PHYSICAL REVIEW C VOLUME 42, NUMBER 4 OCTOBER 1990

Annihilation shifts and widths of the $\bar{p}$-d atomic levels

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(Received 26 March 1990)

The strong interaction shifts and widths of the antiproton-deuteron atom are calculated within a simple three-body model. The nuclear $\bar{p}$-d optical potential is obtained from a projected form of the Faddeev equations with rank-one, separable, S-wave two-body potentials. The $N-\bar{N}$ potential parameters are fit to the zero-energy scattering results of the Graz potential. The Coulomb potential is combined with the nuclear optical potential to obtain the atomic levels from the Schrödinger equation. The truncation of the Faddeev model optical potential to the level of the standard $t\rho$ impulse approximation overestimates the shifts and widths by up to 10% and 30%, respectively. The use of first-order perturbation theory for the atomic eigenvalue overestimates the widths by 1 order of magnitude.

The shifts and widths of the atomic levels of exotic atoms formed from captured antiprotons offer valuable information on the nucleon-antinucleon ($N\bar{N}$) interaction at threshold. The analysis is usually made in terms of the complex energy eigenvalue of a two-body problem involving the $\bar{p}$-nucleus optical potential constructed from a phenomenological $N\bar{N}$ interaction in addition to the attractive Coulomb potential.[1] For heavy nuclear cores that are spin saturated, the spin dependence of the underlying $N\bar{N}$ interaction is not accessible. At the other extreme, the $p\bar{p}$ atom, or protonium, provides direct access to a single spin component of the effective $p\bar{p}$ interaction without the uncertainties associated with folding in the effects of a many-nucleon core.[2] The case of a deuteron core, in which the atom is antiprotonic deuterium, offers the possibility for study of a more complete spin structure of the $N\bar{N}$ interaction while the many-body effects of the nuclear core can be treated more explicitly. The $\bar{p}$-d atom has a Bohr radius of about 44 fm. One would expect that the nuclear distortion of the deuteron core would be negligible and that the simple impulse approximation ($t\rho$) will suffice for the $\bar{p}$-d optical potential.

In this work we calculate the $\bar{p}$-d optical potential in a simple separable potential model of the $NN\bar{N}$ system, add the two-body Coulomb potential, and then solve for the complex atomic energy eigenvalues as a two-body problem. To our knowledge, only one previous estimate has been made for the shifts and widths of the $\bar{p}$-d atom.[3] The approach taken in that work involved the use of the Trueman formula[4] in the form where the complex energy shift is related in first-order perturbation to the atomic state matrix element of the nuclear $\bar{p}$-d scattering amplitude. The latter quantity is estimated by an approximate summation of the three-body multiple scattering series with separable S-wave interactions. Large corrections to the $t\rho$ or impulse approximation for the $\bar{p}$-d amplitude were found to be present and to lead to reductions in the shifts and widths by factors of about 2 and 10, respectively. In this work we avoid these approximations by a direct solution of the model Faddeev equations for the $\bar{p}$-d optical potential, and also by the solution of a two-body eigenvalue problem for the shifted atomic state. We find a much reduced role for three-body dynamical corrections to the nuclear impulse approximation. We also find that, due to strong absorption, the atomic eigenvalue problem is not amenable to first-order perturbative estimates.

The complex energy shifts of the $\bar{p}$-d atomic levels are defined by

$$\Delta E \equiv \Delta E_R - i\Gamma/2 = E_{CN} - E_C \,, \tag{1}$$

where Γ is the full width, E_{CN} is the eigenvalue due to both Coulomb and nuclear interactions, and E_C is the pure Coulomb eigenvalue. We project the three-particle Hamiltonian onto the deuteron ground state channel so that E_{CN} is the eigenvalue of the effective $\bar{p}$-d two-particle Hamiltonian K_o+U. Here K_o is the $\bar{p}$ kinetic energy operator and U is the exact $\bar{p}$-d optical potential for the three-particle model. We ignore the possibility of Coulombic excitation and deexcitation of the deuteron core so that the optical potential can be approximated by

