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Temperature dependence of fusion barriers

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We calculate the free interaction energy of two finite-temperature nuclei in the sudden approximation by using realistic semiclassical energy-density functionals. The fusion barriers for intermediate-size systems come over their ground-state values for higher temperatures in such a way that there is a "critical temperature" above which fusion cannot occur. We discuss the implications of our results for the interpretation of empirical data.

Properties of equilibrated hot and compressed nuclei are important ingredients for astrophysical investigations. This static picture of a finite-temperature nucleus is also useful for the description of some aspects of high-energy heavy-ion collisions although these reactions are rather complicated time-dependent processes (see, e.g., Ref. 1, and references quoted therein).

Theoretical information on finite-temperature nuclei usually stems from a description of the nucleus either as (i) a nuclear liquid in equilibrium with its vapor[2] or as (ii) a heated finite metastable system in the vacuum. For temperatures below about one-half of the Maxwell critical temperature the thermostatic properties of nuclei derived from these two descriptions, however, do not differ much.[3,4]

Fission of hot nuclei may serve as a guide to our understanding of the thermostatic properties of nuclei in question. From early attempts done in the framework of the liquid-drop model the fission barrier was found to decrease with temperature.[5] Microscopic treatments of fission with realistic effective interactions lead also to the same conclusion.[6-8] Intuitively, these results can be explained by the fact that the surface tension (specific free-surface energy) decreases with temperature, whereas the Coulomb repulsion is not affected much since the thermal expansion of nuclei in all calculations turns out not to be important.

As a first approach to the study of the temperature dependence of fusion barriers we try to take over the results for the fission barrier since both fission and fusion have some common overlap of qualitative features. In both cases, the two fragments for large distances only feel the Coulomb repulsion. If their distance is in the range where overlap sets in, the nuclear attraction from the surface tension bends this Coulomb potential in such a way that in the fission potential as well as in the fusion potential—for small and intermediate size systems—a barrier is built up. The shape of the fission barrier of a hypothetical nucleus $(2N,2Z)$ is expected to have similar characteristics as the fusion barrier of the binary system $(N,Z)+(N,Z)$. In particular, if two large nuclei do not have a fusion barrier, the hypothetical compound system is supposed to be unstable against fission, i.e., has no fission barrier at all. For very large overlap during the fusion process, depending on the scattering energy, there may be a repulsion between the two systems which comes mainly from compressing nuclear matter above the saturation density. This effect clearly has no analog in the fission process.

The curves for the fission and fusion potentials, plotted each in its standard diagram, differ in their absolute values. The fission deformation energy is normalized asymptotically, i.e., for large distances of the fission products, to the free energy of the separated fragments, which means that it reaches the sum of the free energies of the fragments whereas the fusion barrier curves are normalized in such a way that all of them approach zero asymptotically. Thus, although the shapes of fusion and fission barriers might look similar it may not be right to conclude that the fusion barriers decrease with temperature in the same way as the fission barriers for the composite system might decrease. In order to draw conclusions about the behavior of fusion barriers from that of fission barriers one first would have to shift all fission barriers from their asymptotic values (which equal the free energies of the fragments) up to zero. Since the free energies of the fragments also depend on temperature this procedure could result unexpectedly in an increase of the barriers with temperature after the new normalization. From the intuitive argument which explained the behavior of the fission barriers, namely from the decrease of the surface tension with temperature, just this behavior of the fusion barrier is expected. The attraction between the two nuclei at a given distance becomes smaller with temperature, and this effect could increase the fusion barrier. There are competing effects from the overlap of the two density distributions. However, then the entropy terms in the free energy also come into play, which results in a repulsion. It is the aim of the present paper to study all of these effects quantitatively in a microscopic approach.

