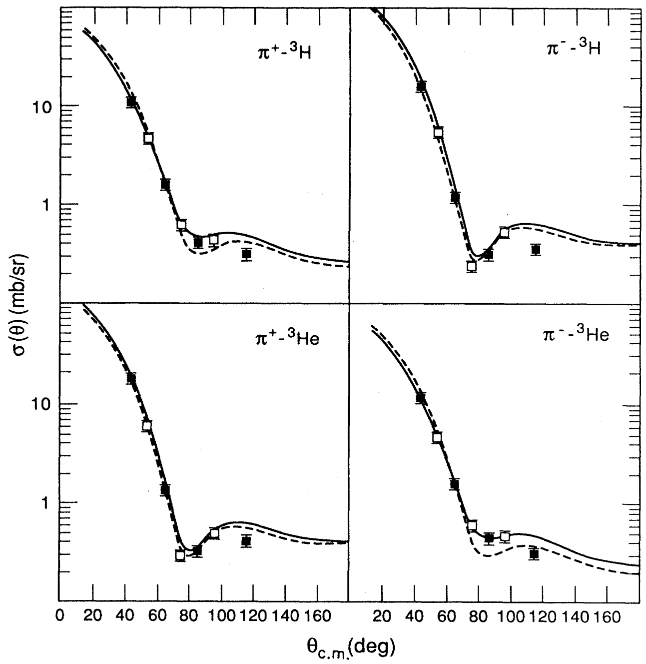

FIG. 1. Angular distributions for the four nuclei considered. The agreement with the data is moderate. The solid curves correspond to case "I" and the dotted to case "A." While there is not much to distinguish the two to the eye, the case "I" gives a much better representation of ρ^+ and ρ^- at 45°.

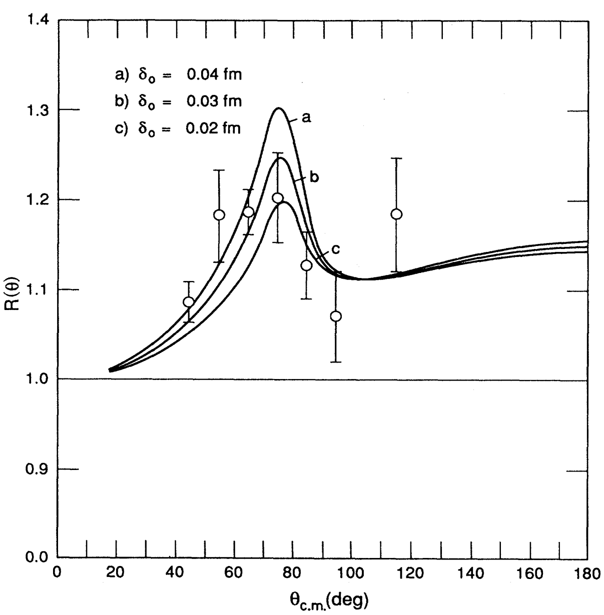

FIG. 2. Example variations with respect to the odd-neutron radius of the calculated "R" compared with the data. It is clear that the minimum region (where the "spin-flip" term, depending only on the odd-nucleon form factor, dominates) controls the fit to the data. The curves are calculated for r_n^e fixed at 1.720 fm and r_n^0 taking values of 1.58, 1.59, and 1.60 fm for case "A."

no effect [i.e., $r_o(^3\text{He}) \equiv r_o(^3\text{H})$, etc.] is the upper left-hand corner of the graph at zero. It is clear that there is definite evidence for a difference in the two radii and that a value can be extracted with a specified error.

To estimate the error in our determination of δ_e and δ_o, a conservative approach is to compute the average and variance of all of the central values and to convolute the error obtained with the average statistical error determined from the ellipses. This procedure gives

$$\delta_e = -0.030 \pm 0.010 \text{ fm} \quad (0.005),$$

$$\delta_0 = 0.038 \pm 0.007 \text{ fm} \quad (0.004).$$

The number in parenthesis is our estimate of the contribution to the error due to theoretical uncertainties. A more optimistic approach in estimating the error entails the elimination of those cases which do not give a good fit to ρ^+ and ρ^- (A, B, C, and G) and those cases which have a large χ^2 (E, F, and F_2). This leaves cases D, I, I_2, J, J_2, and K. An average for these cases alone yields