$$U \approx \langle \phi_d | U_N | \phi_d \rangle + V_C \,. \tag{2}$$

Here V_C is the $\bar{p}$-d Coulomb interaction, $|\phi_d\rangle$ is the deuteron ground state, and U_N is the pure nuclear $\bar{p}$-d optical potential operator given formally by

$$U_N(E) = V_N + V_N Q G_d(E) U_N(E) \,. \tag{3}$$

In Eq. (3), V_N is the sum of effective two-body nuclear potentials

$$V_N \equiv v_{\bar{p}n} + v_{\bar{p}p} \,, \tag{4}$$

Q is the projector onto the excited states of the deuteron, $G_d = (E - H_d - K_o)^{-1}$ where E is the total three-body energy, and H_d is the deuteron Hamiltonian. We use rank-one S-wave separable forms for the three nuclear potentials and solve for the first term of Eq. (2) within a momentum space Faddeev formulation. Both the NN and

$N\bar{N}$ potentials are described by a strength parameter (real for NN, complex for $N\bar{N}$) and a real range parameter for the Yamaguchi form factor. We use the isospin formalism so that there are two spin-isospin channels for NN and four for $N\bar{N}$. The NN parameters are fit to the relevant scattering lengths and effective ranges as well as the deuteron binding energy. For the $N\bar{N}$ case, the parameters are fit to the zero energy scattering results obtained from the Graz potential[5] which describes the available phase shifts below 200 MeV and contains an annihilation mechanism to a phenomenological channel of two spinless bosons. We have previously employed the present rank-one model to study nuclear states of the $NN\bar{N}$ system[6] and the potential parameters are to be found tabulated in that work.

Three-body integral equations for U_N follow from Eq. (3) by elimination of potentials in favor of t matrices for each pair, and in Alt-Grassberger-Sandhas (AGS) form,[7] they are

$$U^{\alpha 3}=\bar{\delta}_{\alpha 3}G_0^{-1}+\sum_{\gamma=1}^{3}\bar{\delta}_{\alpha\gamma}\tilde{T}_\gamma G_0 U^{\gamma 3}, \tag{5}$$

where $U^{33}\equiv U_N$. Here particles 1 and 2 are nucleons and particle 3 is the antiproton, and each interacting pair is labeled by the spectator particle. In Eq. (5), $\bar{\delta}_{\alpha 3}\equiv(1-\delta_{\alpha 3})$, and G_0 is the three-body noninteracting propagator. In terms of the usual t matrices T_γ for each two-body subsystem, the quantities $\tilde{T}_\gamma$ that enter Eq. (5) are $\tilde{T}_\gamma=T_\gamma$ for $\gamma=1,2$ ($N\bar{N}$ channels), but for the NN channel the relation is

$$\tilde{T}_3\equiv T_3-G_0^{-1}PG_d(E)G_0^{-1}, \tag{6}$$

where $P=|\phi_d\rangle\langle\phi_d|$ is the projector onto the ground state of the deuteron ($P=1-Q$). The subtraction term occurs only in the np spin-triplet channel and has the effect of canceling the deuteron pole of the t matrix. The particular off-shell extension of the pole residue of the subtraction term is necessary to reproduce the optical potential of Eq. (3) from the three-body AGS equations. Under the replacement $Q\rightarrow 1$, the optical potential U_N becomes the elastic $\bar{p}d$ scattering amplitude, and Eq. (5) becomes the standard AGS formulation of the Faddeev treatment of this system.

With the indistinguishability of the two nucleons, the three coupled equations (5) reduce to the pair

$$U^{13}=G_0^{-1}+E_{12}T_1G_0U^{13}E_{12}+\tilde{T}_3G_0U_N \tag{7}$$

and

$$U_N=T_1G_0U^{13}+E_{12}T_1G_0U^{13}E_{12}, \tag{8}$$

where E_{12} is the operator that exchanges all of the coordinates of particles 1 and 2. Multiple scattering contributions can be removed from the nuclear optical potential U_N by retaining just the first term of U^{13}, and this yields the single scattering or impulse approximation

$$U_N\approx T_1+E_{12}T_1E_{12}. \tag{9}$$

The results from this approximation will be compared with the full solution from the three-body model.