Fusion and fission barriers for excited nuclear systems normally are plotted as isotherms in the deformation-energy diagram. That does not mean necessarily a restriction to isothermal processes. The isotherms in a pressure-volume diagram for the equation of state contain the full thermodynamical information. Similarly, also information about nonisothermal processes is contained in the plot of isotherms for the fission barriers. It is some kind of contour plot for a function of two variables. Naturally, for a description of some specific process for a finite temperature system it is convenient to use the specific thermodynamic potential with natural variables that are suited

for this process. If, e.g., the process to be studied is isothermal, the adequate potential is the free energy F as a function of the temperature T and distance R, whereas for an isentropic process the relevant potential will be the internal energy E as a function of R and the entropy S.

The force K between two fusing nuclei at a distance R and for some temperature T (not necessarily constant during the process) or some entropy S can be calculated from the general thermodynamic relations[9]

$$K=-\left.\frac{\partial F}{\partial R}\right|_T=-\left.\frac{\partial E}{\partial R}\right|_S, \tag{1}$$

where $F=E-TS$ is the free energy and E the internal energy of the interacting systems. Both relations will give the correct value for the force at a given thermodynamical state if the relevant quantities are inserted properly. For theoretical calculations, where the temperature is given as a parameter in a natural way, it is convenient to calculate first the free energy. The internal energy E could then be obtained as a function of its natural variables S and R if the temperature $T=T(S,R)$ is known from the entropy $S=S(T,R)$ (see, e.g., Ref. 10 for the calculation of the internal interaction energy as a function of S and R). We prefer to use the free interaction energy because it is the easiest to obtain theoretically.

The problem of the temperature dependence of the fusion barrier is a topic in the literature for more than ten years. In the earliest investigation in the Thomas-Fermi approach,[10] the fusion barrier was found to increase with excitation energy (both free energy as a function of temperature as well as the internal energy as a function of entropy increasing). The free energy, however, was calculated in the low-temperature approximation and therefore the quantitative conclusions from there are questionable at higher temperatures. The deficiency of this T^2 approximation was pointed out in Ref. 4 in the context of fission barriers and later in Ref. 11 in the context of fusion barriers. Several recent papers,[12–15] in contrast to Ref. 10, predicted that the fusion barrier decreases with temperature. Some of these papers calculated the internal interaction energy $E=E(T,R)$ as the interaction potential which, however, is a starting point not consistent with the thermodynamic relation (1).

The free interaction energy between two nuclei at a separation R having temperature T is given in the sudden approximation as

$$F_{12}(R,T)=\int d^3r[\mathcal{F}(T,\rho_1+\rho_2)-\mathcal{F}(T,\rho_1)-\mathcal{F}(T,\rho_2)], \tag{2}$$

where ρ_1 and ρ_2 are the densities of the respective nuclei taken as Woods-Saxon (WS) distributions. We evaluate the free-energy density functionals $\mathcal{F}(T,\rho)$ in the temperature-dependent extended Thomas-Fermi approximation[4] (TETF). The kinetic energy and the entropy density functionals are evaluated up to the second order. The SkM* force[4] is taken as the effective interaction that allows for a variable effective nuclear mass dependent on density. The central density ρ_0 in the WS density expression is taken as 0.1603 fm^{-3} consistent with the SkM* interaction. The neutron and proton densities are scaled according to the neutron-proton ratio in the respective nuclei. The half-density radius C is given by the well-known expression depending on the nuclear mass A, the surface diffuseness a, and the radius parameter r_0. The temperature dependence of r_0 and a, to a good approximation, is given by[5,8,16] $r_0(T)=r_0(0)[1+\alpha T^2]$ and $a(T)=a(0)[1+\beta T^2]$. The values of α and β are chosen as $\alpha=0.0011$ MeV^{-2} and $\beta=0.01$ MeV^{-2}, consistent with previous calculations. The Coulomb potential in the free-energy functional is a sum of direct and exchange terms. The direct part is evaluated for two overlapping spheres of homogeneous charge distributions[17] and the exchange part is evaluated in the Slater approximation (see also Ref. 10).

