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Isospin forbidden $E1$ transitions in ^{16}O and ^{40}Ca and the applicability of Siegert's theorem

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We reexamine the well-known problem of isospin forbidden $E1$ decays in self-conjugate nuclei in the light of a recent claim suggesting that the $E1$ decay in ^{16}O violates Siegert's theorem. We first discuss the importance of including, through isospin mixing, giant dipole and monopole components if agreement with observed lifetimes has to be achieved. We than show that the same microscopic calculation which yields broad agreement with (e,e') and lifetimes results in ^{16}O and ^{40}Ca also asserts the *validity* of Siegert's theorem. We conclude by suggesting that the claim regarding a violation of Siegert's theorem can only be definitely ascertained once additional data at low momentum transfer become available.

Isospin forbidden $E1$ transitions continue to attract a great deal of attention.[1-5] Historically, two main reasons have been invoked for this interest; in self-conjugate nuclei $T=0 \rightarrow T=0$ $E1$ transitions can only proceed in the long-wavelength limit by a small admixture of isospin-breaking wave function components. Recently, an interesting and novel aspect has been added to this problem by Friedrich and Voegler[6] who suggested that their electroexcitation experiment of the lowest $T=1$ 1^- state in ^{16}O reveals an incompatibility with the lifetime measurement at photon point, i.e., a violation of Siegert's theorem, observed here for the first time. The purpose of this Rapid Communication is to discuss, in a quantitative manner, the mechanisms which in our opinion are at play in producing strong isospin forbidden $E1$ transitions in ^{16}O and ^{40}Ca and to investigate whether the violation of Siegert's theorem claimed above is indeed the only possibility consistent with the analysis of the electroexcitation experiment in ^{16}O.

The plan of our Rapid Communication is as follows. We first concentrate on calculating the isoscalar electric dipole transition densities in $1\hbar\omega$ and $3\hbar\omega$ spaces, emphasizing the importance of including the latter space to describe correctly electron-scattering form factor in ^{16}O and ^{40}Ca. We then discuss the importance of including, through isospin mixing, giant dipole and monopole components if agreement with observed lifetimes has to be achieved. Finally we show that the same microscopic calculation which yields broad agreement with (e,e') and lifetimes results in ^{16}O and ^{40}Ca also asserts the *validity* of Siegert's theorem. Since this conclusion is at variance with the recent photoelectric experiment in ^{16}O we conclude by suggesting that any definitive claim regarding a violation of Siegert's theorem should await the availability of experimental results at low momentum transfer.

Isoscalar dipole transitions between $T=0$ states in self-conjugate nuclei are well known to be forbidden in first order because of translational invariance of the many-body system. Transitions can be induced by virtue of the higher order terms in qr of the transition operator

$$e\sum_i j_1(qr_i)Y_1(\hat{\mathbf{r}}_i)\tfrac{1}{2}[1+t_3(i)] = e\sum_i \tfrac{1}{3}(qr_i - \tfrac{1}{10}q^3r_i^3 + \cdots)Y_1(\hat{\mathbf{r}}_i)\tfrac{1}{2}[1+t_3(i)]\,. \quad (1)$$

In the long wavelength limit, however, isospin mixing in the dominant $T=0$ configuration has been invoked as the predominant mechanism producing the anomalously fast $E1$ transitions observed in closed shell nuclei like ^{16}O and ^{40}Ca (Refs. 2 and 7).

