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Particle-γ correlation determination of static quadrupole moments

E. E. Gross, D. C. Hensley, and J. R. Beene
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6368
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A complete particle-γ-ray angular correlation has been measured for the decay of the first 2^+ state of ^{24}Mg excited by ^{24}Mg(200 MeV)+^{208}Pb. The particle-γ-ray correlation, expressed as alignment tensors, proves to be particularly sensitive to the static quadrupole moment of the 2_1^+ state. The derived value of the 2_1^+ static quadrupole moment is $Q_2=(-29\pm3)\ e\,\text{fm}^2$ which is (1.56 ± 0.16) times larger than the rotational value.

Conventional methods for determining static quadrupole moments rely on relative Coulomb cross-section measurements at different angles,[1] different energies,[2] or with different projectiles.[3] Here we report on a measurement of static quadrupole moments based on a complete particle-γ-angular correlation which reflects the nuclear alignment and is sensitive to the reorientation process.[4]

The particle-γ correlation was made possible by using a new technique for measuring heavy-ion differential cross sections[5] whereby a nearly 4π γ detector, the spin spectrometer,[6] resolved the excited states by their decay γ rays in coincidence with solid-state detection of the scattered ions which provided the information for the differential cross sections. Initial use of the technique was applied to the 2_1^+ (1.37 MeV), 4_1^+ (4.12 MeV), and 2_2^+ (4.24 MeV) states of ^{24}Mg excited by 200 MeV ^{24}Mg ions scattered on a ^{208}Pb target.[5] For that application, only the γ-ray energy resolution aspect of the spin spectrometer was needed. The discrete nature of the spin spectrometer, 72 individual NaI detectors in a close-packed spherical array, allowed the particle-γ correlation $W(\theta_\gamma,\phi_\gamma)$ to be extracted for the decay γ's. However, decay γ's from the 4_1^+ and 2_2^+ states could only be resolved from overlapping decays from ^{208}Pb in about 30% of the NaI detectors, located along the ^{24}Mg recoil direction, so that an extensive particle-γ correlation was not obtainable for these states. In the case of the 2_1^+ state, a complete particle-γ correlation was obtained over a wide range of ^{24}Mg scattering angles, $22°\le\theta_{\text{cm}}\le55°$.

The angular distribution for the $2_1^+\Rightarrow0_1^+$ γ ray, neglecting cascades from higher states, is given by[7]

$$W(\theta_\gamma,\phi_\gamma)=\sum_{\substack{k=0,2,4\\-k\le q\le k}}\left[\frac{4\pi}{(2k+1)}\right]^{1/2} t_{kq}R_kY_{kq}(\theta_\gamma,\phi_\gamma)\,, \tag{1}$$

where the R_k are tabulated γ-ray correlation coefficients[8] and the $(t_{kq})_{\text{exp}}$ are the experimental statistical tensors that describe the nuclear alignment prior to the decay of the 2_1^+ state. We choose a lab coordinate system where θ and ϕ are the polar and azimuthal angles of the detected γ ray with respect to a z axis along the recoil ^{24}Mg direction. In fitting this expression to the observed γ-ray angular correlation to obtain the $(t_{kq})_{\text{exp}}$, we account for the attenuation due to the finite solid angle of each detector. The resulting experimental alignment tensors are shown as data points in Fig. 1.