We have calculated the interaction potential for a few selected systems, namely ^{35}Cl+^{58}Ni, ^{27}Al+^{70}Ge, ^{40}Ca+^{40}Ca, ^{74}Ge+^{74}Ge, ^{100}Mo+^{100}Mo, ^{120}Cd+^{120}Cd, and ^{208}Pb+^{208}Pb. In Fig. 1, we display the free interaction energy as a function of separation R between the nuclei ^{35}Cl and ^{58}Ni, evaluated in the ETF approximation without the second-order gradient corrections.[4] At zero temperature, a well-developed barrier of height $E_B=62.96$ MeV appears at the separation $R=10$ fm. As the temperature increases, the location of the barrier is pushed a little inside and its height increases. When a certain critical temperature is reached, the barrier vanishes completely. We call this temperature "critical fusion temperature" (T_{CF}), above which it is impossible for two hot nuclear systems to fuse. After the inclusion of the second-order terms in both the kinetic energy density and the entropy density in the calculation of the interaction potential, the role of temperature on the fusion barriers is, however, a little different as can be seen from Fig. 2. As the second-order correction to the free-energy density functional gives rise to a net attraction, all the fusion barriers at different temperatures are lowered compared

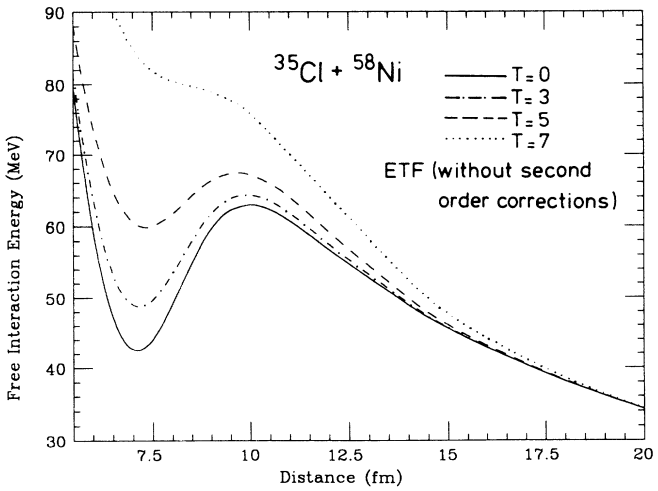

FIG. 1. The free interaction energy for the system ^{35}Cl+^{58}Ni plotted as function of distance R between the nuclei. Here the calculations are done in the ETF approximation without inclusion of second-order correction terms to the free-energy density functional. The solid and dashed-dotted lines correspond to $T=0$ and 3 MeV; the dashed and dotted lines correspond to $T=5$ and 7 MeV, respectively.

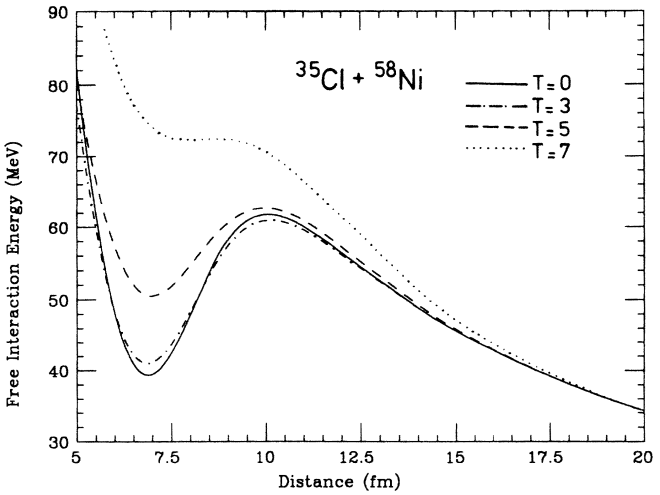

FIG. 2. Same as in Fig. 1; here the calculations are done with the inclusion of second-order corrections in the free-energy density functional.