We calculate the eigenvalues of the Hamiltonian $K_o+U_N+V_c$ in momentum space using the Landé subtraction method as described by Kwon and Tabakin[8] to handle the integrable $1/q^2$ singularity of the Coulomb potential $V_c(q)$. We use the point charge form of V_c and the complex shifts are based on an unperturbed ground state energy $E_C=-16.67$ keV. The energy parameter of $U_N(E)$ is the sum of the deuteron energy and the energy of the atomic level under consideration. However, it is not necessary to calculate $U_N(E)$ self-consistently in this parameter as the variations below the deuteron energy are of no consequence on a nuclear scale. If care is taken in the subtraction of the deuteron pole from intermediate states as required in Eq. (6), we find no important dependence on the atomic component of the energy and we evaluate $U_N(E)$ at $E=-3$ MeV for convenience.

The S-state shifts and widths obtained from the Faddeev model $\bar{p}$-d optical potential are shown by the entries without brackets in Table I for both the doublet and quartet coupling of nuclear spins. If we estimate the size of the spin-doublet $\bar{p}$-d atomic shift as double the $^1S_0$ protonium shift, then recent $\bar{p}p$ experiments would suggest $1.460-i1.130\pm(0.1-i0.09)$ keV,[9] $1.400-i1.600\pm(0.3-i0.4)$ keV,[10] and $1.240-i1.130\pm(0.2-i0.17)$ keV,[11] while recent $\bar{p}p$ theoretical work would suggest, for example, $1.04-i0.89$ keV (Ref. 2) and $1.08-i1.02$ keV.[12] The calculated real shifts in Table I correspond to these estimates while the calculated widths are substantially smaller. This is to be expected since the calculation involves the triplet spin $N\bar{N}$ channel as well as the singlet with the former having an effective annihilation strength which is a factor of 7 less in the present model.[6]

Also shown for comparison are the results from Ref. 3 in which the complex shifts are determined from the Trueman formula[4] in terms of the unperturbed atomic state matrix element of the zero-energy nuclear $\bar{p}$-d scattering amplitude or scattering length. The latter quantity is determined in that work by supplementing the $t\rho$ impulse approximation with estimated corrections for all orders of multiple scattering of the antiproton as well as rescattering of the deuteron constituents. The $N\bar{N}$ potential employed there is the Green and Wycech[13] rank-two separable representation of the Dover-Richard[14] local potential. If we neglect for the moment the differences in the treatment of the nuclear sector of the $NN\bar{N}$ dynamics, the

TABLE I. $\bar{p}$-d atomic S-state energy shifts $\Delta E=\Delta E_R-i\Gamma/2$ (keV) obtained using the Faddeev model optical potential compared to the Wycech-Green-Niskanen (WGN) result (Ref. 3) containing approximate multiple scattering corrections to the impulse approximation. Bracketed values are obtained from a first-order perturbative use of the nuclear optical potential and thus omit nuclear distortion of the atomic state. The WGN result from Ref. 3 uses the Trueman formula (Ref. 4).

Atomic state $^{(2S+1)}L_J$	Faddeev model	WGN (Ref. 3) ($IA+MS$)
$^2S_{1/2}$	$1.48-0.45i$ [$2.61-3.82i$]	$2.14-0.59i$
$^4S_{3/2}$	$1.72-0.36i$ [$4.35-3.94i$]	$2.19-0.64i$

two methods should, in principle, agree if the nuclear distortion of the shifted atomic eigenstate is insignificant. This is because the pole position of the $\bar{Q}$-d T matrix, which is the basis of the Trueman formula, corresponds to the eigenvalue of K_o+U where U is the optical potential. To assess the importance of nuclear distortion of the eigenstate, we also show in brackets in Table I the complex shifts obtained by truncating the present work to first-order perturbation in the nuclear optical potential U_N. That is

$$\Delta E \simeq \langle \psi_c | \langle \phi_d | U_N | \phi_d \rangle | \psi_c \rangle , \tag{10}$$

where ψ_c is the pure Coulomb atomic state. Without nuclear distortion, the widths are overestimated by about an order of magnitude and the real shifts are also significantly overestimated. Within the range of the nuclear interaction, where the atomic state is required, it is not purely Coulombic.