We first discuss the importance of including the higher order term $r^3Y_1(\hat{\mathbf{r}})$ of Eq. (1) in the calculation of the transition density of the isoscalar dipole state. Note that the $r^3Y_1(\hat{\mathbf{r}})$ operator excites *both* $1\hbar\omega$ and $3\hbar\omega$ configurations and that a method has to be devised to remove the spurious components induced through the space extension. We will, in what follows, consider the isoscalar dipole operator to be

$$M_{\lambda=1}(\hat{\mathbf{r}}) = \sum_{i=1}^{A}[r_i^3Y_{10}(\hat{\mathbf{r}}_i) - \eta r_iY_{10}(\hat{\mathbf{r}}_i)]\,, \quad (2)$$

where η will be determined later as satisfying the condition of *translational invariance*. Following Mottelson,[8] let us assume that the system is excited by an external perturbation

$$V(\hat{\mathbf{r}},t) = \alpha(t)M_{\lambda=1}(\hat{\mathbf{r}})\,, \quad (3)$$

where $\alpha(t)$ denotes the time-dependent perturbation amplitude. The transition density we wish to calculate will be related to the external perturbation by the continuity equation

$$\delta\rho(\hat{\mathbf{r}},t) = \alpha(t)\nabla[\rho\nabla M_{\lambda=1}(\hat{\mathbf{r}})]$$
$$= \alpha(t)\left[10r + 3r^2\frac{\partial}{\partial r} - \eta\frac{\partial}{\partial r}\right]\rho_0Y_{10}(\hat{\mathbf{r}})\,, \quad (4)$$

where $\rho_0(r)$ denotes the ground state mass density. We can now use the equation describing translational invari-

ance

$$\int \delta\rho(\hat{\mathbf{r}}) r Y_{10}(\hat{\mathbf{r}}) d^3\hat{\mathbf{r}} = 0 \tag{5}$$

to determine the value of the η coefficient in Eq. (2), namely $\eta = \frac{5}{3}\langle r^2\rangle_A$. The $E1$ transition matrix elements can now be obtained. These are

$$\langle \lambda = 1\ T = 0 | \hat{M}(n=0) | 0\rangle = \frac{\bar{\alpha}}{4\pi} 5Z(\langle r^2\rangle_A - \langle r^2\rangle_p) \tag{6}$$

for the $\hat{M}(n=0) = \sum_{i=1}^{p} r_i Y_{10}(\hat{\mathbf{r}}_i)$ operator and

$$\langle \lambda = 1\ T = 0 | \hat{M}(n=1) | 0\rangle = \frac{\bar{\alpha}}{4\pi}\left(-11Z\langle r^4\rangle_p + \tfrac{50}{3} Z\langle r^2\rangle_A\langle r^2\rangle_p - \tfrac{25}{3} Z\langle r^2\rangle_A^2\right) \tag{7}$$

for the $\hat{M}(n=1) = \sum_{i=1}^{p} [r^3 Y_{10}(\hat{\mathbf{r}}_i) - \eta r Y_{10}(\hat{\mathbf{r}}_i)]$ operator. In Eqs. (6) and (7), we used the transition operator for protons which has both isoscalar and isovector components, while transitional invariance is required for the isoscalar operator by definition. The inclusion of the isovector component is crucial to obtain a finite value for the transition matrix element. In the above the amplitude α is defined as $\alpha \equiv \langle \lambda = 1 | \hat{\alpha} | 0\rangle = (\hbar/2B\omega)^{1/2}$ with the mass parameter

$$B = m\int \rho_p \sum_{\mu} \nabla \hat{M}^{+}_{\lambda\mu}(n=1) \nabla_y \hat{M}_{\lambda\mu}(n=1) d\hat{\mathbf{r}}$$
$$= \frac{3m}{4\pi} Z\left(11\langle r^4\rangle_p - \tfrac{50}{3}\langle r^2\rangle_A\langle r^2\rangle_p + \tfrac{25}{3}\langle r^2\rangle_A^2\right). \tag{8}$$

Note from Eq. (6) above that the matrix element of $\hat{M}(n=0)$ *vanishes* when the mean square radii of proton and neutron distributions are the same, but that $\hat{M}(n=1)$ remains finite when one adopts Hartree-Fock wave functions, which take into account Coulomb and symmetry energies.