$$\delta_e = -0.030 \pm 0.008 \text{ fm} \quad (0.001)\ ,$$

$$\delta_o = 0.035 \pm 0.007 \text{ fm} \quad (0.001)\ .$$

For purposes of comparison with these radius differences extracted from our analysis of pion-^{3}H/^{3}He scattering, we have constructed three separate trinucleon models in which a single parameter (the pion-nucleon form-factor range) was adjusted to yield the experimental ^{3}H and ^{3}He binding energies: (1) We combined the "stiff" Reid-soft-core (RSC) NN potential[43] with the Tucson-Melbourne (TM) two-pion-exchange three-nucleon force.[44] We chose the pion-nucleon form-factor range in the TM three-body force ($\Lambda = 5.49 m_\pi$, where $m_\pi = 139.6$ MeV) such that the triton binding energy was 8.48 MeV. Turning on a point-Coulomb interaction between the two protons in ^{3}He then yielded a binding energy of 7.81 MeV. To reduce this to the experimental 7.72 MeV, Λ was set to $5.41 m_\pi$. That is, we introduce a charge-symmetry-breaking three-body force to decrease the ^{3}He binding to 7.72 MeV. (2) An analogous procedure was followed using the Argonne V_{14} (Av14) NN potential[45] combined with the Brazilian (BR) three-body-force model.[46] Choosing $\Lambda = 5.01 m_\pi$, we obtained a triton binding energy of 8.48 MeV and a ^{3}He energy of 7.81 MeV. Reducing Λ to a value of $4.93 m_\pi$ lowered the ^{3}He binding energy to the desired experimental 7.72 MeV. (3) Both of these three-body-force models are isoscalar. Therefore, we considered a third model in which the $l=0$ spin-singlet central potential strength was increased (multiplied by 1.057) to obtain the desired 8.48-MeV binding of the triton using the AV14 NN force model. (The original AV14 model yields a binding energy for the triton of 7.67 MeV in a 34-channel calculation.[5]) The corresponding ^{3}He binding energy was again 7.81 MeV; 7.72 MeV was attained by introducing an hadronic charge-symmetry-breaking interaction that reduced the spin-singlet enhancement factor to 1.051.

In each of these three cases, one finds $\delta_o = 0.020$ fm and $\delta_e = -0.032$ fm, when the Coulomb interaction between the two protons is the only charge-symmetry-breaking interaction considered; that is, $B(^3\text{H}) = 8.48$ MeV and $E(^3\text{He}) = 7.81$ MeV. This result is indicated by

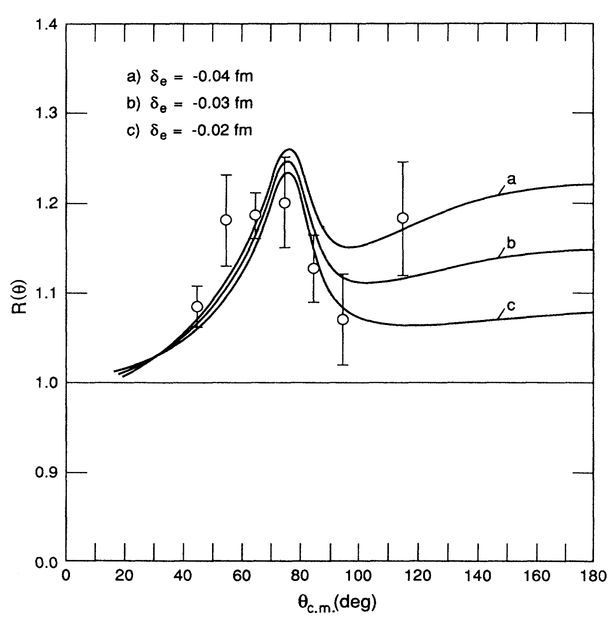

FIG. 3. Example variations with respect to the even-neutron radius of the calculated "R" compared with the data. Here the entire angular range (but especially the part at large angles) is important. The curves are calculated for r_n^o fixed at 1.59 fm and r_n^e taking on values of 1.71, 1.72, and 1.73 fm for case "A."

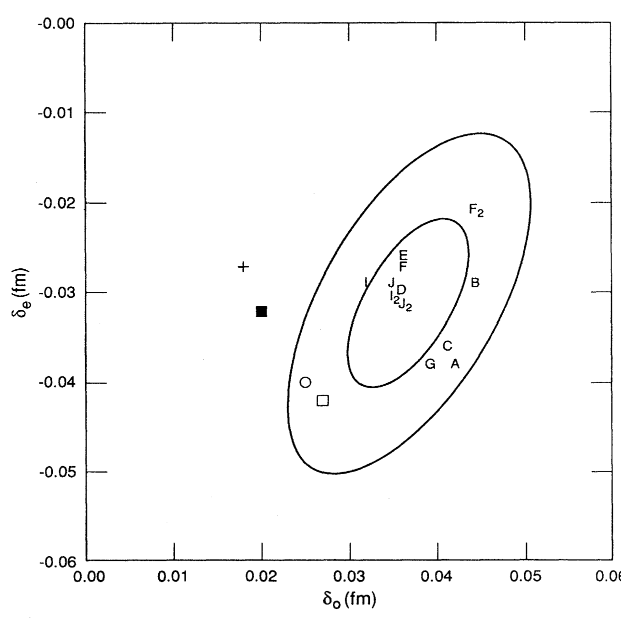

FIG. 4. Locii of the χ^2 surfaces $\chi^2 = \chi^2_{\text{min}} + 1$ and $+4$. The calculated χ^2 is the average of all of the cases shown, hence, the ellipse is spread by the distribution of centroids. Note that the case F_2 is not as aberrant as it might seem since its position along the major axis is ill determined. Case H is not shown and the point K is degenerate with J.

