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Coupling schemes in doubly odd nuclei and identical superdeformed bands

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Using coupling schemes derived for low-lying structures in doubly odd deformed nuclei in the framework of the particle rotor model and properties of orbitals which follow from pseudospin symmetry, likely explanations are given for identical superdeformed bands.

One of the most intriguing and striking recent discoveries in high-spin nuclear physics has been the finding of almost identical superdeformed bands[1-2] in several neighboring nuclei.

On the other hand investigations of band structure in doubly odd nuclei carried out in recent years[3] have lead to the establishment of coupling schemes which imply identical bands[4] in odd, doubly odd, and even-even neighboring nuclei provided the inertia parameters are equal. One of these schemes, called (nonstaggered) semidecoupling,[4] consists of the coupling of a particle occupying an orbital with a given Ω (or a set of states $\{\Omega\}$) and another in an $\Omega=\frac{1}{2}$ orbit with decoupling parameter $a=\pm 1$. In this case the net result is that in the neighboring odd-odd or even-even (in the case of alike particles) nucleus one finds bands identical to the Ω (or $\{\Omega\}$) structure in the odd nucleus (examples for this situation have been shown to exist in the heavier rare-earth region[4]).

Furthermore if two distinguishable (e.g., of different isospin or parity) $\Omega=\frac{1}{2}$, $a=1$ orbits are coupled together one encounters a special case of a doubly decoupled band,[5] namely a two-quasiparticle band of odd spins with alignment one in the odd-odd or even-even system and identical to the ground-state band of the even-even nucleus.[6] In fact, in the framework of the two-quasiparticle plus rotor model[7] the matrix element of the Coriolis interaction coupling these two states is[5]

$$[a_p+(-1)^{(I+1)}a_n][I(I+1)]^{1/2}.$$

For $a_p=a_n=\pm 1$ and odd spins its value is $\pm 2[I(I+1)]^{1/2}$. The two-body part of the recoil term gives a diagonal contribution for the $K=\Omega_n-\Omega_p=\frac{1}{2}-\frac{1}{2}=0$ state which is $(-1)^{I+1}a_n a_p$. Again for the favored signature sequence ($\alpha=1$, odd I) its value is unity. Hence the 2×2 matrix to be diagonalized is (all elements in units of the rotational constant $A=\hbar^2/2\mathcal{I}$):

$$\begin{pmatrix} I(I+1)+1 & 2[I(I+1)]^{1/2} \\ 2[I(I+1)]^{1/2} & I(I+1)-1 \end{pmatrix}. \quad (1)$$

One obtains for the lowest eigenvalue $E_-=I(I+1)-(2I+1)$ (odd I). With the substitution $I=R+1$ (which implies alignment 1), $E_-=R(R+1)-1$ exhibiting explicitly that the spin sequence $I=1,3,5,\ldots$ has identical transition energies to the core states $R=0,2,4,\ldots$. This is a generalization of the well-known case of an $\Omega=\frac{1}{2}$ band with $a=1$, where the rotational energy is

$$I(I+1)-\tfrac{1}{4}+(-1)^{I+1/2}(I+\tfrac{1}{2})=(I-\tfrac{1}{2})(I+\tfrac{1}{2})-\tfrac{1}{2}$$
$$=R(R+1)-\tfrac{1}{2}, \quad (2)$$

with the substitution $I=R+\frac{1}{2}$ (alignment $\frac{1}{2}$). Again one sees that the spin sequence $I=\frac{1}{2},\frac{5}{2},\ldots$ (favored signature $\alpha=\frac{1}{2}$) has transition energies identical to the even-even core.

In fact the excited superdeformed (SD) band[1] in ^{151}Tb, identical to the yrast band of ^{152}Dy has been explained[8] in just these terms invoking the Nilsson proton state labeled $[301\frac{1}{2}]$ (and much better described by the $[\tilde{2}0\tilde{0}\frac{1}{2}]$ asymptotic pseudospin quantum numbers,[9] corresponding to $a=1$). On the other hand the SD yrast band configuration of ^{151}Tb corresponds to the $[642\frac{5}{2}]$ Nilsson orbit (at zero rotational frequency) and denoted as 6_3 in Ref. 8. The excited SD band[1] in ^{150}Gd (identical to the yrast band of ^{151}Tb) can be understood exactly in terms of the semidecoupled scheme utilizing the $\{[\tilde{2}0\tilde{0}\frac{1}{2}];6_3\}$ two-quasiparticle configuration in ^{150}Gd.