Finally one can write two energy weighted sum rules.[9] First let us consider the sum rule for the $\hat{M}_{\lambda=1}(n=1)$ operator which can be expressed as

$$M_{\lambda=1}(n=1) = \sum_{\alpha,\mu} \hbar\omega_\alpha |\langle \alpha | \hat{M}_{\lambda=1,\mu}(n=1) | 0\rangle|^2$$
$$= \tfrac{1}{2}\langle 0 | \sum_{\mu} [\hat{M}^{+}_{\lambda\mu}, [H, \hat{M}_{\lambda\mu}]] | 0\rangle$$
$$= \frac{\hbar^2}{8\pi}\frac{Z}{m}\left(11\langle r^4\rangle_p - \tfrac{50}{3}\langle r^2\rangle_p\langle r^2\rangle_A + \tfrac{25}{3}\langle r^2\rangle_A^2\right). \tag{9}$$

There are two ways to calculate the sum rule (9). The first one is to use the double commutator relation, while the other is to use the matrix element (7) with the amplitude $\bar{\alpha} = (\hbar/2B\omega)^{1/2}$. Both methods give the same answer [cf. Eq. (9)] which means that the collective state $|\lambda = 1, T = 0\rangle$ in Eq. (7) exhausts 100% of the sum rule value. Thus, the sum rule for the operator $\hat{M}_{\lambda=1}(n=0)$ can be calculated by using the matrix element (5) to be

$$M_{\lambda=1}(n=1) = \sum_{\alpha,\mu} \hbar\omega_\alpha |\langle \alpha | \hat{M}_{\lambda=1,\mu}(n=1) | 0\rangle|^2$$
$$= \frac{\hbar^2}{(4\pi)^2}\frac{75}{2B} Z^2(\langle r^2\rangle_A - \langle r^2\rangle_p)^2. \tag{10}$$

We will first make use of our calculated transition density [cf. Eq. (4)] to study the Coulomb form factor of the isoscalar 1^- states in ^{16}O and ^{40}Ca. Hartree-Fock densities calculated with the SGII interaction[10] were employed to determine the proton densities. These are compared with the experimental results of Buti *et al.*[5] obtained at MIT-Bates. As can be seen in Fig. 1, very satisfactory agreement is achieved in both shape and magnitude for the $0^+ \rightarrow 1^-$ transition density in ^{16}O. Similar good agreement is obtained in the ^{40}Ca case. Note that these isoscalar dipole transitions exhaust 9% and 20% of the $\hat{M}_1(n=0)$ sum rule in ^{16}O and ^{40}Ca, respectively. In that context these 1^- states can be viewed as fairly collective. Note also that the inclusion of the $3\hbar\omega$ space is crucial in obtaining reasonable agreement with data since as was noted already by Arima, Manakos, and Strottman[1] calculations in the $1\hbar\omega$ space alone produce form factors which are *1 order of magnitude* smaller than experiment. Note also, however, that, although the Coulomb form factor is non-negligible at the photon point, the calculated transition strengths are too small by nearly a factor of 5 compared to experiment. This is to be expected since the only isospin mixing included so far is due to the difference between proton and neutron density distributions. Isospin

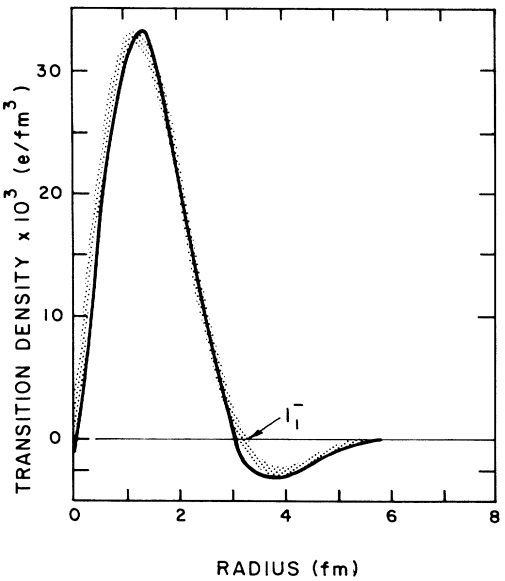

FIG. 1. Comparison of the calculated transition charge density for the $E1$ ($0 \rightarrow 1^-$, $T=0$) transition in ^{16}O (continuous line) with the one extracted from the MIT data (dotted area curve). The agreement between theory and experiment is quite satisfactory except for some deviation at around $R \sim 3.2$ fm.

mixing affecting initial and final wave functions is missing so far and we shall now endeavor to include it specifically in the $E1$ calculation.