The calculated alignment tensors $(t_{kq})_{\text{cal}}$ resulted from a coupled-channels analysis[5] using the symmetric-rotor form-factor option of the program ECIS (Ref. 9) and included all the $E2$ and $E4$ matrix elements coupling the ground state and the three low-lying states of ^{24}Mg as well as a six parameter optical potential. In the analysis, nuclear deformations were related to Coulomb deforma-

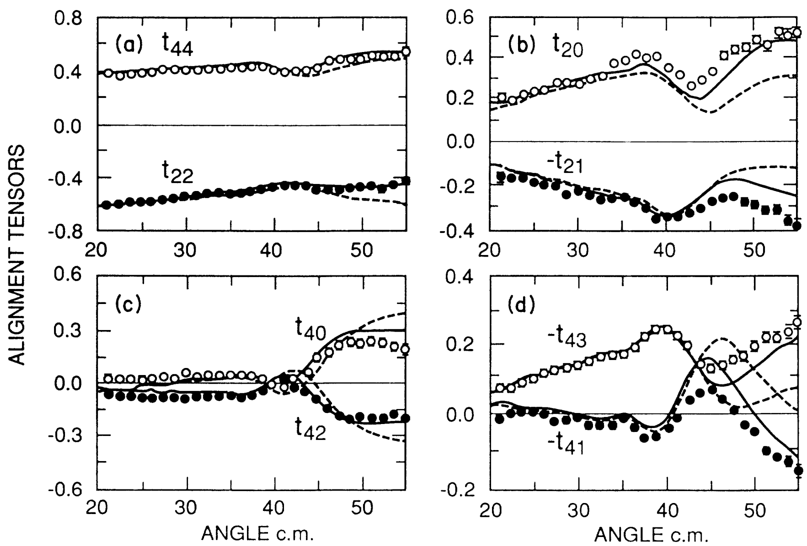

FIG. 1. Alignment tensors t_{kq} for decay of the 2_1^+ (1.37 MeV) state of ^{24}Mg excited by 200 MeV ^{24}Mg+^{208}Pb inelastic scattering as a function of ^{24}Mg cm scattering angle. The data points, opened and closed circles, come from fitting Eq. (1) to the measured particle-γ angular correlation. The dashed curves result from coupled-channel fits to the differential cross sections only (Ref. 5). The solid curves result from a best compromise fit to the differential cross sections and the tensor alignments.

tions by a Hendrie scaling procedure[10] and were not independent parameters. To compare $(t_{kq})_{\rm cal}$ with $(t_{kq})_{\rm exp}$, we must make a correction to $(t_{kq})_{\rm cal}$ for the attenuation due to the precession of the nuclear magnetic moment as a result of the hyperfine interaction prior to the decay.[11] The ^{24}Mg ions exiting from the carbon backing are expected to consist of $\approx 48\%$ 12^+ ions, $\approx 46\%$ 11^+ ions, and $\approx 6\%$ 10^+ ions.[12] The hyperfine precession effect is expected to be dominated by those ^{24}Mg ions in the 11^+ charge state.[13] Assuming the simplest model, i.e., perturbation by a single electronic configuration (single electron ions in the 1s state), the unperturbed alignment tensors are related to the perturbed alignment tensors by a time-integral attenuation coefficient[14] G_k:

$$G_k = 1 - \frac{k(k+1)}{25}\frac{(\omega\tau)^2}{1+(\omega\tau)^2} f(11^+)\,, \qquad (2)$$

where $f(11^+)$ is the fraction of ^{24}Mg scattered ions in the 11^+ charge state, ω is the hyperfine frequency, and τ the mean life of the 2_1^+ state (2.01 ps). From a g-factor measurement[13] on the 2_1^+ state of ^{24}Mg, assuming $f(11^+) \approx 48\%$, and using Eq. (2), we estimate $G_2 \approx 0.90$ and $G_4 \approx 0.66$. This is an approximate result because we have neglected effects of two electron (10^+) ions. We prefer, however, to determine the G_k from our own data by comparing $(t_{kq})_{\rm exp}$ and $(t_{kq})_{\rm cal}$ in the Coulomb dominated region, $\theta_{\rm cm} \leq 35°$. Determining G_2 and G_4 independently in this way we obtain $G_2 = 0.90 \pm 0.01$ and $G_4 = 0.69 \pm 0.01$, whereas if we insist on the relation $(1-G_4)/(1-G_2) = \frac{20}{6}$, as required by Eq. (2), we obtain $G_2 = 0.906 \pm 0.005$ and $G_4 = 0.687 \pm 0.005$. In either case, the G_k are well determined and consistent with the g-factor measurement[13] and Eq. (2). This is possibly an indication that the atomic relaxation processes leading to the 1s state in hydrogenlike 11^+ ions are rather fast, and that most of the 10^+ ions relax quickly to nonperturbing states.