To assess the importance of multiple scattering and rescattering mechanisms in the nuclear $\bar{p}$-d optical potential U_N, we have performed calculations that retain just the single scattering or impulse approximation mechanism as in Eq. (9). This corresponds to the folding of the $N\bar{N}$ T matrices with the deuteron density. The results are displayed in Table II for both the rank-one $N\bar{N}$ potential used for the foregoing Faddeev calculation and also for the $N\bar{N}$ potential[13] employed in the work of Ref. 3. We again use eigenvalues of $K_o+U_N+V_c$ to obtain the complex shifts. Compared to our previous model-exact Faddeev results (Table I), there is an overestimation by 10% for the real shifts and by 30% for the widths when nuclear multiple scattering and rescattering are ignored. The corresponding impulse results of Ref. 3, also displayed in Table II, suggest much larger corrections, e.g., an order of magnitude change in the widths. This is surprising given the peripheral character of the nuclear interaction that bears upon the atomic orbital.

To test whether the differences in the employed $N\bar{N}$ potentials are responsible for this discrepancy, we have repeated the impulse approximation calculation for U_N with the same rank-two Green-Wycech $N\bar{N}$ potential employed in Ref. 3. Two versions of this calculation are performed. The effective two-particle energy of the $N\bar{N}$ t matrices varies with integration over the deuteron distribution. The results obtained with this energy dependence treated correctly according to the underlying three-particle kinematics are displayed under the heading $t[E(k)]$. The column headed by $t(E=0)$ contains the results obtained with this energy frozen at zero. The impulse calculation of Ref. 3 employs a factorized $t\rho$ prescription at a fixed energy $\bar{E}$ calculated from the deuteron distribution. Our results show that this energy dependence is not a major consideration. We also find that the evident $N\bar{N}$ model dependence of our impulse approximation results is at most 15%.

If we further ignore the nuclear distortion of the atomic state by returning to the first-order perturbation estimate of Eq. (10), we obtain agreement with Ref. 3 for impulse results from the same $N\bar{N}$ potential. Our impulse results, without nuclear distortion, are displayed as the bracketed quantities in Table II. In agreement with the findings in the case of the full Faddeev optical potential, the neglect of nuclear distortion of the atomic state overestimates the widths by about an order of magnitude and overestimates the real shifts by up to a factor of 3.

In summary, we have explored the nuclear shifts and widths induced on the S-states of the $\bar{p}$-d atom through use of a simple three-particle $NN\bar{N}$ model. Rank-one separable $N\bar{N}$ optical potentials were employed to produce a Faddeev solution for the $\bar{p}$-d nuclear optical potential containing all orders of multiple scattering and rescattering. The complex shifts are produced from eigenvalues of the effective two-particle Hamiltonian containing the sum of the $\bar{p}$-d pure nuclear optical potential and the Coulomb potential. Nuclear multiple scattering and rescattering effects are found to be much less important here than in the only previously available estimate.[3] We find that the $t\rho$ impulse approximation for the $\bar{p}$-d nuclear optical potential overestimates the real shifts by only 10% while the widths are 30% too large.

This effect of the nuclear distortion of the deuteron core is not as important as the nuclear interaction distortion of the atomic orbital. This latter distortion is omitted if the complex shifts are estimated from the pure atomic state matrix element of the effective nuclear $\bar{p}$-d amplitude rather than from the eigenvalue of the Coulomb plus nuclear Hamiltonian. Such a first-order perturbative approach is the basis of the simple Trueman[4] formula for

TABLE II. $\bar{p}$-d atomic S-state complex energy shifts $\Delta E = \Delta E_R - i\Gamma/2$ (keV) obtained using the impulse approximation to the optical potential. Values without brackets are obtained from eigenvalues of the effective $\bar{p}$-d Hamiltonian. Bracketed values are obtained from a first-order perturbative use of the nuclear optical potential and thus omit nuclear distortion of the atomic state. The WGN result from Ref. 3 uses the Trueman formula (Ref. 4).