Recently two excited SD bands have been reported[10] in ^{147}Gd. One of them has transition energies identical to the yrast band in ^{146}Gd while the other has transitions with energies equal to the average of the nearest-lying transitions in ^{146}Gd (also yrast). An attempt to explain these bands in terms of a single $\Omega=\frac{1}{2}$ orbital encounters two types of difficulties. The first one is that a single $\Omega=\frac{1}{2}$ orbit, whether $a=1$ or -1, gives a single set of transition energies. For $a=1$ one obtains, as discussed above, transition energies equal to the core for the favored signature branch $(\frac{1}{2},\frac{5}{2},\ldots)$ while the $\alpha=-\frac{1}{2}$ sequence $(\frac{3}{2},\frac{7}{2},\ldots)$ is degenerate[5] with $(\frac{5}{2},\frac{9}{2},\ldots)$ giving again transition energies equal to those of the core. For $a=-1$ the structure of both signature components consists of a sequence of degenerate doublets: $(\frac{1}{2},\frac{3}{2});(\frac{5}{2},\frac{7}{2}),\ldots$ and the level energies follow the expression $E(I)=I(I+2)+\frac{1}{4}$ for $\alpha=\frac{1}{2}$. Here the energy for the $\Delta I=2$ transition which originates in the state $I=R+\frac{1}{2}$ (even R) equals $4R+2$ which turns out to be equal to the average:

$$\tfrac{1}{2}[(R+2)(R+3)-R(R+1)+R(R+1)-(R-2)(R-1)]=4R+2. \quad (3)$$

The second difficulty is that for the neutron numbers and deformations involved, the only $\Omega=\frac{1}{2}$ state near the Fermi surface is the $[411\frac{1}{2}]$ orbit. (Again, this is much better described by $[\tilde{3}\tilde{1}\tilde{0}\frac{1}{2}]$ having $a=(-1)^3=-1$, namely the pseudospin limit. The calculated values of the decoupling parameter for this state change from -0.9 to -0.7 for a quadrupole deformation going from 0.3 to 0.54, respectively.)

We have now to turn our attention to the concept of pseudospin[9] which is being currently[1,11] discussed in connection with the identical superdeformed bands. The natural parity single-particle spectrum within a given major shell is dominated[9] by the presence of quasidegenerate doublets labeled in a Nilsson diagram as $(Nn_3,\Lambda,\Omega=\Lambda+\frac{1}{2})$ and $(Nn_3\Lambda+2,\Omega=\Lambda+\frac{3}{2})$ which alternatively can be described[9] as pseudospin-orbit partners with quantum numbers $(\tilde{N}=N-1,\ n_3,\ \tilde{\Lambda}=\Lambda+1,\ \Omega=\tilde{\Lambda}\pm\frac{1}{2})\equiv|\pm\rangle$. These doublets have the following asymptotic properties: $\langle\pm|\tilde{S}_3|\pm\rangle=\pm\frac{1}{2}$ and $\langle+|j_+|-\rangle=1$. In addition, if one of the partners has $\Omega=\frac{1}{2}$ its decoupling parameter is zero. If we assume that these doublets are isolated energetically from the rest, something which is particularly true for very large deformations, then the Coriolis coupling problem reduces to the diagonalization of a 2×2 matrix. If we neglect the splitting between the two components of the doublet and work in a high-spin regime $(I\gg K)$ the particle plus rotor matrix can be written as

$$\begin{pmatrix} I(I+1) & -[I(I+1)]^{1/2} \\ -[I(I+1)]^{1/2} & I(I+1) \end{pmatrix}. \tag{4}$$

The lowest eigenvalue turns out to be $E_-=(I-\frac{1}{2})\times(I+\frac{1}{2})$. Again with the substitution $I=R+\frac{1}{2}$ (alignment $\frac{1}{2}$ for $a=\frac{1}{2}$ and $-\frac{1}{2}$ for $a=-\frac{1}{2}$) it goes over to $E_-=R(R+1)$. In many respects this excitation has the properties of two $\Omega=\frac{1}{2}$ orbitals, one with $a=1$ and the other with $a=-1$ [see Eq. (2)]. The matrix (4) is valid for all values of $I(I\geq\tilde{\Lambda}+\frac{1}{2})$ unlike matrix (1) and Eq. (2). We obtain here alignments $\pm\frac{1}{2}$ and both signature components. The $a=\frac{1}{2}$ trajectory follows the core spacings while the $a=-\frac{1}{2}$ component has transition energies which are the average of the two nearest transitions in the $a=\frac{1}{2}$ sequence. This excitation provides a natural explanation for the two excited SD bands in ^{147}Gd. A suitable pseudospin-orbit doublet which lies near to the Fermi surface and seems to be particularly degenerate is $\{[530\frac{1}{2}],[532,\frac{3}{2}]\}$ (in the pseudospin representation $\{[\tilde{4}\tilde{3}\tilde{1}\frac{1}{2}],[\tilde{4}\tilde{3}\tilde{1}\frac{3}{2}]\}$. The Coriolis matrix element coupling these two orbitals varies from 1.1 to 0.8 for a quadrupole deformation changing from 0.3 to 0.54).