With these values for the G_k, the calculated alignment tensors representing the best fit to the differential cross sections alone[5] are shown as the dashed curves in Fig. 1. This calculation qualitatively accounts for the general behavior of the tensor alignments but there is considerable room for improvement, especially at back angles in the Coulomb-nuclear interference region. Sensitivity of the $(t_{kq})_{\rm cal}$ to the nuclear optical potential was investigated by calculations using the shallow and the deep potentials of Ref. 5. Both potentials give identical alignment tensors. Sensitivity of the $(t_{kq})_{\rm cal}$ to the $M(E4;\, 0_1^+ \leftrightarrow 4_1^+)$ and the $M(E2; 2_1^+ \leftrightarrow 2_2^+)$ matrix elements were also investigated. For values of 41 $e\,{\rm fm}^4 \leq M(E4; 0_1^+ \leftrightarrow 4_1^+) \leq 142$ $e\,{\rm fm}^4$, there is no sensitivity provided $M(E2; 0_1^+ \leftrightarrow 2_1^+)$ is adjusted to maintain the fit to the 2_1^+ differential cross section. This range of $M(E4; 0_1^+ \leftrightarrow 4_1^+)$ covers the range of reported values.[5,15] For values of -11 $e\,{\rm fm}^2 \leq M(E2; 2_1^+ \leftrightarrow 2_2^+) \leq +11$ $e\,{\rm fm}^2$, covering the range of reported values in sign as well as in magnitude,[5,16] there also is little or no sensitivity to the 2_1^+ alignment tensors. We also examined the effect of using βR scaling rather than Hendrie scaling to relate the nuclear matter shape to the Coulomb charge shape. Although this scaling procedure leads to quite different nuclear deformation parameters for the highly deformed ^{24}Mg nucleus, the calculated differential cross sections and alignment tensors were essentially unchanged.

All the sensitivity to the alignment tensors appears to reside in the $E2$ reorientation matrix element $M(E2; 2_1^+ \leftrightarrow 2_1^+) = [(5/16\pi)\frac{35}{2}]^{1/2} Q_2$, where Q_2 is the static quadrupole moment. While the 2_1^+ differential cross section is dominated by the basic $E2$ matrix element $M(E2; 0_1^+ \leftrightarrow 2_1^+)$, there is a small contribution from the $E2$ reorientation matrix element $M(E2; 2_1^+ \leftrightarrow 2_1^+)$. This is illustrated in Fig. 2(a) which shows the variation of χ^2 as a function of these two matrix elements in least-squares fits to the differential cross section. As explained in Ref. 5, the experimental errors on the differential cross section were uniformly scaled so that the minimum χ^2 was approximately equal to the number of data points. From Fig. 2(a), a 1% change in the basic $M(E2;0_1^+ \Longleftrightarrow 2_1^+)$ matrix element causes a 40% movement in the location of the χ^2 minimum on the Q_2 axis. The χ^2 behavior of the alignment tensors is quite different as is shown in Fig. 2(b) which reveals that the alignment tensors are quite sensitive to the reorientation matrix element and prefer $Q_2 \approx -32$ $e\,\text{fm}^2$, irrespective of the value of $M(E2;0_1^+ \Longleftrightarrow 2_1^+)$. In obtaining a value of χ^2 for the fit to the alignment tensor data, the experimental errors were scaled in the same manner as for the differential cross section except that the eight alignment tensors were considered as a single alignment determination weighted on an equal footing with the differential cross section. Treating relative χ^2 in this way, the sum of χ^2, shown in Fig. 2(c), indicates that a good compromise between fitting the differential cross section and the alignment tensors is $Q_2 = (-29 \pm 3)$ $e\,\text{fm}^2$ and $M(E2;0_1^+ \Longleftrightarrow 2_1^+) = (-20.60 \pm 0.3)$ $e\,\text{fm}^2$ where the error quoted for Q_2 has been increased by 50% to account for the uncertainty in the χ^2 weighting criteria used. The alignment tensors for these values are shown by the solid curves in Fig. 1. These values account for a larger body of data and therefore supplant our previous analysis[5] based solely on the differential cross section. If only the tensor alignment data in the Coulomb dominated region is used to determine Q_2, the value extracted would be $Q_2 = (-29 \pm 5)$ which indicates that the Coulomb dominated alignment tensor data also favor a large Q_2 value, although most of the sensitivity comes from the Coulomb-nuclear interference region.