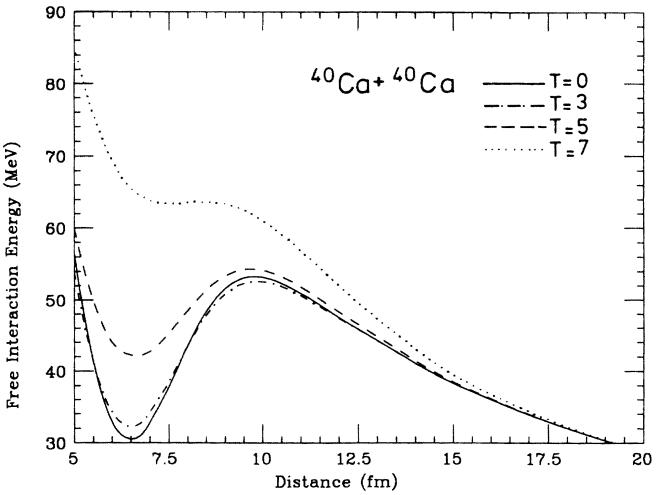

FIG. 3. The free interaction energy for the system $^{40}Ca + ^{40}Ca$ with the inclusion of second-order corrections.

to those evaluated with the TF approximation. We also find that with an initial increase in temperature (up to ~ 4 MeV), the barrier decreases; with further temperature increase, however, the barrier increases up to the critical fusion temperature when the barrier dissolves. This strange behavior of the fusion barrier with temperature is to be understood as arising from a delicate interplay of the increased attraction from larger overlap between heated nuclei, a loss of attraction due to a decrease in the surface tension with temperature and a further repulsion coming from the entropy of mixing.[10,11] We have also repeated the calculation for the interaction energy with only the internal energy term dependent on T [without the entropy term in Eq. (2)]. A monotonic decrease of the internal energy barrier with temperature is found there, as is observed in previous calculations.[12–15]

All calculations reported here are done with WS density distributions with fixed values of the central density ($\rho_0 = 0.1603$ fm^{-3}) and surface diffuseness ($a = 0.56$ fm) at zero temperature. In principle, the densities should be self-consistently determined with the particular effective interaction chosen (SkM* here) which one wishes to employ for the evaluation of the free-energy density functional. For the calculation of the free interaction energy, the WS density distribution with the same ρ_0 as obtained for the SkM* force, is a very reasonable approximation. The surface diffuseness may, however, change from system to system. To see the sensitivity of the fusion barrier to the choice of the parameters, we have made calculations for $^{40}Ca + ^{40}Ca$ with two choices of the diffuseness parameter ($a = 0.56$ and 0.47 fm that results from the self-consistent calculation[4]), with different central densities and also with two different expansion coefficients for the temperature dependence of surface diffuseness ($\beta = 0.01$ and 0.015 MeV^{-2}). They have some influence on the barrier heights and locations, but the relative pattern of fusion barriers with increasing temperature does not change appreciably. In Figs. 3 and 4, we display the fusion barriers for $^{40}Ca + ^{40}Ca$ for $a = 0.56$ and 0.47 fm. The influence of not too unreasonably different parameters on the critical fusion temperature is found to be rather inappreciable. The location and heights of the fusion barriers at zero temperature of the few systems studied here are given in Table I. Wherever experimental results[18,19] are available, our calculations seem to be in fair agreement with them. For the system $^{208}Pb + ^{208}Pb$, there is no fusion barrier, in agreement with what is obtained by Faessler *et al.*[11]