Like most recent theoretical estimates of isospin mixing, we will use first order perturbation theory to include the Coulomb interaction V_c. As far as we are aware, however, no serious attempts have been made to include isospin mixing in the *ground state* wave function, although, as we shall see, this proves to be a major contribution to $E1$ rates. We then write for the initial *and* final states

$$|\widetilde{T=0, J^\pi}\rangle = |T=0, J^\pi\rangle + C|T=1, J^\pi\rangle , \tag{11}$$

with the isospin mixing amplitude C determined by

$$C = \frac{\langle T=1, J^\pi | V_c | T=0, J^\pi\rangle}{E(T=0)-E(T=1)} . \tag{12}$$

In order to avoid double counting with Hartree-Fock states, we use the harmonic oscillation model to determine the excited state wave functions, taking into account configurations corresponding to $1\hbar\omega$ and $3\hbar\omega$ excitations.

The $E1$ transition matrix element now contains contributions from coupling to the giant dipole and giant monopole states, i.e.,

$$\langle \widetilde{T=0,1^-} | \hat{0}(E1) | \widetilde{T=0,0^+}\rangle = \langle T=0,1^-|\hat{0}(E1)|T=0,0^+\rangle + C(1^-)\langle T=1,1^-|\hat{0}(E1)|T=0,0^+\rangle + C(0^+)\langle T=0,1^-|\hat{0}(E1)|T=1,0^+\rangle + \cdots . \tag{13}$$

The transition matrix element of the first term on the right-hand side (rhs) of Eq. (13) is estimated by using the Hartree-Fock transition density instead of the harmonic oscillator one. In this way, we can separate the effects of isospin mixing due to the single particle wave functions from those of the random-phase approximation amplitudes due to the Coulomb interaction.

Using a simplified Skyrme interaction

$$V(\mathbf{r}_1,\mathbf{r}_2) = t_0(1+x_0P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2) + \tfrac{1}{6}t_3\rho_0(1+x_3P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_0) ,$$

with a t_0 coefficient chosen to bring the energy of the spurious state to zero, we find the perturbation amplitudes to be $C(1^-)=8.1\times10^{-4}$ and $C(0^+)=3\times10^{-2}$ for ^{16}O and $C(1^-)=7.2\times10^{-4}$ and $C(0^+)=1.4\times10^{-2}$ for ^{40}Ca. The larger values for the $C(0^+)$ mixing amplitudes should translate into a fairly large contribution to $E1$ rates coming from monopole resonance coupling. Indeed as we observe in Table I, the monopole mixing contribution is by far the largest and is instrumental in bringing the total calculated $E1$ matrix element within a factor of 2 from experiment. This is all the more remarkable if one notices that this result comes from a strong cancellation between the two isospin mixing contributions. On the other hand, these isospin mixing contributions do not appreciably modify the calculated form factors of Fig. 2.

Let us now finally come back to electron scattering form factor problems and the purported violation of Siegert's theorem. As is well known, Siegert's theorem states that at small q the transverse electric form factor is directly proportional to the Coulomb form factor and assumes for this that the magnetization current contribution to the transverse component is negligible. Siegert's theorem then allows one to convert the longitudinal form factor (or transition probabilities) at the correspondingly small q to a lifetime. To be more precise we have derived from the general expression[11] for the transverse form factor E_T its expression in our particle-hole basis to be