We have shown that a complete particle-γ correlation is directly sensitive to the reorientation matrix element. Although previous measurements of Q_2 for the 2_1^+ state of ^{24}Mg have ranged from the symmetric rotor value[17] to twice the rotational value,[18] the present determination clearly indicates that Q_2 is some (1.56 ± 0.16) times larger than the rotational value, which poses a problem for shell model,[19] Hartree-Fock,[20] and potential-energy surface model calculations.[21] While this result is in the context of a model which relates Coulomb and nuclear shapes, the result appears to be independent of the specifics of the model, i.e., Hendrie or βR scaling.

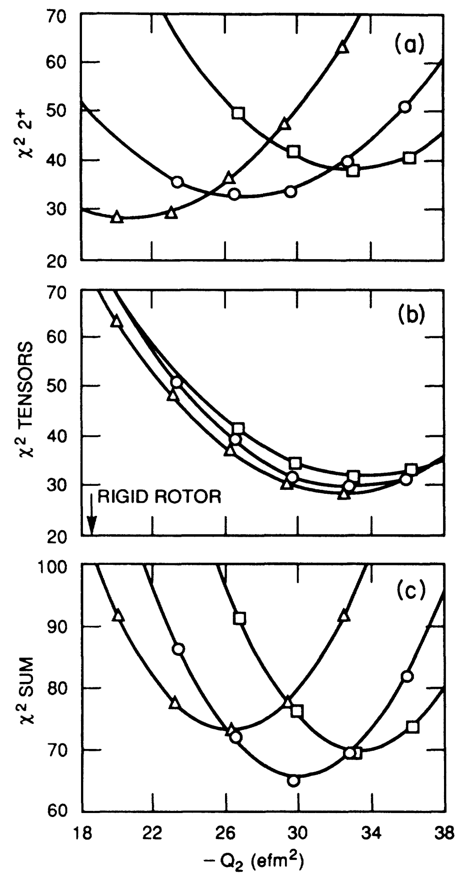

FIG. 2. The calculated behavior of χ^2 for fitting: (a), the 2_1^+ differential cross section; (b), the 2_1^+ tensor alignments; and (c), the sum of (a) and (b), for decay of the 2_1^+ (1.37 MeV) state of ^{24}Mg excited by 200 MeV ^{24}Mg+^{208}Pb inelastic scattering as a function of the static quadrupole moment Q_2 of ^{24}Mg. The triangles are calculations for $M(E2;0_1^+ \Longleftrightarrow 2_1^+) = -20.44$ $e\,\text{fm}^2$, the circles for $M(E2;0_1^+ \Longleftrightarrow 2_1^+) = -20.60$ $e\,\text{fm}^2$, and the squares for $M(E2;0_1^+ \Longleftrightarrow 2_1^+) = -20.75$ $e\,\text{fm}^2$. The value of Q_2 for a rigid rotor is indicated by the arrow in (b).

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