	$V_{N\bar{N}}$ LT (Ref. 6)		$V_{N\bar{N}}$ GW (Ref. 13)	
Atomic state $^{(2S+1)}L_J$	This work $t[E(k)]$	This work $t[E(k)]$	This work $t(E=0)$	WGN (Ref. 3) $t^{ON}(\bar{E})$
$^2S_{1/2}$	$1.63-0.64i$ $[1.18-5.20i]$	$1.59-0.54i$ $[4.57-4.88i]$	$1.38-0.48i$ $[3.55-2.22i]$	$4.48-4.90i$
$^4S_{3/2}$	$1.92-0.47i$ $[3.74-7.12i]$	$1.63-0.53i$ $[3.65-5.20i]$	$1.43-0.42i$ $[3.56-2.41i]$	$3.90-5.24i$

complex shifts of the levels of exotic atoms when they are taken to be proportional to the pure nuclear scattering length. We find the perturbative treatment overestimates the widths by an order of magnitude and overestimate the real shift by factors of 2 or 3. The Trueman formula is known to be quite adequate for pionic atoms[4] but about 20% too large for the stronger absorbing kaonic atoms[15] and protonium.[2] In the present $\bar{p}$-d case the absorption is much stronger and the interaction is of longer range. Within the range of the nuclear interaction where the atomic state is required to be known, it is not purely Coulombic. This is found to account for the difference between our results and those of the only previous work on the $\bar{p}$-d atom at the impulse approximation level. Experimental results for $\bar{p}$-d atomic shifts and widths would help stimulate the further theoretical work needed to develop the $\bar{p}$-d optical potential in terms of a more realistic $N\bar{N}$ interaction so that spin-dependences can be addressed with more confidence.

Partial support for this work was provided by the National Science Foundation under Grants. No. PHY85-05736 and No. PHY88-05633, and by the Los Alamos National Laboratory under Grant No. 9X25-Q3282-1.

[1]A. M. Green, W. Stepien-Rudzka, and S. Wycech, Nucl. Phys. **A399**, 307 (1983).
[2]R. H. Landau and J. Schnick, Phys. Rev. C **36**, 1942 (1987).
[3]S. Wycech, A. M. Green, and J. A. Niskanen, Phys. Lett. **152B**, 308 (1985).
[4]T. L. Trueman, Nucl. Phys. **26**, 57 (1961).
[5]W. Schweiger, J. Haidenbauer, and W. Plessas, Phys. Rev. C **32**, 1261 (1985).
[6]G. P. Latta and P. C. Tandy, Phys. Lett. B **209**, 14 (1988).
[7]E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. **B2**, 167 (1967).
[8]Y. R. Kwon and F. Tabakin, Phys. Rev. C **18**, 932 (1978).
[9]C. W. E. Van Eijk *et al.*, Nucl. Phys. **A486**, 604 (1988).
[10]M. Ziegler *et al.*, Phys. Lett. B **206**, 151 (1988).
[11]R. Bacher, P. Blüm, D. Gotta, K. Heitlinger, D. Rohmann, M. Schneider, J. Egger, L. M. Simons, and K. Elsener, Z. Phys. A **334**, 93 (1989).
[12]J. M. Richard and M. E. Sainio, Phys. Lett. **110B**, 349 (1982).
[13]A. M. Green and S. Wycech, Nucl. Phys. **A377**, 441 (1982).
[14]C. B. Dover and J. M. Richard, Phys. Rev. C **21**, 1466 (1980).
[15]R. H. Landau and B. Cheng, Phys. Rev. C **33**, 734 (1986).