For all the studied systems that have a fusion barrier at zero temperature, it is found that there exists a critical fusion temperature T_{CF} above which it is no longer possible for the system to fuse. The Coulomb repulsion is then just sufficient to overcome the nuclear attraction weakened due to the reduced surface tension so that the system cannot hold together. In Table II, we show T_{CF} as a function of the mass A ($=A_1+A_2$) of the composite systems that are studied here. With increasing mass, T_{CF} decreases, mainly due to the larger influence of the Coulomb force. It is interesting to see that the disappearance of the fusion barrier above T_{CF} is quite consistent with the re-

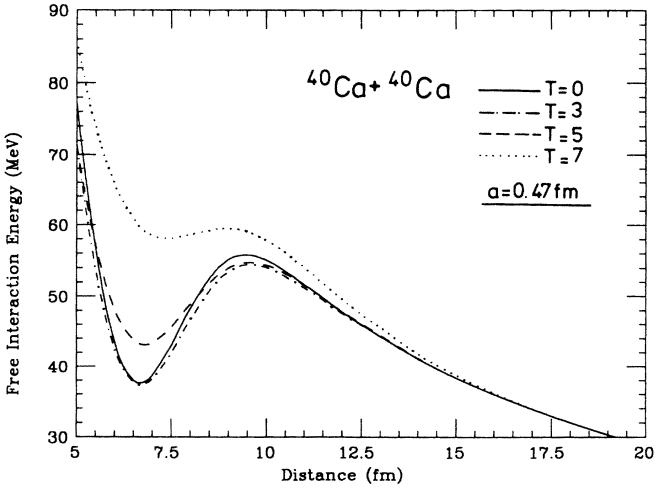

FIG. 4. Same as in Fig. 3; here the calculations are done with a different surface diffuseness ($a = 0.47$ fm) in contrast to that in Fig. 3. For further details, see the text.

TABLE I. The heights V_B and positions R_B of the fusion barriers at zero temperature obtained in second order ETF.

System	V_B (MeV)	R_B (fm)
$^{40}Ca+^{40}Ca$	53.30	9.8
	55.75[a]	9.5[a]
	55.03[b]	9.74[b]
$^{35}Cl+^{58}Ni$	61.75	10.1
$^{27}Al+^{70}Ge$	53.53	10.2
	55.1[c]	10.2[c]
$^{74}Ge+^{74}Ge$	120.99	11.1
$^{100}Mo+^{100}Mo$	197.07	11.7
$^{120}Cd+^{120}Cd$	247.19	12.2

[a]Values calculated with $a=0.47$ fm.
[b]Values quoted in Ref. 18 to have a good fit with the subbarrier fusion cross section for $^{40}Ca+^{40}Ca$.
[c]Experimental values quoted in Ref. 19.

cently observed vanishing of the fusion cross section[20,21] at bombarding energies around the Fermi energy domain. Since, at such bombarding energies, the temperature in a symmetric system may be ~5–7 MeV,[22] it may be worthwhile to explore the possible connection between the disappearance of the fusion cross section and the temperature of the heated interacting nuclei. It is also noteworthy that in this mass range, T_{CF} nearly coincides with the limiting temperature T_{lim} (the temperature above which no phase equilibrium[23] between the finite nuclear liquid and the surrounding vapor exists) obtained in a thermodynamic model[24] of liquid-gas coexistence for the same nuclear masses along the β-stability line (see Table II). Our values for T_{CF} are also consistent with the value of the limiting temperature ("flash temperature") obtained for ^{208}Pb in the picture of metastable equilibrium.[25] For a given composite system, T_{CF} is the lowest for symmetric configurations, but it has been checked numerically that, not too far away from symmetry, the change in T_{CF} is inappreciable (as an example, for $^{27}Al+^{70}Ge$, T_{CF} is 7.2 MeV, for the symmetric configuration for the same composite system, T_{CF} is 7.05 MeV). In the present calculation as well as in the calculations invoking liquid-gas phase equilibrium, the Coulomb potential and the surface tension play a central role, but whether this agreement between T_{CF} and the limiting temperatures[24,25] is purely coincidental or has deeper physical reasons is not known and further investigation on their interrelation may be called for.