The next finding[10] to be explained is an excited SD band in ^{148}Gd which appears identical to the $a=-\frac{1}{2}$ trajectory in ^{147}Gd. Since we are dealing most likely with a two-quasiparticle band in ^{148}Gd let us explore the coupling of an $\Omega=\frac{1}{2}$ orbit (e.g., the $[\tilde{3}\tilde{1}\tilde{0}\frac{1}{2}]$) with a doublet, for instance the one just suggested for ^{147}Gd. The configuration space is spanned by the following basis:

$$\{\Omega_b,\Omega_b-1\}\otimes\{\Omega_a=\tfrac{1}{2}\}=\{\Omega_b+\Omega_a,\Omega_b-\Omega_a,\Omega_b-1+\Omega_a,\Omega_b-1-\Omega_a\}.$$

For $a_a=-1$ the resulting 4×4 matrix is (again neglecting splittings and taking $I\gg K$):

$$\begin{pmatrix} I(I+1) & -[I(I+1)]^{1/2} & -[I(I+1)]^{1/2} & 0 \\ -[I(I+1)]^{1/2} & I(I+1) & 0 & -[I(I+1)]^{1/2} \\ -[I(I+1)]^{1/2} & 0 & I(I+1) & -[I(I+1)]^{1/2} \\ 0 & -[I(I+1)]^{1/2} & -[I(I+1)]^{1/2} & I(I+1) \end{pmatrix}. \tag{5}$$

We obtain for the lowest eigenvalue $E_-=I(I+1)-2[I(I+1)]^{1/2}$. The substitution $[I(I+1)]^{1/2}=[R(R+1)]^{1/2}+1$ gives $E_-=R(R+1)-1$. This implies that the even spin sequence in ^{148}Gd will be equal to the $a=-\frac{1}{2}$ component in ^{147}Gd while the odd spin sequence should equal the $a=\frac{1}{2}$ band. This last point is in fact a prediction. It is worth mentioning that the quasiexact problem (i.e., neglecting only the diagonal single-particle splitting) admits closed solutions with the lowest one being $E_-=I(I+1)-2I-2$, which after the substitution $I=R+1$ goes over into $E_-=R(R+1)-2$.

Still another possibility would be the coupling of two doublets for example the one already used for ^{147}Gd and $\{[402\frac{5}{2}],[404\frac{7}{2}]\}\rightarrow\{[\tilde{3}\tilde{0}\tilde{3}\frac{5}{2}],[\tilde{3}\tilde{0}\tilde{3}\frac{7}{2}]\}$. If we order the basis as $\Omega_b+\Omega_a,\Omega_b+(\Omega_a-1),(\Omega_b-1)+\Omega_a,(\Omega_b-1)+(\Omega_a-1),\ \Omega_b-\Omega_a,\ \Omega_b-(\Omega_a-1),\ (\Omega_b-1)-\Omega_a,(\Omega_b-1)-(\Omega_a-1)$ we obtain a block diagonal 8×8 matrix with two blocks identical to matrix (5). The decoupling of the two blocks is due to the fact that the decoupling parameter of the $[\tilde{4}\tilde{3}\tilde{1}\frac{1}{2}]$ orbit is zero. In this case we would have two solutions of the kind described above.

It is important to realize that this scheme works, provided there is a mechanism which gives the equality of the moments of inertia in the different neighboring systems and in fact, some ideas have been put forward in this respect.[11–13] It would seem likely that since blocking effects are not apparent, pairing correlations should be reduced in the superdeformed structures.

One may envisage even more complicated schemes involving more doublets (or $\Omega=\frac{1}{2}$ orbits) and also other states with no pseudospin alignment to explain different situations. It is evident however that the Coriolis coupling of these excitations provides a natural framework for the understanding of identical superdeformed bands.

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[6]The doubly decoupled band discussed so far in the literature (Ref. 3) corresponds to the $\pi h_{9/2}([541\,\frac{1}{2}])\otimes\nu[521\,\frac{1}{2}]$ structure. Here only the neutron orbit has, experimentally and theoretically, a decoupling parameter $a\approx 1$ and is well described by the pseudospin limit (see text), while the proton has a much larger decoupling parameter. In general, and if only the two $\Omega=\frac{1}{2}$ states are considered, the alignment is $\frac{1}{2}(a_p+a_n)$.

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