$$F_T = \langle (j_2 j_1^{-1}) J || T_\lambda || 0\rangle \alpha \frac{1}{q[\lambda(\lambda+1)]^{1/2}} \left(-\omega_{fi} e_i (\lambda+1)\langle j_2 | \frac{(qr)^\lambda}{(2\lambda+1)!!} - \frac{1}{\lambda+1}\frac{(qr)^{\lambda+2}}{(2\lambda+3)!!} | j_1\rangle \right)$$
$$+ \frac{e\hbar}{2mc} q^2 g_l \langle j_2 | r \frac{(qr)^\lambda}{(2\lambda+1)!!} \frac{\partial}{\partial r}(\rightarrow) - \frac{\partial}{\partial r}(\leftarrow) r \frac{(qr)^\lambda}{(2\lambda+1)!!} | j_1\rangle$$
$$- \tfrac{1}{2}\frac{e\hbar}{2mc} q^2 g_s \langle j_1 | \frac{(qr)^\lambda}{(2\lambda+1)!!} | j_1\rangle [(-)^{j_1-l_1-1/2}(j_1+\tfrac{1}{2}) - (-)^{j_2-l_2-1/2}(j_2+\tfrac{1}{2})] .$$

The first term on the rhs is the Siegert term and its ratio to the Coulomb form factor can be calculated from[11]

$$F_T(q) \rightarrow \left(\frac{\lambda+1}{\lambda}\right)^{1/2} \frac{\omega_{fi}}{q} F_c(q) .$$

In our case $\omega_{fi}=0.036$ fm^{-1}, $\lambda=1$ and in the range $q\approx 0.2$–0.3 fm we obtain $2(\omega_{fi}/q)^2\sim 4\%$. The second term (with g_l as multiplicative factor) yields a negligible contribution due to the cancellation of two derivative terms. Finally one could expect a larger contribution coming from the g_s term since $g_s^{IV}=4.7$, however, our calculation shows that this third term is less than 10% of Siegert's term at $q\sim 0.25$ fm^{-1} and much smaller at the photon point because of the q^2 dependence.

TABLE I. Calculated contributions to the isoscalar $E1$ matrix elements in ^{16}O and ^{40}Ca compared to the value extracted from the lifetime measurements. All units are in e fm.

	H-F density	Dipole mixing	Monopole mixing	Sum	Experiment
^{16}O	-3.15×10^{-3}	8.08×10^{-3}	-1.11×10^{-2}	-6.17×10^{-3}	12×10^{-3}
^{40}Ca	-2.52×10^{-2}	6.30×10^{-3}	-1.12×10^{-1}	-13.1×10^{-2}	7.1×10^{-2}

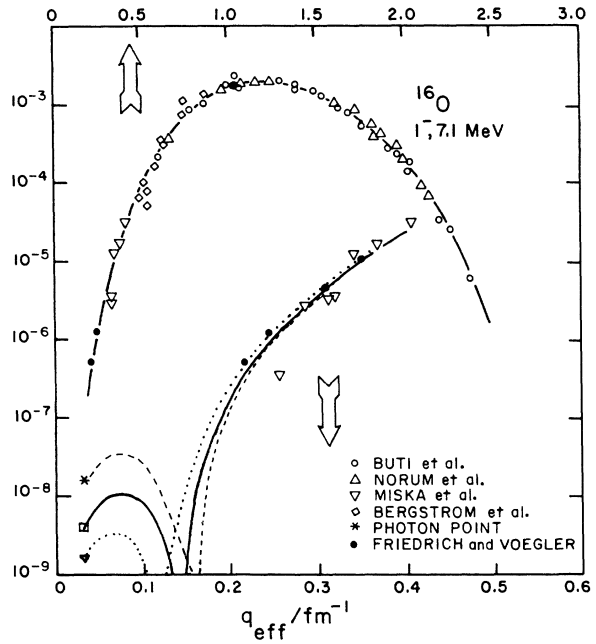

FIG. 2. Calculated and experimental cross sections (divided by the Mott cross section) for longitudinal excitation of the (1^-, $T=0$) level at 7.117 MeV (data and calculation are normalized at $q=0.4$ fm^{-1}). The lower scale is valid for the low-q data, the upper scale for the whole q range. The calculated cross section is denoted by a continuous line, together with two parametrizations of the fit to Friedrich and Voegler data: the dashed curve represents a transition density extending out to a cutoff radius $R_c=14$ fm with the photon point and $\chi^2=105$, whereas the dotted curve corresponds to $R_c=8$ fm, without the photon point and $\chi^2=101$.