the black square in Figs. 4 and 5. When short-range (strong-interaction) charge-symmetry-breaking forces were introduced so that the models yielded the experimental $E(^3\mathrm{He})=7.72$ MeV, then for each of the first two models one finds $\delta_o=0.027$ fm and $\delta_e=-0.042$ fm, whereas for the pure *NN* force model one finds $\delta_0=0.025$ fm and $\delta_e=-0.040$ fm. That is, charge symmetry breaking in the three-body force produces a slightly larger radius difference effect for these models. These results are represented by the open square and circle in Figs. 4 and 5.

One can postulate another type of (charge-symmetry-conserving) three-body force which could account for at least a part of the missing energy difference. If the effect of this three-body force were to increase the $T=1$ attraction and decrease the $T=0$ attraction, then the protons in ^{3}He (and the neutrons in ^{3}H) would be drawn closer together leading to a greater Coulomb repulsion in ^{3}He while the odd-nucleon radius would become larger. For example, if one multiplies the singlet potential by 1.209 and the triplet potential by 0.935, then the triton binding energy remains the same but Coulomb repulsion increases by 31 keV reducing the discrepancy from 90 to 60 keV. The even (proton) radius is decreased to 1.719 fm and the odd (proton) radius is increased to 1.650 fm, both within the electron-scattering experimental errors. Thus, such a model could not be ruled out experimentally at this point by electron scattering. However, if we look at the values of δ_o and δ_e, it is found that they *decrease* (from the pure Coulomb values) to 0.018 and -0.027 fm. (These are shown as the cross in Figs. 4 and 5.) That is, a determination of the even- and odd-radius differences appears capable of eliminating a certain class of models which might otherwise be an alternative to CSB.

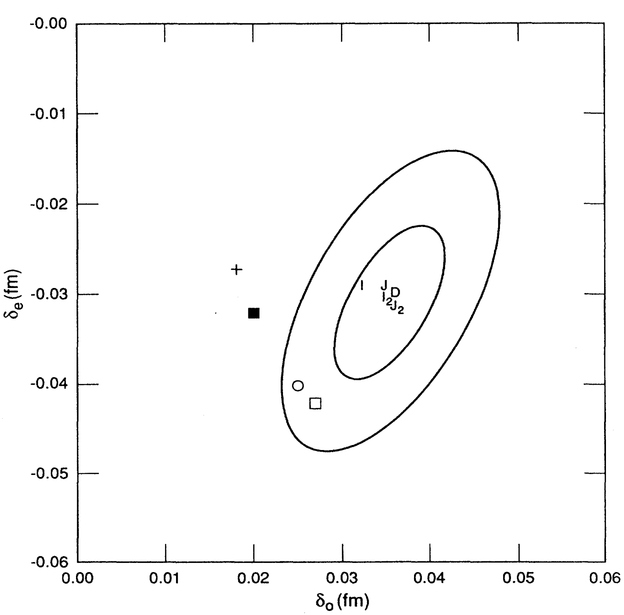

FIG. 5. Same as Fig. 4 except that it includes the reduced "optimistic" set only.

B. The ratios r_1 and r_2

While we have observed that the results of the analysis of the ratio R are rather stable against model variations, such is not the case for the separate ratios r_1 and r_2. They are particularly sensitive to the factor g. They are also somewhat sensitive to the Coulomb energy shifts. In most instances the minimum in r_1 lies at smaller (absolute) values of δ_o and δ_e than it does for R, and the minimum for r_2 lies at larger values than for R. However, for case E (no Coulomb shifts) all three differences are in essential agreement although χ^2 from R is large. We note that the prescription that we have used for estimating the shifts gives a maximum effect and the magnitude of the shift probably should be reduced.

The inferences drawn from these two ratios cannot be pushed too far because of the experimental problem of flux normalization. The measurements were made relative to deuterium and the quoted results are only the true ratios if π^+ and π^- scattering from the deuteron are equal. From recent measurements[47] one can estimate that this ratio is about 1.02 ± 0.02. The value chosen for this correction impacts, to some extent, any conclusions drawn from r_1 and r_2. Nevertheless, they do provide a school for investigating multiple scattering and do indicate which uncertainties are likely to be important for other nuclei where the double-ratio measurement is not possible.