TABLE II. The critical fusion temperatures T_{CF} calculated for the studied systems. Also shown are the limiting temperatures T_{lim} as calculated in Ref. 24 in a thermodynamical model for the nuclei having $A=A_1+A_2$ along the β-stability line.

System	T_{CF} (this work) (MeV)	T_{lim} (Ref. 21) (MeV)
$^{40}Ca+^{40}Ca$	7.1	7.0
$^{35}Cl+^{58}Ni$	7.15	7.0
$^{27}Al+^{70}Ge$	7.2	7.0
$^{74}Ge+^{74}Ge$	6.4	6.3
$^{100}Mo+^{100}Mo$	5.3	5.2
$^{120}Cd+^{120}Cd$	4.2	4.2

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[1]E. Suraud, C. Grégoire, and B. Tamain, *Progress in Particle and Nuclear Science* (Pergamon, Oxford, 1989), Vol. 23, p. 357.
[2]P. J. Siemens, Nature (London) **305**, 410 (1983).
[3]W. Stocker, Phys. Lett. **142B**, 319 (1984).
[4]M. Brack, C. Guet, and H.-B. Hakansson, Phys. Rep. **123C**, 276 (1985).
[5]R. W. Haase and W. Stocker, Phys. Lett. **44B**, 26 (1973).
[6]J. Bartel and P. Quentin, Phys. Lett. **152B**, 29 (1985).
[7]D. Dalili, J. Nemeth, and C. Ngô, Z. Phys. **321**, 335 (1985).
[8]C. Guet, E. Strumberger, and M. Brack, Phys. Lett. B **205**, 427 (1988).
[9]A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, London, 1975), Vol. 2, p. 372.
[10]U. Gaehde and W. Stocker, Nucl. Phys. A **278**, 177 (1977).
[11]A. Faessler, M. Rashdan, M. Ismail, N. Ohtsuka, and W. Wadia, Z. Phys. A **333**, 153 (1989).
[12]E. Tomasi, X. S. Chen, S. Leray, C. Ngô, M. Barranco, X. Vinas, and H. Ngô, Nucl. Phys. A **389**, 69 (1982).
[13]G. La Rana, C. Ngô, A. Faessler, L. Rikus, R. Sartor, M. Barranco, and X. Vinas, Nucl. Phys. A **414**, 309 (1984).
[14]M. Rashdan, A. Faessler, M. Ismail, and N. Ohtsuka, Nucl. Phys. A **468**, 168 (1987).
[15]L. Guo Qiang and Xu Gong-Ou, Phys. Rev. C **41**, 169 (1990).
[16]X. Campi and S. Stringari, Z. Phys. A **309**, 239 (1983).
[17]J. E. Poling, E. Norbeck, and R. R. Carlson, Phys. Rev. C **13**, 648 (1976).
[18]H. Esbensen, S. H. Fricke, and S. Landowne, Phys. Rev. C **40**, 2046 (1989).
[19]E. F. Aguilera, J. J. Vega, J. J. Kolata, A. Morsad, R. G. Tighe, and X. J. Kong, Phys. Rev. C **41**, 910 (1990).
[20]G. Auger *et al.*, Phys. Lett. **169B**, 161 (1986).
[21]B. Borderie and M. F. Rivet, Z. Phys. A **321**, 703 (1985).
[22]S. Bhattacharya, K. Krishan, S. K. Sammadar, and J. N. De, Phys. Rev. C **37**, 2916 (1988).
[23]S. Levit and P. Bonche, Nucl. Phys. A **437**, 426 (1985).
[24]D. Bandhyopadhyay, C. Samanta, S. K. Sammadar, and J. N. De, Nucl. Phys. A **511**, 1 (1990).
[25]A. H. Blin and M. Brack, Nucl. Phys. A **504**, 300 (1989).