Thus, altogether, our calculation for the ratio between magnetization and convection currents yields *a value of the order of a few percent* for both ^{16}O and ^{40}Ca 1^- form factors, i.e., no detectable violation of Siegert's theorem. Note that this result is in agreement with the Darmstadt ^{40}Ca data[7] which also asserts the validity of Siegert's theorem in that case. Our result also agrees with the ^{40}Ca data as far as *the magnitude of the convection current* is concerned since the Darmstadt group reports an effective charge due to convection current of 0.5, whereas our ^{40}Ca calculation which exhausts 20% of the sum rule, yields e_{eff} (convection) $=(0.2)^{1/2}=0.45$.

Let us now examine the possible reasons for differing with Friedrich and Voegler on whether Siegert's theorem applies to the isoscalar $E1$ decay in ^{16}O or not. Prior to the photoelectric experiment of Friedrich and Voegler a measurement by the Darmstadt group[7] had revealed a destructive interference of $T=1$ and $T=0$ contributions leading to a zero in the form factor at $q=0.17$ fm^{-1} (see Fig. 2). Because of this zero one cannot extrapolate the measured form factor smoothly down to the photon point at $q_\omega=0.036$ fm^{-1}. Friedrich and Voegler have performed a flexible Fourier-Bessel expansion of their data using different cutoff radii. They make the valid point that their analysis depends as expected on the cutoff radii; using $R_c=14$ fm allows, for instance, one to describe the photon point results and (e,e') data simultaneously but demands a cutoff radius which these authors deem unrealistic. Using $R_c=8$ fm, however, yields a situation where the lifetime data determined from (ee') cross sections (including Friedrich and Vogler's results) are deemed incompatible with the (γ,γ) measurements. Also, one should notice that Friedrich and Voegler have adopted the MIT-Bates results in performing the longitudinal-transverse separation. Although as stated by these authors, this procedure could cast some doubt on the details of their analysis, it does not modify their general conclusion regarding Siegert's theorem since the transverse contribution they used is less than 5% for all data points. Still in view of the fact that the lowest momentum available to Friedrich and Vogler's analysis corresponded to $q=0.218$ fm^{-1} it would seem appropriate before casting final judgement of the validity of Siegert's theorem to solicit measurements *at small momentum transfer* with the specific aim at determining independently the longitudinal-transverse separation. In that regard a recent (γ,γ) experiment by Moreh *et al.*[12] which yields satisfactory agreement with the direct lifetime measurement is of interest.

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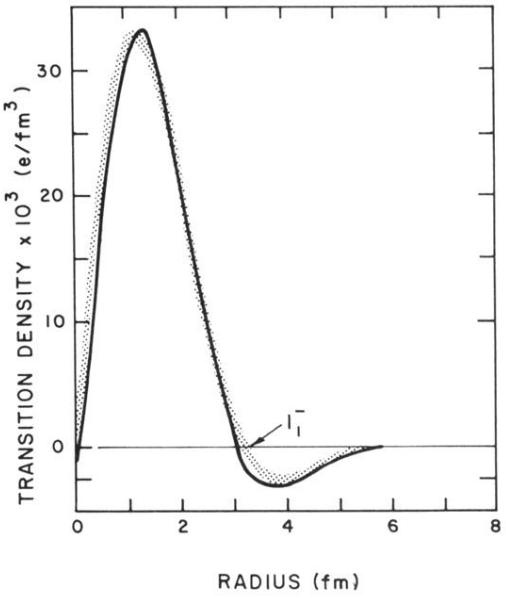

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