VII. CONCLUSIONS

Calculations for the trinucleon system have established that there is a Coulomb anomaly in terms of the trinucleon binding-energy difference as is the case for a number of other nuclei. The difference in binding between ^{3}He and the triton is 760 keV. Theoretical estimates only yield some 650 keV due to Coulomb repulsion between the two protons. The remaining 100 keV is attributed to some other charge-symmetry-breaking effect. It is natural to ask if an analogous effect is visible in the radii. The present treatment lends itself well to answering this question. What we have extracted is the difference in "odd size" (o) and "even size" (e) between ^{3}He and ^{3}H. That is, we find

$$\delta_e=r_n(^3\mathrm{H})-r_p(^3\mathrm{He})=-0.030\pm0.008 \text{ fm}$$

and

$$\delta_o=r_n(^3\mathrm{H})-r_p(^3\mathrm{H})=0.035\pm0.007 \text{ fm} .$$

Model Faddeev calculations imply differences of the order of -0.042 fm for the even radii and 0.027 fm for the odd radii.

From Fig. 4 or 5 we see, because of the form of the correlated errors, that the general magnitude of the effect (its nonzero size) is better determined than either of the individual radius differences. The overlap with the theoretical charge-symmetry-breaking δ_0 and δ_e is nonnegligible and, based on present data, it seems unlikely that the experimental radius differences are smaller than the theoretical predictions. Measurements at larger angles, and possibly improved precision in the forward-angle data, are needed in order to provide a definitive answer. Smaller differences would perhaps indicate that the like pair of nucleons reside closer to one another than predicted by the present Faddeev calculations.

The present analysis indicates that a significant charge-symmetry-breaking effect (beyond the Coulomb interaction) has been observed in the differences of the $^3H/^3He$ radii.

ACKNOWLEDGMENTS

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APPENDIX: KMT FOR SYSTEMS OF NONIDENTICAL NUCLEONS

In deriving their result, KMT (Ref. 39) assumed that the interaction of the projectile with each nucleon is the same. The situation involving different interactions for protons and neutron has been analyzed by Garcilazo[48] and requires a coupled-channel calculation. Since such a calculation for the three-body case is beyond the scope of the present paper, we present an heuristic argument based on the (almost) geometric series generated by the multiple-scattering expansion. For the strict KMT assumption of identical nucleon interactions (we set the Green's functions equal to 1 in this schematic representation to focus on the algebraic properties of the series) we have

$$\begin{aligned} T &= At + A(A-1)t^2 + A(A-1)^2t^3 + \cdots \\ &= At/[1-(A-1)t] \\ &= [A/(A-1)]\{(A-1)t/[1-(A-1)t]\} . \end{aligned} \tag{A1}$$

The quantity in curly brackets is the amplitude resulting from the solution of the Schrödinger equation with a potential, $(A-1)t$. Thus, we formally recover the KMT result.

For the three-nucleon case in which there is one pion-nucleon interaction strength for the odd nucleon (t_1) and a second strength for the even nucleons (t_2), we can write the first few terms of the multiple-scattering series

$$\begin{aligned} T &= 2t_2 + t_1 + 2t_2^2 + 4t_1t_2 + 2t_2^3 + 6t_1t_2^2 + 4t_1^2t_2 + \cdots \\ &= 1/g\,[g(2t_2+t_1) + g(2t_2^2+4t_1t_2) \\ &\qquad + g(2t_2^3+6t_1t_2^2+4t_1^2t_{2)+} \cdots] , \end{aligned} \tag{A2}$$

where we have introduced the factor "g" and grouped the terms by scattering order. Since there are only three nucleons and at least one of the interactions is "weak," we may expect that the series converges rapidly enough that the first few terms will give some guidance in choosing the best value of g. We consider three cases to obtain some intuitive feeling for the size of g.

(1) If we require that the second-order term be the square of the first, we obtain

$$g = 2t_2(t_2+2t_1)/(2t_2+t_1)^2 . \tag{A3}$$

For $t_1 = t_2$ we have $g = \frac{2}{3}$, as we must to recover the KMT result. For $t_2 = 3t_1$, $g = \frac{30}{49} \simeq 0.61$ while for $t_1 = 3t_2$, $g = \frac{14}{25} = 0.56$, both less than $\frac{2}{3}$.

(2) Requiring the third-order term to be the cube of the first, we arrive at

$$g^2 = 2t_2(t_2+t_1)(t_2+2t_1)/(2t_2+t_1)^3 , \tag{A4}$$

which gives, for $t_1 = 3t_2$, $g = \sqrt{56/125} \simeq 0.67$ and for $t_2 = 3t_1$, $g = \sqrt{120/343} \simeq 0.59$.

(3) If we assume for the case in which $t_2 = 3t_1$ that the scattering from the pair completely dominates and that we can neglect t_1 completely, then we obtain $g = \frac{1}{2}$.

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